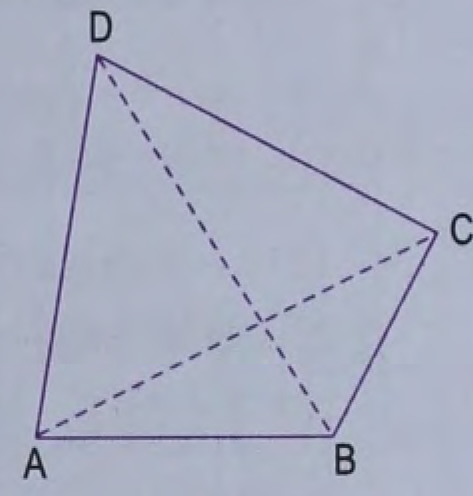
Chapter 23

QUADRILATERALS AND POLYGONS

QUADRILATERALS

A closed plane figure bounded by four line segments is called a ***quadrilateral***.

In the adjoining diagram, ABCD is a quadrilateral.



It has

four sides — AB, BC, CD and DA

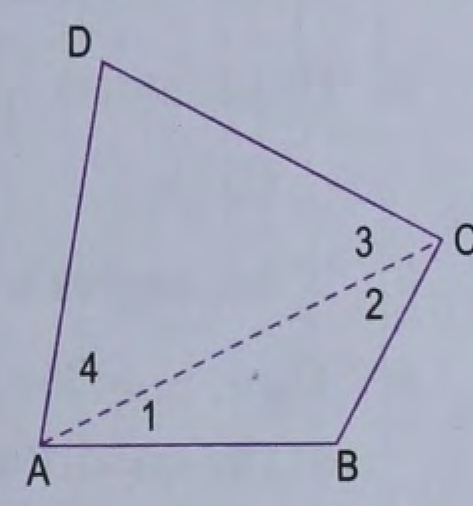
four (interior) angles — ∠A, ∠B, ∠C and ∠D

four vertices — A, B, C and D

two diagonals — AC and BD.

Sum of (interior) angles of a quadrilateral is 360°

In the adjoining figure, ABCD is *any* quadrilateral. Diagonal AC divides it into two triangles. We know that the sum of angles of a triangle is 180°,



in ΔABC, ∠1 + ∠B + ∠2 = 180° ...(*i*)

in ΔACD, ∠4 + ∠D + ∠3 = 180° ...(*ii*)

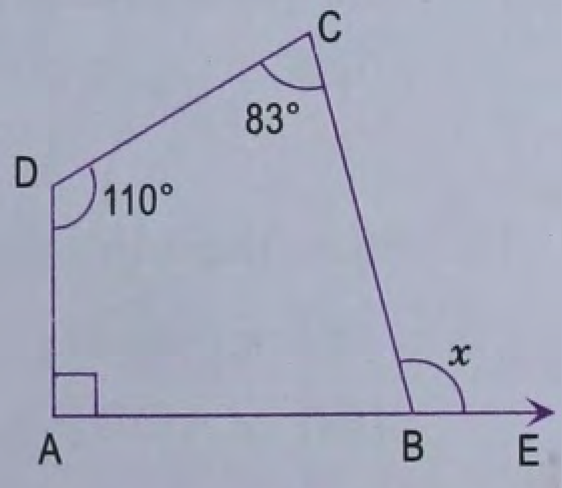
On adding (*i*) and (*ii*), we get

∠1 + ∠4 + ∠B + ∠D + ∠2 + ∠3 = 360°

⇒ ∠A + ∠B + ∠D + ∠C = 360° (from figure)

Hence, the sum of (interior) angles of a quadrilateral is 360°.

Example 1. From the adjoining diagram, calculate the value of x.



Solution. As the sum of (interior) angles of a quadrilateral is 360°,

90° + 110° + 83° + ∠ABC = 360°

⇒ ∠ABC = 360° – 90° – 110° – 83° = 77°.

As ABE is a straight line,

x + ∠ABC = 180°

⇒ x = 180° – 77° (∵ ∠ABC = 77°)

⇒ x = 103°.

Example 2. If the angles of a quadrilateral are in the ratio 5 : 8 : 11 : 12, find the angles.

Solution. Since the angles of the quadrilateral are in the ratio 5 : 8 : 11 : 12, let these angles be $5x$, $8x$, $11x$ and $12x$.

As the sum of angles of a quadrilateral is 360°,

$5x + 8x + 11x + 12x = 360°$

⇒ $36x = 360°$

$\Rightarrow \quad x = 10°$

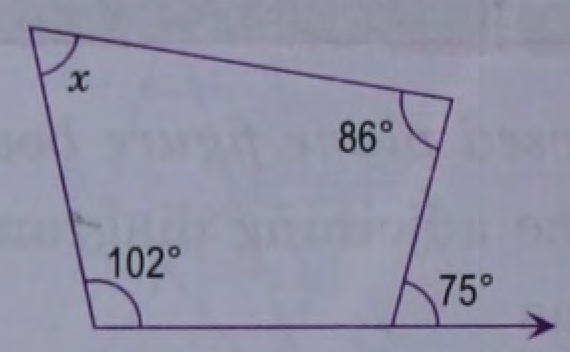
$\therefore$ The angles of the quadrilateral are

$5 \times 10°$, $8 \times 10°$, $11 \times 10°$ and $12 \times 10°$ *i.e.* 50°, 80°, 110° and 120°.

Exercise 23.1

1. If three angles of a quadrilateral are 70°, 83° and 112°, find the fourth angle.

2. From the adjoining diagram, find the value of x.



3. If two angles of a quadrilateral are 76° and 138° and the other two angles are equal, find the measure of equal angles.

4. A quadrilateral has three interior angles each equal to 95°. Find the size of the fourth interior angle.

5. If one of the angles of a quadrilateral is 210° and the remaining three angles are equal, find the measure of the equal angles.
   Note. It is a re-entrant quadrilateral.

6. If the angles of a quadrilateral are $x°$, $(x - 20)°$, $(x - 30°)$ and $(x + 10°)$, find
   (*i*) x (*ii*) the angles of the quadrilateral.

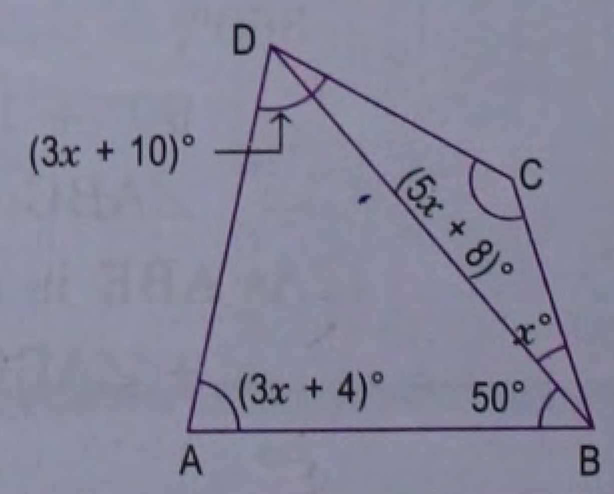
7. If the angles of a quadrilateral are in the ratio 2 : 3 : 4 : 6, find the angles.

8. Three angles of a quadrilateral are in the ratio 3 : 5 : 6. If the fourth angle is 80°, find the other angles of the quadrilateral.

9. Two angles of a quadrilateral are 78° and 87°. If the other two angles are in the ratio 5 : 8, find the size of each of them.

10. In a quadrilateral ABCD, AB || DC. If $\angle A : \angle D = 2 : 3$ and $\angle B : \angle C = 7 : 8$, find the measure of each angle.

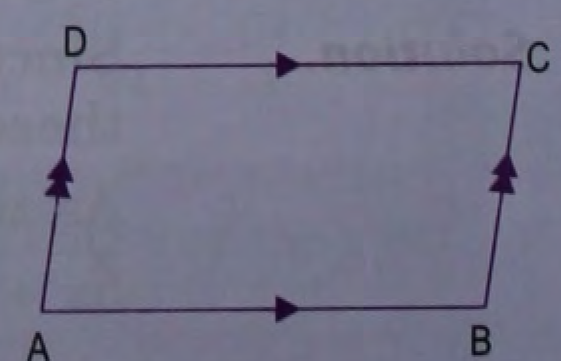
11. From the adjoining figure, find
    (*i*) x (*ii*) $\angle DAB$
    (*iii*) $\angle ADB$



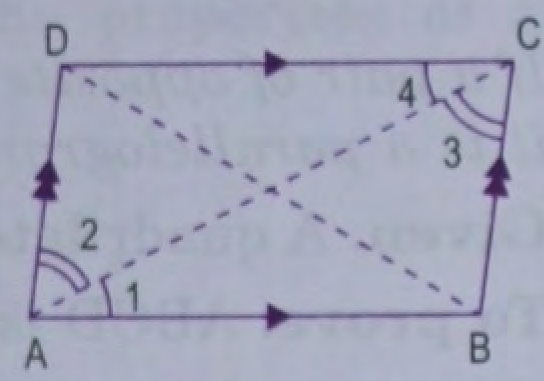
PARALLELOGRAM

A quadrilateral in which both pairs of opposite sides are parallel is called a ***parallelogram****.*

In the adjoining quadrilateral, AB || DC and AD || BC, so ABCD is a parallelogram.



Theorem 1



(i) The opposite sides of a parallelogram are equal.

(ii) The opposite angles of a parallelogram are equal.

(iii) Each diagonal bisects the parallelogram.

Given. A parallelogram ABCD.

