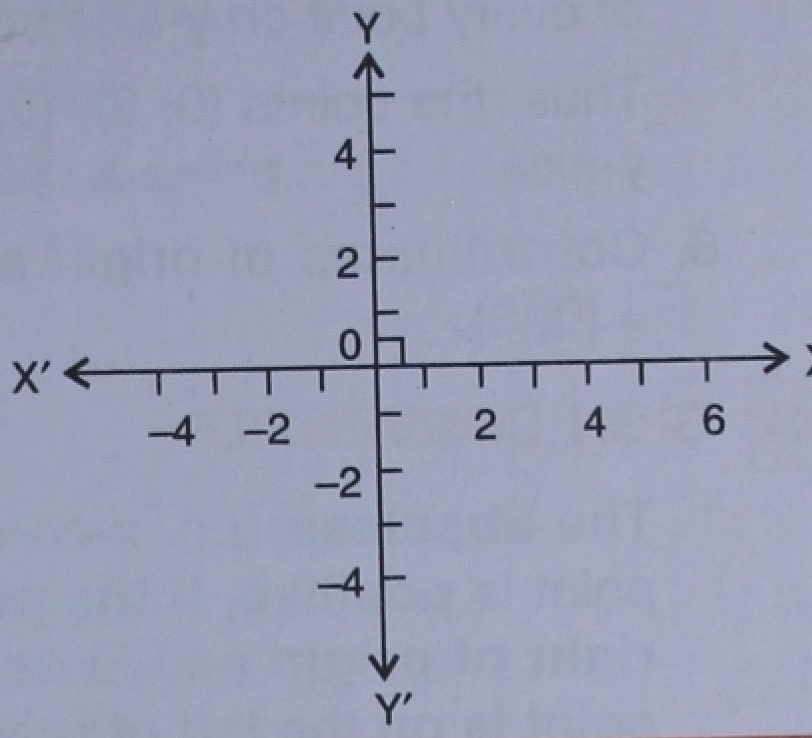
LINEAR GRAPHS

22.1 INTRODUCTION

A graph consists of two mutually perpendicular number lines intersecting each other at zero. See the adjoining figure.



The given figure shows two number lines XOX′ and YOY′ such that :

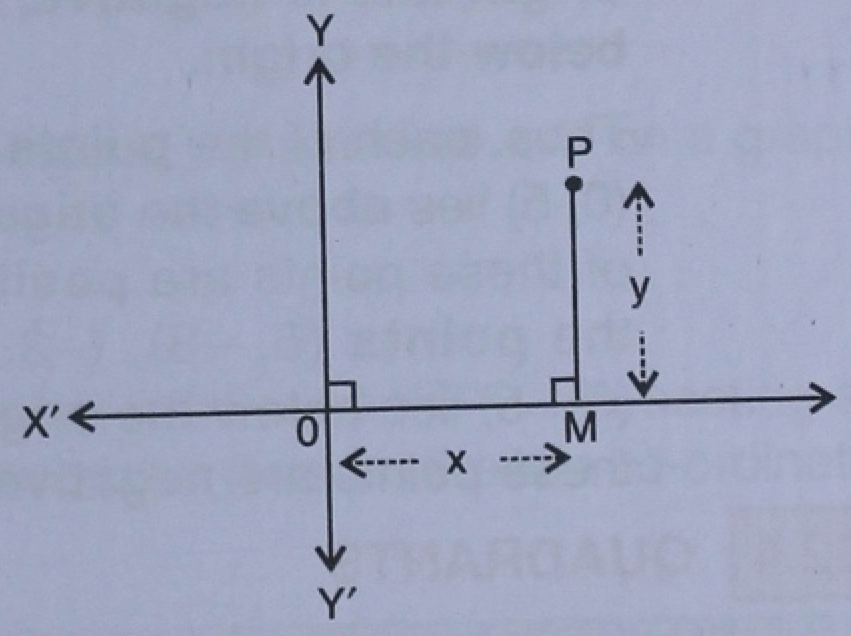
(i) the lines are perpendicular to each other.

(ii) the lines intersect each other at zero (0).

x-axis	The horizontal number line XOX′ is called **x-axis.**
y-axis	The vertical number line YOY′ is called **y-axis.**
Co-ordinate axes	Taking together, the number lines XOX′ and YOY′ are called **co-ordinate axes.** Axes is plural of axis.
Origin	The point at which the two axes intersect, is called **origin** and is denoted by letter 'O'.
Co-ordinate plane	The plane which contains both the co-ordinate axes, is called **co-ordinate plane.**

22.2 CO-ORDINATES OF A POINT

Consider a point P in the co-ordinate plane and PM perpendicular to x-axis.



1. The distance of point P, taken along x-axis and starting from origin 0, is called the x-co-ordinate or *abscissa* of the point P.

   i.e. OM = abscissa (or x-co-ordinate) of point P
   = x(let)

2. The distance of point P, taken along y-axis and starting from origin 0, is called the y-co-ordinate or ordinate of the point.

   i.e. PM = ordinate (or y-co-ordinate) of point P
   = y(let)

3. The abscissa (x-co-ordinate) and ordinate (y-co-ordinate) of a point together are called the co-ordinates of the point.

