Chapter 28

PERIMETER AND AREA OF PLANE FIGURES

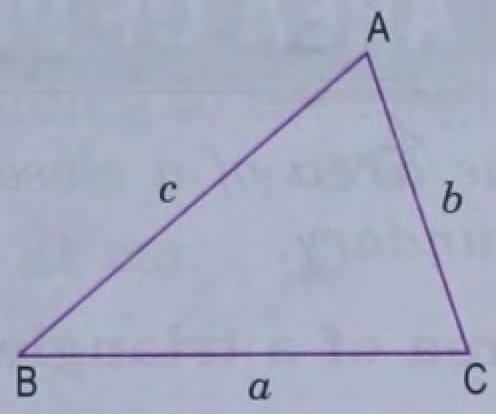
PERIMETER OF PLANE FIGURES

*The **perimeter** of a closed plane figure is the length of its boundary i.e. the sum of lengths of its sides.*

Perimeter of a triangle

If a, b and c are the lengths of the sides of any triangle ABC, then its

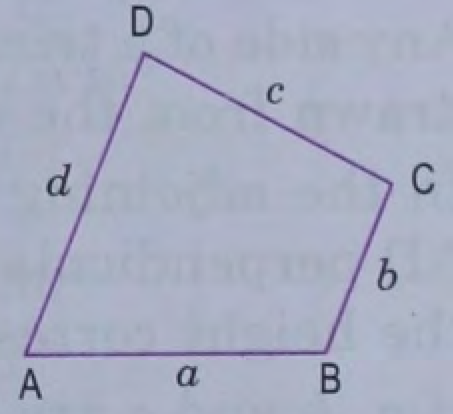
perimeter $= a + b + c$.



Perimeter of quadrilateral

If a, b, c and d are the lengths of the four sides of any quadrilateral ABCD, then its

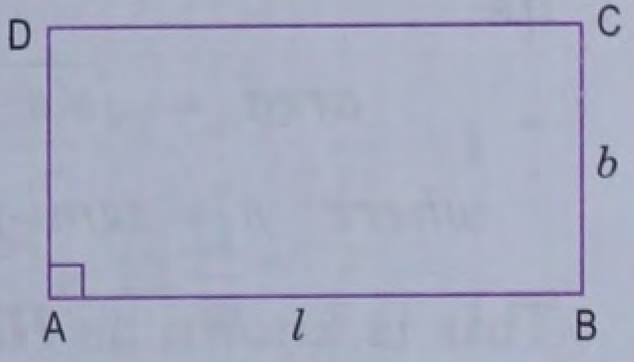
perimeter $= a + b + c + d$.



Perimeter of a rectangle

If l and b are the length and breadth of any rectangle ABCD, then its

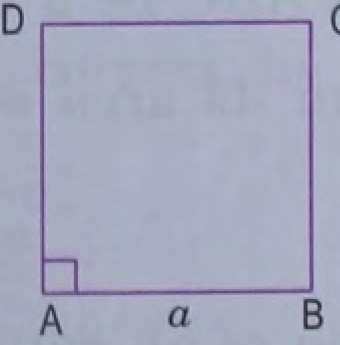
perimeter $= 2(l + b)$.



Perimeter of a square

If a is the length of a side of any square ABCD, then its

perimeter $= 4a$.



Length of diagonal of a rectangle

If d is the length of a diagonal and l, b are the length and the breadth of any rectangle ABCD, then

$$d = \sqrt{l^2 + b^2}.$$

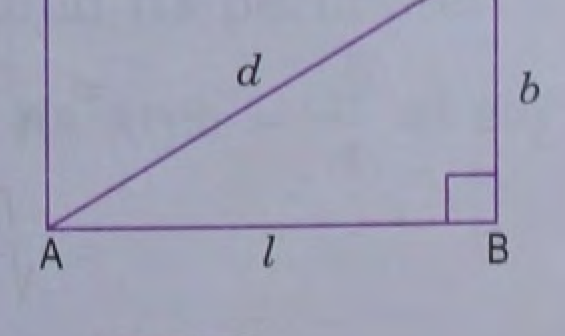
In the adjoining diagram, $\angle B = 90°$.

From Δ ABC, by Pythagoras theorem, we get

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad d^2 = l^2 + b^2$$

$$\Rightarrow \quad d = \sqrt{l^2 + b^2}.$$

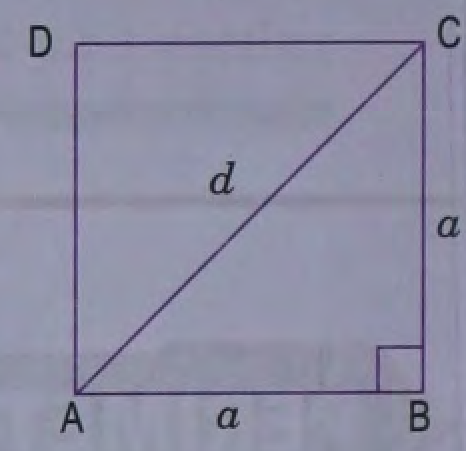


In fact, a rectangle has two diagonals and length of each diagonal $= \sqrt{l^2 + b^2}$.

Length of diagonal of a square

If d is the length of a diagonal and a is the length of a side of any square ABCD, then

$$d = \sqrt{2}\,a.$$



In the adjoining diagram, $\angle B = 90°$.

From ΔABC, by Pythagoras theorem, we get

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad d^2 = a^2 + a^2 = 2a^2$$

$$\Rightarrow \quad d = \sqrt{2}\,a.$$

In fact, a square has two diagonals and length of each diagonal = $\sqrt{2}\,a$.

AREA OF PLANE FIGURES

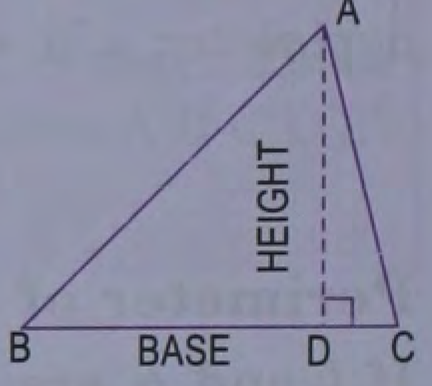
The ***area*** *of a closed plane figure is the measure of the region (surface) enclosed by its boundary.*

Area of a triangle

- *Area of a triangle* = $\frac{1}{2}$ × *base* × *height*

  Any side of a triangle can be taken as its **base** and the length of perpendicular (altitude) drawn from the opposite vertex to the base is called its (**corresponding**) **height**.

  In the adjoining diagram, BC is the base of ΔABC. From A, draw AD perpendicular to BC, then the length of the line segment AD is the height corresponding to the base BC.

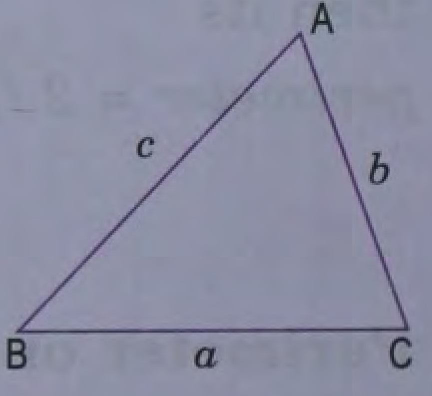


- If a, b and c are the lengths of the sides of any triangle ABC; then its

  $$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

  $$\text{where} \quad s = \text{semi-perimeter} = \frac{a+b+c}{2}.$$

  This is known as ***Heron's Formula***.

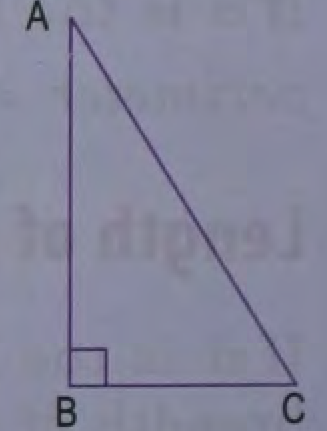


- *Area of a right angled triangle*

  Let ABC be a triangle in which $\angle B = 90°$,

  then its area = $\frac{1}{2} \times BC \times AB$

  $= \frac{1}{2}$ (product of sides containing right angle).



- *Area of an equilateral triangle*

  Let ABC be an equilateral triangle with side a, then

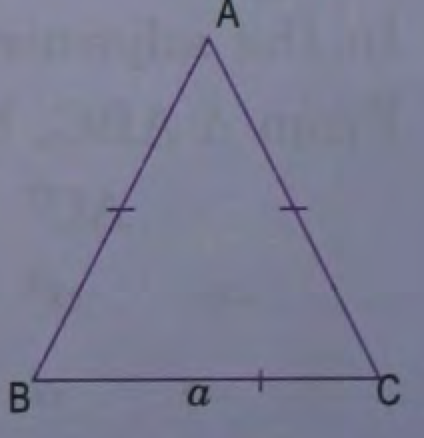
  $$s = \text{semi-perimeter} = \frac{a+a+a}{2} = \frac{3a}{2}.$$

  By Heron's formula,

  $$\text{area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

  $$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

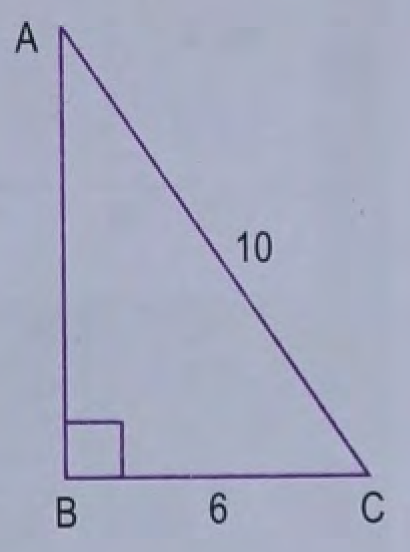
  $$= \sqrt{\frac{3a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2}} = \frac{\sqrt{3}}{4}a^2$$



Hence, area of an equilateral triangle with side $a = \frac{\sqrt{3}}{4}a^2$.

Example 1. If the base of a right-angled triangle is 6 units and the hypotenuse is 10 units, find its area.

Solution. Let ABC be a right-angled triangle at B *i.e.* $\angle B = 90°$ and its base BC = 6 units and hypotenuse AC = 10 units.



From ΔABC, by Pythagoras theorem, we get

$$AC^2 = BC^2 + AB^2 \quad \Rightarrow \quad 10^2 = 6^2 + AB^2$$

$$\Rightarrow \quad AB^2 = 10^2 - 6^2 = 100 - 36 = 64$$

$$\Rightarrow \quad AB = \sqrt{64} \text{ units} = 8 \text{ units.}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \left(\frac{1}{2} \times 6 \times 8\right) \text{ sq. units} = 24 \text{ sq. units.}$$

Example 2. Calculate the area of a triangle whose sides are 29 cm, 20 cm and 21 cm. Hence find the length of the altitude corresponding to the longest side.

Solution. Since the sides of the triangle are 29 cm, 20 cm and 21 cm.

$$\therefore \quad s = \text{semi-perimeter} = \frac{29 + 20 + 21}{2} \text{ cm} = 35 \text{ cm.}$$

$$\therefore \quad \text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{35(35-29)(35-20)(35-21)} \text{ cm}^2$$

$$= \sqrt{35 \times 6 \times 15 \times 14} \text{ cm}^2$$

$$= \sqrt{5 \times 7 \times 2 \times 3 \times 3 \times 5 \times 7 \times 2} \text{ cm}^2$$

$$= (5 \times 7 \times 2 \times 3) \text{ cm}^2 = 210 \text{ cm}^2$$

The longest side of the triangle is 29 cm, let h cm be the length of the corresponding altitude, then

$$\text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\Rightarrow \quad 210 = \frac{1}{2} \times 29 \times h \quad \Rightarrow \quad h = \frac{420}{29} = 14\frac{14}{29}.$$

Hence, the length of the altitude corresponding to the longest side = $14\frac{14}{29}$ cm.

