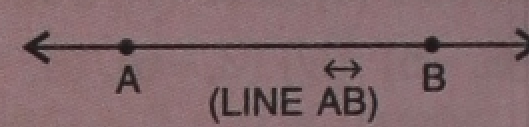
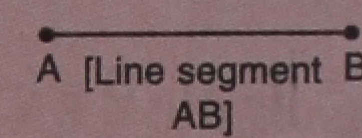
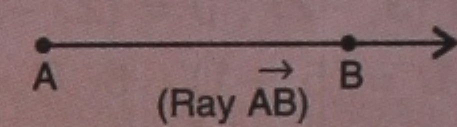
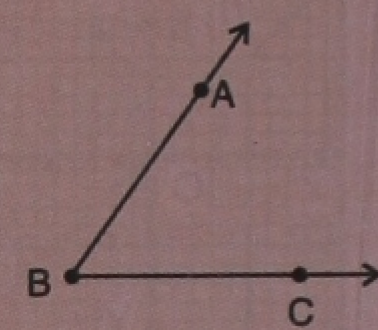
UNIT – 4
GEOMETRY

CHAPTER 23

FUNDAMENTAL CONCEPTS

23.1 REVIEW

1. Point	**A point is a mark of position.** It has neither length nor width nor thickness. So it occupies no space.
2. Line	**A line has only length.** It has neither width nor thickness. It has an infinite length. 
3. Line segment	A **line segment** is a part of a line whose both ends are fixed. It has a definite length. 
4. Ray	A **ray** is a part of a line whose one end is fixed and the other end can be extended infinitely. 
5. Plane	**A plane is a flat surface.** It has length and width, but no thickness.
6. Space	The **space** is made up of every thing which exists in the universe. Infact, every surface, etc. is a part of the space.
7. Angle	An **angle** is formed when two line segments or two rays have a common end point. The two line segments (or rays), forming the angle, are called the arms of the angle whereas their common end-point is called the vertex of the angle.  In the given figure, BA and BC are arms of the angle and the common point B is the vertex of the angle.

23.2 TYPES OF ANGLES

1. **Acute angle :** It is the angle which measures between 0° and 90°.

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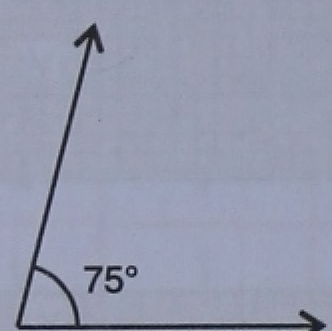

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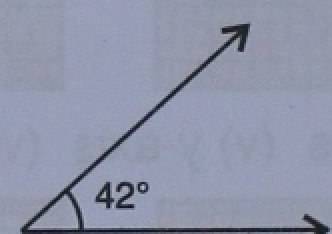

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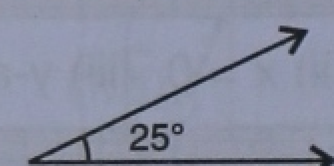

, etc.

2. **Right angle :** It is the angle which measures 90°.

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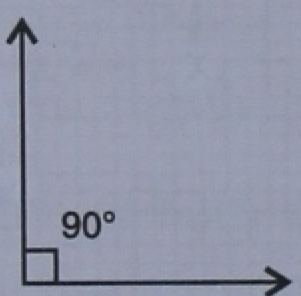

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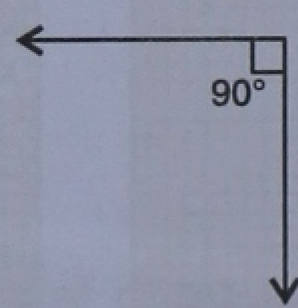

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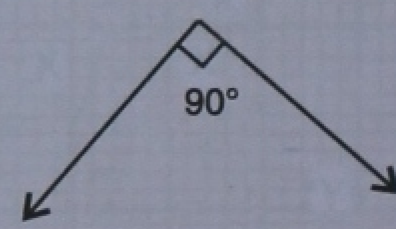

, etc.

3. **Obtuse angle :** It is the angle which measures between 90° and 180°.

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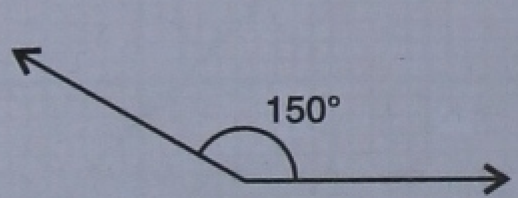

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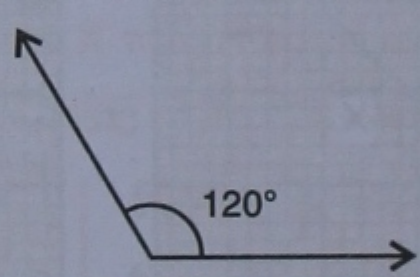

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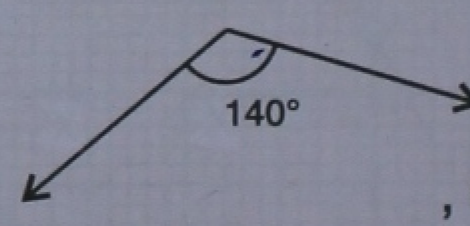

, etc.

4. **Straight angle :** It is the angle which measures 180°.

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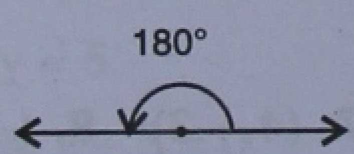

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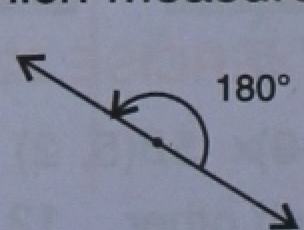

, etc.

5. **Reflex angle :** It is the angle which measures between 180° and 360°.

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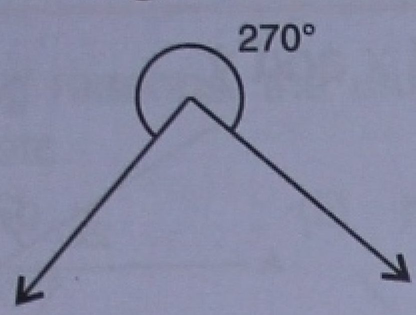

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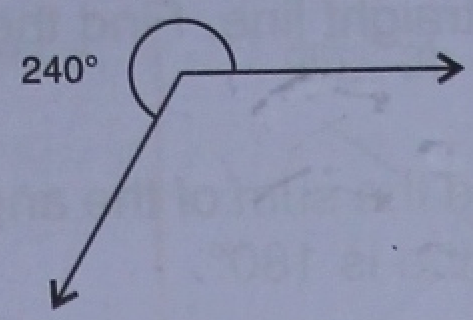

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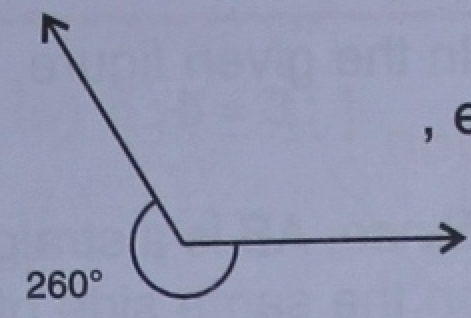
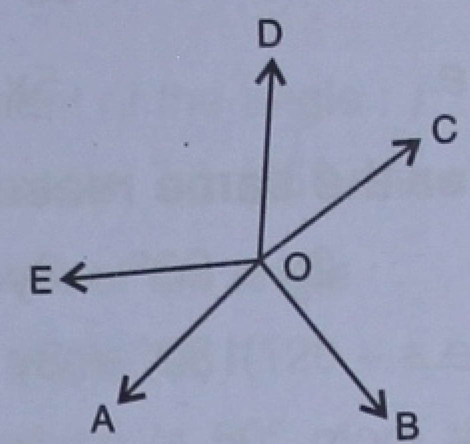


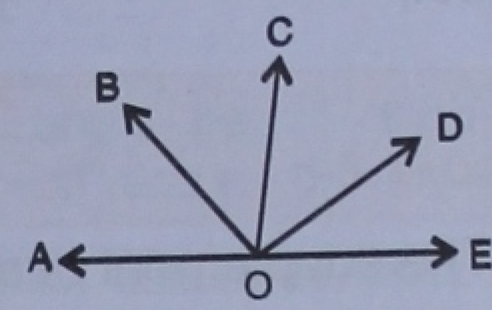
, etc.

23.3 MORE ABOUT ANGLES

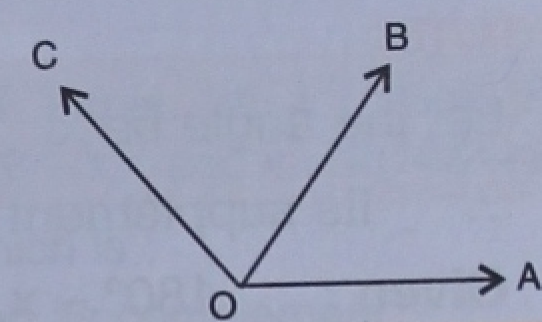
1. **Angles on a point :** Whatever be the number of angles formed at a point, their sum is always 360°.
   i.e., $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$.