   Thus, co-ordinates of a point = (its abscissa, its ordinate).

   i.e. co-ordinates of the point P = (x, y)

> In stating the co-ordinates of a point, the abscissa precedes the ordinate and both are enclosed in a bracket after being separated by a comma.
> *e.g.*, if the abscissa of point is 2 and its ordinate is –3, then its co-ordinates = (2, –3). *Conversely*, if the co-ordinates of a point are (a, b), then its abscissa = a and its ordinate = b.

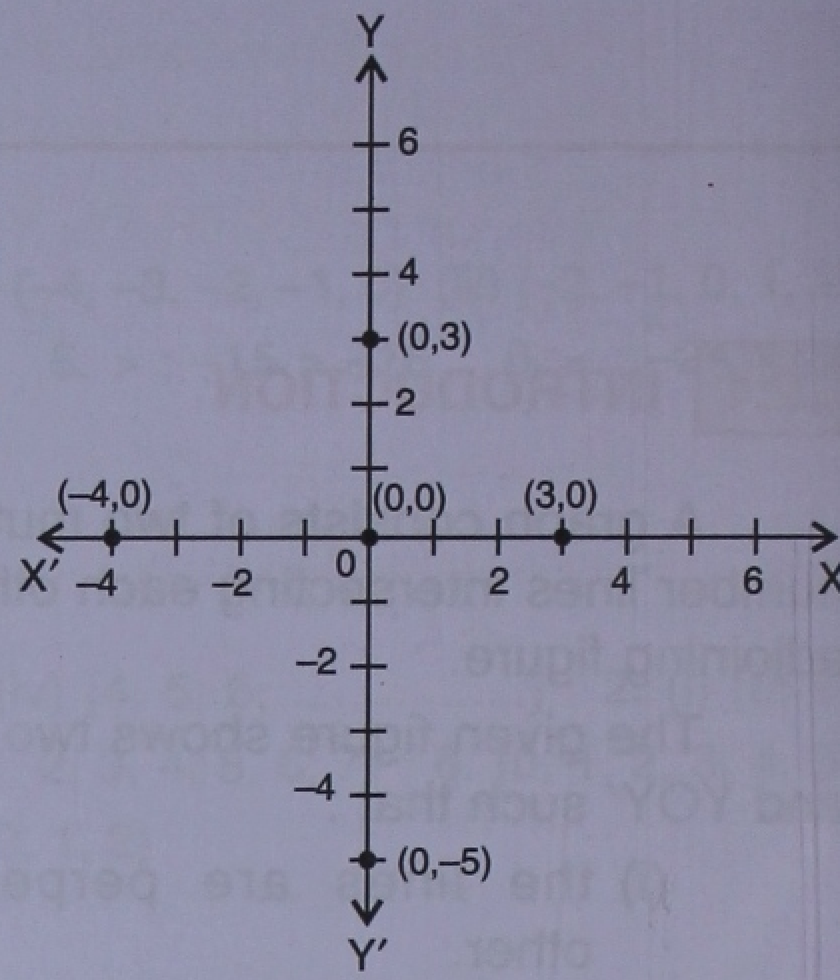
4. For every point on x-axis, the value of its ordinate (*i.e.* y-co-ordinate) is zero and so the co-ordinates of every point on x-axis are of the form (x, 0).

Thus, the points (3, 0), (–4, 0), (0, 0), etc. lie on x-axis.

5. For every point on y-axis, the value of its abscissa (*i.e.* x-co-ordinate) is zero and so the co-ordinates of every point on y-axis are of the form (0, y).

   Thus, the points (0, 3), (0, –5), (0, 0), etc. lie on y-axis.