To prove. (*i*) AB = DC and AD = BC

(*ii*) ∠B = ∠D and ∠A = ∠C

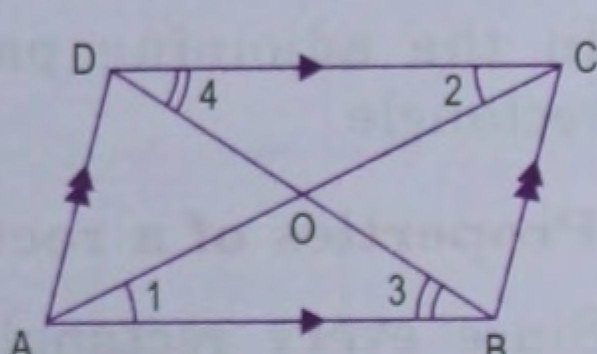
(*iii*) Area of Δ ABC = area of Δ ACD and area of Δ ABD = area of Δ BCD

Construction. Join AC and BD.

Proof.

Statements	*Reasons*
In ΔABC and ΔCDA	
1. ∠1 = ∠4	1. Alt. ∠s, as AB ∥ DC and AC cuts them
2. ∠3 = ∠2	2. Alt. ∠s, as AD ∥ BC and AC cuts them
3. AC = AC	3. Common
4. Δ ABC ≅ Δ CDA	4. A.S.A. axiom of congruency
(*i*) AB = DC and AD = BC	'c.p.c.t.'
(*ii*) ∠B = ∠D	'c.p.c.t.'
and ∠A = ∠C	Adding 1 and 2, ∠1 + ∠2 = ∠3 + ∠4
(*iii*) area of Δ ABC = area of Δ CDA $= \frac{1}{2}$(area of parallelogram ABCD)	ΔABC ≅ Δ CDA and congruent triangles have equal area.
⇒ AC bisects parallelogram ABCD	
Similarly, Δ ABD ≅ Δ CDB	
⇒ area of ΔABD = area of Δ CDB	
⇒ BD bisects parallelogram ABCD	
Q.E.D.	

Theorem 2



The diagonals of a parallelogram bisect each other.

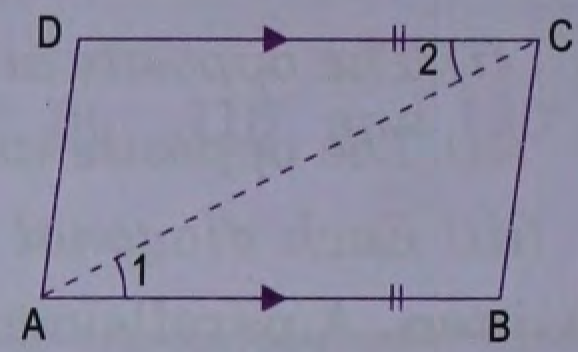
Given. A parallelogram ABCD whose diagonals AC and BD intersect at O.

To prove. OA = OC and OB = OD.

Proof.

Statements	*Reasons*
In ΔOAB and ΔOCD	
1. ∠1 = ∠2	1. Alt. ∠s, as AB ∥ DC and AC cuts them
2. ∠3 = ∠4	2. Alt. ∠s, as AB ∥ DC and BD cuts them
3. AB = DC	3. Opp. sides of a ∥ gm are equal, Theorem 1
4. Δ OAB ≅ Δ OCD	4. A.S.A. axiom of congruency
∴ OA = OC and OB = OD	'c.p.c.t.'
Q.E.D.	

Theorem 3



If a pair of opposite sides of a quadrilateral are equal and parallel, it is a parallelogram.

Given. A quadrilateral ABCD in which AB || DC and AB = DC.

To prove. ABCD is a parallelogram.

Construction. Join AC.

Proof.

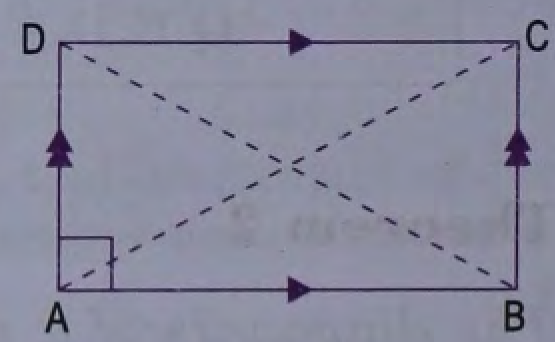
Statements	*Reasons*
In ΔABC and ΔCDA	
1. ∠1 = ∠2	1. Alt. ∠s, as AB || DC and AC cuts them
2. AB = DC	2. Given
3. AC = AC	3. Common
4. Δ ABC ≅ Δ CDA	4. S.A.S. (axiom of congruency)
5. ∠ACB = ∠CAD	5. 'c.p.c.t.'
6. AD || BC	6. AC cuts AD and BC, and alt. ∠s are equal
Hence, ABCD is a parallelogram **Q.E.D.**	By definition

Properties of a parallelogram

- Both pairs of opposite sides are parallel (by definition).
- Both pairs of opposite sides are equal.
- Both pairs of opposite angles are equal.
- The diagonals bisect each other.
- Each diagonal bisects the parallelogram.

Some special parallelograms

Rectangle



*If one angle of a parallelogram is a right angle then it is called a **rectangle**.*

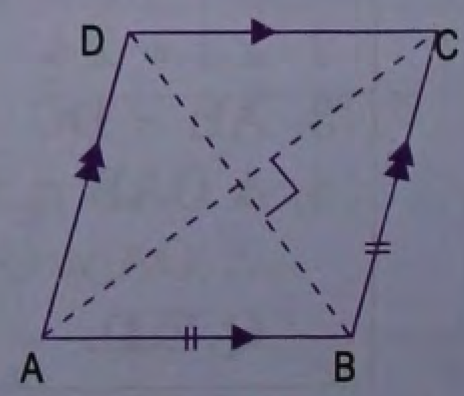
In the adjoining parallelogram ABCD, ∠A = 90°, so it is a rectangle.

Properties of a rectangle

Since every rectangle is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rectangle are:

- All the (interior) angles of a rectangle are right angles.
  In the above diagram, ∠A = ∠B = ∠C = ∠D = 90°.
- The diagonals of a rectangle are equal.
  In the above diagram, AC = BD.

Rhombus



*If two adjacent sides of a parallelogram are equal, then it is called a **rhombus**.*

In the adjoining parallelogram, AB = BC, so ABCD is a rhombus.

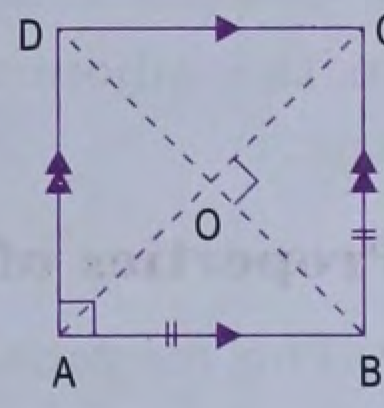
Properties of rhombus

Since every rhombus is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a rhombus are :

- All the sides of a rhombus are equal.
  In the above diagram, AB = BC = CD = DA.
- The diagonals of a rhombus intersect at right angles.
  In the above diagram, AC ⊥ BD.
- The diagonals bisect the angles of a rhombus.
  In the above diagram, diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Square

If two adjacent sides of a rectangle are equal, then it is called a ***square****. Alternatively, if one angle of a rhombus is a right angle, then it is called a* ***square****.*



In the adjoining rectangle, AB = BC, so ABCD is a square.

Properties of a square

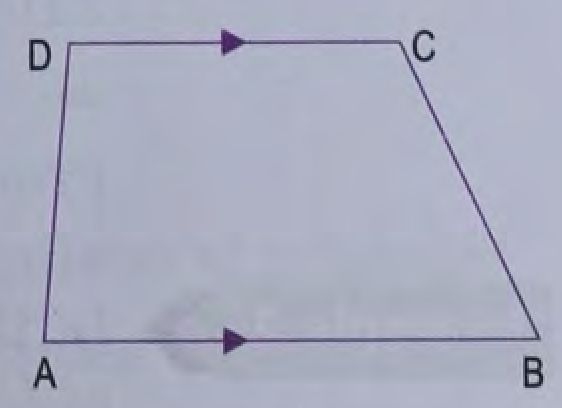
Since every square is a parallelogram, therefore, it has all the properties of a parallelogram. Additional properties of a square are :

- All the interior angles of a square are right angles.
  In the above diagram, ∠A = ∠B = ∠C = ∠D = 90°.
- All the sides of a square are equal.
  In the above diagram, AB = BC = CD = DA.
- The diagonals of a square are equal.
  In the above diagram, AC = BD.
- The diagonals of a square intersect at right angles.
  In the above diagram, AC ⊥ BD.
- The diagonals bisect the angles of a square.
  In the above diagram, diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

In fact, a square is a rectangle as well as rhombus, so it has all the properties of a rectangle as well as that of a rhombus.

Trapezium

A quadrilateral in which one pair of opposite sides is parallel is called a ***trapezium****.*



The parallel sides are called the **bases** of the trapezium.

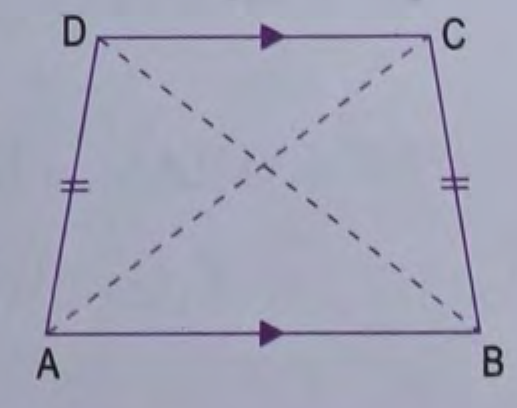
In the adjoining quadrilateral, AB || DC, so ABCD is a trapezium.