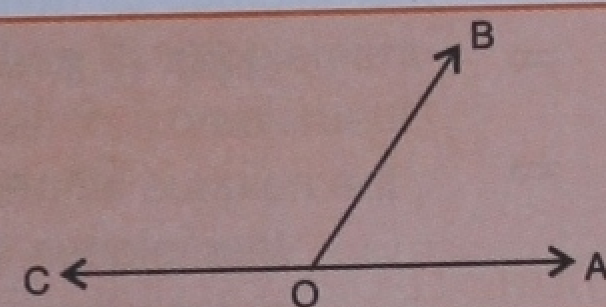
2. **Angles on the same side of a straight line :** Whatever be the number of angles formed at a point of a straight line on the same side of it, their sum is always 180°.
   i.e., $\angle AOB + \angle BOC + \angle COD + \angle DOE = 180°$.



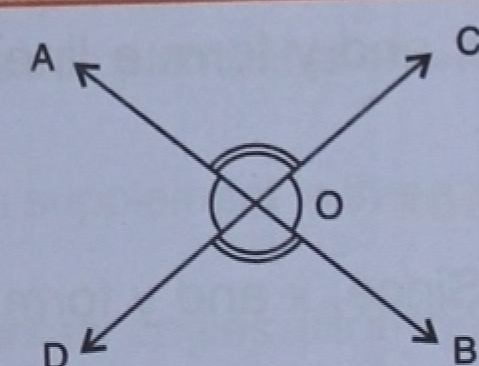
3. **Adjacent angles :** Two angles are said to be adjacent angles if they have a common vertex, a common arm and the other arms of the two angles lie on the opposite sides of the common arm.



If the sum of two adjacent angles is 180°, their outer arms are in a straight line. Such adjacent angles are said to form a **linear pair of angles.**



4. **Vertically opposite angles :** When two lines intersect each other, four angles are formed. The pair of angles which lie on the opposite sides of the point of intersection are called **vertically opposite angles.**



Vertically opposite angles are always equal.

$\therefore$ $\angle AOD = \angle BOC$ and $\angle AOC = \angle BOD$.

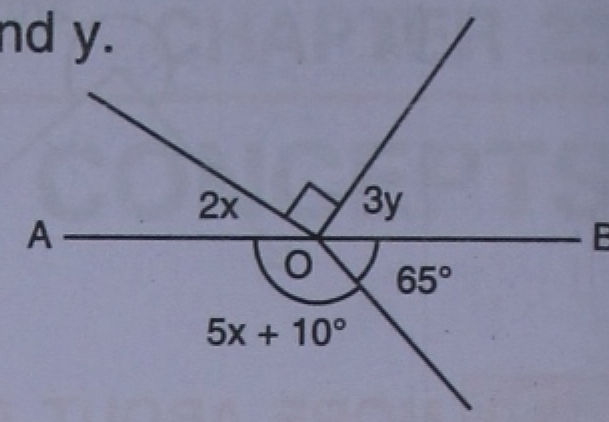
5. **Complementary angles :** Two angles are said to be complementary, if their sum is one right angle *i.e.* 90°. **Each angle is called complement of the other.**
6. **Supplementary angles :** Two angles are said to be supplementary, if their sum is two right angles *i.e.* 180°. **Each angle is called supplement of the other.**

TEST YOURSELF

1. In a linear pair of adjacent angles, one angle is 110°, then the other angle is ....
2. Two adjacent angles are 90° each, they form a ....
3. Two vertically opposite angles are (x – 20)° and 85°; then .... = ... and x = ...
4. Complement of 80° = ... and its supplement = ...
5. Supplement of 70° = ... and its complement = ...

Example 1 :

In the given figure, AB is a straight line. Find the values of x and y.



Solution :

Since, AB is a straight line and the sum of the angles at a point on the same side of straight line is 180°.

$\therefore \quad 5x + 10° + 65° = 180° \Rightarrow \quad 5x = 180° - 75°$

i.e., $\quad 5x = 105° \Rightarrow \quad x = \frac{105°}{5} = 21°$

For the same reason :

$2x + 90° + 3y = 180° \Rightarrow \quad 2 \times 21° + 90° + 3y = 180°$

i.e., $\quad 132° + 3y = 180° \Rightarrow \quad 3y = 180° - 132° = 48°$

i.e., $\quad y = \frac{48°}{3} = 16°$

$\therefore \quad x = 21°$ and $y = 16°$ **(Ans.)**

Example 2 :

The supplement of an angle is 10° more than three times its complement. Find the angle.

Solution :

Let the angle be x.

$\therefore$ Its supplement $= 180° - x$ and its complement $= 90° - x$

Given : $\quad 180° - x = 3(90° - x) + 10°$

$\Rightarrow \quad 180° - x = 270° - 3x + 10°$ *i.e.,* $3x - x = 280° - 180°$

$\Rightarrow \quad 2x = 100°$ and $x = \frac{100°}{2} = 50°$ **(Ans.)**

Example 3 :

x and y form a linear pair of two adjacent angles. If y = 3x – 12°; find the values of x and y.

Solution :

Since, x and y form a linear pair of two adjacent angles,

$x + y = 180°$

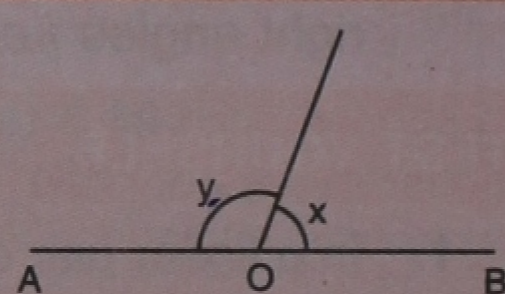
$\Rightarrow x + 3x - 12° = 180° \quad [\because y = 3x - 12°]$

$\therefore \quad 4x = 180° + 12° \Rightarrow \quad 4x = 192°$ and $x = \frac{192°}{4} = 48°$

$x + y = 180° \quad \Rightarrow \quad 48° + y = 180°$ *i.e.* $y = 180° - 48° = 132°$

$\therefore \quad x = 48°$ and $y = 132°$ **(Ans.)**

TEST YOURSELF

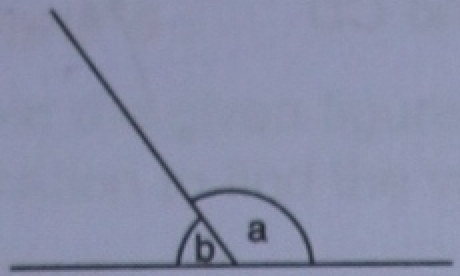


6. In the given figure, x + y = ..., if AOB is ....
7. The adjacent angles on a straight line are in the ratio 7 : 5; the angles are .... and ....
8. If x + 10° and 2x – 40° are complementary ⇒ .... = 90° and x = ...
9. If 3x + 40° and 5x + 20° are supplentary, ⇒ .... = 180° and x = ...
10. The angle between bisectors of two adjacent complementary angles is ....
11. The angle between bisectors of two adjacent supplementary angles is ....

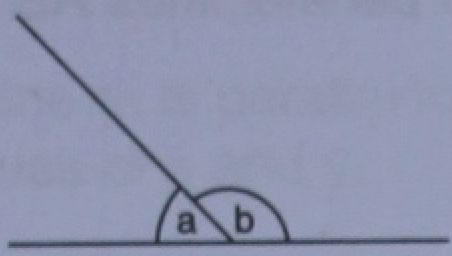
EXERCISE 23 (A)

