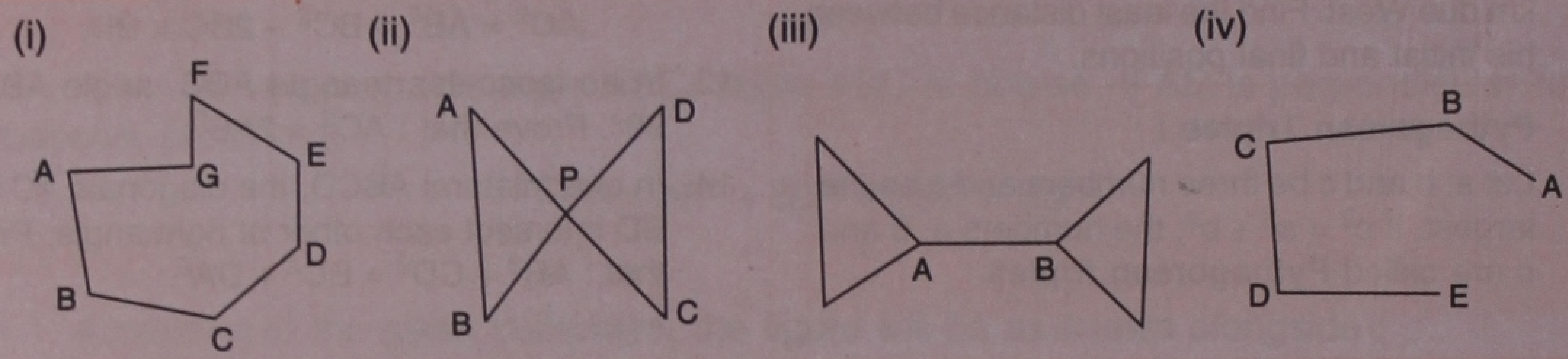
CHAPTER 26

POLYGON

26.1 INTRODUCTION

Polygon	It is a closed plane figure, bounded by straight-line segments. The line segments forming a polygon intersect only at end-points and each end-point is shared by only two line segments.