Example 3. Find the area of an equilateral triangle of side 8 m correct to three significant figures. Use $\sqrt{3} = 1{\cdot}732$.

Solution. Given side of an equilateral triangle = a = 8 m.

$$\text{Area of the equilateral triangle} = \frac{\sqrt{3}}{4} a^2$$

$$= \left(\frac{\sqrt{3}}{4} \times 8 \times 8\right) \text{m}^2 = 16\sqrt{3} \text{ m}^2$$

$$= (16 \times 1{\cdot}732) \text{ m}^2 = 27{\cdot}712 \text{ m}^2$$

$= 27{\cdot}7$ m^2, correct to 3 significant figures.

Example 4. If the area of an equilateral triangle is $49\sqrt{3}$ cm^2, find its perimeter.

Solution. Let a side of the equilateral triangle be a cm, then its area = $\frac{\sqrt{3}}{4}a^2$ cm^2.

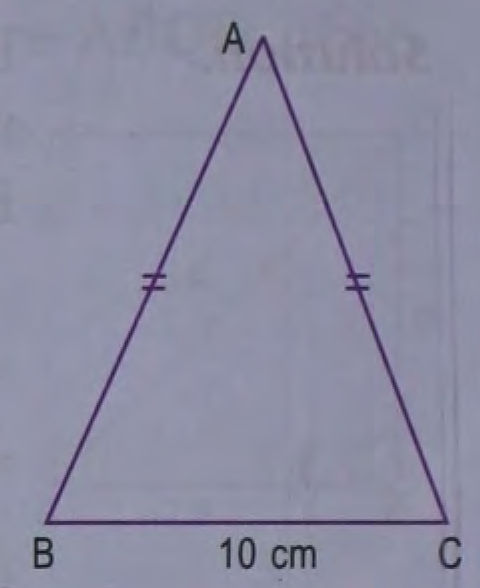
According to given information, $\frac{\sqrt{3}}{4} a^2 = 49\sqrt{3}$

$$\Rightarrow a^2 = 4 \times 49 \Rightarrow a = \sqrt{4 \times 49} = 14.$$

$\therefore$ The perimeter of the triangle = $3a$ cm = (3×14) cm = 42 cm.

Example 5. If the base of an isosceles triangle is 10 cm and its perimeter is 36 cm, find the area of the triangle.

Solution. Let ABC be an isosceles triangle with BC as its base and AB, AC its equal sides, then BC = 10 cm. Let the length of each of its two equal sides be x cm, then perimeter of $\Delta ABC = (x + x + 10)$ cm = 36 cm (given)



$\Rightarrow \quad 2x + 10 = 36 \quad \Rightarrow \quad 2x = 36 - 10 = 26$

$\Rightarrow \quad x = 13.$

$\therefore$ Thus, sides a, b and c of the triangle ABC are 10 cm, 13 cm and 13 cm respectively.

s = semi-perimeter = $\frac{1}{2}$ of 36 cm = 18 cm

$\therefore \quad s - a = (18 - 10)$ cm = 8 cm,

$s - b = (18 - 13)$ cm = 5 cm

and $\quad s - c = (18 - 13)$ cm = 5 cm

$\therefore$ The area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18 \times 8 \times 5 \times 5}$ cm^2 = $\sqrt{144 \times 25}$ cm^2

$= 12 \times 5$ cm^2 = 60 cm^2.

Exercise 28.1

1. If the diagonal of a square is $10\sqrt{2}$ cm, find its perimeter.
2. Calculate the length of the diagonal of the following rectangles with dimensions:
   (*i*) l = 16 cm, b = 12 cm (*ii*) l = 40 cm, b = 9 cm
3. Fill in the following blanks :

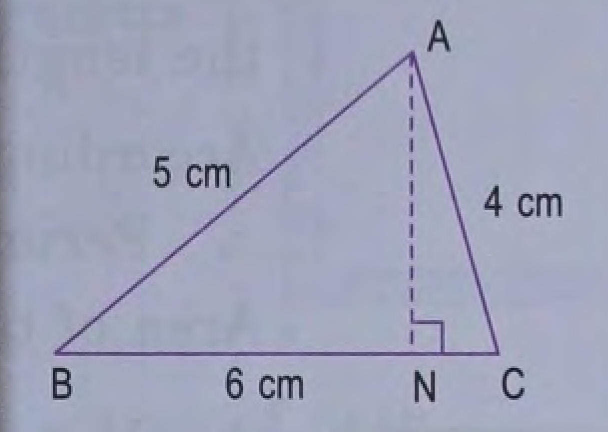
	Base	*Height*	*Area of triangle*
(*i*)	18 cm	10 cm	...
(*ii*)	10·5 cm	12·6 cm	...
(*iii*)	1·4 m	85 cm	...
(*iv*)	...	13 cm	104 cm^2
(*v*)	14 cm	...	77 cm^2
(*vi*)	13·6 cm	...	63·92 cm^2
(*vii*)	...	1·2 m	5220 cm^2

4. The base of a right angled triangle is 12 cm and its hypotenuse is 13 cm. Find the area of the triangle.
5. Find the area of a triangle whose sides are :
   (*i*) 3 cm, 4 cm and 5 cm
   (*ii*) 12 cm, 9·6 cm and 7·2 cm.
6. Find the area of a triangle whose sides are 34 cm, 20 cm and 42 cm. Hence find the length of the altitude corresponding to the shortest side.
7. Two sides of a triangle are 15 cm and 12 cm. The length of the altitude to the smaller side is 10 cm. Find
   (*i*) the area of the triangle.
   (*ii*) the length of the altitude to the other side.

8. From the adjoining diagram, find

(*i*) the area of ΔABC correct to two places of decimal.

(*ii*) the length of the altitude from A to side BC correct to one decimal place.



9. Find the area of an equilateral triangle of side 6 cm correct to three significant figures.

10. If the area of an equilateral triangle is $81\sqrt{3}$ cm², find its perimeter.

11. If the perimeter of an equilateral triangle is 36 cm, calculate its area and height correct to one decimal place.

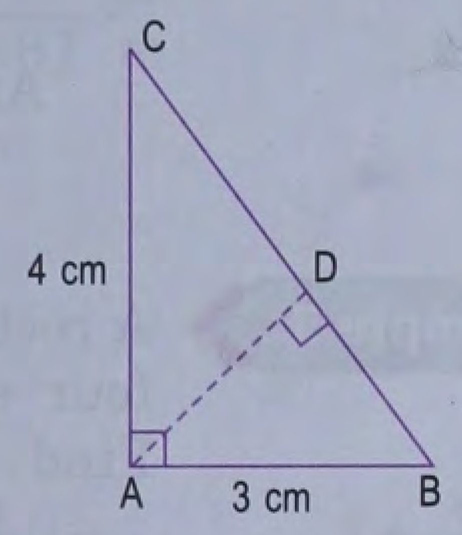
12. The lengths of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 48 cm. Find its area.

13. From the adjoining figure, find

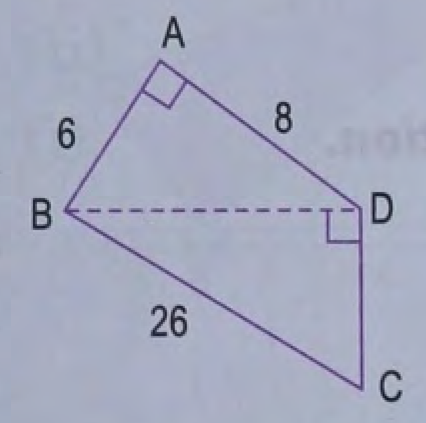
(*i*) the area of ΔABC

(*ii*) length of BC

(*iii*) the length of altitude from A to BC.



14. In the adjoining quadrilateral ABCD, $\angle BAD = 90°$ and $\angle BDC = 90°$. All measurements are in centimetres. Find the area of the quadrilateral ABCD.
[Hint. From ΔABD, $BD^2 = 6^2 + 8^2 = 100$.**]**



15. If one of the two equal sides of an isosceles triangle is 10 cm and its perimeter is 32 cm, find the area of the triangle.

16. A park is in the shape of an isosceles triangle and its base and altitude are in the ratio 3 : 2. If the cost of levelling the park at the rate of ₹ 18·50 per metre square is ₹ 88800, find the sides of the park.

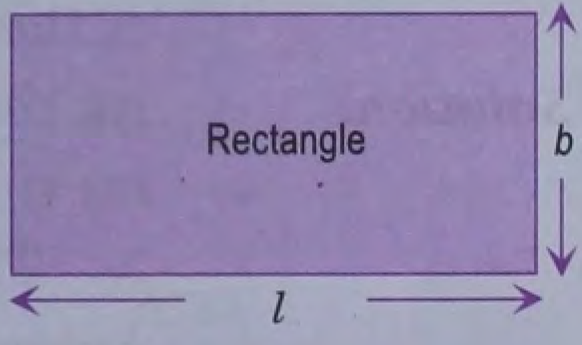
AREA OF RECTANGLE AND SQUARE

Area of a rectangle

Area of a rectangle = length × breadth

Thus, $A = l \times b$ where A = area, l = length and b = breadth

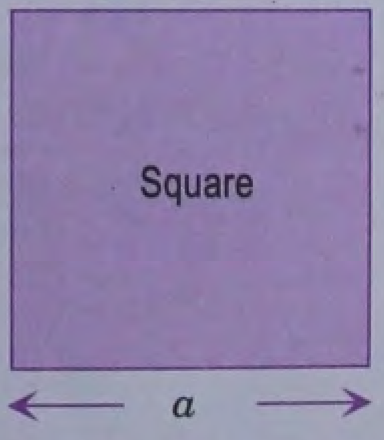
$\therefore\ l = \frac{A}{b}$ and $b = \frac{A}{l}$.



Area of a square

Area of a square = (length of a side)²

Thus, $A = a^2$ where A = area and
a = length of a side of the square.



Example 1. Find the perimeter and the area of a square whose diagonal is $7\sqrt{2}$ cm long.

Solution. Let the length of a side of the square be a cm, then the length of diagonal = $\sqrt{2}\,a$ cm.

According to given information, $\sqrt{2}\,a = 7\sqrt{2} \Rightarrow a = 7$.

$\therefore$ Perimeter of the square = $4a$ cm = (4×7) cm = 28 cm

Area of the square = a^2 cm² = (7×7) cm² = 49 cm².

Example 2. Find the area of a rectangle whose diagonal is 15 cm and breadth is 9 cm.

Solution. Given, length of diagonal = d = 15 cm and breadth = b = 9 cm.

Let the length of the rectangle be l cm, then

$$d = \sqrt{l^2 + b^2} \Rightarrow d^2 = l^2 + b^2$$

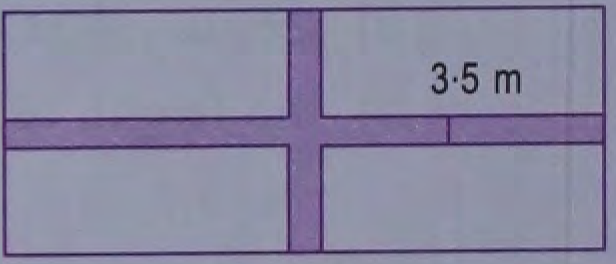
$$\Rightarrow 15^2 = l^2 + 9^2 \Rightarrow l^2 = 15^2 - 9^2 = 225 - 81 = 144$$

$$\Rightarrow l = \sqrt{144} = 12.$$

$\therefore$ Area of the rectangle = length × breadth

= (12×9) cm² = 108 cm².