1. Find, giving reasons, the unknown angles a, b, x, y, z, etc.

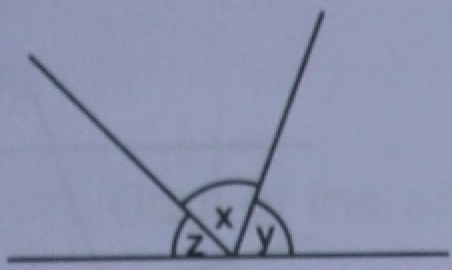
(i) 4a = 5b



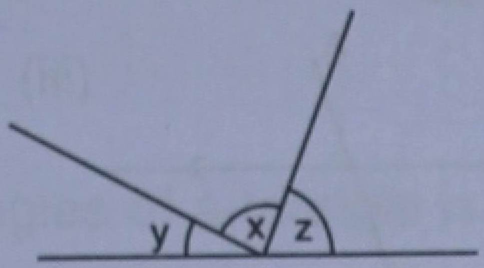
(ii) b = 2a – 15



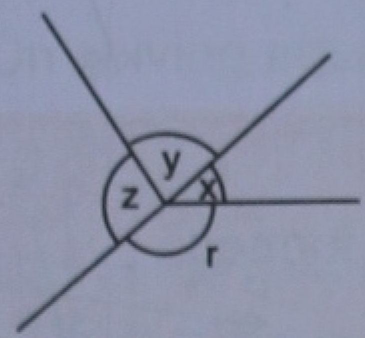
(iii) x : y : z = 2 : 4 : 3



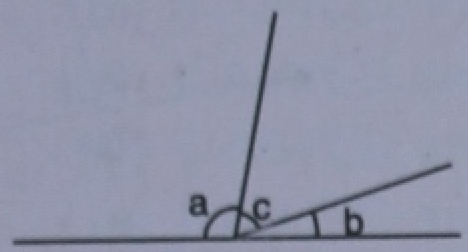
(iv) x = 3y and 3z = 5x



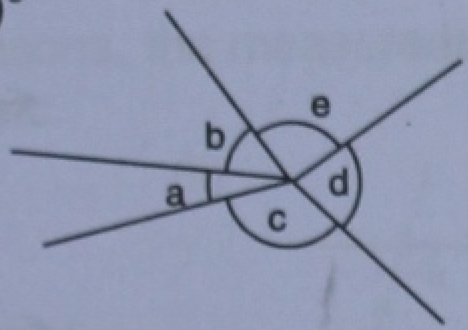
(v) y = 2x, z = 3x and r = 4x



(vi) c = 3b and a = 5b



(vii) b = (2a + 5)°, c = 6a°, d = (3a + 20)° and e = (5a – 5)°



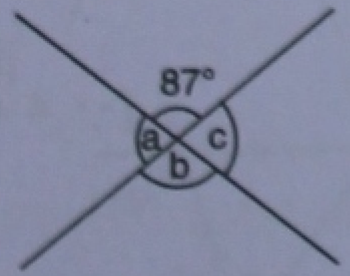
2. Two adjacent angles on a straight line are (5x – 6)° and 7(x + 6)°. Find the value of x and magnitudes of both the angles.

3. The measures of two adjacent angles on a straight line are x° and (2x – 27)°. Find each angle.

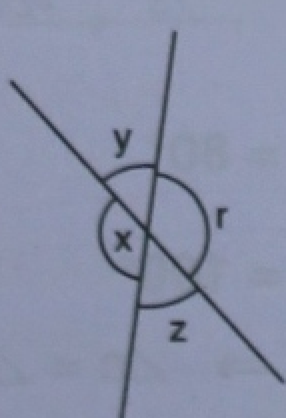
4. Two adjacent angles on a straight line are in the ratio 5 : 4. Find the measure of each angle.

5. Each of the following figures shows the intersection of two lines. Calculate the unknown angles.

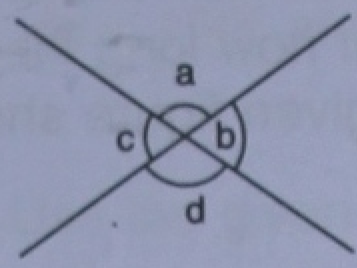
(i)



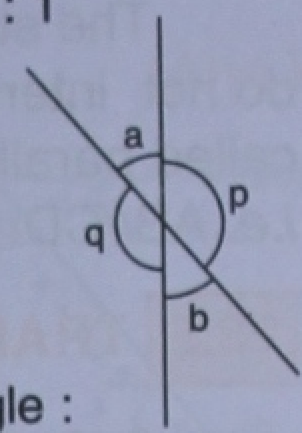
(ii) x = 3y



(iii) 3b = 2d



(iv) p : b = 3 : 1



6. Find the complement of the angle :

(i) $\frac{1}{4}$ of a right angle (ii) (150 – a + b)°

7. Find the supplement of the angle :

(i) $\frac{2}{5}$ of a right angle (ii) (120 + a – 2b)°

8. Find the angle, which is 60° more than its complement.

> Let the required angle be x°, then its complement = (90 – x)° and x – (90 – x) = 60

9. Find the angle which measures twice of its supplement.

> Let the required angle be x°; then its supplement = (180 – x)° and x = 2(180 – x)

10. Find the angle which is :

(i) 80° more than its complement

(ii) 20° more than its supplement

(iii) 30° less than its complement

(iv) 56° less than its supplement

(v) equal to its supplement

(vi) equal to its complement.

11. Find the angle, if its complement is one-fourth of its supplement.

12. Find the angle, if its supplement is three times of its complement.

13. (i) Two supplementary angles are in the ratio 5 : 7. Find the angles.

> 1st angle = $\frac{5}{12} \times 180°$ and
>
> 2nd angle = $\frac{7}{12} \times 180°$

(ii) Find two supplementary angles, if they are in the ratio 7 : 11.

14. (i) The ratio between two complementary angles is 2 : 3; find the angles.

(ii) Find two complementary angles, if they are in the ratio 3 : 7.

15. Show that the bisectors of two adjacent supplementary angles include a right angle.

16. Find the measure of an angle, if :

(i) five times of its complement is 24° less than twice of its supplement.

(ii) four times of its complement is 12° more than twice the difference between its supplement and complement.

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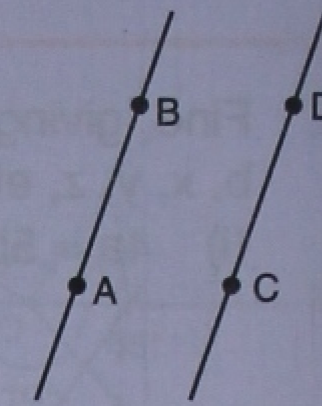
23.4 PARALLEL LINES

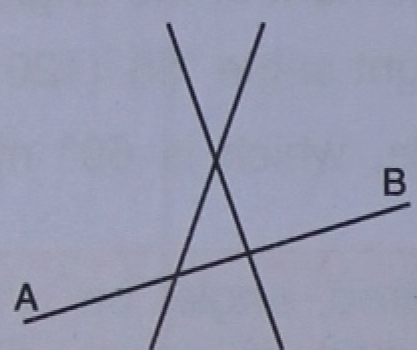


The straight lines, which are coplanar (*i.e.*, lie in the same plane) and do not intersect, no matter how long they are produced on either side, are called parallel lines. The given figure shows two parallel lines AB and CD *i.e.* AB//CD.

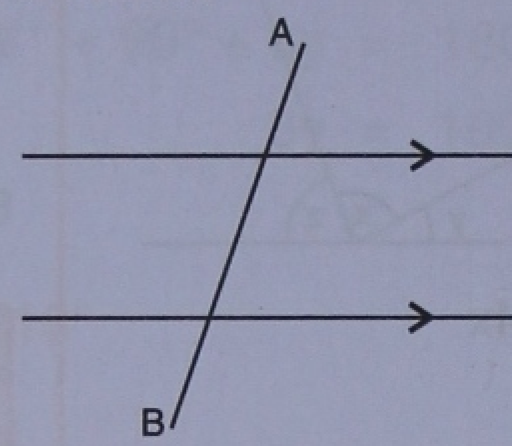
23.5 TRANSVERSAL

A straight line, that intersects two or more coplanar lines, is called a transversal. In each of the following figures; AB represents a transversal.

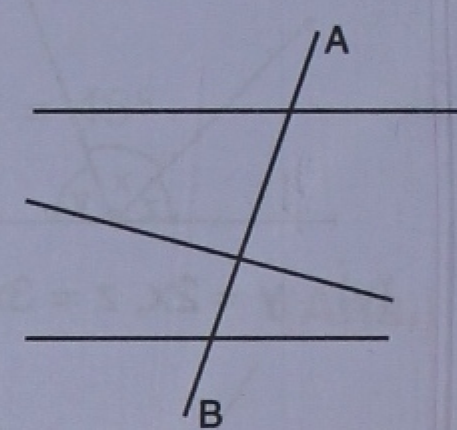
(i)



(ii)



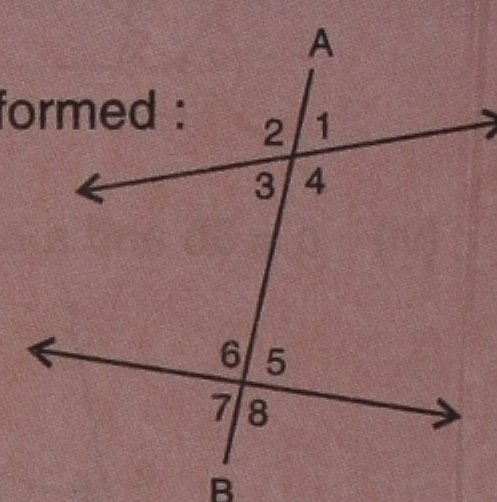
(iii)



Special pairs of angles formed by two lines and a transversal :

When two lines are cut by a transversal, the following pairs of angles are formed :

1. **Two pairs of alternate interior angles** : ∠3, ∠5 and ∠4, ∠6.
2. **Two pairs of alternate exterior angles :** ∠1, ∠7 and ∠2, ∠8.
3. **Four pairs of corresponding angles :** ∠1, ∠5; ∠2, ∠6; ∠3, ∠7 and ∠4, ∠8.
4. **Two pairs of co-interior angles :** ∠3, ∠6 and ∠4, ∠5.