6. Co-ordinates of origin are (0, 0) *i.e.* origin = (0, 0).



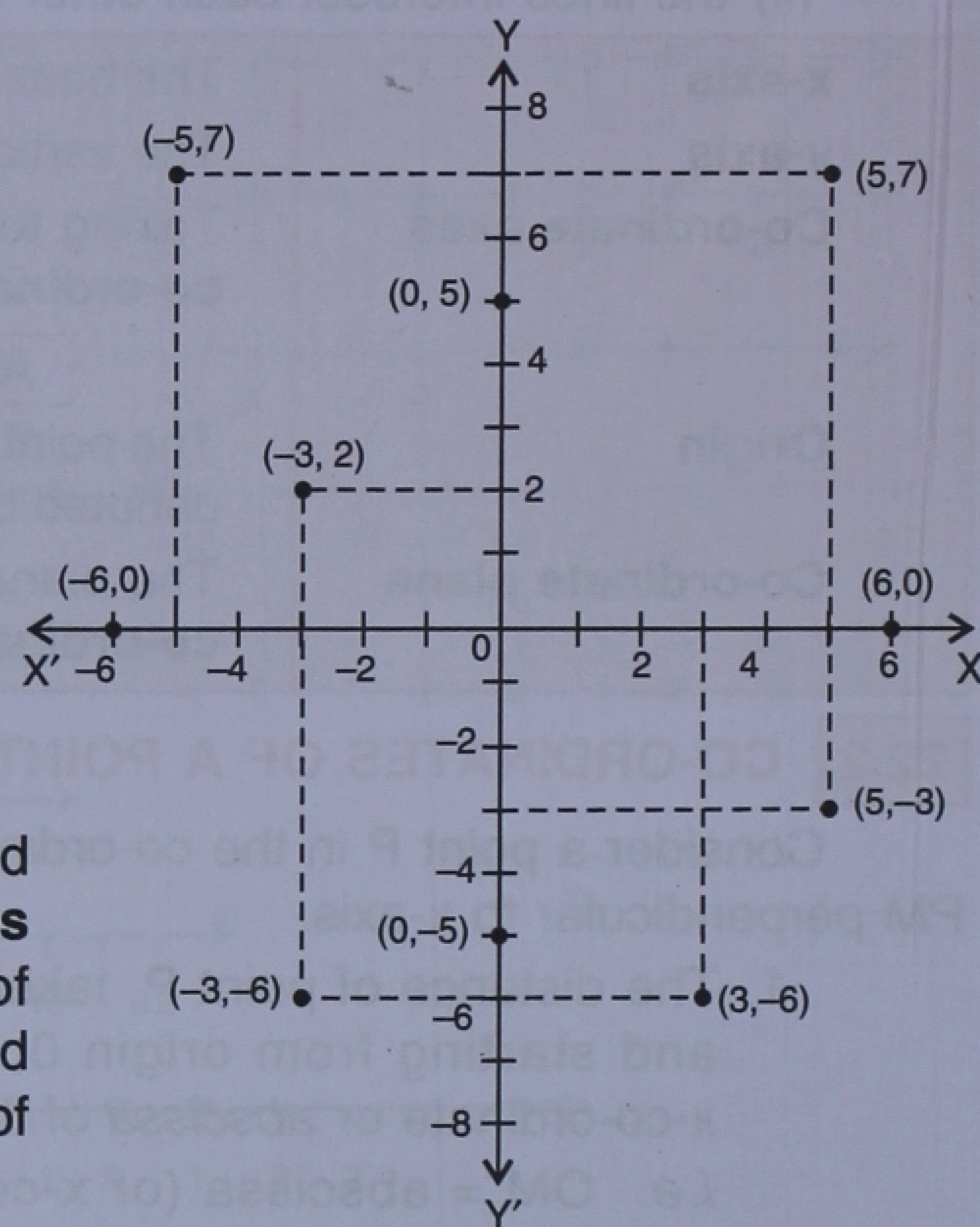
22.3 SIGN CONVENTION

1. The **abscissa** (*i.e.* x-co-ordinate) of a point is **positive, if the point is on the right of origin** and is **negative, if the point is on the left of origin.**

   Thus, each of the **points** (5, 7), (3, – 6) and (6, 0) lies **on the right side of origin** as **abscissae** of these points are **positive.** And, each of the **points** (–5, 7), (–3, –6), (–3, 2) and (–6, 0) lies **on the left side of origin** as **abscissae** of these points are **negative.**

2. The **ordinate** (*i.e.* y-co-ordinate) of a point is **positive, if the point is above the origin** and is **negative, if the point is below the origin.**

   Thus, each of the **points** (5, 7), (–3, 2) and (0, 5) lies **above the origin** as the **ordinates** of these points are **positive.** And, each of the **points** (5, –3), (–3, –6), (3, –6) and (0, –5) lies **below the origin as ordinates** of these points are **negative.**



22.4 QUADRANTS

The two co-ordinate axes (x-axis and y-axis) divide the co-ordinate plane into four parts which are called **quadrants.**

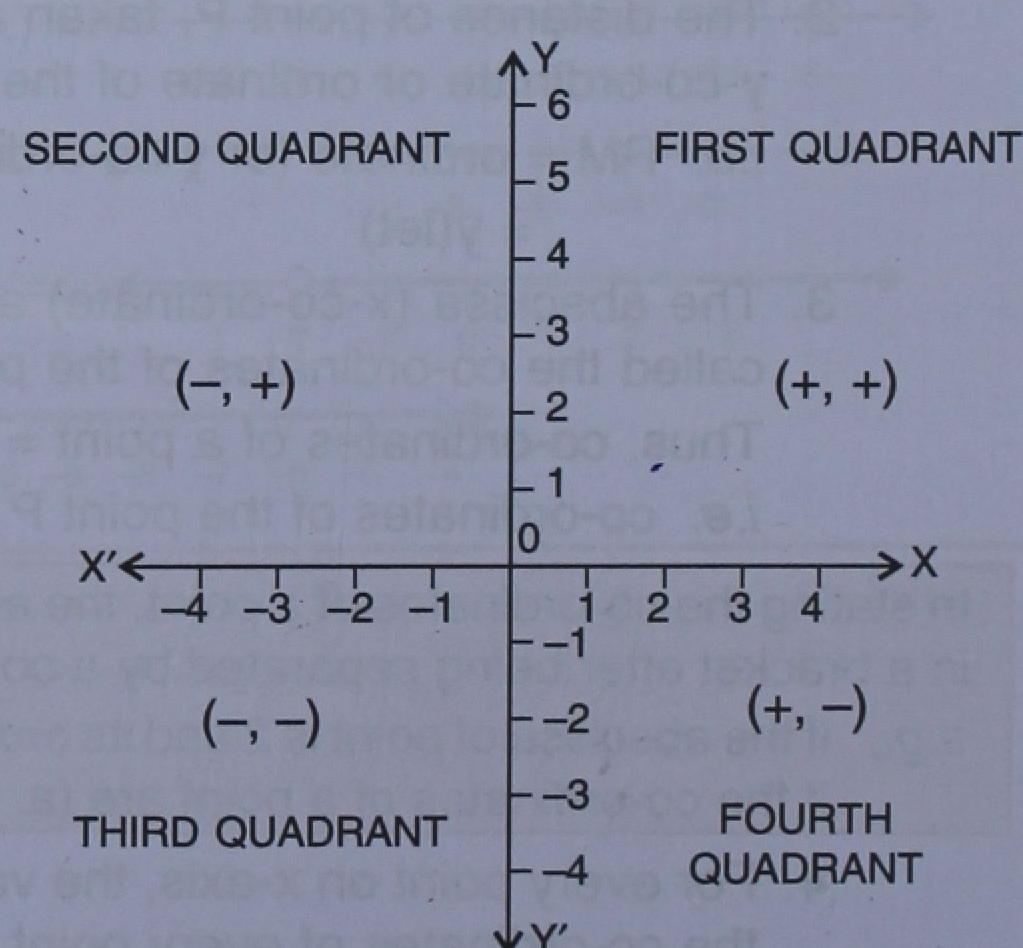
As shown in the adjoining figure :