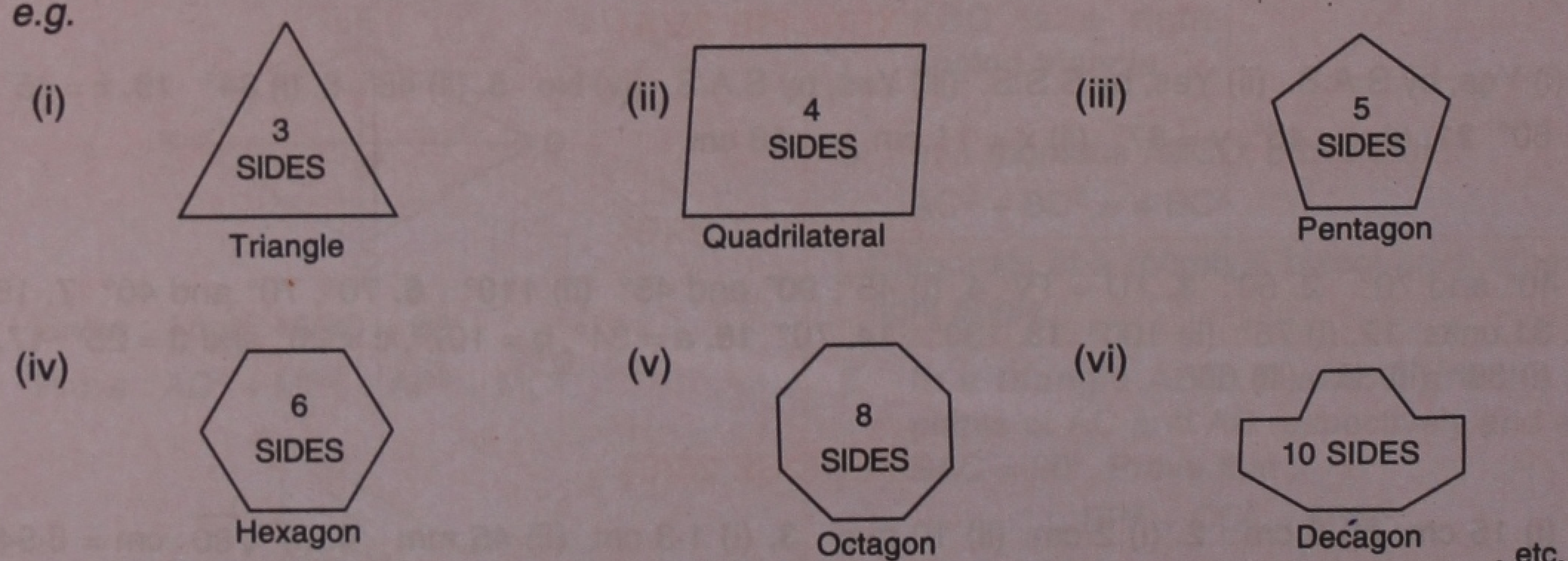


1. The figure (i), given above, **represents a polygon.**
2. In figure (ii), given above, the line segments AC and BD intersect at point P which is not an end-point, therefore, the figure **does not represent a polygon.**
3. The figure (iii), given above, **does not represent a polygon** as points A and B are the end-points of three line segments.
4. The figure (iv), given above, **does not represent** a **polygon** as it is not a closed figure. Also, the end-points A and E are not shared by two line segments.

The segments which make up a polygon are called the *sides* of the polygon and the end-points of the segments are called the vertices of the polygon.

Polygons are named according to the number of sides they contain.

e.g.



, etc.

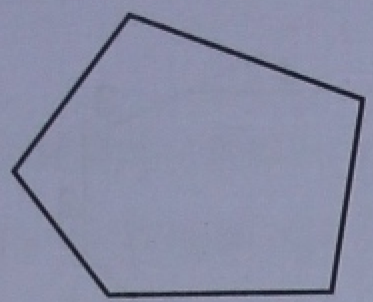
26.2 TYPES OF POLYGONS

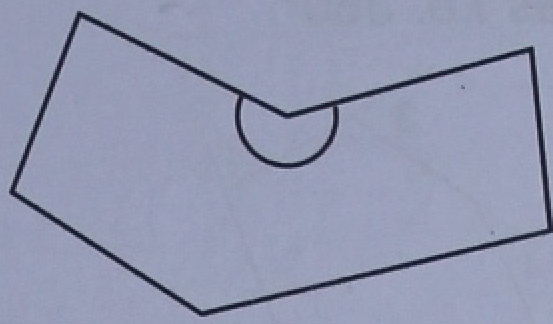
1. ***Convex polygon :***

   If *each angle* of a polygon *is less than 180°*, it is called a *convex polygon.*

2. *Concave polygon :*

If *at least one angle* of a polygon *is more than 180°*, it is called a *concave* or re-entrant polygon.

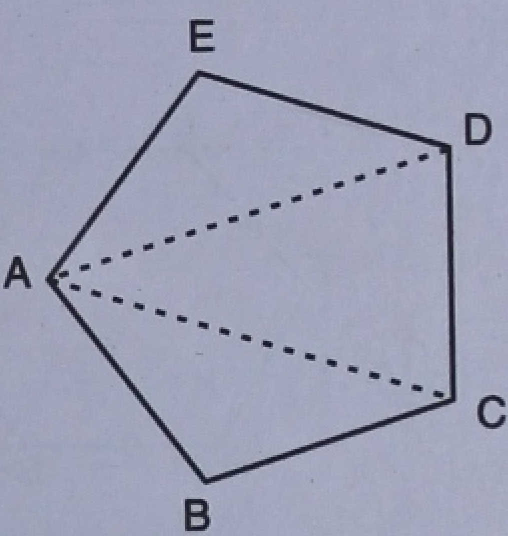

Convex polygon


Concave polygon

Unless it is stated, a polygon means a convex polygon.

Remember :

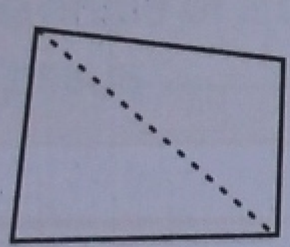
A line segment joining any two non-consecutive vertices of a polygon is called its diagonal.