23.6 PARALLEL LINES AND TRANSVERSAL

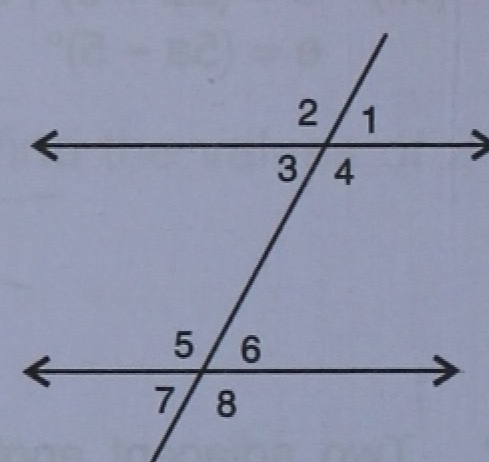
If two parallel lines are intersected by a transversal :

(i) *alternate interior angles are equal*
i.e. ∠3 = ∠6 and ∠4 = ∠5.

(ii) *alternate exterior angles are equal*
i.e. ∠1 = ∠7 and ∠2 = ∠8.

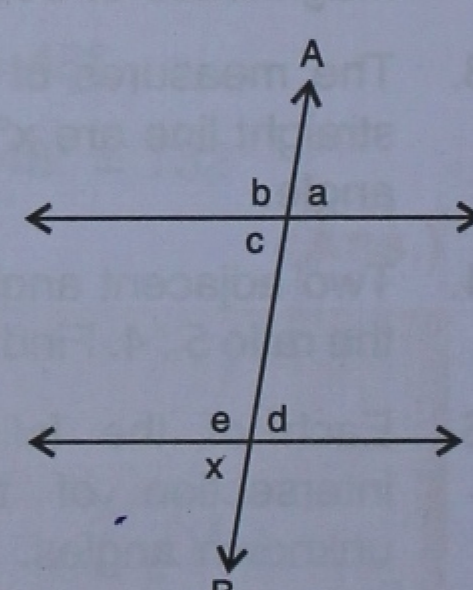
(iii) *corresponding angles are equal*
i.e. ∠1 = ∠6, ∠2 = ∠5, ∠3 = ∠7 and ∠4 = ∠8.

(iv) *co-interior* (or allied or conjoined) *angles are supplementary*
i.e. ∠3 + ∠5 = 180° and ∠4 + ∠6 = 180°.



Example 4 :

The given diagram shows two parallel lines cut by the transversal AB. If ∠a : ∠b = 4 : 5; find angles a, b, c, d, e and x.



Solution :

Since, ∠a and ∠b form a linear pair of angles therefore,

∠a + ∠b = 180°

Also, given that : ∠a : ∠b = 4 : 5

∴ **∠a** = $\frac{4}{9}$ × 180° = **80°** [∵ 4 + 5 = 9]

and, **∠b** = $\frac{5}{9}$ × 180° = **100°**

∵ Vertically opposite angles are equal ⇒ ∠c = ∠a = **80°**

∵ Alternate interior angles are equal ⇒ ∠d = ∠c = **80°**

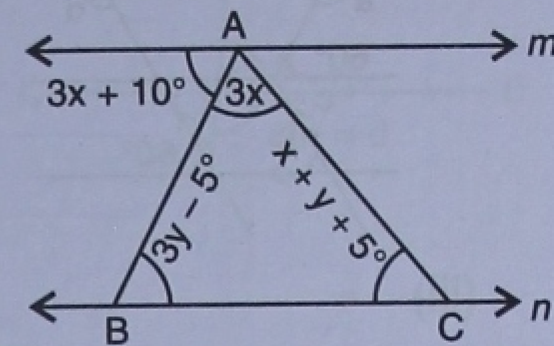
$\because$ Corresponding angles equal $\Rightarrow \angle e = \angle b = 100°$

$\because$ Alternate exterior angles are equal $\Rightarrow \angle x = \angle a = 80°$

$\therefore$ $\angle a = 80°, \angle b = 100°, \angle c = 80°, \angle d = 80°, \angle e = 100°$ and $\angle x = 80°$ **(Ans.)**

Example 5 :

In the given figure, line m is parallel to line n. Use the given information to find the values of x and y.



Solution :

$\because$ Alternate angles are equal $\Rightarrow 3x + 10° = 3y - 5°$

$\Rightarrow 3x - 3y = -5° - 10°$

$\Rightarrow 3x - 3y = -15°$ and $x - y = -5°$ I

$\because$ Sum of the angles of a triangle is 180° $\Rightarrow 3x + 3y - 5° + x + y + 5° = 180°$

$\Rightarrow 4x + 4y = 180°$

and, $x + y = 45°$ II

On solving equations I and II, we get : $x = 20°$ and $y = 25°$ **(Ans.)**

TEST YOURSELF

12. In the given figures,

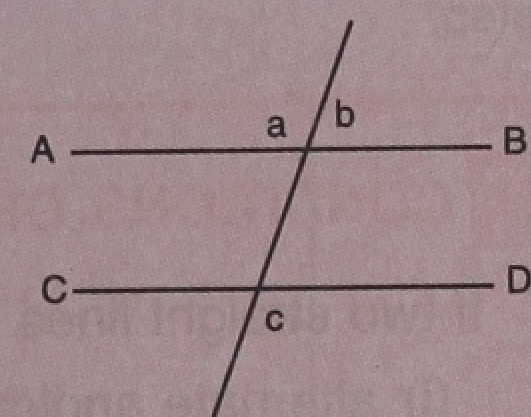
(i) If a = c, the line AB and CD are ....

(ii) If AB//CD, the relation between b and c is ....

(iii) If AB//CD and b = c; then b = ...

(iv) If AB//CD and b : c = 4 : 5, c = ...

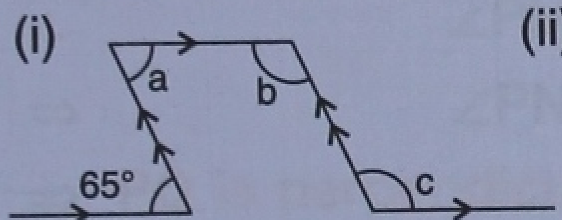
(v) If AB//CD and b = c – 20°, then c = ...



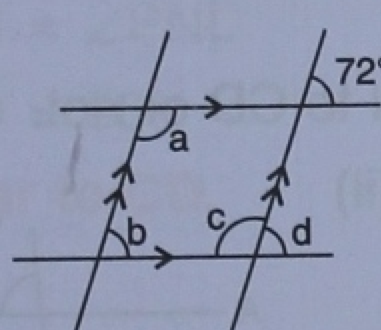
EXERCISE 23 (B)

1. Find, giving reasons, the measures of angles a, b, c, d, etc.

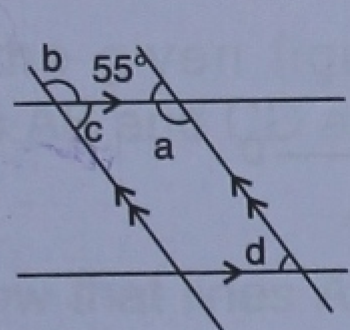
(i)



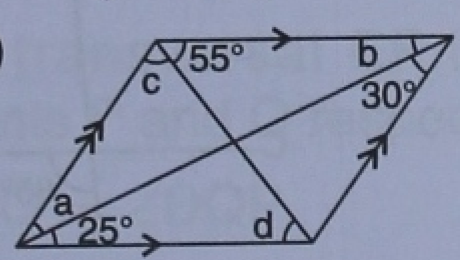
(ii)



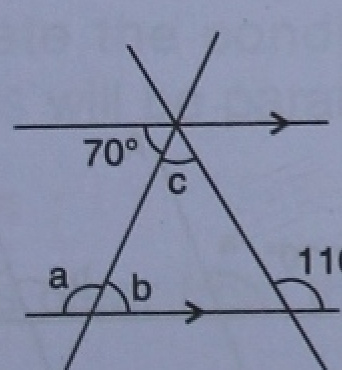
(iii)



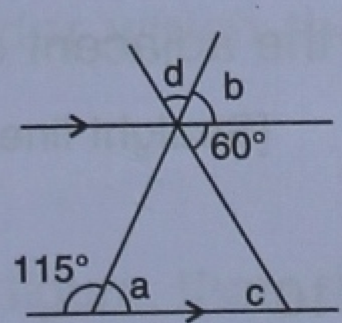
(iv)



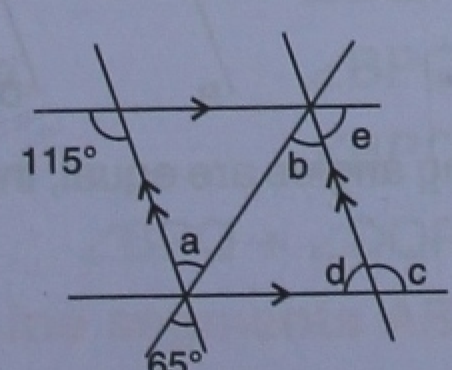
(v)



(vi)

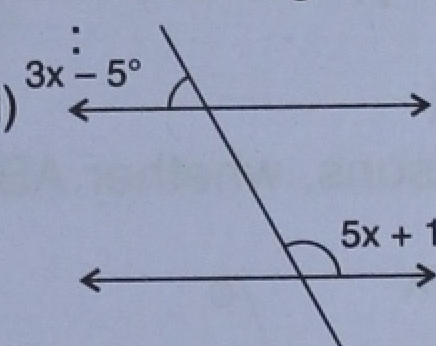


(vii)