Example 3. A rectangular garden 90 m by 50 m is divided into four equal parts by two cross-paths 3·5 m wide. Find



(*i*) the area of the cross-paths.

(*ii*) the area of the unshaded portion.

Solution. (*i*) Area of the path parallel to length = $(90 \times 3{\cdot}5)$ m² = 315 m²

Area of the path parallel to breadth = $(50 \times 3{\cdot}5)$ m² = 175 m²

Area of the common portion of the paths = $(3{\cdot}5 \times 3{\cdot}5)$ m² = 12·25 m²

$\therefore$ The area of the cross-paths = 315 m² + 175 m² – 12·25 m²

= 477·75 m².

(*ii*) The area of the unshaded portion

= total area of garden – area of path

= (90×50) m² – 477·75 m²

= 4500 m² – 477·75 m² = 4022·25 m².

Example 4. The length and breadth of a rectangular plot are in the ratio 5 : 3. If the area of the plot is 2940 square metres, find the cost of fencing the boundary of the plot at the rate of ₹ 5·70 per metre.

Solution. As the ratio of length : breadth = 5 : 3,

let the length of the plot be $5x$ metres, then its breadth = $3x$ metres.

$\therefore$ The area of the plot = $(5x \times 3x)$ m² = $15x^2$ m²

According to given information, $15x^2 = 2940$

$$\Rightarrow x^2 = \frac{2940}{15} = 196$$

$$\Rightarrow x = \sqrt{196} = 14.$$

$\therefore$ The length of plot = (5×14) m = 70 m and

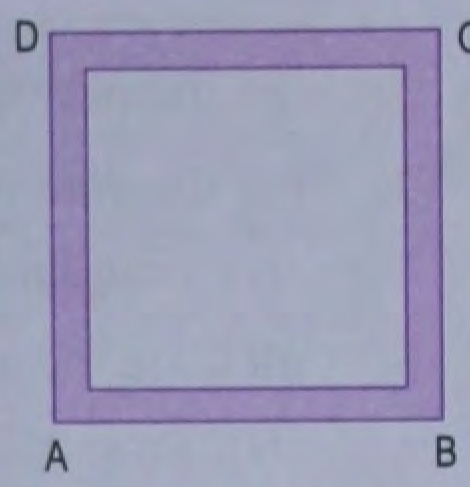
breadth = (3×14) m = 42 m.

$\therefore$ The length of the boundary = 2 (70 + 42) m = (2×112) m = 224 m.

$\therefore$ The cost of fencing the boundary of the given plot

= ₹ $(224 \times 5{\cdot}70)$ = ₹ 1276·80.

Example 5. The area of a square garden ABCD is 2025 m². Inside the garden, a path of uniform width runs all around its boundary. If the area of the path is 581 m², find



(*i*) the area of the unshaded portion.
(*ii*) perimeter of the unshaded portion.
(*iii*) the width of the path.

Solution. (*i*) Area of the unshaded portion = total area – area of path

$= 2025\ m^2 - 581\ m^2 = 1444\ m^2.$

(*ii*) As ABCD is a square and the width of the path is uniform, therefore, the unshaded portion is also a square.

$\therefore$ Side of this square $= \sqrt{1444}\ m = \sqrt{4 \times 361}\ m$

$= (2 \times 19)\ m = 38\ m$

$\therefore$ Perimeter of the unshaded portion $= (4 \times 38)\ m = 152\ m.$

(*iii*) Side of the square ABCD $= \sqrt{2025}\ m = \sqrt{25 \times 81}\ m$

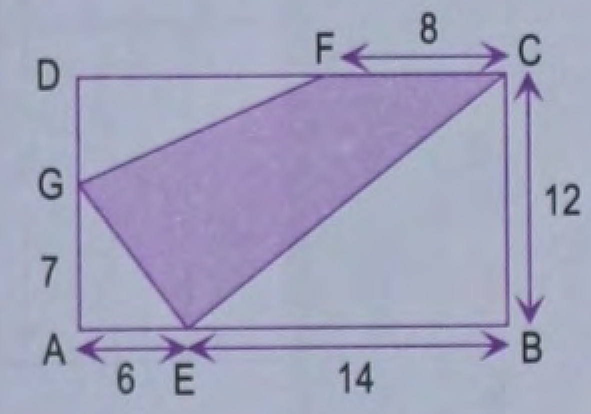
$= (5 \times 9)\ m = 45\ m$

Let x metres be the width of the path, then

$2x + 38 = 45 \quad \Rightarrow \quad 2x = 7 = x = 3{\cdot}5$

$\therefore$ The width of the path = 3·5 m.

Example 6. In the adjoining figure, ABCD is a rectangle and all measurements are in centimetres. Find the area of the shaded region.



Solution. As ABCD is a rectangle, AB = DC and AD = BC.

From figure, AB = AE + EB = 6 + 14 = 20

DF = DC – FC = AB – 8 = 20 – 8 = 12

GD = AD – AG = BC – 7 = 12 – 7 = 5

$\therefore$ Area of the shaded region

= area of rect. ABCD – area of ΔEBC – area of ΔGAE – area of ΔDGF

$= (20 \times 12 - \frac{1}{2} \times 14 \times 12 - \frac{1}{2} \times 6 \times 7 - \frac{1}{2} \times 12 \times 5)\ cm^2$

$= (240 - 84 - 21 - 30)\ cm^2 = 105\ cm^2.$

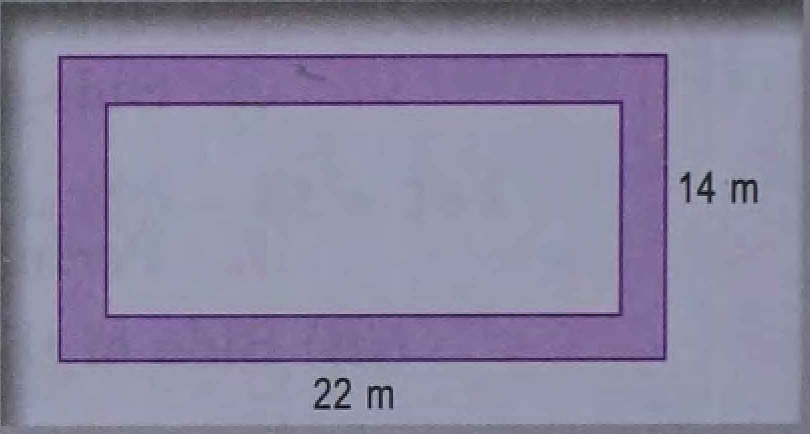
Exercise 28.2

1. (*i*) Find the area of a square whose perimeter is 52 cm.
   (*ii*) Find the perimeter of a square whose area is 784 cm².

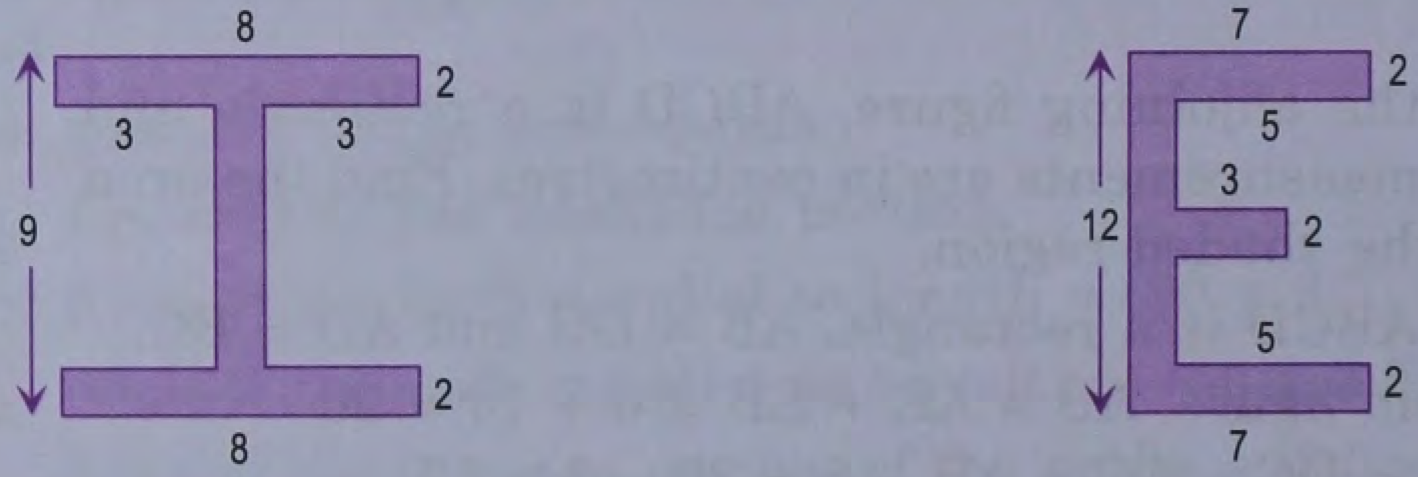
2. For a rectangle, complete the following table :

Length	Breadth	Perimeter	Area
2·3 m	90 cm	... m	... m²
2·7 m	85 cm	... cm	... cm²
15 m	...	...	165 m²
...	7·4 cm	...	71·04 cm²
21 cm	...	76 cm	...
...	8·6 cm	38 cm	...

3. Find the perimeter and the area of a square whose diagonal is $8\sqrt{2}$ cm.
4. Find the area of the following rectangles :
   (*i*) diagonal = 26 cm, breadth = 10 cm (*ii*) diagonal = 41 cm, length = 40 cm.
5. Calculate the area of the following rectangles :
   (*i*) length : breadth = 3 : 2, perimeter = 40 cm
   (*ii*) length : breadth = 7 : 5, perimeter = 120 cm.
6. The length and the breadth of a rectangular field are in the ratio 9 : 5. If the area of the field is 14580 square metres, find the cost of surrounding the field with a fence at the rate of ₹3·25 per metre.
7. In the adjoining figure, find the area of the path which is of uniform width 1·9 m inside the rectangle.

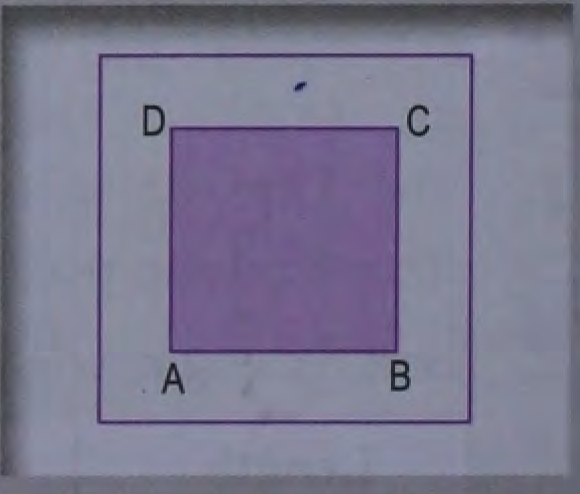


8. Find the area and the perimeter of the following figures. All angles are right angles and all measurements are in centimetres.