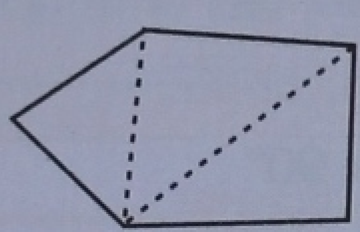


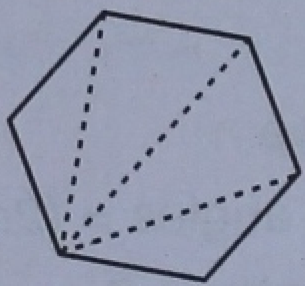
In the adjoining figure, AC is a diagonal of pentagon ABCDE as it joins two non-consecutive vertices A and C of the pentagon. Similarly, AD is also a diagonal. More diagonals can be drawn through the vertices B, C, D and E of the pentagon ABCDE.

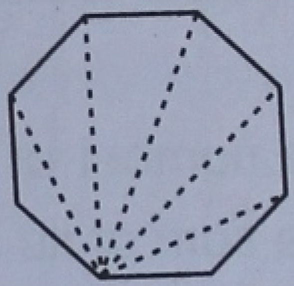
26.3 SUM OF ANGLES OF A POLYGON

Draw all possible diagonals through a single vertex of a polygon to form as many triangles as possible.


4 sides
2 triangles


5 sides
3 triangles


6 sides
4 triangles


8 sides
6 triangles

It is observed that the number of triangles formed is two less than the number of sides in the polygon.

So, if a polygon has n sides, the number of triangles formed will be n – 2.

Since, the sum of angles of a triangle = 180°

∴ The sum of angles of (n – 2) triangles = $(n - 2) \times 180°$

⇒ **Sum of angles** (interior angles) **of a polygon with n sides** = $(n - 2) \times 180°$

= $(2n - 4) \times 90°$

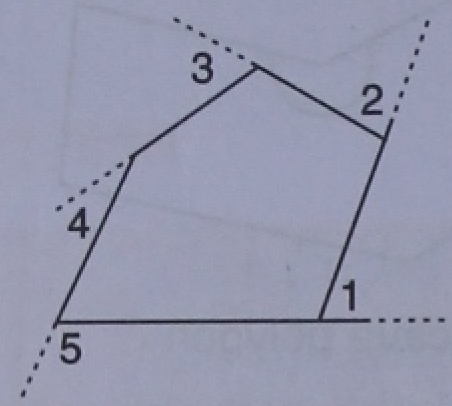
= **(2n – 4) right angles**

TEST YOURSELF

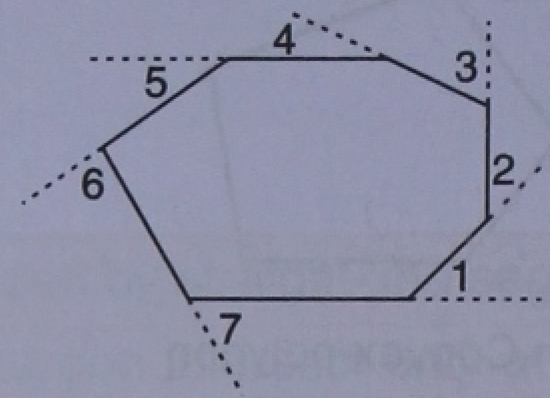
1. If a polygon has 7 sides, it has .... vertices.
2. From each vertex of a ten-sided polygon, ... diagonals can be drawn.
3. If one angle of a polygon is 190°, the polygon is called a .... polygon.
4. A hexagon has ...... sides, and the sum of its interior angles is $(2n - 4) \times 90°$ = ... = ... = ...
5. By drawing maximum number of diagonals from one vertex of an n-sided polygon; .... triangles are formed and sum of the interior angles of these triangles ....

26.4 SUM OF EXTERIOR ANGLES OF A POLYGON

If the sides of a polygon are produced in order, the sum of exterior angles so formed is always 4 right angles *i.e.* 360°.