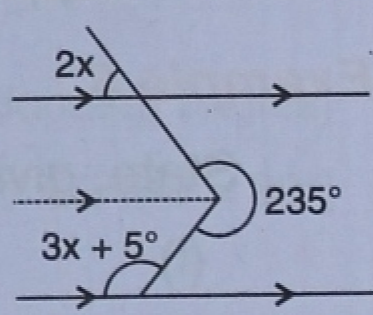


2. In each case, given below, find the value of x :

(i)



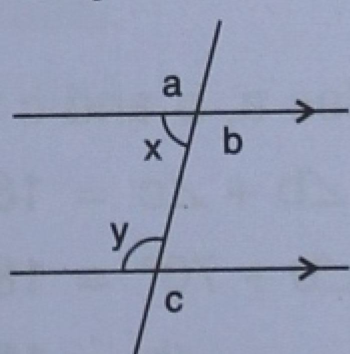
(ii)

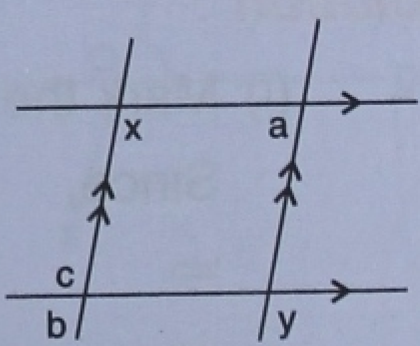


3. In each case, given below, find the values of x and y; then find the angles represented by a, b and c :

(i) Given : x : y = 7 : 11

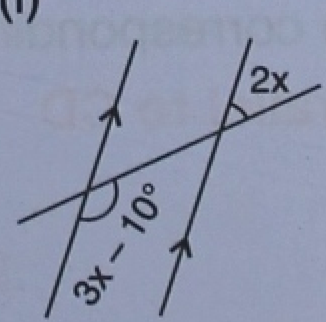
(ii) Given : x + y = 240°



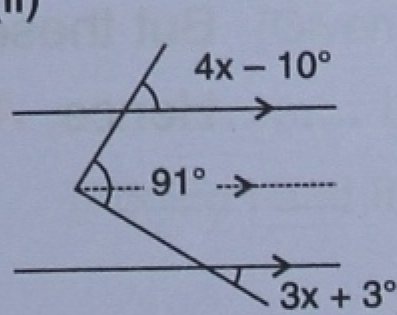


4. Find the value of 'x' from each of the following:

(i)

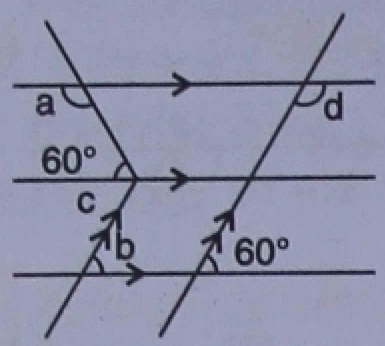


(ii)

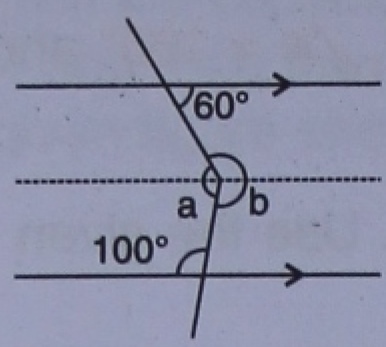


5. Find, giving reasons, the value of angles marked by a, b, c, etc.

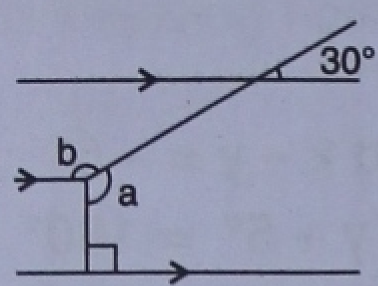
(i)



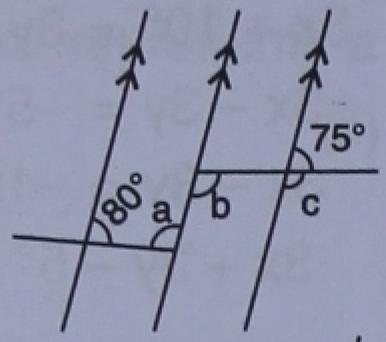
(ii)



(iii)

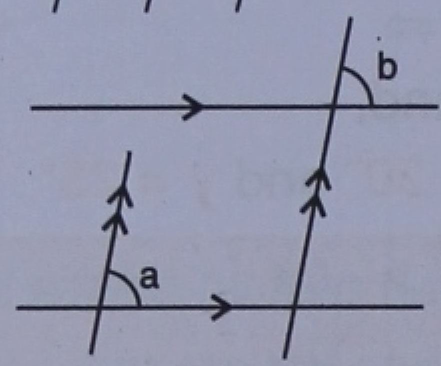


(iv)

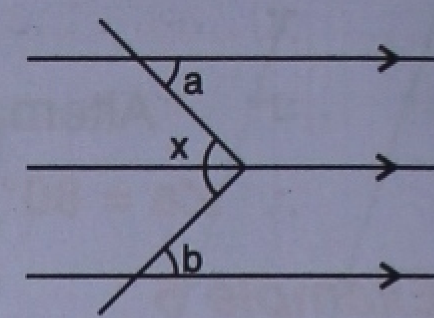


6. Use the information given in the following to show ∠a = ∠b.

Give reason for each step.

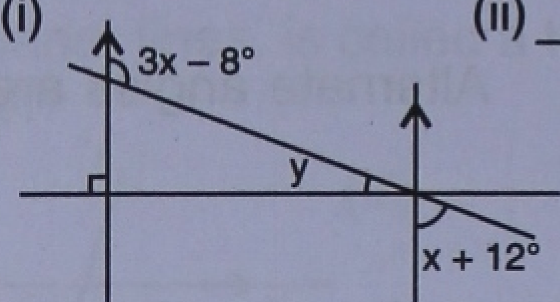


7. Giving reasons, show that ∠x = ∠a + ∠b.

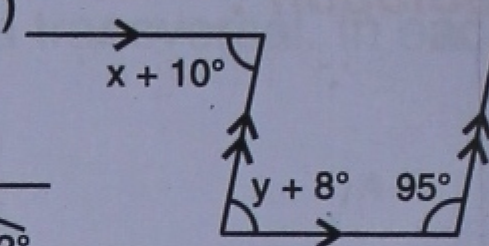


8. In each of the following, find the values of x and y.

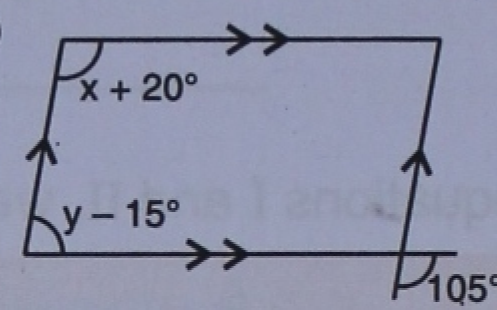
(i)



(ii)



(iii)



23.7 CONDITIONS OF PARALLELISM

If two straight lines are intersected by a transversal, such that :

(i) alternate angles are equal, or

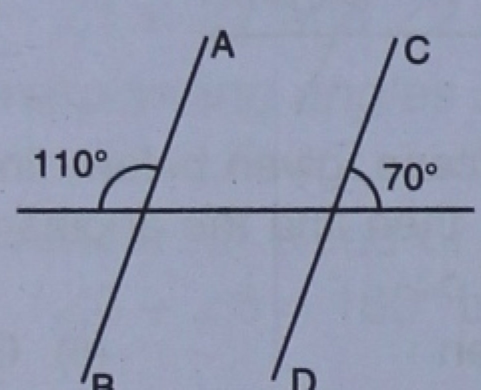
(ii) corresponding angles are equal, or

(iii) co-interior angles (*i.e.* interior angles on the same side of the transversal) are supplementary, then the two lines are parallel to each other.

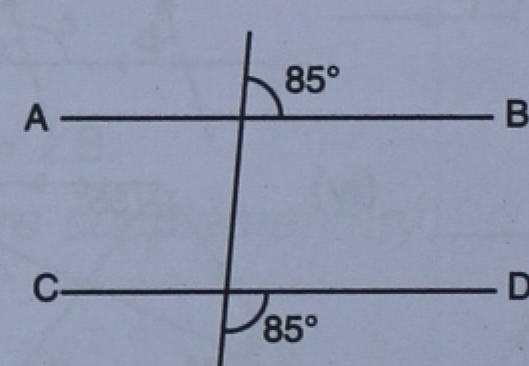
Example 6 :

State, giving reasons, whether AB is parallel to CD or not.

(i)

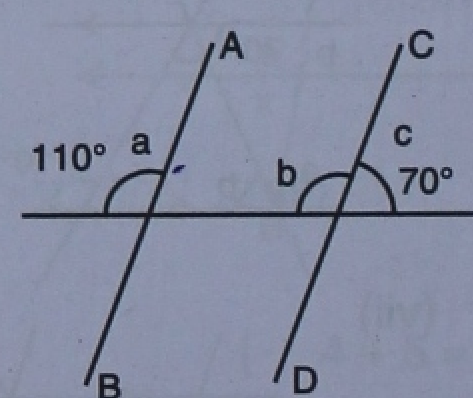


(ii)



Solution :

(i) Mark the angles a, b and c as shown in the adjacent diagram.