(i) In first quadrant, XOY, the abscissa and the ordinate both are positive.

(ii) In the second quadrant, X′OY, the abscissa is negative and the ordinate is positive.

(iii) In the third quadrant X′OY′, the abscissa and the ordinate both are negative.

(iv) In the fourth quadrant, XOY′, the abscissa is positive and the ordinate is negative.



TEST YOURSELF

1. Co-ordinate axes are two mutually .... number ....; which intersect each other at their ....
2. XOX′ is .... number line and is called ....
3. YOY′ is .... number line and is called ....
4. The point of intersection of two number lines (axes) is called ....
5. State, **true** or **false** :
   (i) The point (5, 0) lies on x-axis ....
   (ii) The point (0, –8) lies on y-axis ....
   (iii) The point (0, 6) lies on x-axis ....
   (iv) The point (0, 0) lies on y-axis ....
   (v) If the abscissa of a point is zero, the point lies on x-axis ....
   (vi) If the ordinate of a point is zero, the point lies on x-axis ....
6. Out of the points : (3, 5), (–3, 5), (3, –5), (–3, –5), (7, 8), (5, –4), (–6, 2), (8, 3), (5, –5), (–4, –4), (5, –3), (–6, –5), (–2, 3) and (6, 4) lie in :
   (i) first quadrant : ...., ..., ... and ....
   (ii) second quadrant : ...., ... and ....
   (iii) third quadrant : ...., ... and ....
   (iv) fourth quadrant : ...., ..., ... and ....

22.5 PLOTTING THE POINTS

Example 1 :

Plot the points A (2, 3), B (–3, 2), C (–2, –2), D (1, –3), E (3, 0) and F (0, –1) on a graph paper.

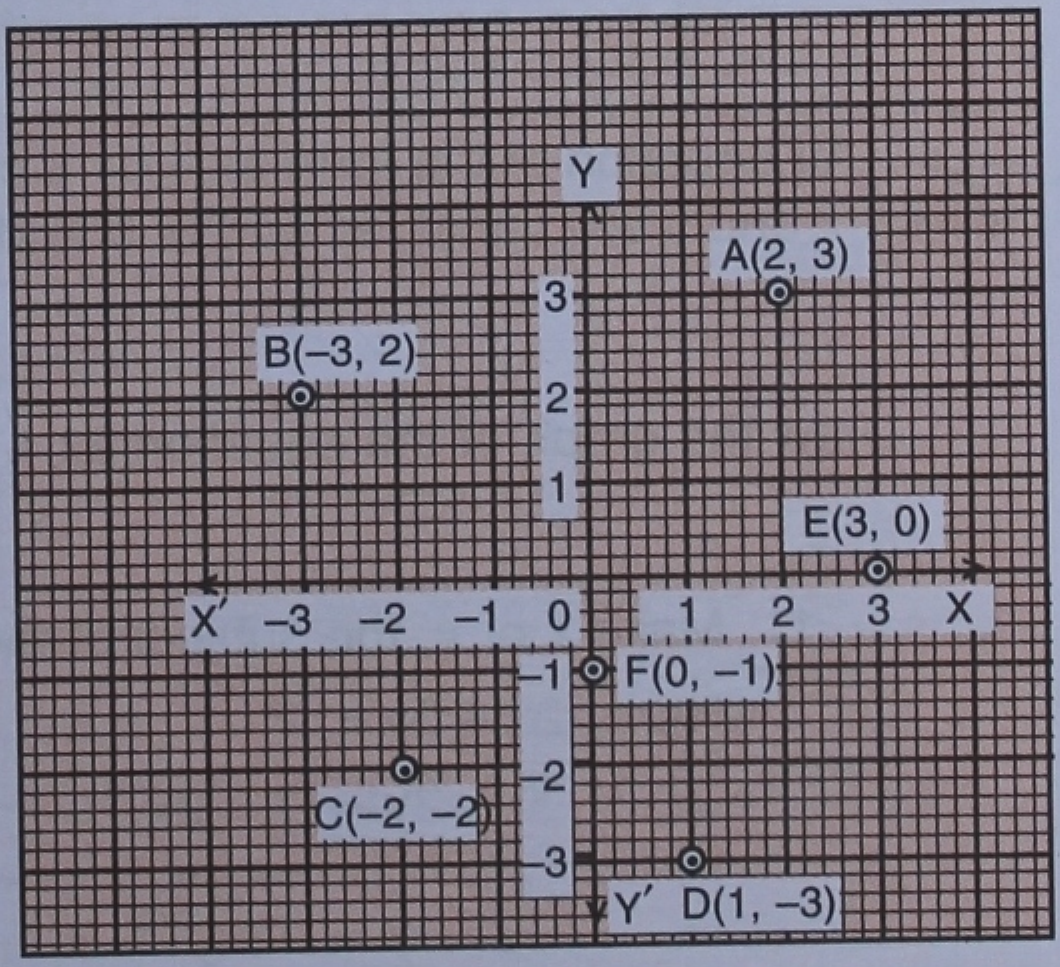
Solution :