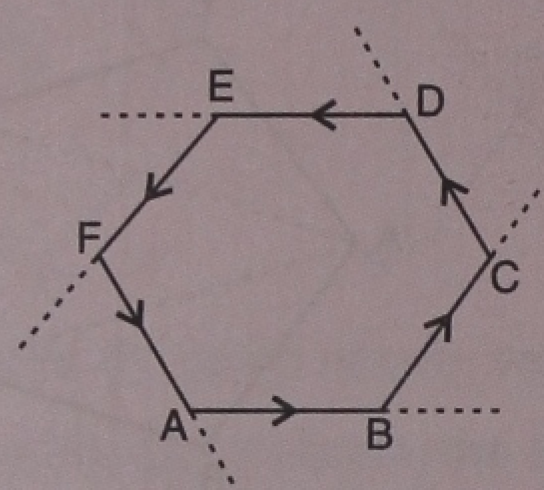
[$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360°$]

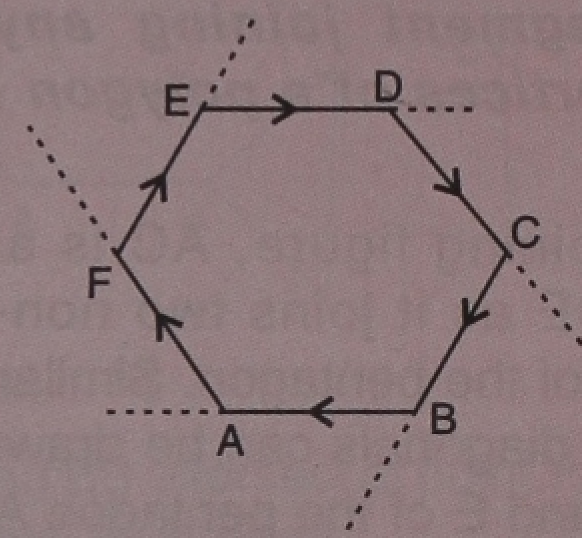


[$\angle 1 + \angle 2 +$ .... $+ \angle 7 = 360°$]

If a man walks along the sides of a polygon and each side of the polygon is produced in the direction of motion of the man, the sides of the polygon are said to be produced in order.

e.g.





In each diagram, the direction of motion of the man is represented by arrows.

Example 1 :

Is it possible to have a polygon, the sum of whose interior angles is 9 right angles.

Solution :

Let the number of sides be n.

$\therefore$ The sum of its interior angles = $(2n - 4) \times 90°$

According to the given statement :

$(2n - 4) \times 90° = 9 \times 90°$ [$\because$ 9 right angles = $9 \times 90°$]

$\Rightarrow$ $2n - 4 = 9$

$\Rightarrow$ $n = 6{\cdot}5$; **which is not possible** **(Ans.)**

1. The number of sides in a polygon is always a natural number and is never in fraction or decimals.
2. The smallest number of sides in a polygon is 3, which is in case of a triangle.

Example 2 :

The sides of a pentagon are produced in order. If the measures of exterior angles so obtained are x°, (2x)°, (3x)°, (4x)° and (5x)°, find all the exterior angles.

Solution :

Since, the sum of exterior angles obtained in the above case = 360°

$\Rightarrow$ $x° + (2x)° + (3x)° + (4x)° + (5x)° = 360°$

$\Rightarrow$ $(15x)° = 360°$ *i.e.* $x = \frac{360}{15} = 24$

$\therefore$ **Exterior angles** = 24°, (2 × 24)°, (3 × 24)°, (4 × 24)° and (5 × 24)°

= **24°, 48°, 72°, 96° and 120°** **(Ans.)**

Example 3 :

One angle of a seven-sided polygon is 114° and each of the other six angles is x°. Find the magnitude of x°.