Since, $\angle b + \angle c = 180°$ [Straight line angle]

$\Rightarrow \angle b + 70° = 180°$

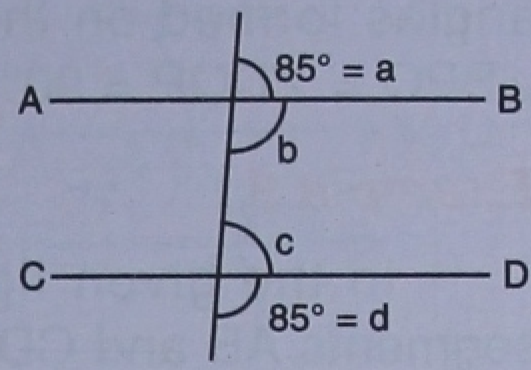
$\Rightarrow \angle b = 180° - 70° = 110°$

$\therefore \angle a = \angle b$ [Each is 110°]

But these are the corresponding angles.

Hence, **AB is parallel to CD** [If corresponding angles are equal, the lines are parallel] **(Ans.)**

(ii) Mark the angles a, b, c and d as shown in the adjacent diagram.



Since, $\angle a + \angle b = 180°$ [Straight line angle]

$\Rightarrow \angle b = 180° - 85°$ [Given, $\angle a = 85°$]

$= 95°$

Again, $\angle c + \angle d = 180°$ [Straight line angle]

$\Rightarrow \angle c = 180° - \angle 85°$ [Given, $\angle d = 85°$]

$= 95°$

Now, $\angle b + \angle c = 95° + 95° = 190°$, which is not equal to 180°

$\therefore$ **AB is not parallel to CD** [Two lines are //, if co-interior angles are supplementary] **(Ans.)**

Alternative method :

Since, $\angle b = 95°$ and $\angle d = 85°$

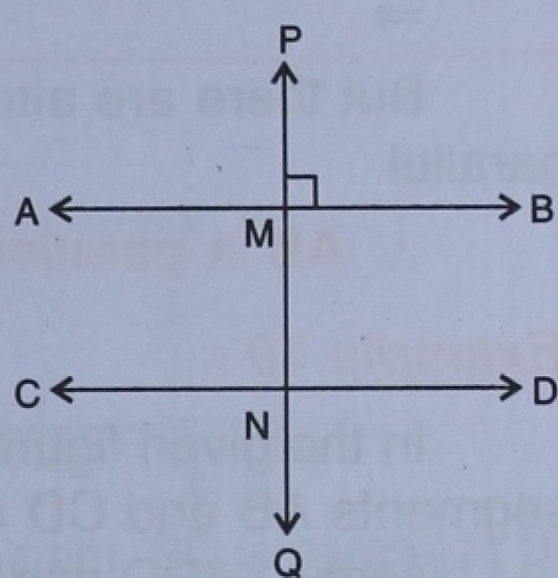
$\therefore \angle b \neq \angle d$ *i.e.* corresponding angles are unequal.

Hence, **AB is not parallel to CD** [Two lines are parallel, if corresponding angles are equal] **(Ans.)**

Example 7 :

In the given figure, line AB is parallel to line CD and line PQ is perpendicular to AB.

Show that line PQ is perpendicular to line CD also.



Solution :

Given : PQ is perpendicular to AB

$\Rightarrow \angle PMB = 90°$

Since, AB is parallel to CD and PQ is transversal

$\therefore \angle PMB = \angle PND$ [Corresponding angles]

$\Rightarrow \angle PND = 90°$ [$\because \angle PMB = 90°$]

$\Rightarrow$ **PQ is perpendicular to CD.**

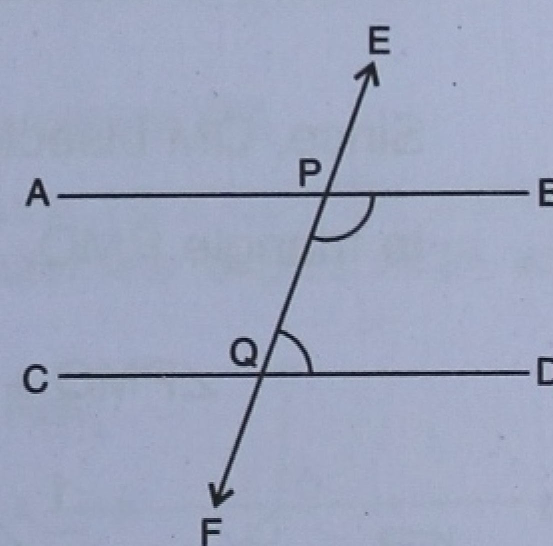
Example 8 :

In the given figure, transversal EF intersects line segments AB and CD at points P and Q respectively such that

$\angle BPQ = \angle DQP$

Show that lines AB and CD are not necessarily parallel.

State the condition under which the two given line segments will be parallel.



Solution :

It is clear from the given figure, that $\angle DQP$ is acute

i.e. $\angle DQP < 90°$ I

Since, $\angle BPQ = \angle DQP$ [Given]

$\therefore \angle BPQ < 90°$ II

$\therefore \angle BPQ + \angle DQP < 180°$ [Adding I and II]

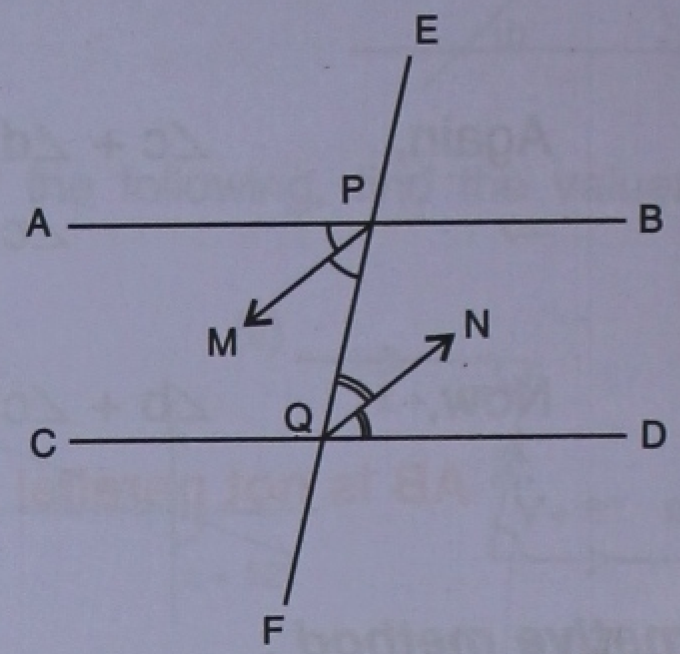
$\Rightarrow$ **Line segments AB and CD are not parallel.**

The two given line segments AB and CD will be parallel, when the sum of the interior angles formed on the same side of the transversal EF is 180°. For this we must have $\angle BPQ = \angle DQP = 90°$. **Ans.**

Example 9 :

In the given figure, transversal EF cuts line segments AB and CD at points P and Q respectively. PM bisects $\angle APQ$ and QN bisects $\angle PQD$.

If PM is parallel to QN, show that AB and CD are also parallel to each other.



Solution :

Since, PM bisects $\angle APQ \Rightarrow \angle MPQ = \frac{1}{2}\angle APQ$ I

Since, QN bisects $\angle PQD \Rightarrow \angle PQN = \frac{1}{2}\angle PQD$ II

Since, PM is parallel to QN and PQ is transversal

$\therefore \quad \angle MPQ = \angle PQN$ .... III [Alternate angles]

$\Rightarrow \quad \angle APQ = \angle PQD$ [From I, II and III]

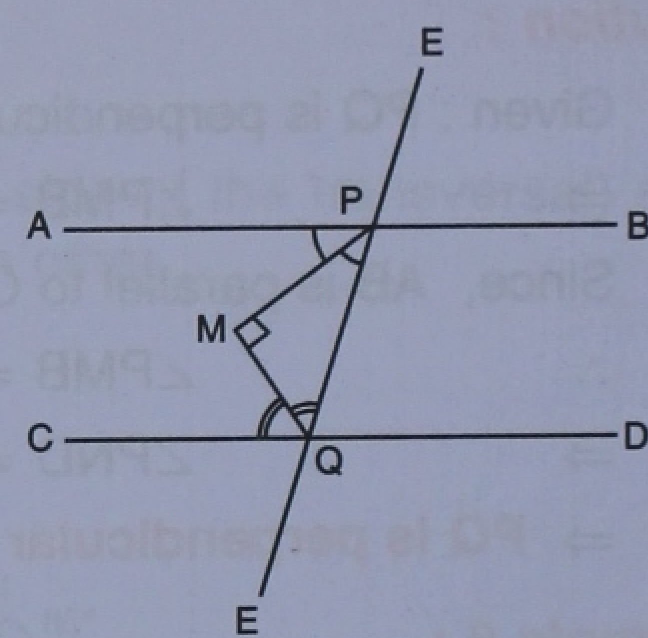
But there are alternate angles and whenever alternate angles are equal, the lines are parallel

$\therefore$ **AB is parallel to CD.**

Example 10 :

In the given figure, transversal EF intersects line segments AB and CD at points P and Q respectively. PM bisects $\angle APQ$ and QM bisects $\angle PQC$.