Take a graph paper and on it, draw the co-ordinate axes XOX′ and YOY′ intersecting at origin 0 (as shown in the diagram). With a proper scale, mark the numbers on the two co-ordinate axes.

(i) For plotting A (2, 3); starting from origin O, move 2 units (abscissa) along x-axis, on the right of 0, and then from there move 3 units (ordinate) along y-axis, above 0. Mark the resulting point as A and write its co-ordinates (2, 3) near it.

(ii) For plotting B (–3, 2); starting from origin 0, move 3 units along x-axis, on the left of 0, and then from there, 2 units along y-axis, above 0. Mark the resulting point as B and write its co-ordinates near it.

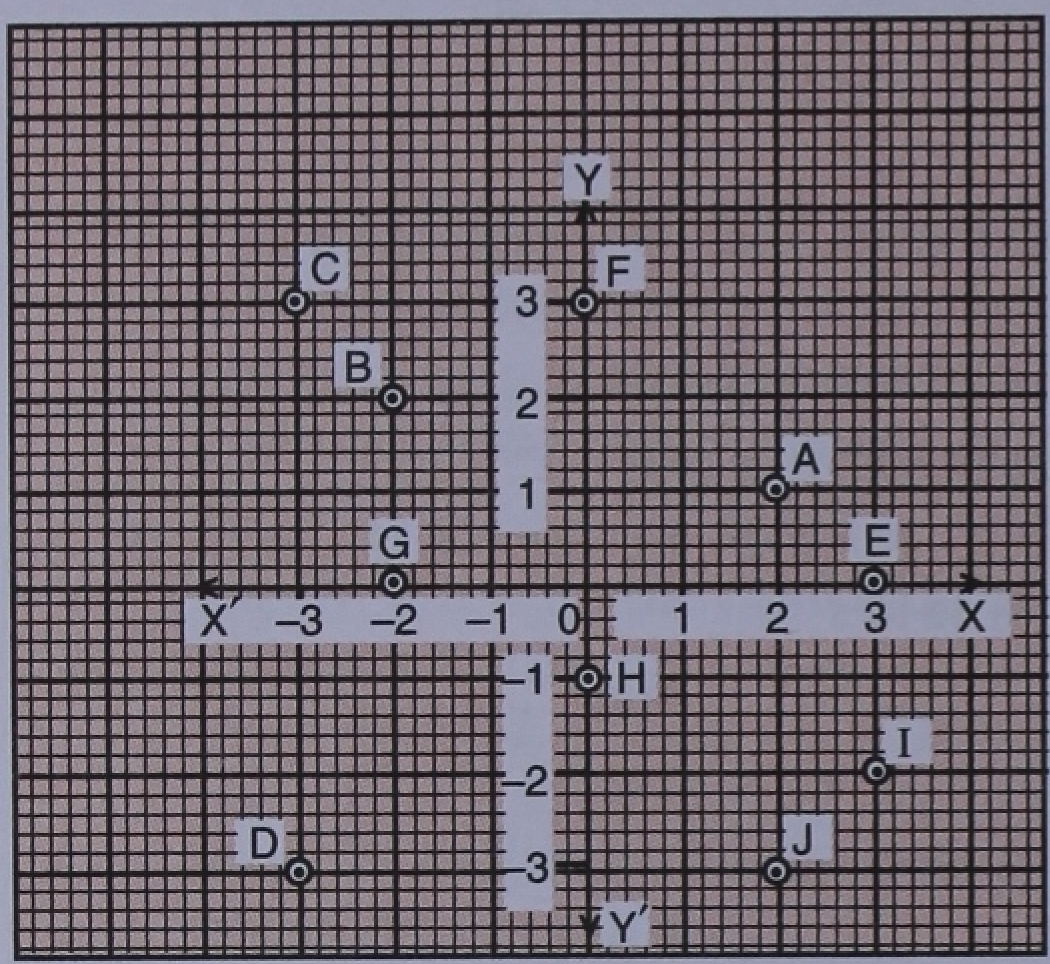
Similarly, mark the other points as shown.