Solution :

Since, each of the other six angles is x°

$\Rightarrow$ Sum of these six angles = 6x°

$\Rightarrow$ Sum of all the seven angles = 114° + 6x° I

According to the formula :

Sum of interior angles of the seven-side polygon

$= (2n - 4) \times 90°$

$= (2 \times 7 - 4) \times 90° = 900°$ II

$\therefore$ 114° + 6x° = 900° [From I and II]

$\Rightarrow$ 6x° = 900° – 114° = 786° $\Rightarrow$ **x° = 131°** **(Ans.)**

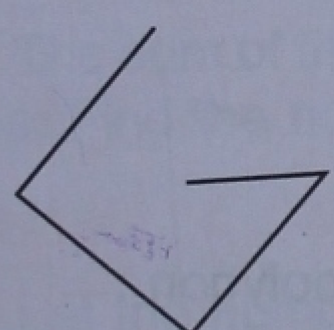
TEST YOURSELF

6. The angles of a quadrilateral are in the ratio 5 : 6 : 3 : 4. The smallest angle of this quadrilateral is .... = ...
7. The anlges of a pentagon are in the ratio 7 : 6 : 5 : 4 : 5; its largest angle is .... = ...
8. Two angles of a quadrilateral are 68° and 107° and the other two angles are in the ratio 2 : 3. Since, 360° – 68° – 107° = ..., the smallest of other two angles is .... = ...
9. One angle of a quadrilateral is 120° and the remaining angles are equal; each of the equal angles is ....

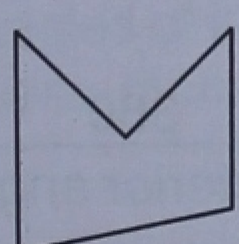
EXERCISE 26 (A)

1. State which of the following are polygons :

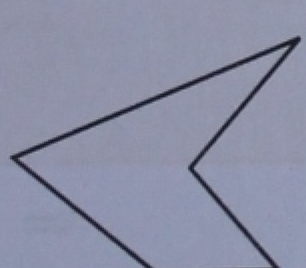
(i)



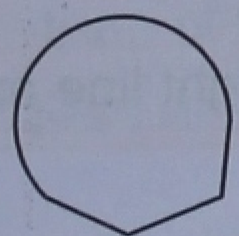
(ii)



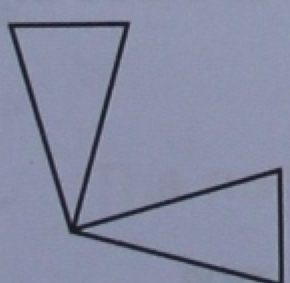
(iii)



(iv)



(v)



If the given figure is a polygon, name it as **convex** or **concave.**

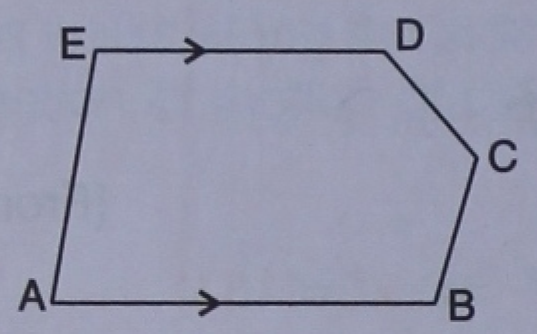
2. Calculate the sum of angles of a polygon with :
   (i) 10 sides (ii) 12 sides
   (iii) 20 sides
3. Find the number of sides in a polygon if the sum of its interior angles is :
   (i) 900° (ii) 1620°
   (iii) 16 right angles
4. Is it possible to have a polygon, whose sum of interior angles is :
   (i) 870° (ii) 2340°
   (iii) 7 right angles ?
5. (i) If all the angles of a hexagon are equal, find the measure of each angle.
   (ii) If all the angles of a 14-sided figure are equal, find the measure of each angle.
6. Find the sum of exterior angles obtained on producing, in order, the sides of a polygon with:
   (i) 7 sides (ii) 10 sides (iii) 250 sides
7. The sides of a hexagon are produced in order. If the measures of exterior angles so obtained

are $(6x - 1)°$, $(10x + 2)°$, $(8x + 2)°$, $(9x - 3)°$, $(5x + 4)°$ and $(12x + 6)°$; find each exterior angle.

8. The interior angles of a pentagon are in the ratio 4 : 5 : 6 : 7 : 5. Find each angle of the pentagon.

9. Two angles of a hexagon are 120° and 160°. If the remaining four angles are equal, find each equal angle.

10. The figure, given below, shows a pentagon ABCDE with sides AB and ED parallel to each other, and ∠B : ∠C : ∠D = 5 : 6 : 7.