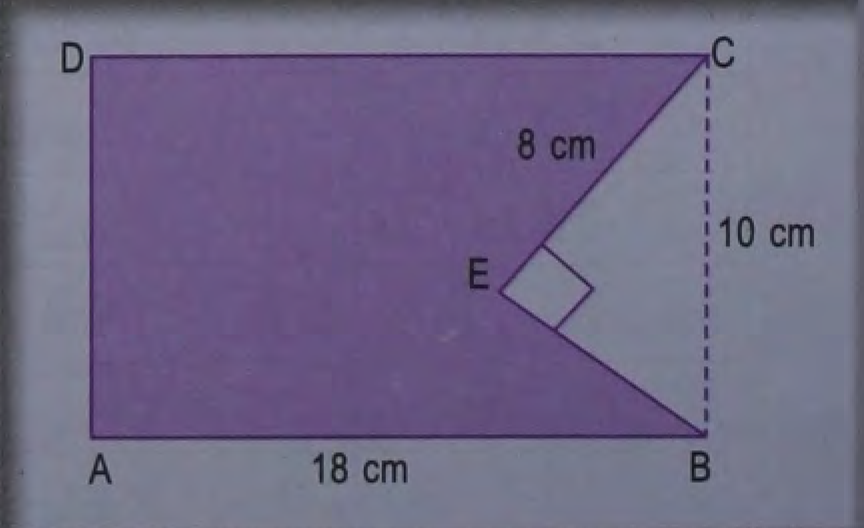


(*i*) (*ii*)

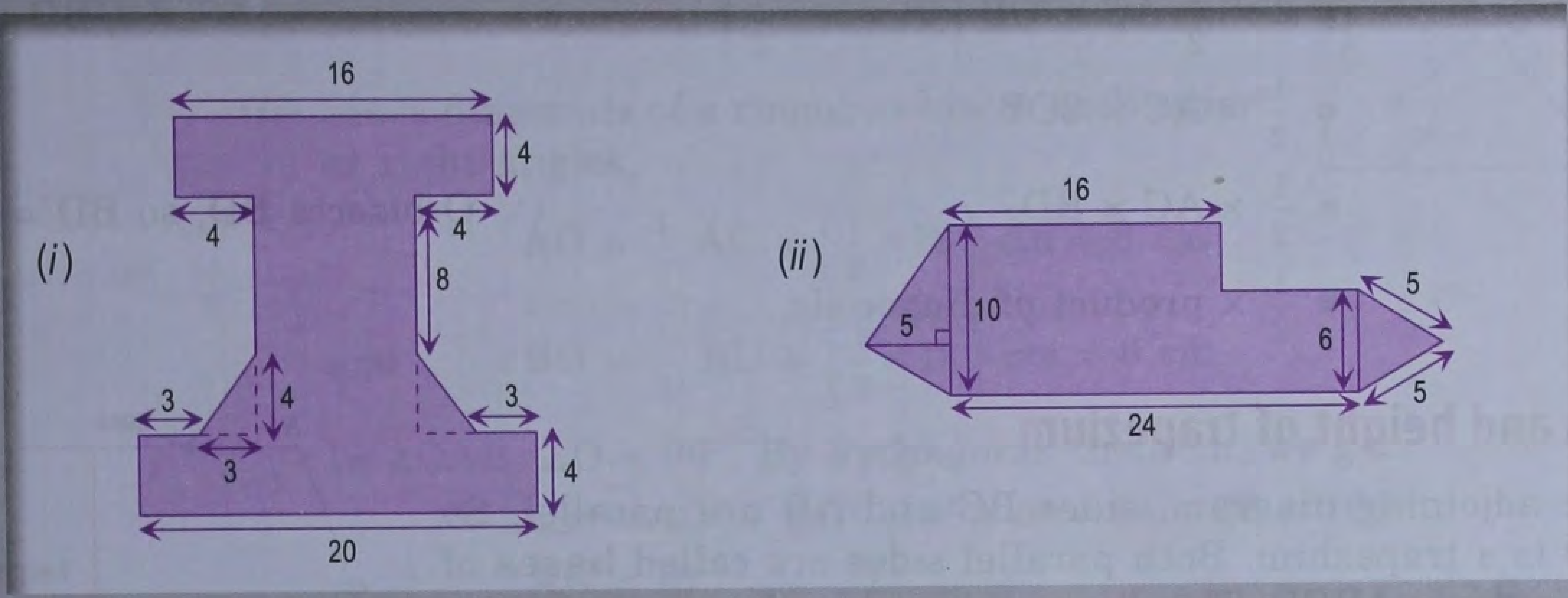
9. Find the cost of flooring a room 6·5 m by 5 m with square tiles of side 25 cm at the rate of ₹1880 per hundred tiles.
10. The cost of levelling a square lawn at ₹2·50 per square metre is ₹13322·50. Find the cost of fencing it at the rate of ₹5 per metre.
11. A rectangle is 16 m by 9 m. Find a side of the square whose area equals the area of the rectangle. By how much does the perimeter of the rectangle exceed the perimeter of the square?
12. In the adjoining figure, ABCD is a square grassy lawn of area 729 m^2. A path of uniform width runs all around it. If the area of the path is 295 m^2, find
   (*i*) the length of the boundary of the square field enclosing the lawn and the path.
   (*ii*) the width of the path.



13. In the diagram, ABCD is a rectangle of size 18 cm by 10 cm. In Δ BEC, ∠E = 90° and EC = 8 cm. Find the area enclosed by the pentagon ABECD.
   [Hint. From Δ EBC, $EB^2 = 10^2 - 8^2$. Area of pentagon = area of rectangle ABCD – area of ΔEBC.**]**

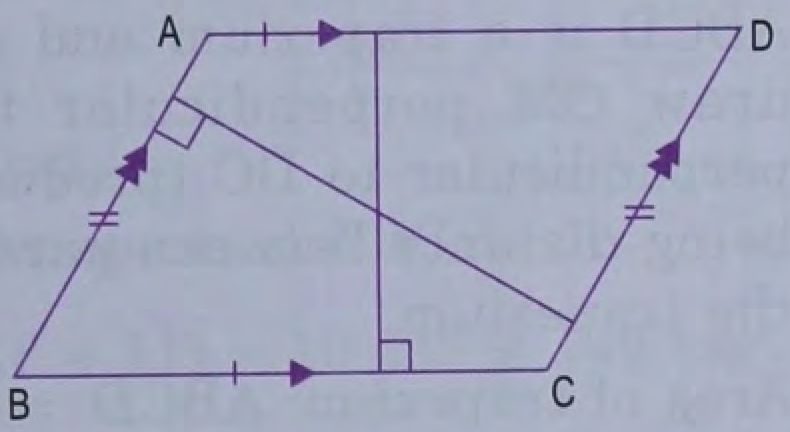


14. Find the area of the shaded region of each of the following figures, given that all measurements are in centimetres :



AREA OF PARALLELOGRAM, RHOMBUS AND TRAPEZIUM

Base and height of a parallelogram

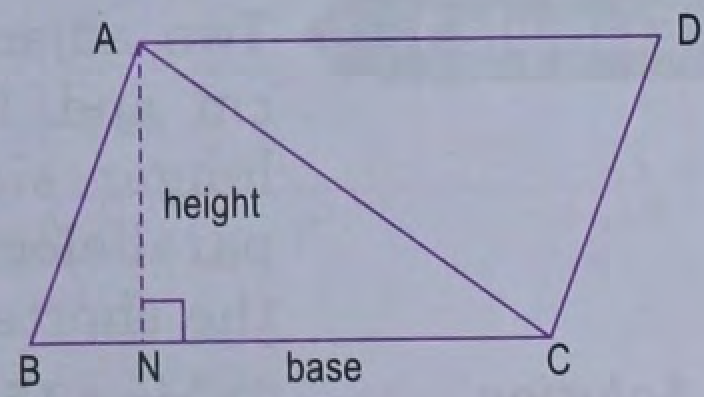


In the adjoining diagram, ABCD is a parallelogram. It has two pairs of opposite sides which are parallel and equal. Any side can be taken as base.

The distance between the parallel sides BC and AD is the **height corresponding to the base BC (or AD)** and the distance between the parallel sides AB and CD is the height corresponding to the base AB (or CD).

Area of a parallelogram

Area of a parallelogram = base × height



In the adjoining diagram, ABCD is a parallelogram. AC is its one diagonal. From A, draw AN perpendicular to BC. If we take BC as a base of the parallelogram ABCD, then AN is the height corresponding to the base BC. Since a diagonal of a parallelogram divides the parallelogram into two congruent triangles,

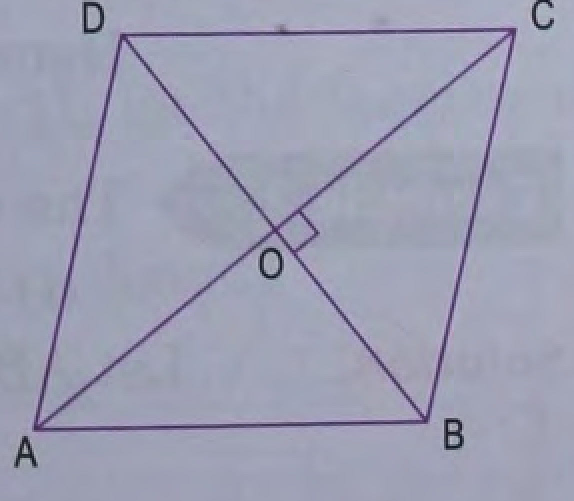
area of parallelogram ABCD $= 2 \times$ area of Δ ABC

$= 2 \times \frac{1}{2} \times BC \times AN = BC \times AN$

$=$ base × height.

Note. You can choose any side as the base of the parallelogram but you must be careful to pair it with the correct perpendicular height.

Area of rhombus



Area of a rhombus $= \frac{1}{2} \times$ *product of diagonals*

In the adjoining diagram, ABCD is a rhombus. We know that the diagonals of a rhombus bisect each other at right angles.

Since a diagonal of a rhombus divides it into two congruent triangles,
area of rhombus = 2 × area of ΔABC

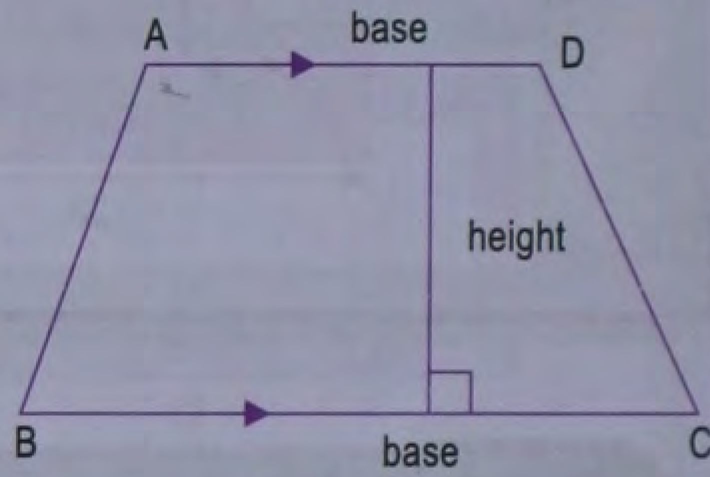
$$= 2 \times \frac{1}{2} AC \times OB \qquad (\because BD \perp AC)$$

$$= \frac{1}{2} \times AC \times 2OB$$

$$= \frac{1}{2} \times AC \times BD \qquad (\because \text{O bisects BD, so } BD = 2OB)$$

$$= \frac{1}{2} \times \text{product of diagonals.}$$

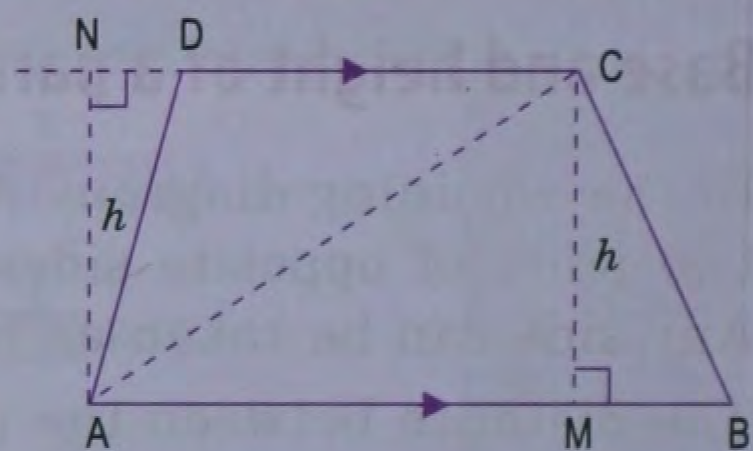
Bases and height of trapezium



In the adjoining diagram, sides BC and AD are parallel. So ABCD is a trapezium. Both parallel sides are called **bases** of the trapezium ABCD. The distance between the parallel sides is called the **height** of the trapezium.