EXERCISE 22 (A)

1. Plot the following points on a graph paper :
   (i) (1, 2) (ii) (–5, 4) (iii) (–3, –4)
   (iv) (5, –3) (v) (0, 2) (vi) (–2, 0)
   (vii) (5, 0) (viii) (0, –4)

2. Write down the co-ordinates of the points A to J marked in the following diagram :



3. In each case, plot the given points on a graph and join them together with a straight line :
   (i) (0, 0), (3, 3), (–2, –2) and (5, 5)
   (ii) (2, 0), (4, 2), (–2, –4) and (0, –2)
   (iii) (1, 2), (3, 3), (–1, 1) and (–3, 0)
   (iv) (0, –1), (2, 3), (–2, –5) and (3, 5)

4. In each case, plot the given points on a graph paper and join them with straight lines. Give a special name to the figure obtained in each case :
   (i) (4, 2), (–1, 2), (–3, –2) and (2, –2)
   (ii) (3, 2), (3, –4), (–2, 2) and (–2, –4)
   (iii) (2, 1), (2, –2), (–1, 1) and (–1, –2)
   (iv) (1, 4), (3, 0) and (–1, 0)

5. A (5, 3), B (–1, 3) and C (–1, –1) are the three vertices of a rectangle ABCD. Plot the given points on a graph paper and then use this graph to find the co-ordinates of the fourth vertex D.

6. A (1, 0), B (–2, 0) and D (1, –3) are the vertices of a square ABCD. By plotting the given points on a graph paper, find the co-ordinates of the unknown vertex C.

7. Plot the points A (4, 5), B (–1, 5), C (–1, –2) and D (4, –2) on a graph paper. Join AB, BC, CD and DA. Give a special name to the quadrilateral ABCD obtained. Also, find its area.

8. Plot the points A (1, –1), B (–1, 4) and C (–3, –1) on a graph paper to obtain the triangle ABC. Give a special name to the triangle ABC and, if possible, find its area.

9. Draw a rectangle OABC; where vertex O is the origin, vertex A is on the positive side of x-axis at a distance of 4 units from origin and vertex C is on the positive side of y-axis at a distance of 5 units from origin. Find the co-ordinates of vertices A, B and C.

10. Square OABC is drawn with vertex O as origin, vertex A on the positive side of x-axis and vertex C on the positive side of y-axis. If each side of the square OABC is of length 6 units, draw OABC on a graph paper and then use the graph to find the co-ordinates of vertices A, B and C.

22.6 TO DRAW A GRAPH OF THE GIVEN LINEAR EQUATION IN TWO VARIABLES

An equation of the form ax + by + c = 0 is called a linear equation in two variables, in which x and y are variables and a, b and c are constants.

Example 2 :

Draw the graph of the equation 2x + 3y = 7.

Solution :

Steps : 1. Make x or y, the subject of the equation.

Here, $2x + 3y = 7$

$\Rightarrow \quad 3y = 7 - 2x$

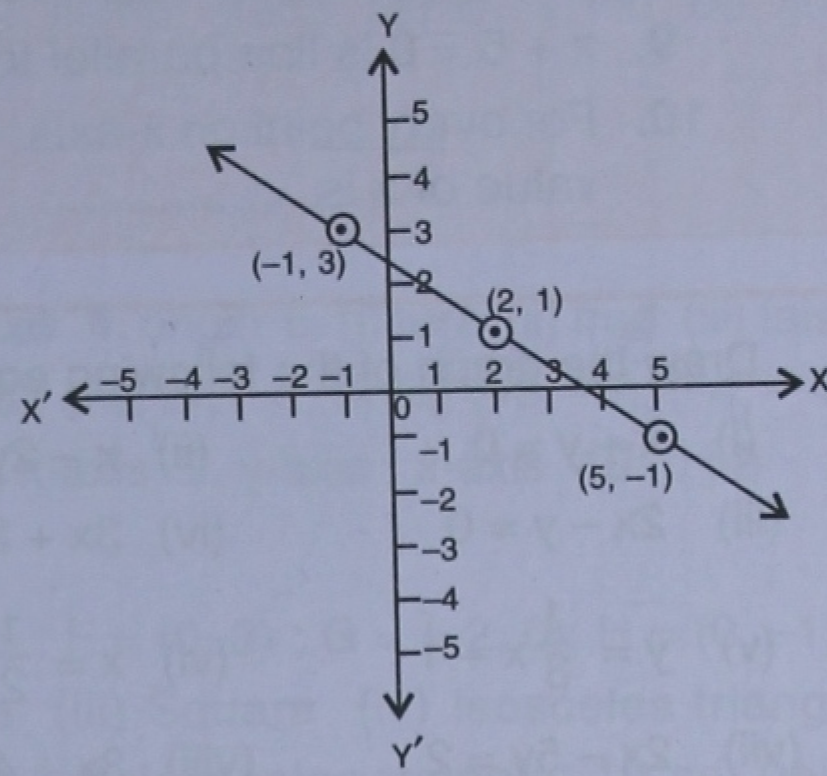
$\Rightarrow \quad y = \dfrac{7 - 2x}{3}$ [Making y, the subject of equation]