(i) Using formula, find the sum of interior angles of the pentagon.

(ii) Write the value of ∠A + ∠E.

(iii) Find angles B, C and D.

11. Two angles of a polygon are right angles and the remaining are 120° each. Find the number of sides in it.

$2 \times 90° + (n - 2) \times 120° = (2n - 4) \times 90°$.

12. In a hexagon ABCDEF, side AB is parallel to side FE and ∠B : ∠C : ∠D : ∠E = 6 : 4 : 2 : 3. Find ∠B and ∠D.

13. The angles of a hexagon are $x + 10°$, $2x + 20°$, $2x - 20°$, $3x - 50°$, $x + 40°$ and $x + 20°$. Find x.

14. In a pentagon, two angles are 40° and 60°, and the rest are in the ratio 1 : 3 : 7. Find the biggest angle of the pentagon.

26.5 REGULAR POLYGON

A polygon is said to be a regular polygon, if all its

(i) interior angles are equal, (ii) sides are equal and

(iii) exterior angles are equal.

(a) If a regular polygon has n sides :

1. The sum of its interior angles = $(2n - 4) \times 90°$

   And, each interior angle = $\frac{(2n-4)\times 90°}{n}$

2. The sum of its exterior angles = 360°

   And, each exterior angle = $\frac{360°}{n}$

3. No. of sides (n) of the regular polygon = $\frac{360°}{\text{Exterior angle}}$

(b) Whether the given polygon is regular or not, at each vertex of the polygon :

exterior angle + interior angle = 180°.

Since, both the angles together form a straight line angle.

Example 4 :

If each interior angle of a regular polygon is 144°, find the number of sides in it.

Solution :

Let the number of sides of the regular polygon be n.

∴ Its each interior angle = $\frac{(2n-4)\times 90°}{n}$ *i.e.* $144° = \frac{(2n-4)\times 90°}{n}$

⇒ $144n = 180n - 360$ *i.e.* $n = 10$

∴ **No. of sides = 10** **(Ans.)**

Alternative method :

Given: Each interior angle = 144°

and we know, interior angle + exterior angle = 180°

⇒ 144° + exterior angle = 180°

i.e. exterior angle = 36°

Since, $\text{no. of sides of a regular polygon} = \dfrac{360°}{\text{exterior angle}}$

∴ **No. of sides in the given polygon** $= \dfrac{360°}{36°} = 10$ **(Ans.)**

Example 5 :

Is it possible to have a regular polygon with each interior angle equal to 105° ?

Solution :

The number of sides in a polygon is always a whole number which is greater than or equal to 3.

Let the number of sides in the regular polygon be n.

$$\therefore \frac{(2n-4)\times 90°}{n} = 105° \Rightarrow 180\,n - 360 = 105\,n$$

$$\Rightarrow 180\,n - 105\,n = 360$$

$$\Rightarrow 75\,n = 360$$

$$\text{and,} \quad n = \frac{360}{75} = 4\frac{4}{5}$$

Since, $n = 4\frac{4}{5}$ is not a whole number.

∴ **No regular polygon is possible with each interior angle equal to 105°. (Ans.)**

Example 6 :

The sum of the interior angles of a regular polygon is equal to six times the sum of exterior angles. Find the number of sides of the polygon.

Solution :

Let the number of sides of the regular polygon be n.

∴ Sum of interior angles = (2n – 4) × 90°

∴ Sum of exterior angles = 360°

$$\Rightarrow (2n-4)\times 90° = 6\times 360°$$

$$\Rightarrow 2n - 4 = \frac{6\times 360}{90} = 24$$

$$\Rightarrow 2n = 28 \text{ and } n = 14$$

∴ **The number of sides of the regular polygon = 14** **(Ans.)**

Example 7 :

An exterior angle and an interior angle of a regular polygon are in the ratio 2 : 7. Find the number of sides in the polygon.