If $\angle PMQ = 90°$, show that AB and CD are parallel to each other.



Solution :

Since, PM bisects $\angle APQ \Rightarrow \angle MPQ = \frac{1}{2}\angle APQ$ I

Since, QM bisects $\angle PQC \Rightarrow \angle MQP = \frac{1}{2}\angle PQC$ II

In triangle PMQ,

$$\angle PMQ + \angle MPQ + \angle MQP = 180° \quad \textit{i.e.} \quad 90° + \frac{1}{2}\angle APQ + \frac{1}{2}\angle PQC = 180°$$

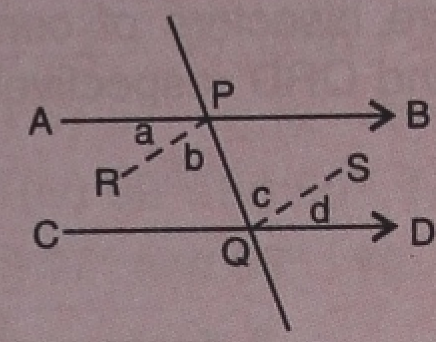
$$\Rightarrow \quad \frac{1}{2}\angle APQ + \frac{1}{2}\angle PQC = 90° \quad \textit{i.e.} \quad \angle APQ + \angle PQC = 180°$$

i.e. sum of the interior angles on the same side of the transversal EF is 180° and we know, whenever the sum of interior angles on the same side of the transversal is 180°, the lines are parallel.

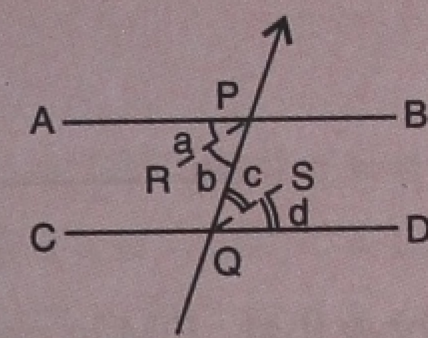
$\therefore$ **Lines AB and CD are parallel to each other.**

TEST YOURSELF

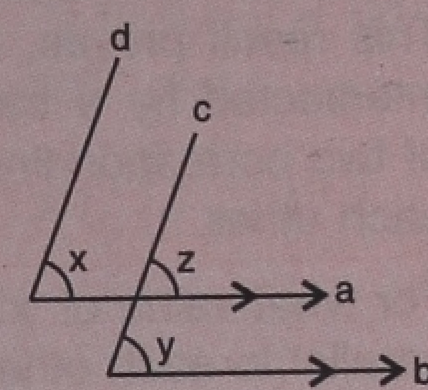
13. (i) In the given figure, if AB//CD; then .... and if PR//QS; then ....
(ii) If AB//CD, PR is not parallel to QS and b : c = 7 : 8, then d : a = ...



14. In the given figure, if AB//CD.
(i) The relation between a, b, c and d is ....
(ii) If PR//QS, the relation between b and c is .... also the relation between a and d is ....



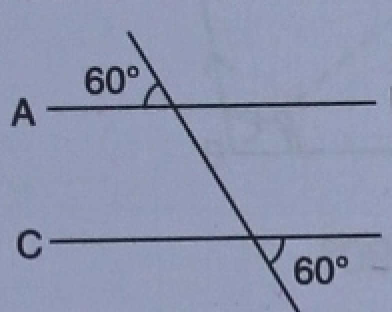
15. In the given figure, lines a and b are parallel ⇒ y = ... Also, if x = y, then x = ... But x and .... are correspoing .... and so lines c and d are ....



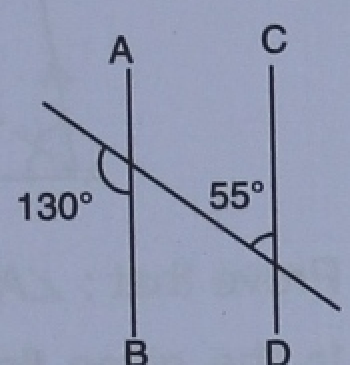
EXERCISE 23 (C)

1. State, giving reasons, whether AB is parallel to CD or not :

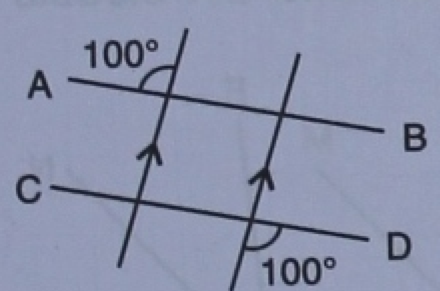
(i)



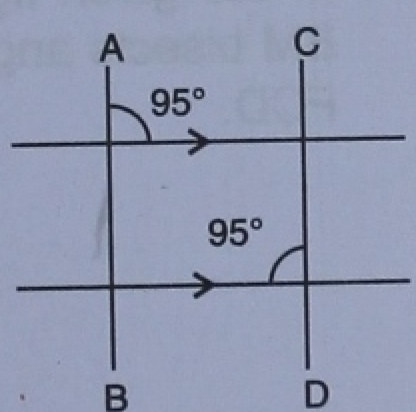
(ii)



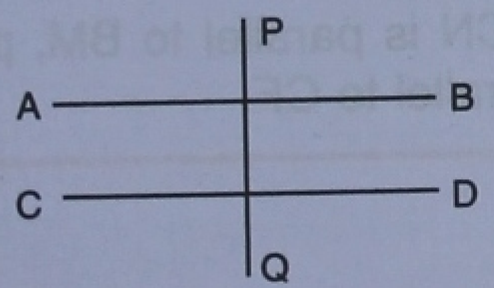
(iii)



(iv)

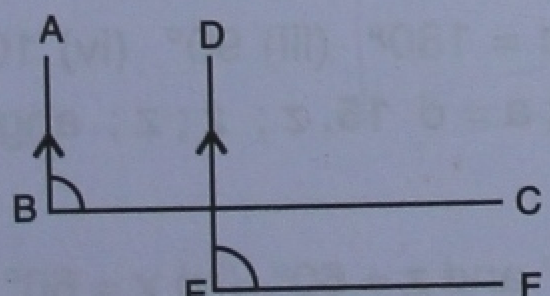


2. In the following figure, AB and CD both are perpendicular to the same line PQ. Show by giving suitable reasons, that AB is // to CD.

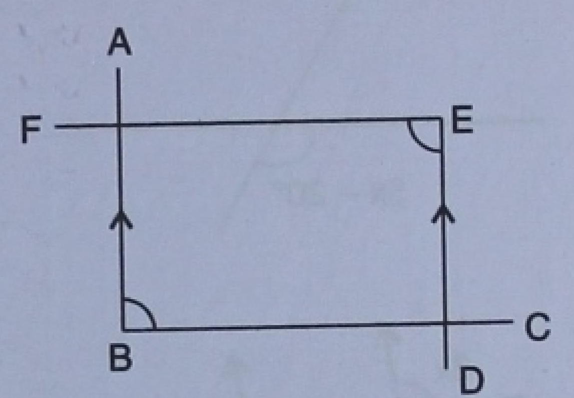


3. In each of the following cases; AB is parallel to DE and ∠B = ∠E. Show that : BC is parallel to EF.

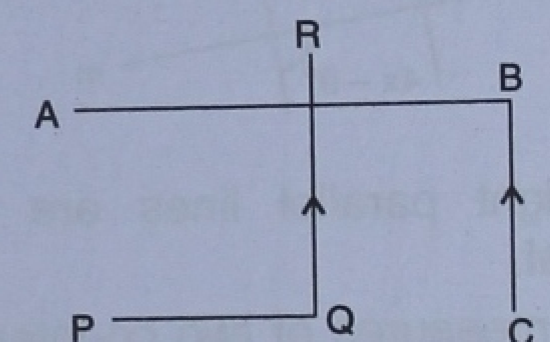
(i)



(ii)



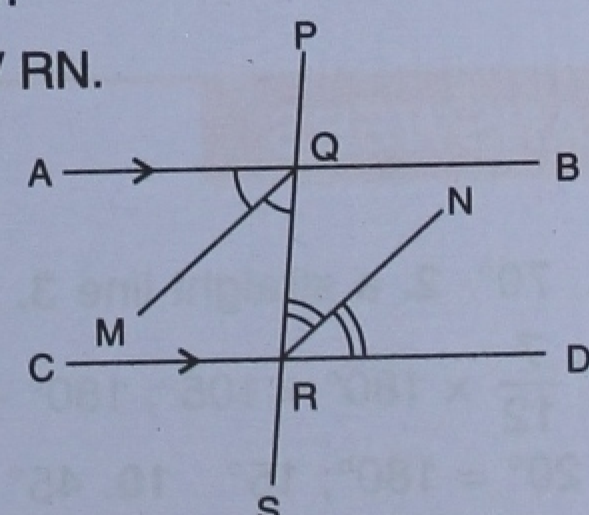
4. In the following diagram, RQ is parallel to BC and ∠Q and ∠B are supplementary.