Property of a trapezium

- Co-interior angles of a trapezium are supplementary angles.
  In the above diagram, ∠A + ∠D = 180° and ∠B + ∠C = 180°.

Isosceles trapezium

If two non-parallel sides of a trapezium are equal then it is called an ***isosceles trapezium****.*



In the adjoining quadrilateral, AB || DC and AD = BC, so ABCD is an isosceles trapezium.

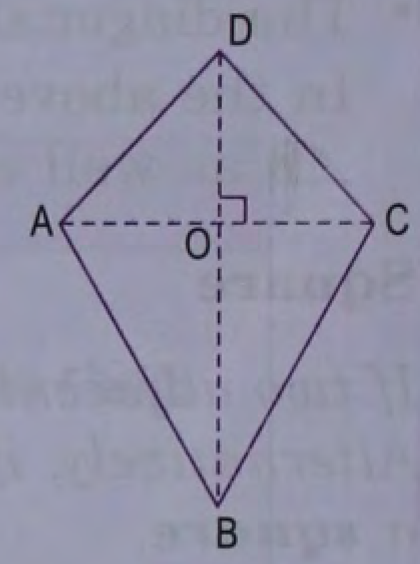
Properties of an isosceles trapezium

- Co-interior angles are supplementary angles.
  In the above diagram, ∠A + ∠D = 180° and ∠B + ∠C = 180°
- Angles on the same base are equal.
  In the above diagram, ∠A = ∠B and ∠C = ∠D.
- Diagonals are equal (in length).
  In the above diagram, AC = BD.

Kite

A quadrilateral in which two pairs of adjacent sides are equal is called a ***kite***.

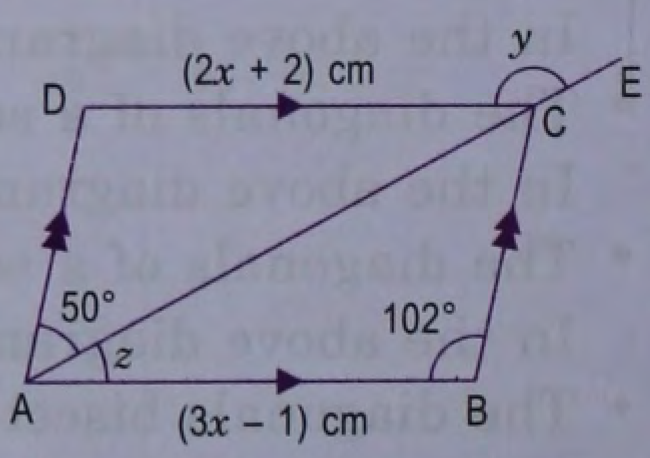
In the adjoining quadrilateral, AB = BC and AD = CD, so ABCD is a kite.



Properties of a kite

- The diagonals of a kite intersect at right angles.
  In the above diagram, AC ⊥ BD.
- In the above diagram, ∠A = ∠C.
- In the above diagram, OA = OC.
- In the above diagram, diagonal BD bisects ∠B as well as ∠D.
- In the above diagram, diagonal BD divides the kite into congruent triangles. Here Δ ABD ≅ Δ CBD.

Example 1. In the adjoining figure, ABCD is a parallelogram. Find the values of x, y and z.



Solution. Given, ABCD is a parallelogram.

$3x - 1 = 2x + 2$ (opp. sides are equal)

$\Rightarrow x = 3$

$\angle D = \angle B = 102°$. (opp. ∠s are equal)

For ΔADC, ∠DCE is an exterior angle

$\therefore y = 50° + \angle D$ (ext. ∠ = sum of two int. opp. ∠s)

$= 50° + 102° = 152°$.

$\angle DAB + 102° = 180°$ (AD || BC, sum of co-int. ∠s = 180°)

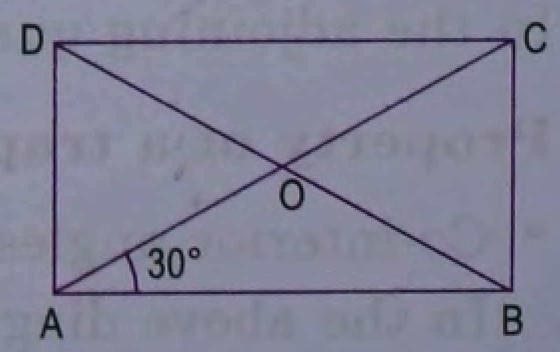
$\Rightarrow \angle DAB = 180° - 102° = 78°$

From figure, $z = \angle DAB - \angle DAC = 78° - 50° = 28°$.

Example 2. In the adjoining rectangle ABCD, diagonals intersect at O. If ∠OAB = 30°, find

(*i*) ∠ACB (*ii*) ∠ABO

(*ii*) ∠COD (*iv*) ∠BOC.



Solution. Given, ABCD is a rectangle.

(*i*) $\angle ABC = 90°$ (each angle of a rectangle = 90°)

$\angle ACB + 30° + 90° = 180°$ (sum of angles in ΔABC)

$\Rightarrow \angle ACB = 180° - 30° - 90° = 60°$.

(*ii*) AC = BD (diagonals are equal)

$\Rightarrow 2AO = 2BO$ (diagonals bisect each other)

$\Rightarrow$ AO = OB

$\Rightarrow$ ∠ABO = ∠OAB (angles opp. equal sides in ΔOAB)

$\Rightarrow$ ∠ABO = 30°. (∵ ∠OAB = 30° given)

(*iii*) ∠AOB + 30° + 30° = 180°
(sum of angles in ΔAOB)

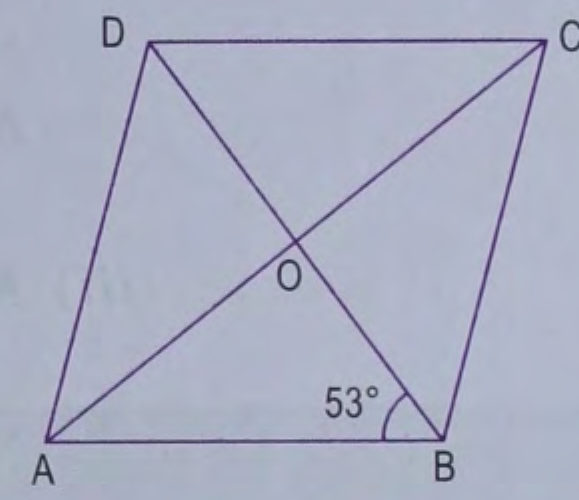
$\Rightarrow$ ∠AOB = 180° – 30° – 30° = 120°

∠COD = ∠AOB = 120°. (vert. opp. ∠*s*)

(*iv*) ∠BOC + 120° = 180° (angles on a straight line)

$\Rightarrow$ ∠BOC = 180° – 120° = 60°.

Example 3. In the adjoining rhombus ABCD, diagonals intersect at O. If ∠ABO = 53°, find



(*i*) ∠OAB
(*ii*) ∠ADC
(*iii*) ∠BCD.

Solution. Given, ABCD is a rhombus.

(*i*) ∠AOB = 90° (diagonals intersect at right angles)

∠OAB + 53° + 90° = 180° (sum of angles in ΔOAB)

$\Rightarrow$ ∠OAB = 180° – 53° – 90° = 37°.

(*ii*) As diagonal BD bisects ∠ABC,

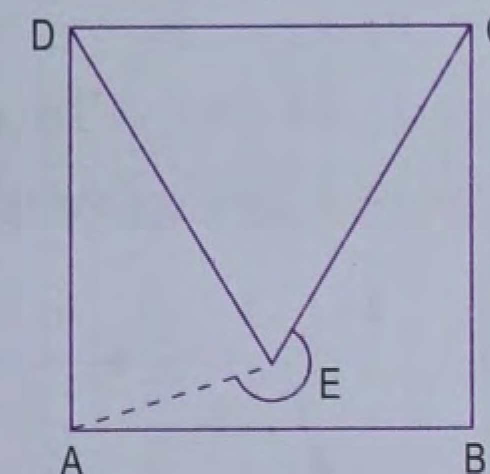
∠ABC = 2 ∠ABO = 2 × 53° = 106°

∴ ∠ADC = ∠ABC = 106°. (opp. ∠*s* are equal)

(*iii*) ∠BCD + 106° = 180° (AD ∥ BC, sum of co-int. ∠*s* = 180°)

$\Rightarrow$ ∠BCD = 180° – 106° = 74°.

Example 4. In the adjoining figure, ABCD is a square and CDE is an equilateral triangle. Find



(*i*) ∠AED (*ii*) ∠EAB
(*iii*) reflex ∠AEC.

Solution. Given, ABCD is a square and CDE is an equilateral triangle. We know that each angle in a square = 90° and each angle in an equilateral triangle is 60°.

(*i*) From figure, ∠ADE = 90° – 60° = 30°

ED = DC (sides of an equilateral triangle)

AD = DC (sides of a square)

$\Rightarrow$ ED = AD

$\Rightarrow$ ∠DAE = ∠AED (angles opp. equal sides in ΔAED)

∠DAE + ∠AED + ∠ADE = 180° (sum of angles in ΔAED)

$\Rightarrow$ 2 ∠AED = 180° – 30° = 150° (∵ ∠ADE = 30°)

$\Rightarrow$ ∠AED = 75°.