Solution :

Given : Exterior angle : Interior angle = 2 : 7

$\Rightarrow$ If exterior angle = 2x, the interior angle = 7x

Since, an exterior angle + interior angle = 180°

$\Rightarrow 2x + 7x = 180°$ *i.e.* $9x = 180°$ and $x = 20°$

$\therefore$ Exterior angle of the given regular polygon = $2x = 2 \times 20° = 40°$

And, **the no. of sides in the polygon** = $\frac{360°}{\text{exterior angle}}$

$= \frac{360°}{40°} = \mathbf{9}$ **(Ans.)**

Example 8 :

The ratio of the number of sides of two regular polygons is 1 : 2, and the ratio of the sum of their interior angles is 3 : 8. Find the number of sides in each polygon.

Solution :

Since, the ratio between the number of sides of the two polygons is 1 : 2.

Let the number of sides be x and 2x.

Since, the sum of interior angles of a polygon = $(2n - 4) \times 90°$

$\therefore$ The sum of interior angles of the 1st polygon = $(2x - 4) \times 90°$

and, the sum of interior angles of the 2nd polygon = $(2 \times 2x - 4) \times 90° = (4x - 4) \times 90°$

Given, the ratio of the sum of interior angles of the two regular polygons is 3 : 8.

$\Rightarrow \frac{(2x-4)\times 90°}{(4x-4)\times 90°} = \frac{3}{8}$ *i.e.* $\frac{2x-4}{4x-4} = \frac{3}{8}$

$\Rightarrow 16x - 32 = 12x - 12$ *i.e.* $4x = 20$

$\Rightarrow x = \frac{20}{4} = 5$

$\therefore$ **The number of sides in the two polygons** = x and 2x = **5 and 10** **(Ans.)**

TEST YOURSELF

10. In case of regular polygon : all its :

(i) ....

(ii) ....

(iii) ....

11. In case of a regular polygon :

(i) number of sides = $\frac{360°}{\text{............}}$

(ii) each exterior angle = ...

12. If an interior angle of a regular polygon is 135°; its exterior angle = ... = ... and the number of sides in the polygon is .... = ...

13. The ratio between the exterior angles of two regular polygons is 5 : 7. The ratio between the number of their sides is ....

EXERCISE 26 (B)

1. Fill in the blanks :

In case of regular polygon, with :		
no. of sides	each exterior angle	each interior angle
(i) ..8.....	....	....
(ii) ..12..	....	....
(iii)	.......72°.....	....
(iv)	......45°.....	....
(v)	....	.......150°....
(vi)	....	......140°......

2. Find the number of sides in a regular polygon, if its each interior angle is :

   (i) 160° (ii) 135°

   (iii) $1\frac{1}{5}$ of a right angle.

3. Find the number of sides in a regular polygon, if its each exterior angle is :

   (i) $\frac{1}{3}$ of a right angle

   (ii) two-fifths of a right angle

4. Is it possible to have a regular polygon whose each interior angle is :

   (i) 170° (ii) 138°

5. Is it possible to have a regular polygon whose each exterior angle is :

   (i) 80° (ii) 40% of a right angle

6. Find the number of sides in a regular polygon, if its interior angle is equal to its exterior angle.

7. The exterior angle of a regular polygon is one-third of its interior angle. Find the number of sides in the polygon.

8. The measure of each interior angle of a regular polygon is five times the measure of its exterior angle. Find :

   (i) measure of each interior angle,

   (ii) measure of each exterior angle and

   (iii) number of sides in the polygon

9. The ratio between the interior angle and the exterior angle of a regular polygon is 2 : 1. Find :

   (i) each exterior angle of the polygon,

   (ii) number of sides in the polygon.

10. The ratio between the exterior angle and the interior angle of a regular polygon is 1 : 4. Find the number of sides in the polygon.

11. The sum of interior angles of a regular polygon is twice the sum of its exterior angles. Find the number of sides of the polygon.

12. AB, BC and CD are three consecutive sides of a regular polygon. If angle BAC = 20°, find:

    (i) its each interior angle

    (ii) its each exterior angle

    (iii) the number of sides in the polygon.

13. Two alternate sides of a regular polygon, when produced, meet at the right angle. Calculate the number of sides in the polygon.