Show that AB is parallel to PQ.

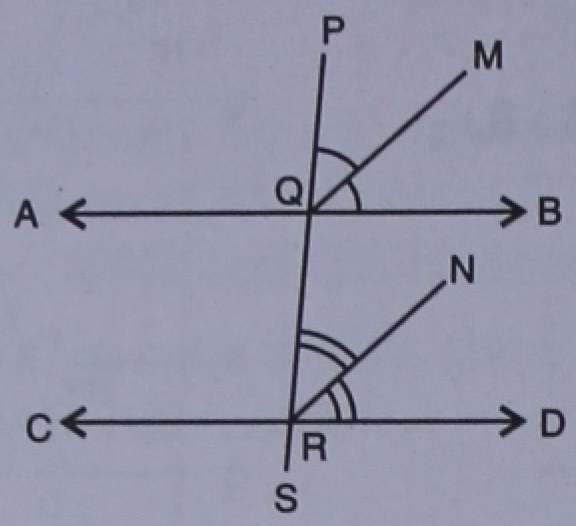
5. In the following figure, AB // CD; QM and RN are bisectors of alternate angles AQR and QRD respectively.

Show that : QM // RN.



This result proves, if two parallel lines are intersected by a transversal, the bisectors of two alternate angles are parallel to each other.

6. In the following figure, AB//CD, QM and RN are bisectors of corresponding angles PQB and QRD respectively.

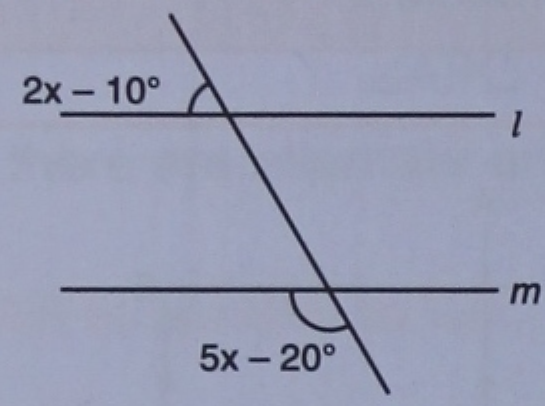


Show that : QM is parallel to RN.

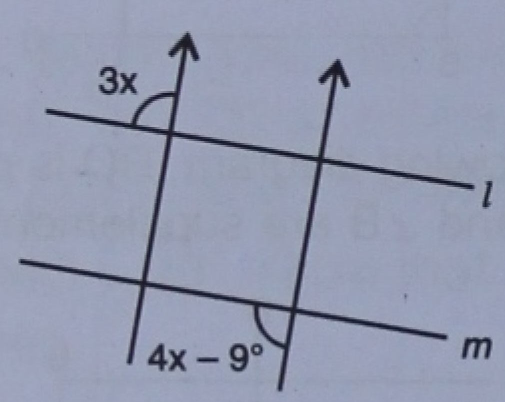
> This result proves, if two parallel lines are intersected by a transversal, the bisectors of two corresponding angles are parallel to each other.

7. For what value of 'x' will the lines *l* and *m* be parallel to each other.

(i)



(ii)



8. Two straight parallel lines are cut by a transversal.
   (i) If the measures of two co-interior angles are (2a)° and (3a – 10)°; find the value of 'a'.
   (ii) If the measures of two corresponding angles are (2b + 15)° and (3b – 7)°; find the value of 'b'.

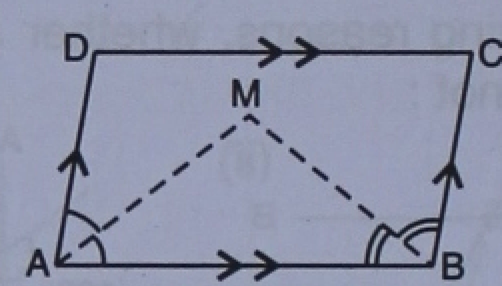
9. (i) Two straight lines are parallel to the same third line; prove that they are parallel to each other also.
   (ii) Two straight lines are perpendicular to the same third line; prove that they are parallel to each other.

10. Two parallel lines are cut by a transversal at points P and Q. The bisectors of interior angles, on the same side of the transversal, intersect at point R.
    Prove that angle PRQ is a right angle.

11. Two straight lines are cut by a transversal. If the bisectors of a pair of co-interior angles are perpendicular to each other, prove that the two straight lines are parallel to each other.

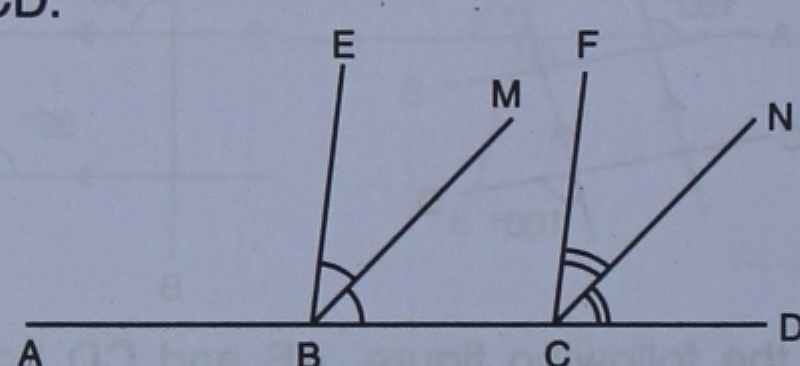
12. In a parallelogram ABCD, the bisectors of angles at B and C intersect each other at point E. Prove that angle BEC is equal to a right angle.

13. In the following figure, AM bisects ∠DAB and BM bisects ∠ABC.



Prove that : ∠AMB = 90°.

14. In the given figure, ABCD is a straight line. BM bisects angle EBC and CN bisects angle FCD.



   (i) If EB is parallel to FC, prove that BM is parallel to CN.
   (ii) If CN is parallel to BM, prove that BE is parallel to CF.

ANSWERS

TEST YOURSELF

1. 70° **2.** a straight line **3.** (x – 20)° ; 85° ; 105 **4.** 10°; 100° **5.** 110° ; 20° **6.** 180° ; a straight line **7.** $\frac{7}{12}$ × 180° = 105°; 180° – 105° = 75°; 105° ; 75° **8.** x + 10° + 2x – 40° = 90; 40° **9.** 3x + 40° + 5x + 20° = 180°; 15° **10.** 45° **11.** 90° **12.** (i) parallel (ii) b + c = 180° (iii) 90° (iv) 100° (v) 100° **13.** (i) a + b = c + d ; b = c (ii) 7 : 8 **14.** (i) a + b = c + d (ii) b = c ; a = d **15.** z ; z ; z ; angles ; parallel.

EXERCISE 23(A)

1. (i) a = 100° ; b = 80° (ii) a = 65° ; b = 115° (iii) x = 40° ; y = 80° and z = 60° (iv) x = 60°, y = 20° and z = 100° (v) x = 36°, y = 72°, z = 108° and r = 144° (vi) a = 100°, b = 20° and c = 60°

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(vii) a = 20°, b = 45°, c = 120°, d = 80° and e = 95° **2.** x = 12; 54° and 126° **3.** 69° and 111° **4.** 100° and 80° **5.** (i) a = 93°, b = 87° and c = 93° (ii) y = 45°, x = 135°, z = 45° and r = 135° (iii) a = d = 108° and b = c = 72° (iv) a = b = 45° and p = q = 135° **6.** (i) 67·5° (ii) (a – b – 60)° **7.** (i) 144° (ii) (60 – a + 2b)° **8.** 75° **9.** 120° **10.** (i) 85° (ii) 100° (iii) 30° (iv) 62° (v) 90° (vi) 45° **11.** 60° **12.** 45° **13.** (i) 75° and 105° (ii) 70° and 110° **14.** (i) 36° and 54° (ii) 27° and 63° **16.** (i) 38° (ii) 42°

EXERCISE 23(B)

1. (i) a = 65° and b = c = 115° (ii) a = 108° = c and b = d = 72° (iii) a = b = 125° and c = d = 55° (iv) a = 30°, b = 25°, c = 70° and d = 55° (v) a = 110°, b = 70° and c = 40° (vi) a = b = 65°, c = 60° and d = 55° (vii) a = 65° = b = d = e and c = 115° **2.** (i) 23° (ii) 50° **3.** (i) x = 70° and y = 110°, a = 110° = b = c (ii) x = y = 120°, a = 60° = b and c = 120° **4.** (i) 38° (ii) 14° **5.** (i) a = d = 120° and b = c = 60° (ii) a = 140° and b = 220° (iii) a = 120° and b = 150° (iv) a = 100° and b = c = 105° **8.** (i) x = 44° and y = 34° (ii) x = 75° and y = 77° (iii) x = 85° and y = 90°

EXERCISE 23(C)

1. (i) Yes (ii) No (iii) Yes (iv) No **7.** (i) 30° (ii) 27° **8.** (i) a = 38° (ii) b = 22°