(*ii*) ∠EAB = 90° – 75° = 15°. (∵ ∠DAE = ∠AED = 75°)

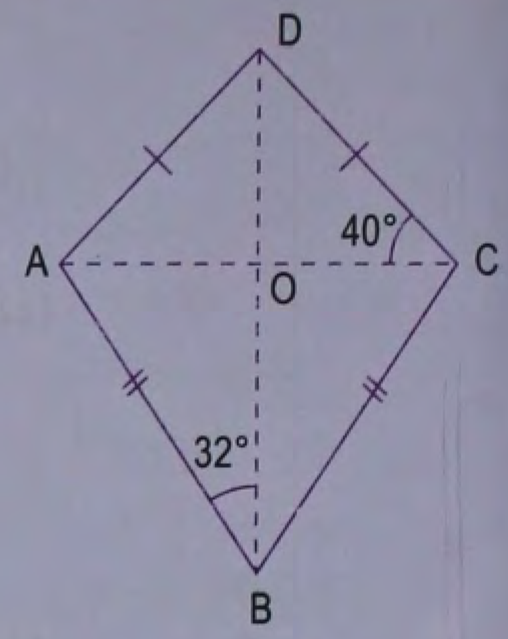
(*iii*) ∠AEC = ∠AED + ∠DEC = 75° + 60° = 135°

∴ Reflex ∠AEC = 360° – 135° = 225°.

Example 5. In the adjoining kite, diagonals intersect at O. If $\angle ABO = 32°$ and $\angle OCD = 40°$, find

(*i*) $\angle ABC$ (*ii*) $\angle ADC$

(*iii*) $\angle BAD$.



Solution. Given, ABCD is a kite.

(*i*) As diagonal BD bisects $\angle ABC$,

$\angle ABC = 2\ \angle ABO = 2 \times 32° = 64°$.

(*ii*) $\angle DOC = 90°$ (diagonals intersect at right angles)

$\angle ODC + 40° + 90° = 180°$ (sum of angles in ΔOCD)

$\Rightarrow \quad \angle ODC = 180° - 40° - 90° = 50°$

As diagonal BD bisects $\angle ADC$,

$\angle ADC = 2\ \angle ODC = 2 \times 50° = 100°$.

(*iii*) As diagonal BD bisects $\angle ADC$,

$\angle ODA = \angle ODC \quad \Rightarrow \quad \angle ODA = 50°$ ($\because \angle ODC = 50°$)

Now $\angle BAD + \angle ABD + \angle BDA = 180°$ (sum of angles in ΔABD)

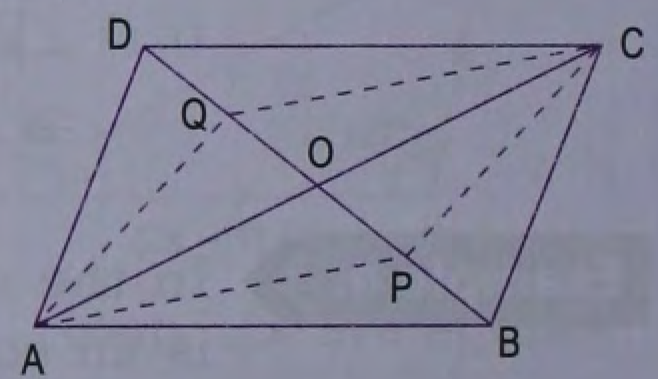
$\Rightarrow \quad \angle BAD + \angle ABO + \angle ODA = 180°$

$\Rightarrow \quad \angle BAD + 32° + 50° = 180°$

$\Rightarrow \quad \angle BAD = 180° - 32° - 50° = 98°$.

Example 6. In the adjoining figure, ABCD is a parallelogram. If P and Q are points on the diagonal BD such that BP = DQ, prove that APCQ is a parallelogram.

Solution. Since diagonals of a parallelogram bisect each other,



OA = OC and OB = OD

BP = DQ (given)

$\therefore$ OB – BP = OD – DQ $\Rightarrow$ OP = OQ.

In ΔOAP and ΔOCQ

OA = OC

OP = OQ (proved above)

$\angle AOP = \angle COQ$ (vert. opp. $\angle s$)

$\therefore \quad \Delta OAP \cong \Delta OCQ$ (S.A.S. axiom of congruency)

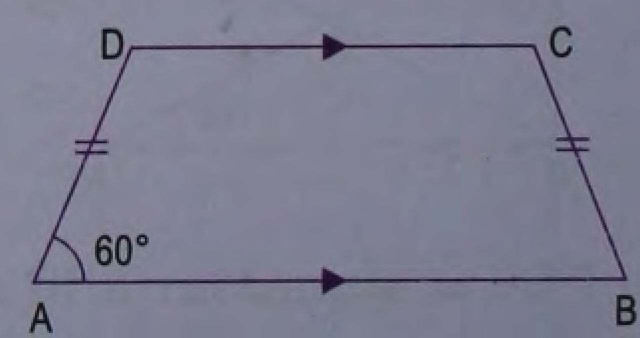
$\therefore$ AP = CQ (c.p.c.t.)

and $\angle OAP = \angle OCQ$ (c.p.c.t.)

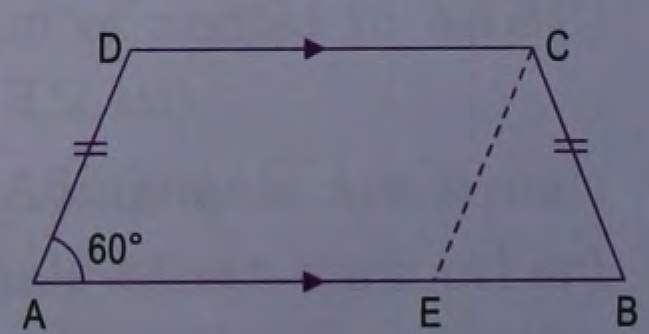
$\Rightarrow$ AP $\parallel$ CQ. [$\because$ line AC cuts the lines AP and CQ, and alt $\angle s$ are equal]

Thus, in quadrilateral APCQ, AP = CQ and AP $\parallel$ CQ, therefore, APCQ is a parallelogram (Theorem 3).

Example 7. In the adjoining figure, ABCD is an isosceles trapezium. If $\angle A = 60°$, DC = 20 cm and AD = 15 cm, find the length of AB.



Solution. Through C, draw a straight line parallel to DA to meet AB at E. Then AECD is a parallelogram, so



AE = DC

$\Rightarrow$ AE = 20 cm ($\because$ DC = 20 cm given)

Also $\angle CEB = \angle A$ (∵ CE ∥ DA. corres. ∠s are equal)

⇒ $\angle CEB = 60°$ (∵ ∠A = 60° given)

$\angle B = \angle A$ (∵ In an isosceles trap., base angles are equal)

⇒ $\angle B = 60°$ (∵ ∠A = 60° given)

$\angle ECB + \angle CEB + \angle B = 180°$ (sum of angles of a Δ = 180°)

⇒ $\angle ECB + 60° + 60° = 180°$

⇒ $\angle ECB = 60°$

⇒ ∠CEB is an equilateral triangle.

∴ $EB = BC$

⇒ $EB = 15$ cm (∵ BC = AD = 15 cm given)

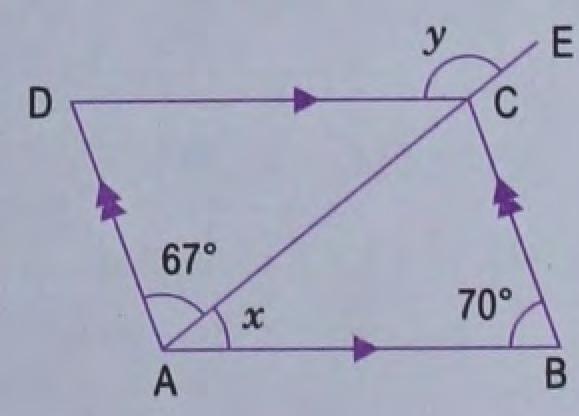
From figure,

$AB = AE + EB = 20 \text{ cm} + 15 \text{ cm} = 35$ cm.

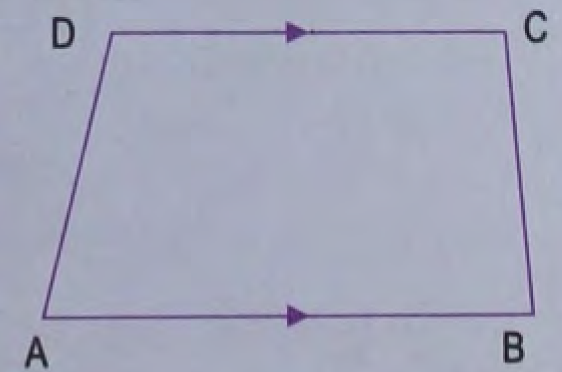
Hence, length of AB = 35 cm.