Area of a trapezium

Area of a trapezium = $\frac{1}{2}$ *(sum of parallel sides) × height.*



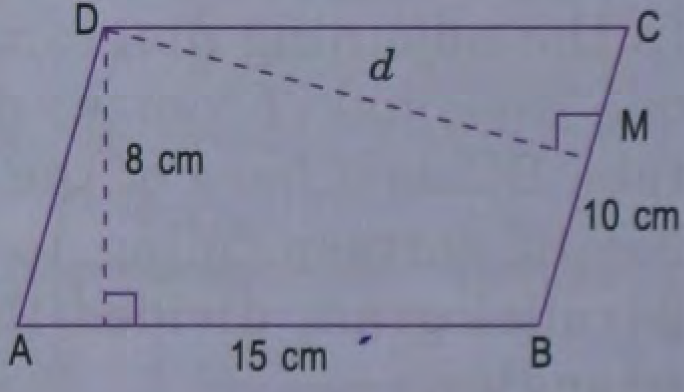
In the adjoining diagram, sides AB and DC are parallel. So ABCD is a trapezium and AB, DC are its bases. From C, draw CM perpendicular to AB and from A, draw AN perpendicular to DC (produced). Then CM = AN = h (each being distance between parallel lines), so h is the height of the trapezium.

Area of trapezium ABCD = area of Δ ABC + area of Δ ACD

$$= \frac{1}{2} \times AB \times h + \frac{1}{2} \times DC \times h = \frac{1}{2}(AB + DC) \times h$$

$$= \frac{1}{2} \text{ (sum of parallel sides)} \times \text{height}$$

Example 1. Two adjacent sides of a parallelogram are 15 cm and 10 cm. If the distance between the longer sides is 8 cm, find the area of the parallelogram. Also find the distance between the shorter sides.



Solution. Taking 15 cm as the base of the parallelogram, its height is 8 cm.

Area of the parallelogram = base × height

$= (15 \times 8)$ cm^2

$= 120$ cm^2.

Let d cm be the distance between the shorter sides, then

area of the parallelogram = $(10 \times d)$ cm^2

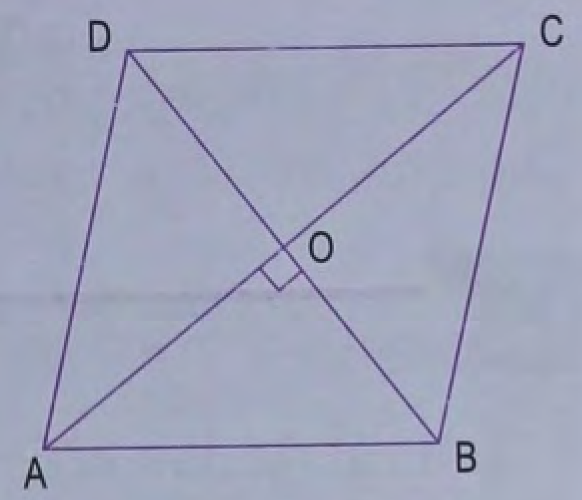
$\Rightarrow \quad 10\,d = 120 \quad \Rightarrow \quad d = 12.$

Hence, the distance between the shorter sides = 12 cm.

Example 2. The diagonals of a rhombus are 16 cm and 12 cm. Find :

(*i*) its area (*ii*) length of a side (*iii*) perimeter.

Solution. Let ABCD be a rhombus with diagonals AC = 16 cm and BD = 12 cm.



(*i*) Area of rhombus = $\frac{1}{2}$ × product of diagonals

$$= \left(\frac{1}{2} \times 16 \times 12\right) \text{ cm}^2 = 96 \text{ cm}^2.$$

(*ii*) Since diagonals of a rhombus bisect each other at right angles,

$$AO = \frac{1}{2} AC = \left(\frac{1}{2} \times 16\right) \text{ cm} = 8 \text{ cm}$$

and $$BO = \frac{1}{2} BD = \left(\frac{1}{2} \times 12\right) \text{ cm} = 6 \text{ cm}.$$

In ΔOAB, ∠O = 90°. By Pythagoras theorem, we get

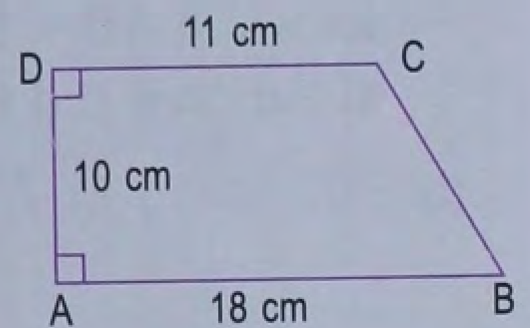
$$AB^2 = AO^2 + BO^2 = 8^2 + 6^2 = 64 + 36 = 100$$

$$\Rightarrow \quad AB = 10.$$

Hence, the length of a side of the rhombus = 10 cm.

(*iii*) Perimeter of the rhombus = (4 × 10) cm = 40 cm.

Example 3. Calculate the area enclosed by the adjoining shape.



Solution. ∠A + ∠D = 90° + 90°

⇒ sum of co-interior angles = 180°

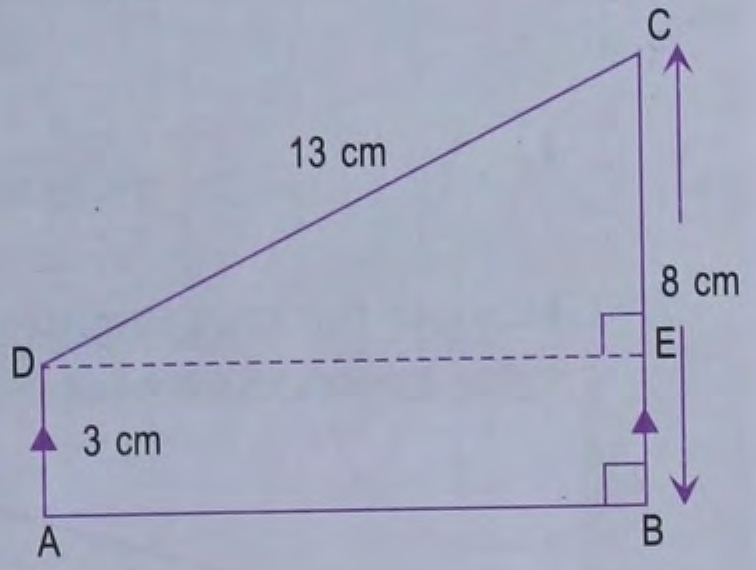
⇒ AB and DC are parallel.

So, ABCD is a trapezium with height = 10 cm.

Area enclosed by the given shape = area of trapezium ABCD

$= \frac{1}{2}$ (sum of parallel sides) × height $= \frac{1}{2}$ (18 + 11) × 10 cm² = 145 cm².

Example 4. In the adjoining sketch, AD is parallel to BC. Find the area of the trapezium ABCD.



Solution. From figure,

$$EC = BC - BE = BC - AD$$
$$= 8 \text{ cm} - 3 \text{ cm} = 5 \text{ cm}$$

From ΔDEC, by Pythagoras theorem, we get

$$CD^2 = DE^2 + EC^2 \Rightarrow 13^2 = DE^2 + 5^2$$

$$\Rightarrow DE^2 = 13^2 - 5^2 = 169 - 25 = 144$$

$$\Rightarrow DE = \sqrt{144} = 12.$$

∴ Height of trapezium ABCD = 12 cm.

Area of trapezium ABCD = $\frac{1}{2}$ (sum of parallel sides) × height

$$= \frac{1}{2} (3 + 8) \times 12 \text{ cm}^2 = 66 \text{ cm}^2.$$

Example 5. Two parallel sides of a trapezium are in the ratio 7 : 11 and the distance between them is 17 cm. If the area of the trapezium is 306 cm², find the lengths of its parallel sides.

Solution. As the lengths of parallel sides are in the ratio 7 : 11, let the lengths of parallel sides be $7x$ cm and $11x$ cm.

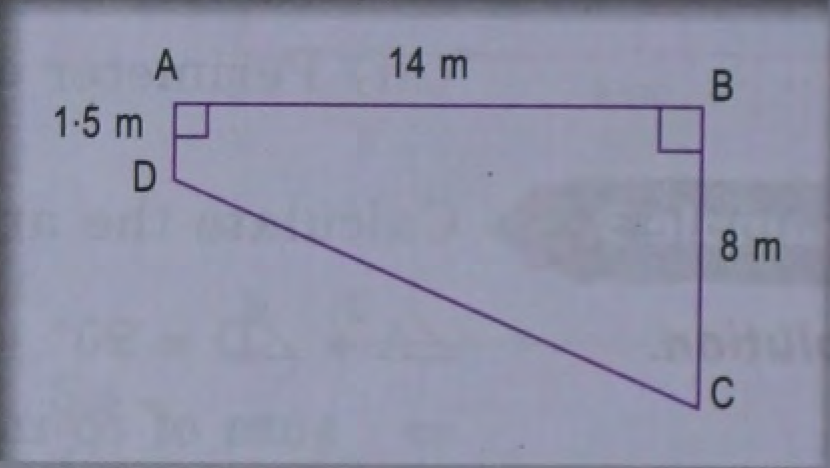
Since area of trapezium = $\frac{1}{2}$ (sum of parallel sides) × height

$$\Rightarrow 306 = \frac{1}{2}(7x + 11x) \times 17 \Rightarrow 306 = 153x \Rightarrow x = 2.$$

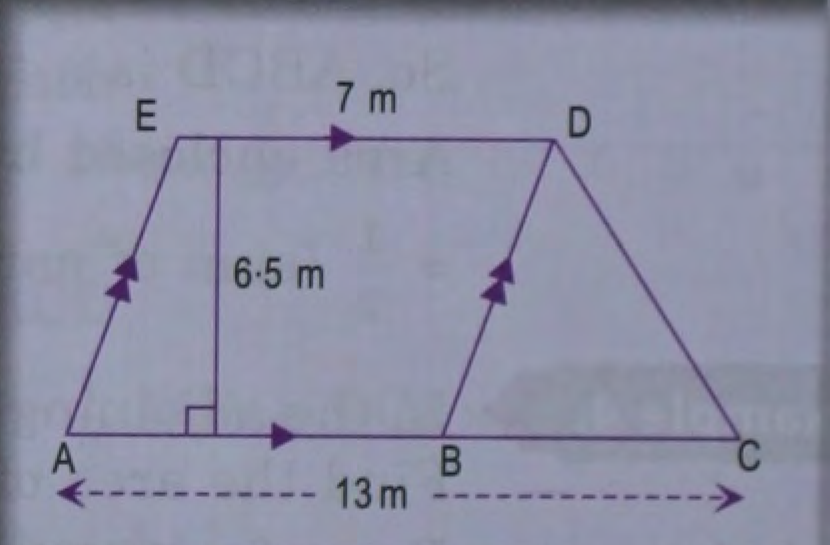
$\therefore$ The lengths of parallel sides are (7×2) cm and (11×2) cm
i.e. 14 cm and 22 cm.