2. Give atleast three suitable values to the variable on the right side (*i.e.* x) and find the corresponding values of y (the variable on left-side). Here,

if, x = 2, $y = \frac{7-2\times2}{3} = \frac{3}{3} = 1$

if, x = 5, $y = \frac{7-2\times5}{3} = \frac{-3}{3} = -1$

if, x = –1, $y = \frac{7-2\times-1}{3} = \frac{9}{3} = 3$



3. Construct a table for different pairs of values of x and y as shown below :

x	2	5	–1
y	1	–1	3

4. Plot the points, from the table, on a graph paper and draw a straight line passing through the points plotted on the graph.

The graph of a linear equation in two variables is always a straight line.

Alternative method : If instead of y, we make, x the subject of the equation, we shall be getting the same straight line. In this case :

Steps : 1. 2x + 3y = 7

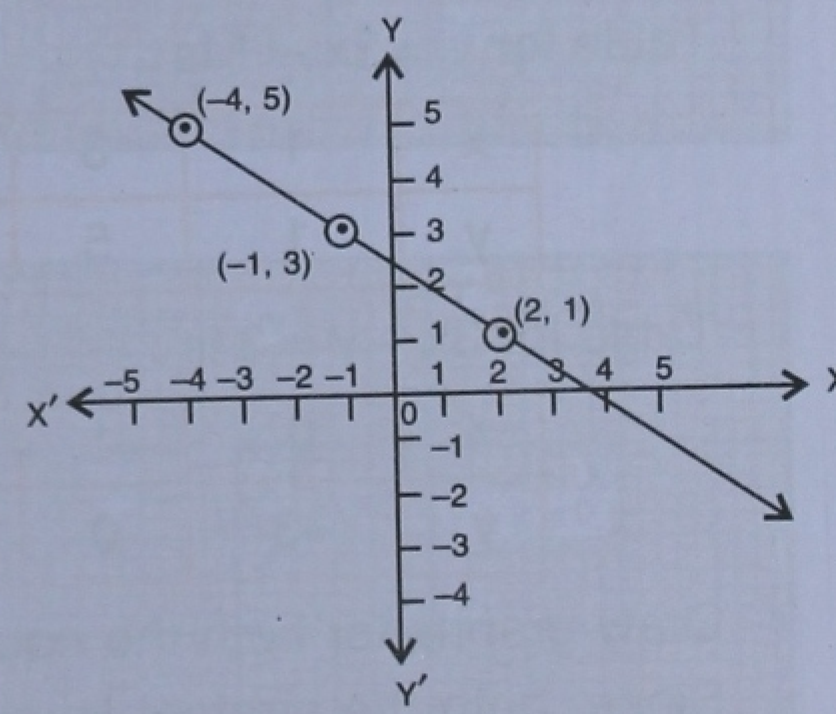
$\Rightarrow \quad 2x = 7 - 3y$

$\Rightarrow \quad x = \frac{7-3y}{2}$

2. If, y = 1, $x = \frac{7-3\times1}{2} = \frac{4}{2} = 2$ [Giving suitable values to y and to find the corresponding values of x]

If, y = 3, $x = \frac{7-3\times3}{2} = -1$

If, y = 5, $x = \frac{7-3\times5}{2} = -4$

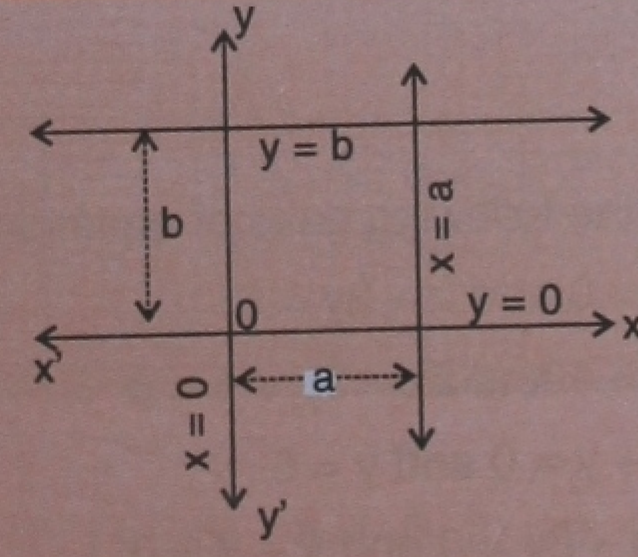


3. Constructing a table for different values of x and y.

x	2	–1	–4
y	1	3	5

4. Plotting the points from the table and then drawing the straight line.

1. The graph of equation x = 0 is the y-axis.
2. The graph of equation y = 0 is the x-axis.
3. The graph of equation x = a is a straight line parallel to y-axis and at a distance 'a units' from it.
4. The graph of equation y = b is a straight line parallel to x-axis and at a distance 'b units' from it.