Exercise 23.2

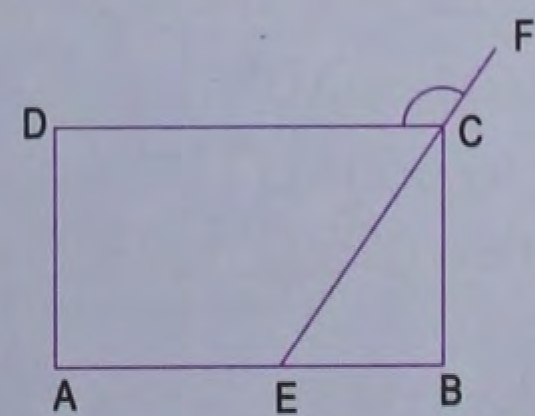
1. State whether the following statements are true or false :
   (*i*) Every rectangle is a rhombus.
   (*ii*) Every square is a rhombus.
   (*iii*) Every square is a rectangle.
   (*iv*) Every square is a parallelogram.
   (*v*) Every rectangle is a square.
   (*vi*) Every rectangle is a parallelogram.
   (*vii*) Every rhombus is a square.
   (*viii*) Every rhombus is a parallelogram.
   (*ix*) Every parallelogram is a rhombus.
2. In a parallelogram ABCD, $\angle A = (4x - 5)°$ and $\angle C = (3x + 10)°$. Find ∠A and ∠B.
3. If in a square ABCD, $AB = (2x + 3)$ cm and $BC = (3x - 5)$ cm, find BD.
   [Hint. $BD^2 = AB^2 + AD^2$ by Pythagoras theorem, AD = BC.]
4. If the ratio of two conjoined angles of a parallelogram is 5 : 7, find the angles of the parallelogram.
5. In the adjoining figure, ABCD is a parallelogram. Find the values of x and y.



6. In the adjoining figure, ABCD is a trapezium.
   If $\angle A : \angle D = 5 : 7$, $\angle B = (3x + 11)°$ and $\angle C = (5x - 31)°$, then find all the angles of the trapezium.



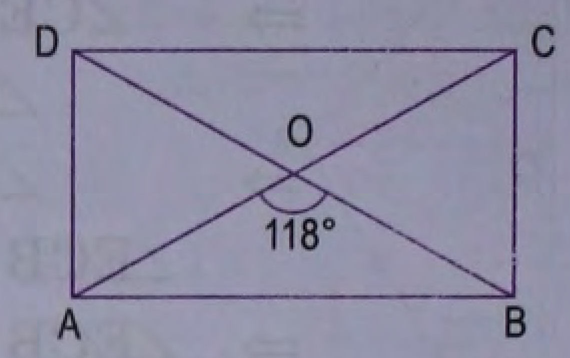
7. In the adjoining figure, ABCD is a rectangle.
   If $\angle CEB : \angle ECB = 3 : 2$, find
   (*i*) ∠CEB (*ii*) ∠DCF.



8. In the adjoining figure, ABCD is a rectangle and diagonals intersect at O. If $\angle AOB = 118°$, find

(*i*) $\angle ABO$ (*ii*) $\angle ADO$

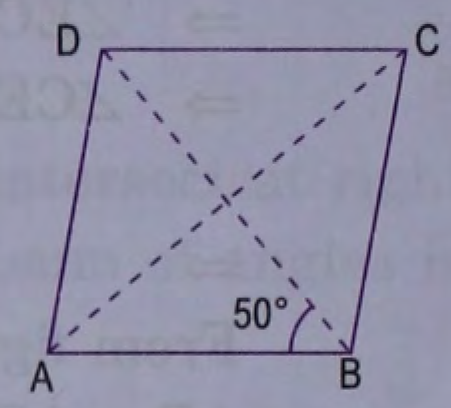
(*ii*) $\angle OCB$.



9. In the adjoining figure, ABCD is a rhombus and $\angle ABD = 50°$. Find

(*i*) $\angle CAB$ (*ii*) $\angle BCD$

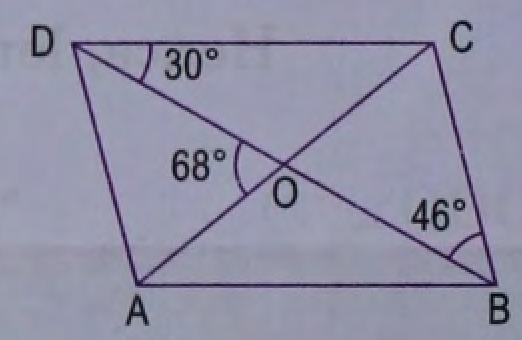
(*iii*) $\angle ADC$.



10. In the adjoining figure, ABCD is a parallelogram and diagonals intersect at O. Find

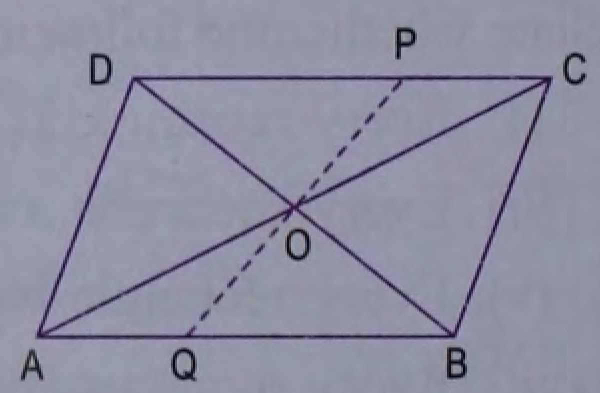
(*i*) $\angle CAD$ (*ii*) $\angle ACD$

(*iii*) $\angle ADC$.



11. In the adjoining figure, ABCD is a parallelogram and diagonals intersect at O. Prove that O is mid-point of PQ.

[Hint. Show that $\Delta AOQ \cong \Delta COP$.**]**



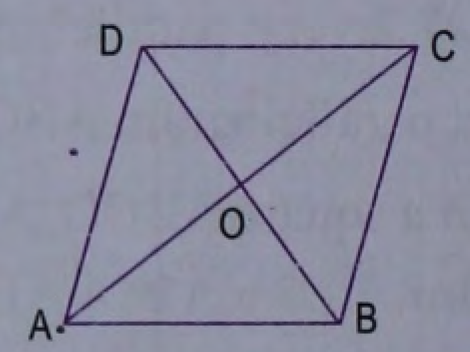
12. In the adjoining figure, ABCD is a rhombus and its diagonals intersect at O. Prove that

(*i*) the diagonals bisect each other.

(*ii*) the diagonals are at right angles.

[Hint. (*i*) Show that $\Delta AOB \cong \Delta COD$

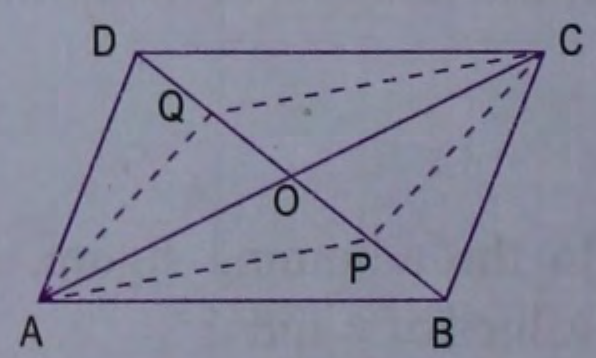
(*ii*) Show that $\Delta AOB \cong \Delta COB$.**]**



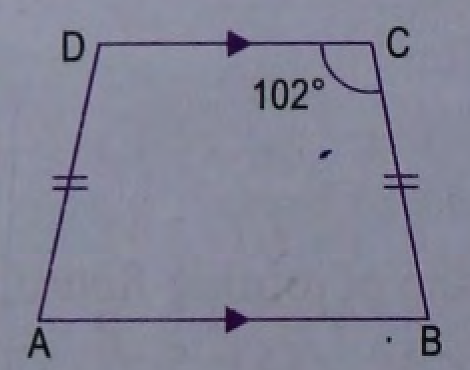
13. In the adjoining figure, ABCD is a parallelogram and AP || CQ. Prove that

(*i*) $\Delta OAP \cong \Delta OCQ$ (*ii*) AP = CQ

(*iii*) APCQ is a parallelogram.



14. In the adjoining isosceles trapezium ABCD, $\angle C = 102°$. Find all the remaining angles of the trapezium.

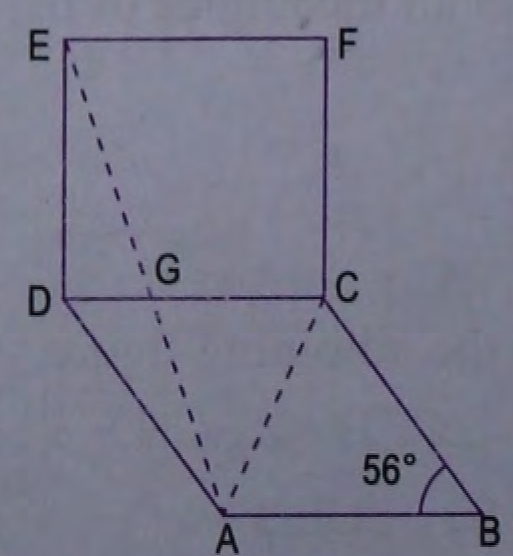


15. In the adjoining figure, ABCD is a rhombus and DCFE is a square. If $\angle ABC = 56°$, find

(*i*) $\angle DAG$ (*ii*) $\angle FEG$

(*iii*) $\angle GAC$ (*iv*) $\angle AGC$.

[Hint. (*i*) $\angle EDA = 90° + 56° = 146°$, ED = AD.**]**



POLYGONS

*A closed plane figure bounded by line segments is called a **polygon**.*

The line segments are called its **sides** and the points of intersection of consecutive sides are called its **vertices**. An angle formed by two consecutive sides of a polygon inside the polygon is called an **interior angle** or simply an **angle** of the polygon.