Exercise 28.3

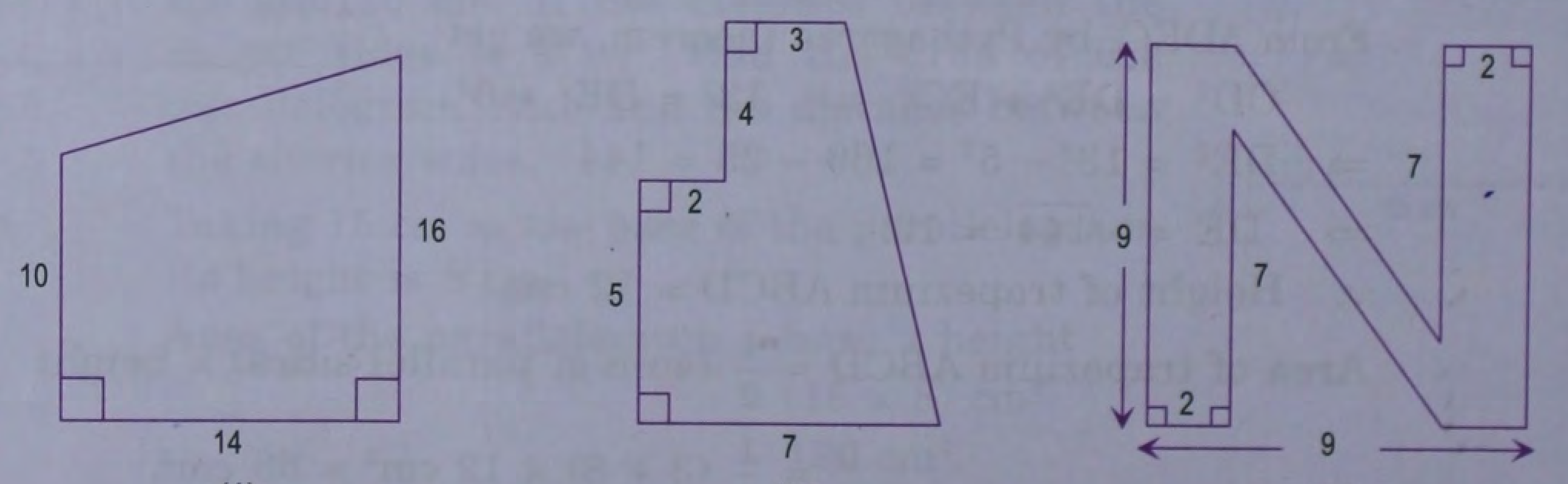
1. Find the area of a parallelogram whose :
   (*i*) base = 2 m and height = 1·5 m (*ii*) base = 3·4 m and height = 90 cm.
2. Two adjacent sides of a parallelogram are 24 cm and 18 cm. If the distance between the longer sides is 12 cm, find the distance between the shorter sides.
3. Each side of a rhombus is 13 cm and one diagonal is 10 cm. Find
   (*i*) the length of its other diagonal (*ii*) the area of the rhombus.
4. The parallel sides of a trapezium are 17 cm and 25 cm. If the distance between them is 13 cm, find the area of the trapezium.
5. The cross-section ABCD of a swimming pool is a trapezium. Its width AB = 14 m, depth at the shallow end is 1·5 m and at the deep end is 8 m. Find the area of the cross-section.



6. From the adjoining diagram, calculate
   (*i*) the area of trapezium ACDE
   (*ii*) the area of parallelogram ABDE
   (*iii*) the area of triangle BCD.

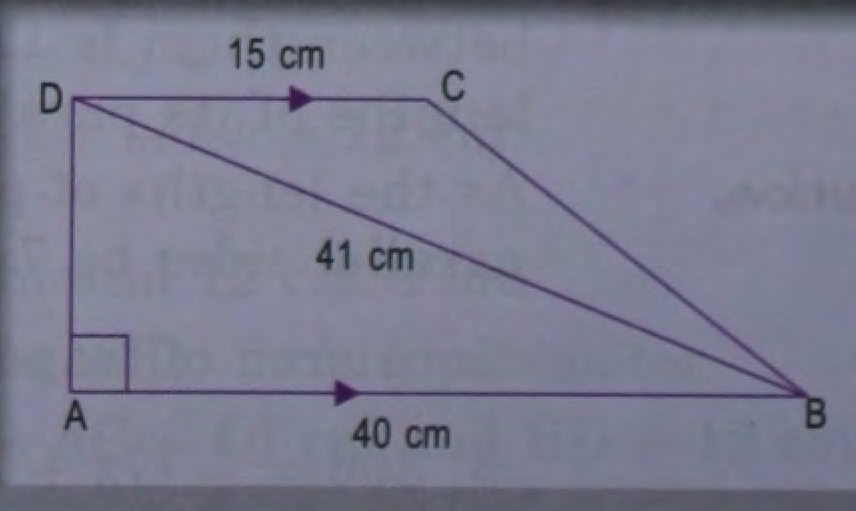


7. Calculate the area enclosed by the following shapes. All measurements are in centimeters.



(*i*) (*ii*) (*iii*)

8. From the adjoining sketch, calculate
   (*i*) the length AD
   (*ii*) the area of trapezium ABCD
   (*iii*) the area of triangle BCD.



9. The area of a rhombus is equal to the area of a triangle whose base and the corresponding altitude are 24·8 cm and 16·5 cm respectively. If one of the diagonals of the rhombus is 22 cm, find the length of the other diagonal.

10. The perimeter of a trapezium is 52 cm. If its non-parallel sides are 10 cm each and its altitude is 8 cm, find the area of the trapezium.

11. The area of a trapezium is 540 cm^2. If the ratio of parallel sides is 7 : 5 and the distance between them is 18 cm, find the lengths of parallel sides.

12. The area of a trapezium is 729 cm^2 and the distance between two parallel sides is 18 cm. If one of its parallel sides is 3 cm shorter than the other parallel side, find the lengths of its parallel sides.

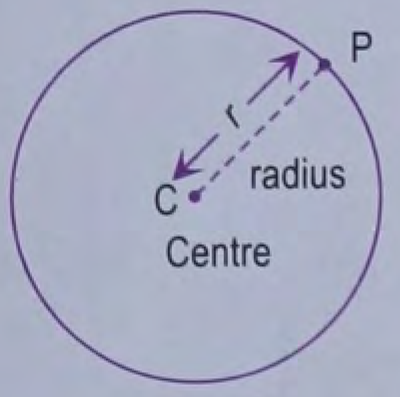
13. The area of a parallelogram is 98 cm^2. If one altitude is half the corresponding base, determine the base and the altitude of the parallelogram.

CIRCUMFERENCE AND AREA OF A CIRCLE

Circle

*A **circle** is the set of all those points, say P, in a plane, each of which is at a constant distance from a fixed point in that plane.*

The fixed point is called the **centre** and the constant distance is called the **radius**. Note that a circle is a closed curve and its radius is always positive. The adjoining figure shows a circle with centre C and radius r. Two circles are called **concentric** if they have same centre.



Diameter

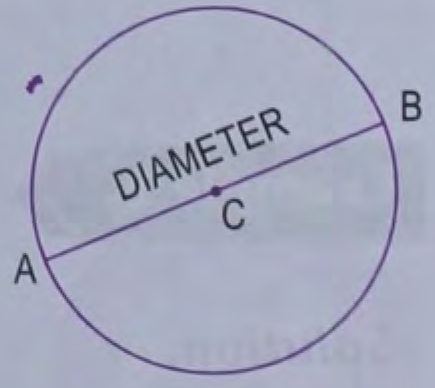
A chord of a circle passing through its centre is called a **diameter** of the circle.

In the adjoining figure, AB is a diameter of the circle with centre C. As CA and CB are both radii of the circle,

$$CA = CB = r \text{ (radius)}$$

$$\therefore \quad AB = 2r = 2 \times \text{radius}$$

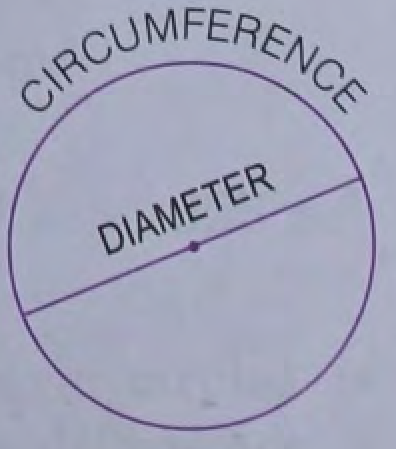
Length of a diameter = 2 × radius.



Circumference

The whole arc of a circle is called the **circumference** of the circle. Usually, the term circumference of a circle refers to the length of the boundary of the circle.

The ratio of circumference of any circle to its diameter is constant and this constant is denoted by π (read as 'Pie').



Thus, $\dfrac{\text{circumference}}{\text{diameter}} = \pi$

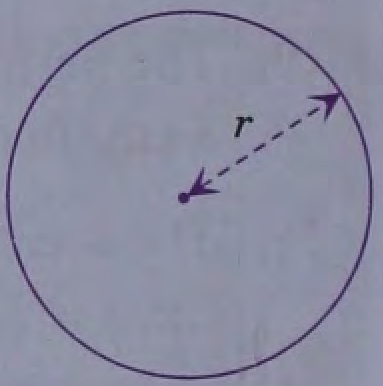
$\Rightarrow$ circumference $= \pi \times d$, where d is a diameter of the circle.

Note. π is an irrational number and its approximate value is $\frac{22}{7}$. In the subsequent work, we shall take $\pi = \frac{22}{7}$ (unless stated otherwise).

Area of a circle

***Area** of a circle is the measure of the region (surface) enclosed by its boundary.*

Formulae

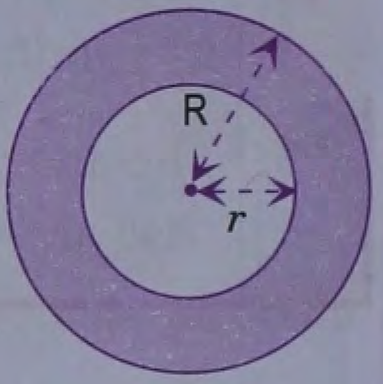


Circumference and area of a circle :

If r is the radius of a circle, then

- *the circumference of the circle* $= 2\pi r$
- *the area of the circle* $= \pi r^2$.

Area of a circular ring (track) :



If R and r are the radii of the bigger and smaller (concentric) circles, then the area of the circular ring (shaded portion)

$= \pi (R^2 - r^2)$.

Example 1. Find the circumference and the area of a circle of

(*i*) radius = 14 cm (*ii*) diameter = 3·5 cm.

Solution. (*i*) Given, radius of the circle = r = 14 cm

$$\text{Circumference} = 2\pi r = \left(2 \times \frac{22}{7} \times 14\right) \text{cm} = 88 \text{ cm}.$$

$$\text{Area of the circle} = \pi r^2 = \left(\frac{22}{7} \times 14 \times 14\right) \text{cm}^2 = 616 \text{ cm}^2.$$

(*ii*) Given, diameter of the circle = 3·5 cm

$$\therefore \quad \text{radius} = r = \frac{3{\cdot}5}{2} \text{ cm} = \frac{7}{4} \text{ cm}$$

$$\text{Circumference} = 2\pi r = \left(2 \times \frac{22}{7} \times \frac{7}{4}\right) \text{cm} = 11 \text{ cm}$$

$$\text{Area of the circle} = \pi r^2 = \left(\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}\right) \text{cm}^2 = \frac{77}{8} \text{ cm}^2 = 9{\cdot}625 \text{ cm}^2.$$

Example 2. A circular radar screen has a circumference of 176 cm. Only 90% of its area is effective. Calculate the effective area of the screen.