14. In a regular pentagon ABCDE, draw a diagonal BE and then find the measure of :

    (i) ∠BAE (ii) ∠ABE (iii) ∠BED

15. The difference between the exterior angles of two regular polygons, having the sides equal to (n – 1) and (n + 1) is 9°. Find the value of n.

> When number of sides of a regular polygon = n –1,
>
> the value of its each exterior angle = $\frac{360°}{n-1}$
>
> And, when number of sides of a regular polygon = n + 1,
>
> the value of its each exterior angle = $\frac{360°}{n+1}$
>
> Given : $\frac{360°}{n-1} - \frac{360°}{n+1} = 9°$
>
> On solving, we get **n = 9**

16. If the difference between the exterior angle of a n sided regular polygon and an (n + 1) sided regular polygon is 12°, find the value of n.

17. The ratio between the number of sides of two regular polygons is 3 : 4 and the ratio between the sum of their interior angles is 2 : 3. Find the number of sides in each polygon.

18. Three of the exterior angles of a hexagon are 40°, 51° and 86°. If each of the remaining exterior angles is x°, find the value of x.

19. Calculate the number of sides of a regular polygon, if :

    (i) its interior angle is five times its exterior angle.

(ii) the ratio between its exterior angle and interior angle is 2 : 7.

(iii) its exterior angle exceeds its interior angle by 60°.

20. The sum of interior angles of a regular polygon is thrice the sum of its exterior angles. Find the number of sides in the polygon.

ANSWERS

TEST YOURSELF

1. 7 **2.** 7 **3.** concave **4.** six ; $(2 \times 6 - 4) \times 90°$; $8 \times 90°$; 720° **5.** $n - 2$; $(n - 2) \times 180°$ **6.** $\frac{3}{18} \times 360°$; 60° **7.** $\frac{7}{27} \times 540°$, 140° **8.** 185°, $\frac{2}{5} \times 185°$; 74° **9.** $\frac{360° - 120°}{3} = 80°$ **10.** (i) interior angles are equal (ii) exterior angles are equal (iii) sides are equal **11.** (i) exterior angle (ii) $\frac{360°}{\text{no. of sides}}$ **12.** 180° – 135°, 45°, $\frac{360°}{45°}$, 8 **13.** $\frac{360°}{5}$; $\frac{360°}{7} = \frac{360°}{5} \times \frac{7}{360°} = 7 : 5$

EXERCISE 26(A)

1. (ii), (iii) and (v). (ii) concave (iii) concave **2.** (i) 1440° (ii) 1800° (iii) 3240° **3.** (i) 7 (ii) 11 (iii) 10 **4.** (i) No (ii) Yes (iii) No **5.** (i) 120° (ii) $\left(154\frac{2}{7}\right)^\circ$ **6.** (i) 360° (ii) 360° (iii) 360° **7.** 41°, 72°, 58°, 60°, 39° and 90° **8.** 80°, 100°, 120°, 140°, and 100° **9.** 110° **10.** (i) 540° (ii) $\angle A + \angle E = 180°$ (iii) $\angle B = 100°$, $\angle C = 120°$ and $\angle D = 140°$ **11.** 5 **12.** $\angle B = 216°$, $\angle C = 144°$, $\angle D = 72°$ and $\angle E = 108°$ **13.** $x = 70°$ **14.** 280°

EXERCISE 26(B)

1. (i) 45° and 135° (ii) 30° and 150° (iii) 5 and 108° (iv) 8 and 135° (v) 12 and 30° (vi) 9 and 40° **2.** (i) 18 (ii) 8 (iii) 5 **3.** (i) 12 (ii) 10 **4.** (i) Yes (ii) No **5.** (i) No (ii) Yes **6.** 4 **7.** 8 **8.** (i) 150° (ii) 30° (iii) 12 **9.** (i) 60° (ii) 6 **10.** 10 **11.** 6 **12.** (i) 140° (ii) 40° (iii) 9 **13.** 8 **14.** (i) 108° (ii) 36° (iii) 72° **15.** $n = 9$ **16.** $n = 5$ **17.** 6 and 8 **18.** 61 **19.** (i) 12 (ii) 9 (iii) 3 **20.** 8