A polygon has the same number of angles as it has sides. A polygon is named according to the number of sides/angles it has :

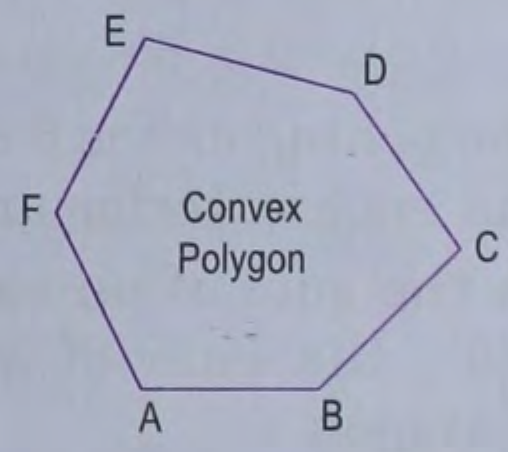
Number of sides/angles	*Name*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon/Septagon
8	Octagon
9	Nonagon
10	Decagon

In general, a polygon having n sides is called ***n*-sided polygon** or ***n*-gon**. Thus, a polygon having 20 sides is called 20-gon.

Diagonal of a polygon

Line segment joining any two non-consecutive vertices of a polygon is called its **diagonal**.

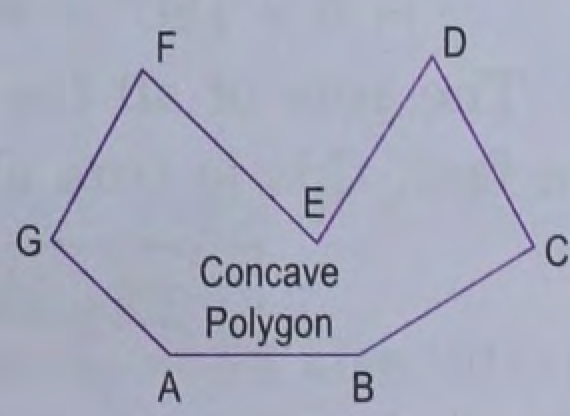
Convex polygon



*If all the (interior) angles of a polygon are less than 180°, it is called a **convex polygon**.*

In the adjoining figure, ABCDEF is a convex polygon. In fact, it is a convex hexagon.

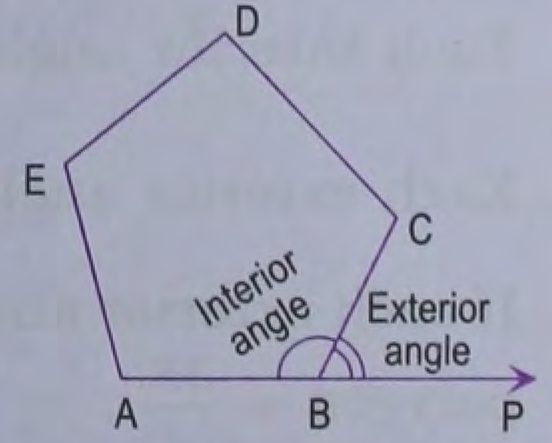
Concave polygon



*If one or more of the (interior) angles of a polygon is greater than 180° i.e. reflex, it is called a **concave** (or **re-entrant**) **polygon**.*

In the adjoining figure, ABCDEFG is a concave (*or* re-entrant) polygon. In fact, it is a concave heptagon.

Exterior angle of a convex polygon



If we produce a side of a convex polygon, the angle it makes with the next side is called an **exterior angle**.

In the adjoining figure, ABCDE is a pentagon. Its side AB has been produced to P, then ∠CBP is an exterior angle.

Notice that corresponding to each interior angle, there is an exterior angle.

Also, as an exterior angle and its adjacent interior angle make a straight line, we have :

An exterior angle + adjacent interior angle = 180°

Regular polygon

*A polygon is called **regular polygon** if all its sides have equal length and all its angles have equal size.*

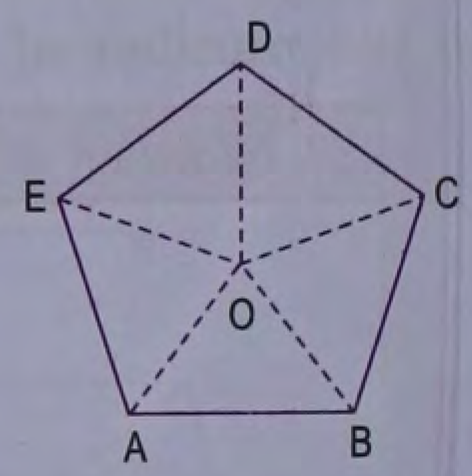
Thus, in a regular polygon :

- all sides are equal in length
- all interior angles are equal in size
- all exterior angles are equal in size.

All regular polygons are convex.

Angle Property of a Polygon

Sum of interior angles of a polygon



In the adjoining figure, ABCDE is a pentagon. It has 5 sides and 5 (interior) angles. Take any point O inside the pentagon and join it with vertices. We notice that 5 triangles are formed.

As the sum of angles of a triangle is 2 right angles, therefore, the sum of all the angles of the 5 triangles

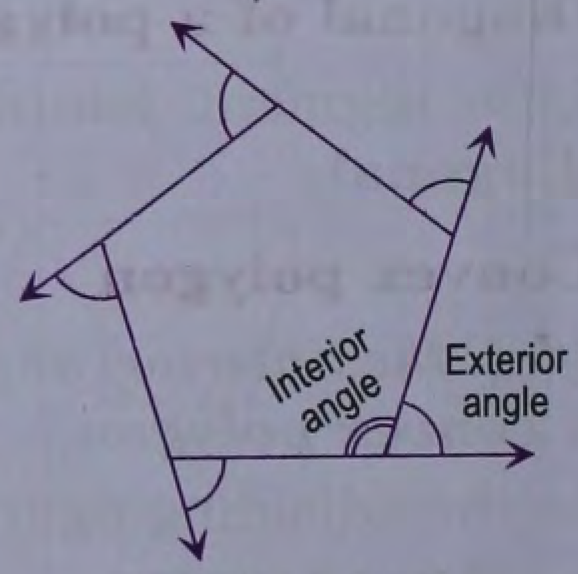
$= (2 \times 5)$ right angles.

Also the sum of angles at the point O = 4 right angles. It follows that the sum of all the (interior) angles of the pentagon ABCDE = $(2 \times 5 - 4)$ right angles.

In fact, this is true about every polygon of n sides. So, we have an important result :

> ***The sum of interior angles of a polygon of n sides = (2n – 4) right angles***

Sum of exterior angles of a convex polygon



5 exterior angles of a pentagon

In the adjoining figure, ABCDE is a convex pentagon. It has 5 sides and 5 interior angles. On putting $n = 5$ in the above formula, sum of interior angles of a pentagon

$= (2 \times 5 - 4)$ right angles $= 6 \times 90° = 540°$.

The pentagon has 5 exterior angles (the sides are produced in order) and each exterior angle has an adjacent interior angle.

As the sum of an exterior angle and its adjacent interior angle is 180°, the sum of all the exterior and the interior angles of a pentagon

$= 5 \times 180° = 900°$.

$\therefore$ The sum of all the exterior angles $= 900° - 540° = 360°$.

In fact, this is true about every convex polygon. So, we have another important result:

> ***The sum of exterior angles of a convex polygon = 360°***

From the above two results, it follows that :

- Each interior angle of a regular polygon of n sides $= \dfrac{2n - 4}{n}$ right angles
- Each exterior angle of a regular polygon of n sides $= \dfrac{360°}{n}$
- If each exterior angle of a regular polygon is $x°$, then the number of sides in the regular polygon $= \dfrac{360}{x}$.

Example 1. (*i*) Find the sum of interior angles of nonagon.

(*ii*) Find the measure of each interior angle of a regular 16-gon.

Solution. (*i*) A nonagon has 9 sides.

Sum of its interior angles $= (2 \times 9 - 4)$ right angles $= 14 \times 90°$

$= 1260°$

(*ii*) Each exterior angle of a regular 16-sided polygon

$$= \frac{360^\circ}{16} = \frac{45^\circ}{2} = 22{\cdot}5^\circ = 22^\circ\ 30'$$

$\therefore$ Each interior angle of regular 16-gon = $180^\circ - 22^\circ\ 30' = 157^\circ\ 30'$.

Example 2. A heptagon has four equal angles each of 132° and three equal angles. Find the size of equal angles.

Solution. A heptagon has 7 sides.

Sum of its interior angles = $(2 \times 7 - 4)$ right angles

$= 10 \times 90^\circ = 900^\circ$.

Let the size of each equal angle be x°, so we have

$4 \times 132^\circ + 3x^\circ = 900^\circ$

$\Rightarrow\ 3x^\circ = 900^\circ - 528^\circ = 372^\circ \quad \Rightarrow\ x = 124$

Hence, the size of each equal angle = 124°.

Example 3. Is it possible to have a regular polygon whose each interior angle is 105°?

Solution. Given each interior angle = 105°,

so each exterior angle = $180^\circ - 105^\circ = 75^\circ$.

$\therefore$ The number of sides of the polygon = $\frac{360}{75} = \frac{24}{5} = 4\frac{4}{5}$, which is not a natural number.

Therefore, no regular polygon is possible whose each interior angle is 105°.

Example 4. The sum of interior angles of a polygon is 2700°. How many sides this polygon has?