Solution. Let r cm be radius of the circular radar screen,

then its circumference = $2\pi r$

According to given information, $2\pi r = 176$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 176 \quad \Rightarrow \quad r = 28$$

$$\text{Area of the screen} = \pi r^2 = \left(\frac{22}{7} \times 28 \times 28\right) \text{cm}^2 = 2464 \text{ cm}^2$$

As only 90% of the area of the radar screen is effective,

$$\therefore \quad \text{the effective area of the screen} = \left(\frac{90}{100} \times 2464\right) \text{cm}^2 = 2217{\cdot}6 \text{ cm}^2.$$

Example 3. A bicycle wheel has a diameter (including the tyre) of 70 cm. How many times would the wheel rotate to cover a distance of 4·4 km?

Solution. Given, diameter of the bicycle = 70 cm

$$\therefore \quad \text{radius} = r = \frac{70}{2} \text{ cm} = 35 \text{ cm}$$

Circumference of the wheel = $2\pi r = \left(2 \times \frac{22}{7} \times 35\right)$ cm = 220 cm

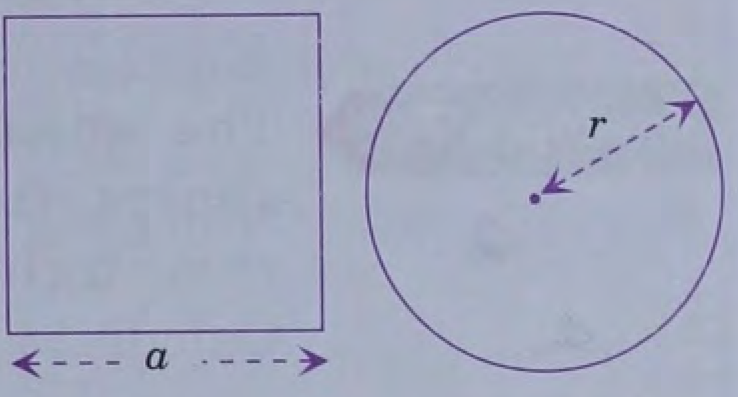
$\therefore$ The distance covered by the wheel in one revolution = 220 cm

Since the distance to be covered by the wheel = 4·4 km

= (4·4 × 1000 × 100) cm = (44 × 10000) cm,

$\therefore$ the number of times the wheel would rotate = $\frac{44 \times 10000}{220}$ = 2000.

Example 4. A copper wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent into the form of a circle, find the area of the circle.



Solution. Let the side of the square be a cm.

Area of square = a^2 cm^2

According to given information, $a^2 = 121$

$\Rightarrow a = \sqrt{121} = 11.$

$\therefore$ The length of the wire = perimeter of the square

= $4a$ cm = (4 × 11) cm = 44 cm

Let r cm be the radius of the circle.

Circumference of the circle = $2\pi r$.

As the same wire is to be bent into the form of circle,

$\therefore$ $2\pi r = 44$ $\Rightarrow$ $2 \times \frac{22}{7} \times r = 44$

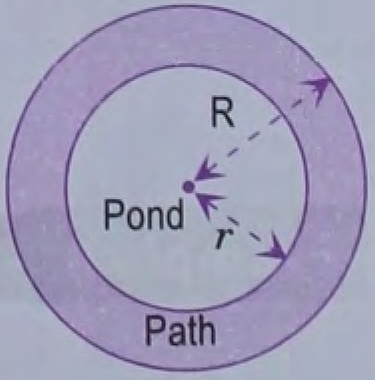
$\Rightarrow$ $r = 7$

$\therefore$ The area of the circle = $\pi r^2 = \left(\frac{22}{7} \times 7 \times 7\right)$ cm^2 = 154 cm^2.

Example 5. A circular fish pond has a diameter of 14 m. The pond is surrounded by a concrete path 1·75 m wide. Find the area of the path.

Solution. Given diameter of the fish pond = 14 m,

radius of the pond = $\left(\frac{1}{2} \times 14\right)$ m = 7 m



Width of the circular path = 1·75 m.

Let R be the radius of the outer circle.

Then R = 7 m + 1·75 m = $\left(7 + \frac{7}{4}\right)$ m = $\frac{35}{4}$ m

Area of the path = $\pi (R^2 - r^2)$

$= \frac{22}{7}\left(\frac{35}{4} \times \frac{35}{4} - 7 \times 7\right) m^2 = 22\left(\frac{175}{16} - 7\right) m^2$

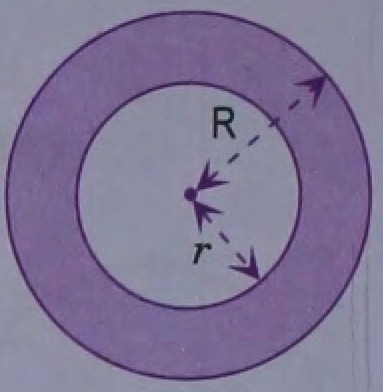
$= 22\left(\frac{175 - 112}{16}\right) m^2 = \frac{11 \times 63}{8} m^2 = \frac{693}{8} m^2 = 86{\cdot}625\ m^2.$

Example 6. The area of a circular ring enclosed between two concentric circles is 286 cm^2. Find the radii of the two circles, given that their difference is 7 cm.

Solution. Let the radii of the outer and inner circles be R cm and r cm respectively.

According to the given information,

$R - r = 7$ ...(i)



and $\pi (R^2 - r^2) = 286$

$\Rightarrow \pi (R - r)(R + r) = 286$

$\Rightarrow \frac{22}{7} \times 7 \times (R + r) = 286$ [using (*i*)]

$\Rightarrow R + r = 13$ ...(*ii*)

Adding (*i*) and (*ii*), we get

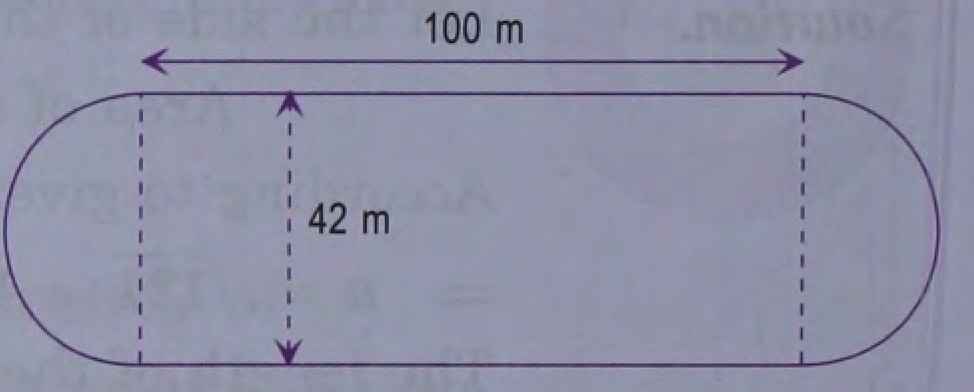
$2R = 20 \Rightarrow R = 10.$

Subtracting (*i*) from (*ii*), we get

$2r = 6 \Rightarrow r = 3.$

$\therefore$ The radii of the two circles are 10 cm and 3 cm.

Example 7. The adjoining sketch represents a sports field which consists of a rectangle and two semicircles. Calculate



(*i*) the area of the field.

(*ii*) the length of the boundary of field.

Solution. The two semicircles put together make a circle of diameter 42 m *i.e.* of radius 21 m.

(*i*) Area of the field = area of rectangle + area of two semicircles

$$= (100 \times 42)\ m^2 + \pi r^2$$

$$= 4200\ m^2 + \left(\frac{22}{7} \times 21 \times 21\right) m^2$$

$$= (4200 + 1386)\ m^2 = 5586\ m^2.$$

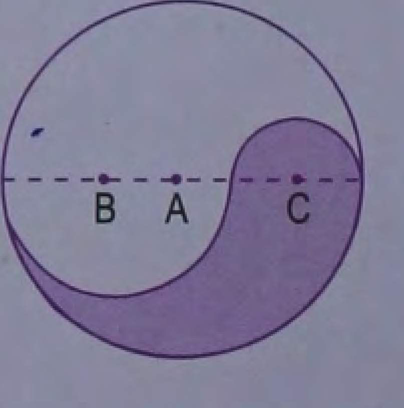
(*ii*) The boundary of the field consists of two line segments, each of length 100 m and two semicircles of radius 21 m.

$\therefore$ The length of the boundary $= (2 \times 100)\ m + 2\pi r$

$$= 200\ m + \left(2 \times \frac{22}{7} \times 21\right) m$$

$$= 200\ m + 132\ m = 332\ m.$$

Example 8. In the adjoining figure, the points A, B and C are centres of arcs of circles of radii 5 cm, 3 cm and 2 cm respectively. Find the perimeter and the area of the shaded region. (Leave the answer in π.)



Solution. Perimeter of the shaded region

$$= \left(\frac{1}{2} \times 2\pi \times 5 + \frac{1}{2} \times 2\pi \times 3 + \frac{1}{2} \times 2\pi \times 2\right) cm$$

$$= \pi (5 + 3 + 2)\ cm = 10\pi\ cm.$$

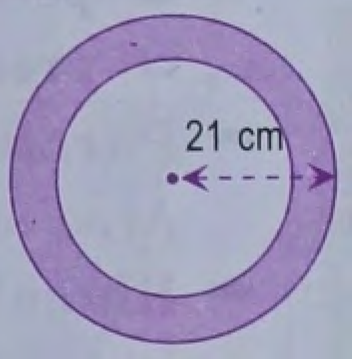
Area of the shaded region

$$= \left(\frac{1}{2} \times \pi \times 5^2 - \frac{1}{2} \times \pi \times 3^2 + \frac{1}{2} \times \pi \times 2^2\right) cm^2$$

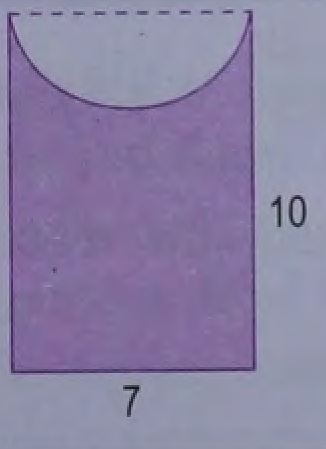
$$= \frac{\pi}{2} (25 - 9 + 4)\ cm^2 = 10\pi\ cm^2.$$

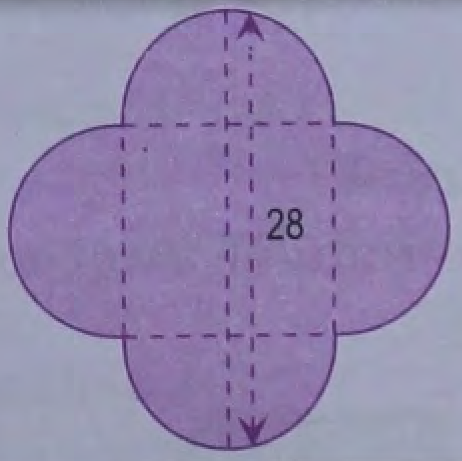
Exercise 28.4

1. Find the circumference and the area of a circle of radius
   (*i*) 7 cm (*ii*) 21 cm (*iii*) 3·5 cm.
2. A circus ring is a circle with diameter 21 m. Find the area of the ring.
3. The minute hand of a tower clock is 1·4 m long. How far does the tip of the hand move in 1 hour?
4. Taking the radius of the Earth to be 6398 km, calculate the length of the equator.
5. Find the length of the diameter of a circle whose circumference is 44 cm.
6. Find the radius and the area of a circle if its circumference is 18π cm.
7. Find the radius and the circumference of a circle whose area is 144π cm^2.
8. Cotton thread is wound on a reel which has a diameter of 2·8 cm. There are 1200 turns of the thread on the reel. What is the total length in metres of the thread on the reel?
9. How many times will the wheel of a car rotate in a journey of 88 km, given that the diameter of the wheel is 56 cm?
10. From a square cardboard of side 21 cm, a circle of maximum area is cut out. Find the area of the cardboard left.
    [Hint. Diameter of circle of maximum area = 21 cm.]
11. A wire is in the form of a square of side 27·5 cm. It is straightened and bent into the shape of a circle. Find the area of the circle.
12. In the adjoining figure, the area enclosed between the concentric circles is 770 cm^2. If the radius of the outer circle is 21 cm, calculate the radius of the inner circle.