Solution. Let the polygon have n sides, then the sum of its interior angles

$= (2n - 4)$ right angles $= (2n - 4) \times 90^\circ$

By the question, $(2n - 4) \times 90^\circ = 2700^\circ$

$\Rightarrow\ 2n - 4 = 30 \quad \Rightarrow\ 2n = 34 \quad \Rightarrow\ n = 17$.

Hence, the polygon has 17 sides.

Example 5. The ratio between an exterior angle and the interior angle of a regular polygon is 1 : 8. Find the number of sides in the polygon.

Solution. In a regular polygon, all exterior angles are equal in size and also interior angles are equal in size.

Let an exterior angle be x, then interior angle is $180^\circ - x$.

According to given information, $\frac{x}{180^\circ - x} = \frac{1}{8}$

$\Rightarrow\ 8x = 180^\circ - x \quad \Rightarrow\ 9x = 180^\circ \quad \Rightarrow\ x = 20^\circ$.

$\therefore$ The number of sides in the polygon = $\frac{360}{20} = 18$.

Example 6. Each interior angle of a regular polygon is 144°. Find the interior angle of a regular polygon which has double the number of sides as the first polygon.

Solution. Each interior angle of the first polygon = 144° (given),

$\therefore$ each exterior angle of the first polygon = $180^\circ - 144^\circ = 36^\circ$

$\therefore$ The number of sides in the first polygon = $\frac{360}{36} = 10$

$\therefore$ The number of sides in the second polygon = $2 \times 10 = 20$

$\therefore$ Each exterior angle in the second polygon = $\frac{360°}{20} = 18°$

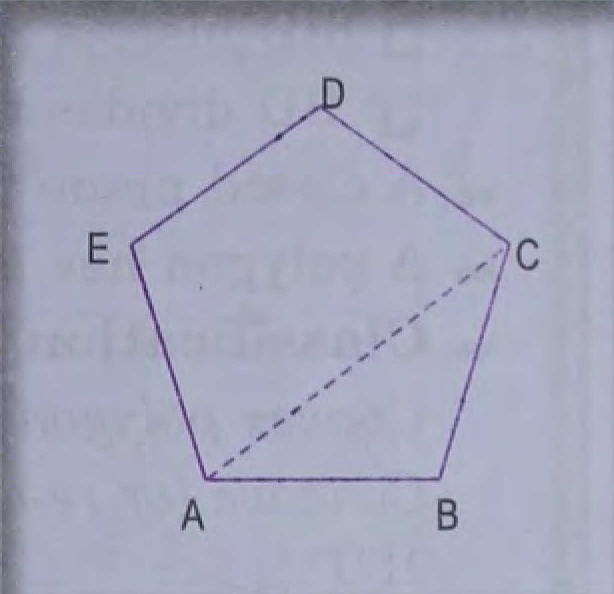
$\therefore$ Each interior angle in the second polygon = $180° - 18° = 162°$.

Exercise 23.3

1. Find the sum of interior angles of a :
   (*i*) hexagon (*ii*) octagon (*iii*) decagon.
2. Find the sum of interior angles of a polygon with
   (*i*) 11 sides (*ii*) 19 sides (*iii*) 25 sides.
3. Find the measure of each interior angle of a regular
   (*i*) hexagon (*ii*) heptagon (*iii*) octagon
   (*iv*) decagon (*v*) 18-gon (*vi*) 24-gon.
4. Find the number of sides of a regular polygon if each of its exterior angles is
   (*i*) 72° (*ii*) 45° (*iii*) 24° (*iv*) $\left(51\frac{3}{7}\right)^{\circ}$.
5. Find the number of sides of a regular polygon if each of its interior angles is
   (*i*) 162° (*ii*) 108° (*iii*) 120° (*iv*) 140° (*v*) $\left(147\frac{3}{11}\right)^{\circ}$.
6. Find the number of sides in a polygon if the sum of its interior angles is:
   (*i*) 1260° (*ii*) 1980° (*iii*) 3420°.
7. Is it possible to have a polygon the sum of whose interior angles is
   (*i*) 1800° (*ii*) 450° (*iii*) 1120° (*iv*) 31 right angles?
8. Is it possible to have a regular polygon each of whose interior angle is
   (*i*) 130° (*ii*) 165° (*iii*) $1\frac{3}{4}$ right angles?
9. The angles of a pentagon are $x°$, $(x - 10)°$, $(x + 20)°$, $(2x - 44)°$ and $(2x - 70)°$. Calculate x.
10. The exterior angles of a pentagon are in the ratio 1 : 2 : 3 : 4 : 5. Find all the interior angles of the pentagon.
    [Hint. Let exterior angles be x, $2x$, $3x$, $4x$, $5x$, then $x + 2x + 3x + 4x + 5x = 360° \Rightarrow x = 24°$.**]**
11. Five angles of a hexagon are each 116°, calculate the size of the sixth angle.
12. A heptagon has three equal angles each of 120° and four equal angles. Find the size of equal angles.
13. The ratio between an exterior angle and the interior angle of a regular polygon is 1 : 5. Find
    (*i*) the measure of each exterior angle
    (*ii*) the measure of each interior angle
    (*iii*) the number of sides in the polygon.
14. Each interior angle of a regular polygon is double of its exterior angle. Find the number of sides in the polygon.

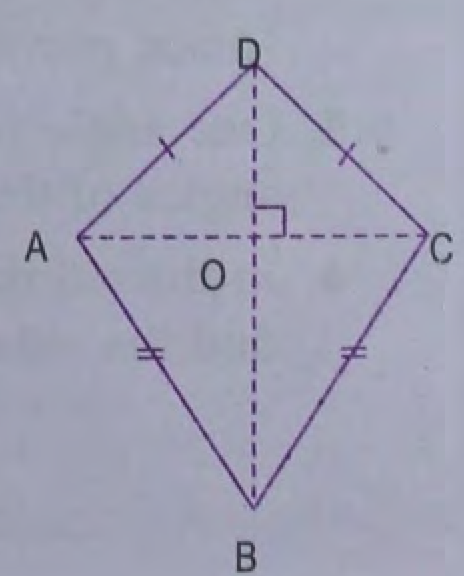
15. Each interior angle of a regular polygon is 150°. Find the interior angle of a regular polygon which has double the number of sides as the given polygon.

16. In the adjoining figure, ABCDE is a regular pentagon. Find
(*i*) ∠ABC
(*ii*) ∠CAB
(*iii*) ∠ACD.



Summary

- A closed plane figure bounded by four line segments is called a quadrilateral. It has four sides, four (interior) angles, four vertices and two diagonals.
- Sum of interior angles of a quadrilateral is 360°.
- **Properties of a parallelogram**
  - Both pairs of opposite sides are parallel.
  - Both pairs of opposite sides are equal.
  - Both pairs of opposite angles are equal.
  - The diagonals bisect each other.
  - Each diagonal bisects the parallelogram.
- If two opposite sides of a quadrilateral are equal and parallel, then it is a parallelogram.
- **Properties of a rectangle**
  It has all the properties of a parallelogram. Its additional properties are:
  - Each (interior) angle = 90°.
  - The diagonals are equal (in length).
- **Properties of a rhombus**
  It has all the properties of a parallelogram. Its additional properties are:
  - All the sides are equal (in length).
  - The diagonals intersect at right angles.
  - The diagonals bisect the angles of a rhombus.
- **Properties of a square**
  It has all the properties of a parallelogram. Its additional properties are:
  - Each (interior) angle = 90°.
  - All the sides are equal (in length).
  - The diagonals are equal (in length).
  - The diagonals intersect at right angles.
  - The diagonals bisect the angles of a square.
- **Properties of an isosceles trapezium**
  - Co-interior angles are supplementary.
  - Angles on the same base are equal.
  - Diagonals are equal (in length).
- **Properties of a kite**
  In the adjoining diagram, ABCD is a kite.
  - The diagonals intersect at right angles.
  - ∠A = ∠C.



- BD bisects ∠B as well as ∠D.
- BD divides the kite into two congruent triangles.

- A closed plane figure bounded by line segments is called a polygon.
- A polygon has the same number of (interior) angles as it has sides.
- **Classification of polygons**

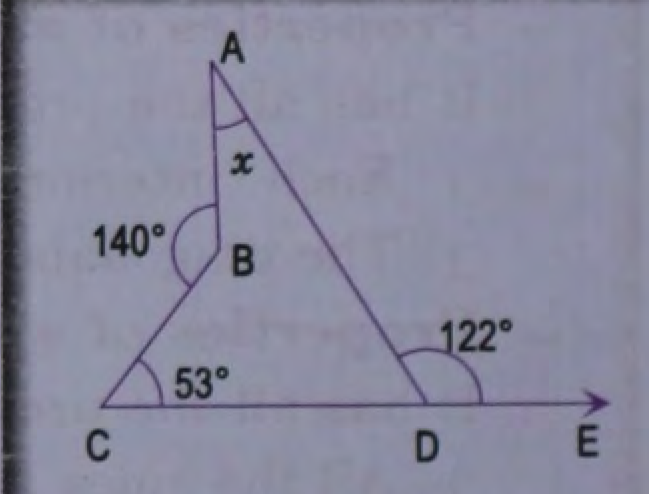
  Convex polygon — all interior angles are less than 180°.

  Concave (or re-entrant) polygon — one or more of the interior angles is greater than 180°.

  Regular polygon — all sides have equal length and all interior angles have equal size. Of course, all exterior angles will also have equal size.
- All regular polygons are convex.
- **Angle property of a polygon**
  - The sum of interior angles of a polygon of n sides = $(2n - 4)$ right angles.
  - The sum of exterior angles of a convex polygon is 360°.
- Each interior angle of a regular polygon of n sides = $\frac{2n-4}{n}$ right angles.
- Each exterior angle of a regular polygon of n sides = $\frac{360°}{n}$.
- If each exterior angle of a regular polygon is $x°$, then the number of sides in the polygon = $\frac{360}{x}$.

Check Your Progress

1. From the adjoining diagram, find the value of x.

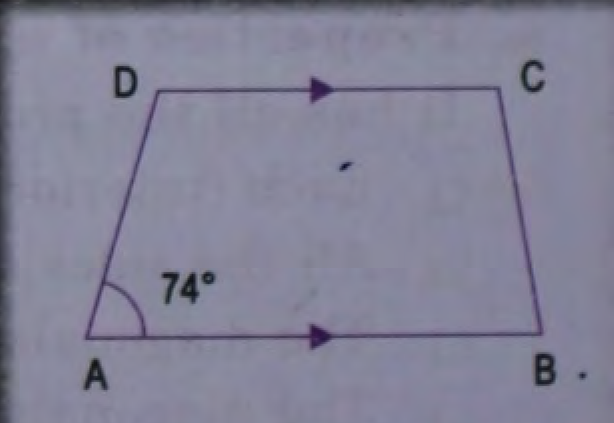


[Hint. Reflex angle B = 220°, ∠ADC = 58°
Sum of interior angles is 360°.]

2. If two angles of a quadrilateral are 76° 37′ and 57° 23′, and out of the remaining two angles, one angle is 10° smaller than the other, find these angles.

3. In the adjoining figure, AB || DC, ∠A = 74° and ∠B : ∠C = 4 : 5. Find

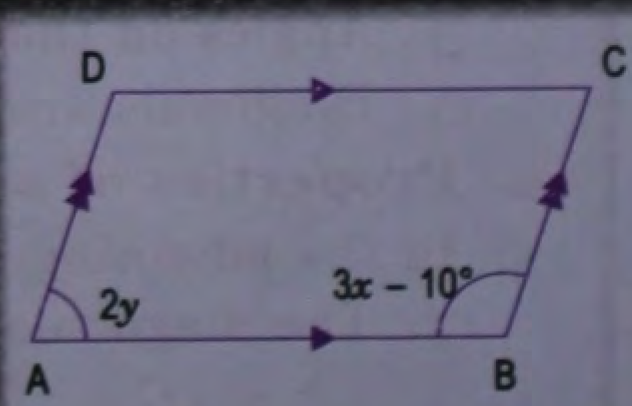
(*i*) ∠D (*ii*) ∠B
(*iii*) ∠C.



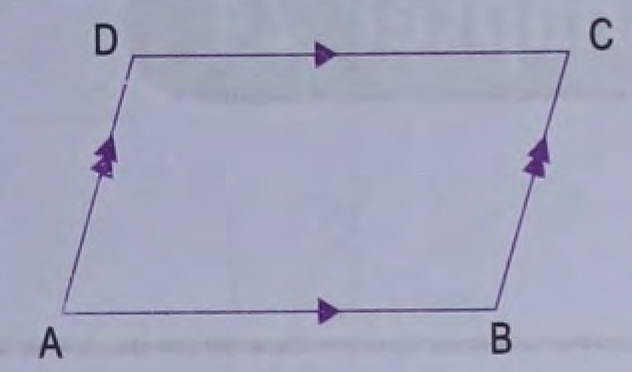
4. In quadrilateral ABCD, ∠A : ∠B : ∠C : ∠D = 3 : 4 : 6 : 7. Find all the angles of the quadrilateral. Hence prove that AB and DC are parallel. Is BC also parallel to AD?

5. One angle of a parallelogram is two-third of the other. Find the angles of the parallelogram.

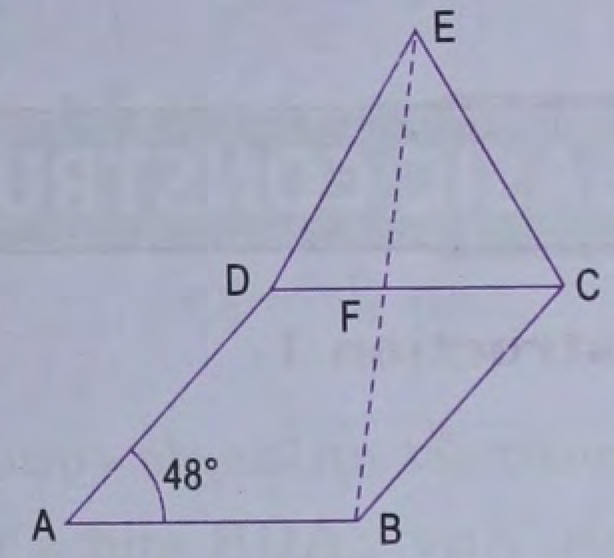
6. In the adjoining figure, ABCD is a parallelogram. If $x - y = 5°$, find the values of x and y.



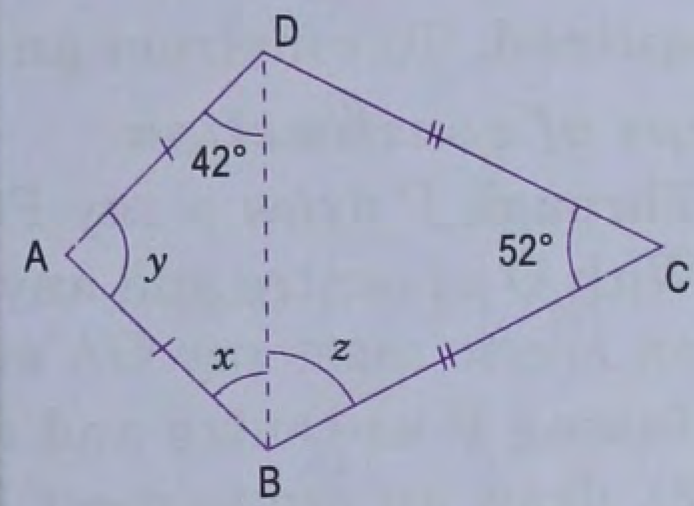
7. In the adjoining figure, ABCD is a parallelogram. If AB = 2x + 5, CD = y + 1, AD = y + 5 and BC = 3x – 4, then find the ratio of AB : BC.



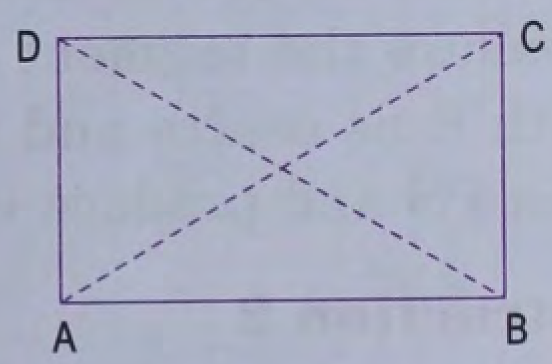
8. In the adjoining figure, ABCD is a rhombus and EDC is an equilateral triangle. If ∠DAB = 48°, find
   (*i*) ∠BEC (*ii*) ∠DEB
   (*iii*) ∠BFC.
   [Hint. ∠BCE = 48° + 60° = 108°, BC = EC.]



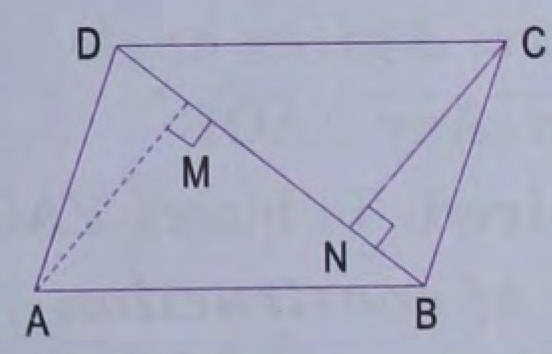
9. In the adjoining figure, ABCD is a kite. If ∠BCD = 52° and ∠ADB = 42°, find the values of *x*, *y* and *z*.
   [Hint. Join AC.]



10. In the adjoining figure, ABCD is a rectangle. Prove that AC = BD.
    [Hint. Δ ABC ≅ ΔBAD.]



11. In the adjoining figure, ABCD is a parallelogram. AM and CN are drawn perpendiculars from A and C respectively on the diagonal BD. Prove that AM = CN.
    [Hint. Prove that ΔADM ≅ ΔCBN.]



12. Find the measure (in degrees) of each interior angle of a regular 40-gon.
13. Find the number of sides of a regular polygon if each of its interior angle is 157° 30′.
14. If the sum of interior angles of a polygon is 3780°, find the number of sides.
15. Find the number of sides in a regular polygon if its interior and exterior angles are equal.
16. Two angles of a polygon are right angles and every other angle is 120°. Find the number of sides of the polygon.
    [Hint. Let the number of sides be n, then $2 \times 90° + (n-2) \times 120° = (2n-4) \times 90°$.]
17. The sum of interior angles of a regular polygon is twice the sum of its exterior angles. Find the number of sides of the polygon.
18. The angles of a hexagon are $(2x+5)°$, $(3x-5)°$, $(x+40)°$, $(2x+20)°$, $(2x+25)°$ and $(2x+35)°$. Find the value of x.
19. An exterior angle of a regular polygon is one-fourth of its interior angle. Find the number of sides in the polygon.
20. The adjoining figure represents a part of the regular octagon ABCD.... with the diagonal AC drawn. Find
    (*i*) ∠ABC
    (*ii*) ∠CAB
    (*iii*) ∠ACD.