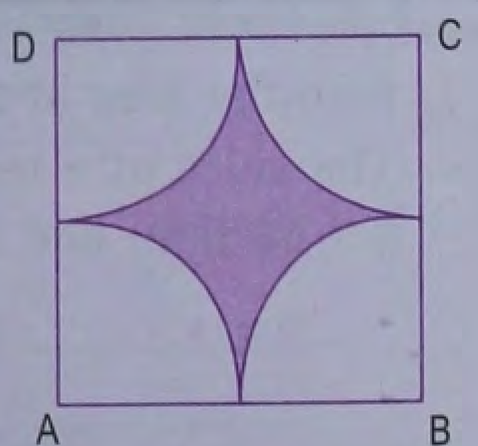


13. The sum of diameters of two circles is 2·8 m and the difference of their circumferences is 0·88 m. Find the radii of two circles.
14. Calculate the length of the boundary and the area of the shaded region in the following diagrams. All measurements are in centimetres.
    (*i*) Unshaded part is a semicircle (*ii*) Four semicircles on a square

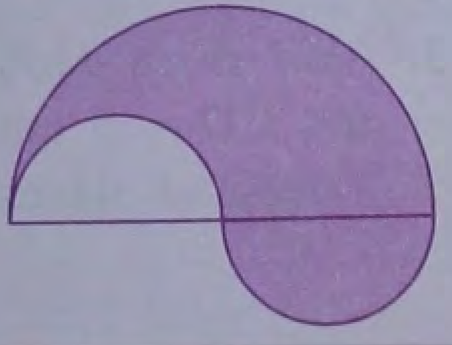




15. In the adjoining figure, ABCD is a square of side 14 cm. A, B, C and D are centres of circular arcs of equal radius. Find the perimeter and the area of the shaded region.



16. The boundary of the shaded region in the given figure consists of three semicircles, the smaller being equal. If the diameter of the larger one is 28 cm, find
    (*i*) the length of the boundary
    (*ii*) the area of the shaded region.



Summary

- Perimeter of a closed plane figure is the length of its boundary.
- Area of a closed plane figure is the measure of the region (surface) enclosed by its boundary.
- **Area of a triangle**
  - Area of a triangle = $\frac{1}{2}$ × base × height
  - Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ where a, b and c are the lengths of the sides and $s = \frac{a+b+c}{2}$.
  - Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2$, where a is the side.
- **Rectangle**

  If l = length and b = breadth of a rectangle, then
  - perimeter = $2(l + b)$
  - length of a diagonal = $\sqrt{l^2 + b^2}$
  - area = $l \times b$.
- **Square**

  If a is the length of side of a square, then
  - perimeter = $4a$
  - length of a diagonal = $\sqrt{2}\,a$
  - area = a^2.
- **Parallelogram**

  Area of a parallelogram = base × height
- **Rhombus**

  Area of a rhombus = $\frac{1}{2}$ × product of diagonals
- **Trapezium**

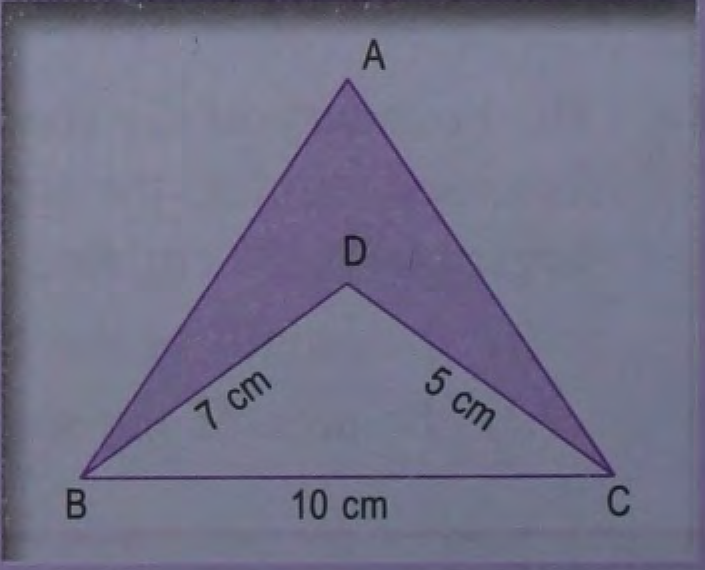
  Area of a trapezium = $\frac{1}{2}$ (sum of parallel sides) × height
- **Circle**

  If r is the radius of a circle, then
  - length of a diameter = $2r$
  - circumference = $2\pi r$
  - area = πr^2
  - Take $\pi = \frac{22}{7}$ (unless given otherwise).
- **Circular ring (track)**

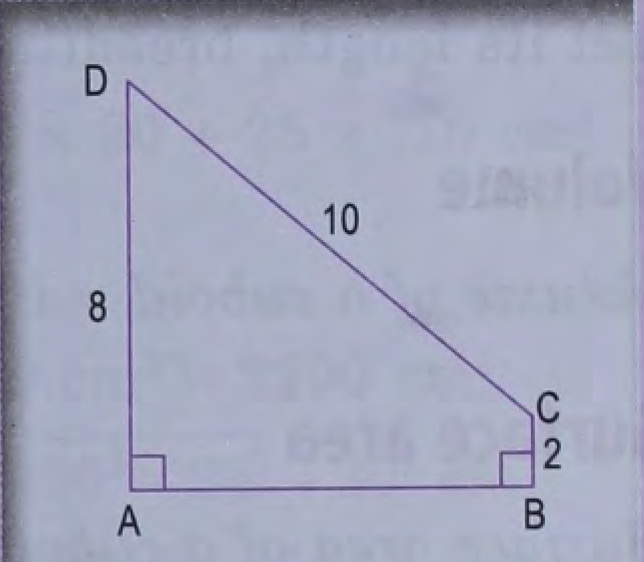
  If R and r are the radii of two concentric circles then the area of the circular ring (track) = $\pi(R^2 - r^2)$.

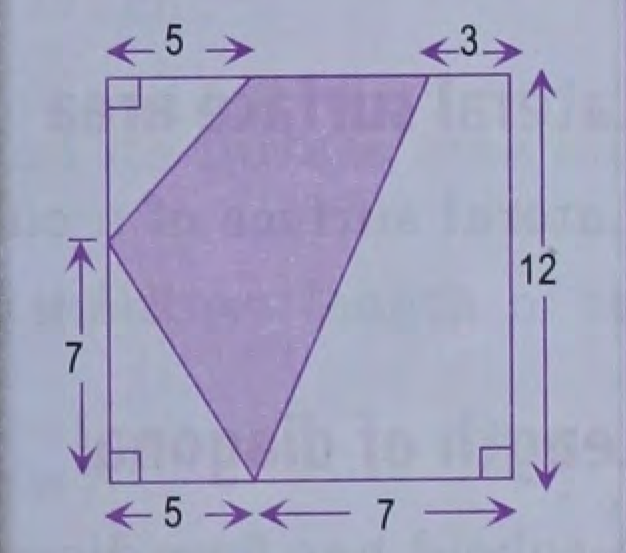
Check Your Progress

1. Find the area of an isosceles right triangle, if one of the equal sides is 14 cm long.
2. The sides of a triangle are 975 m, 1050 m and 1125 m. If the field is sold at the rate of ₹ 2 lakh per hectare, find its selling price. [1 hectare = 10000 m²]
3. ABC is an equilateral triangle with side 10 cm. If BD = 7 cm and CD = 5 cm, find the area of the shaded region correct to 2 decimal places.

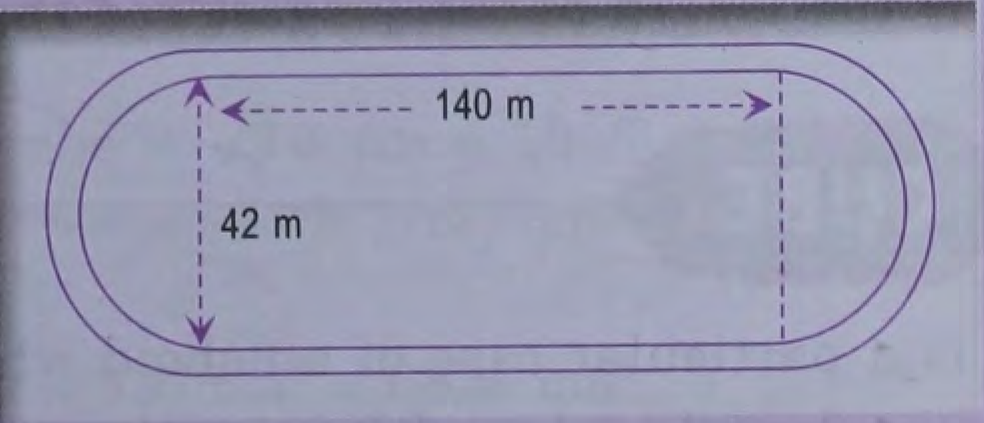


4. The area of a triangle is 48 cm^2. If a side and the corresponding altitude are in the ratio 3 : 2, find their lengths.
5. It costs ₹ 936 to fence a square field at ₹ 7·80 per metre. Find the cost of levelling the field at ₹ 2·50 per square metre.
6. A person walks at 3 km/hr. How long will he take to go round a square ground 5 times, the area of which being 2025 m^2?
7. The area of a square plot is 1764 m^2. Find the length of its one side and one diagonal.
   [Hint. Length of diagonal = $\sqrt{2}$ × length of a side.]
8. ABCD is a parallelogram with sides AB = 12 cm, BC = 10 cm and diagonal AC = 16 cm. Find the area of the parallelogram. Also find the distance between its shorter sides.
   [Hint. Find area of ΔABC by Heron's formula. Area of parallelogram ABCD = 2 × area of ΔABC.]
9. The parallel sides of an isosceles trapezium are in the ratio 2 : 3. If its height is 4 cm and area is 60 cm^2, find
   (*i*) the lengths of parallel sides
   (*ii*) the perimeter of the trapezium.
10. In the adjoining diagram, all measurements are in centimetres. Find
   (*i*) the length of AB.
   (*ii*) the area of the trapezium ABCD.
   [Hint. From C, draw CN perpendicular to AD.]





11. In the adjoining diagram, all measurements are in centimetres. Calculate the area of the shaded region.
12. If the area of a circle is 78·5 cm^2, find its circumference. (Take π = 3·14)
13. Find the circumference of the circle whose area is 16 times the area of the circle with diameter 7 cm.
14. Find the circumference of the circle whose area is equal to the sum of the areas of three circles with radius 2 cm, 3 cm and 6 cm.
15. From a square cardboard, a circle of biggest area was cut out. If the area of the circle is 154 cm^2, calculate the original area of the cardboard.
   [Hint. Side of the square board = diameter of the circle.]
16. A road 3·5 m wide surrounds a circular park whose circumference is 88 m. Find the cost of paving the road at the rate of ₹ 60 per square metre.
17. The adjoining sketch shows a running tract 3·5 m wide all around which consists of two straight paths and two semicircular rings. Find the area of the track.



18. If the radius of circle is increased by 100%, find the percentage increase in the area of the circle.