TEST YOURSELF

7. Graph of line $3x - 2y = 0$ passes through ....
8. $x = 0$ represent .... and $y = 0$ represents ....
9. $x + 5 = 0$ is line parallel to .... and $y - 3 = 0$ is a line parallel to ....
10. For every point on x-axis, the value of y is .... and for every point on y-axis, the value of x is ....

EXERCISE 22 (B)

1. Draw the graph of the following equations :
   (i) $x + y = 0$ (ii) $x - 2y = 0$
   (iii) $2x - y = 0$ (iv) $3x + 2y = 0$
   (v) $y = \frac{1}{3}x + 1$ (vi) $x = \frac{1}{2}y - 3$
   (vii) $2x - 5y = 2$ (viii) $3x + 4y = 1$
2. Fill in the blanks ;
   (i) The equation of x-axis is ....
   (ii) The equation of y-axis is ....
   (iii) The graph of $x = 2$ is a line parallel to .... axis.
   (iv) The graph of $y = 3$ is a line parallel to .... axis.
   (v) The graph of line $x + 2 = 0$ is parallel to .... axis.
   (vi) The graph of line $y + 3 = 0$ is parallel to .... axis.
3. Draw the graph of the following equations :
   (i) $x - 5 = 0$ (ii) $y = 0$
   (iii) $x + 5 = 0$ (iv) $y + 2 = 0$
   (v) $y = 2x + 3$ (vi) $y = x - 6$
   (vii) $x = 3 - \frac{5}{2}y$ (viii) $x = -2 - y$

22.7 SOLVING A PAIR OF SIMULTANEOUS EQUATIONS GRAPHICALLY

Steps : 1. Draw graph (straight line) for each equation.
2. From the graph, read the point of intersection of the two straight lines drawn.

Example 3 :

Solve the given equations graphically : $y = 2x - 1$ and $3x - y = 3$

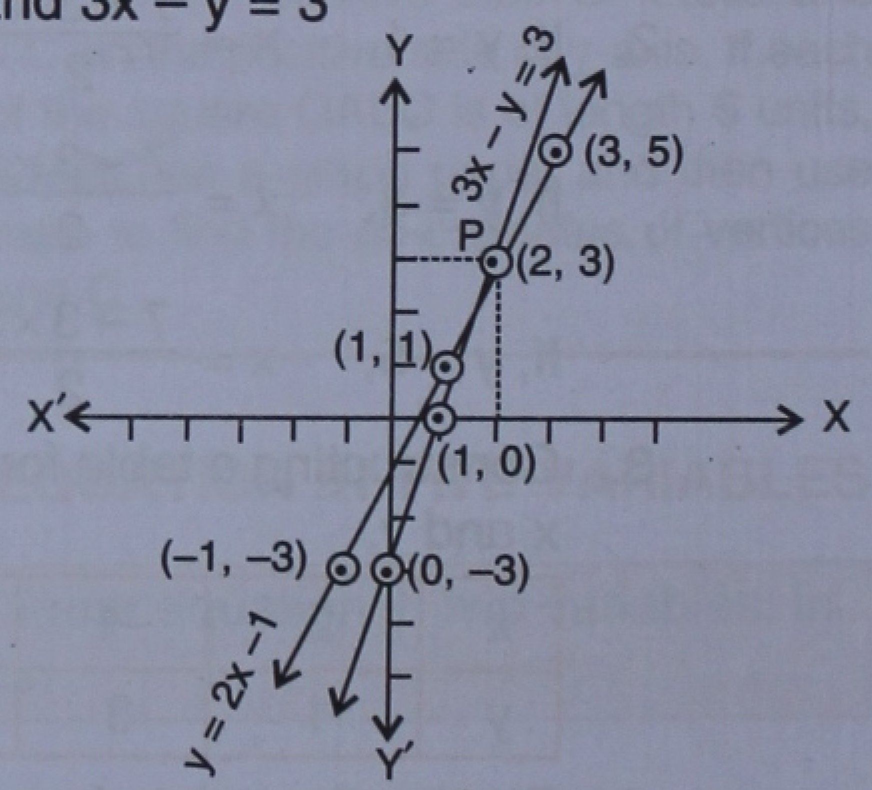
Solution :

Table for $y = 2x - 1$ is :

x	1	3	–1
y	1	5	–3

Table for $3x - y = 3$ is :

x	0	1	2
y	–3	0	3



Draw graphs for both the equations.

Since, both the straight lines intersect at point P and the co-ordinates of point P are (2, 3).

∴ **Solution of the given equations is $x = 2$ and $y = 3$.** **(Ans.)**

EXERCISE 22 (C)

Solve the following pairs of equations, graphically :

1. $x = 0$ and $x + 3y = 6$
2. $x = 4$ and $2x - 3y + 1 = 0$
3. $x + y = 0$ and $y = 5$
4. $3x - 2y = 0$ and $y + 3 = 0$
5. $x + y = 7$ and $x - y = 3$
6. $3x - 4y = 1$ and $x - 2y + 1 = 0$
7. $\frac{x}{2} - \frac{y}{3} = 3$ and $x + y = 1$
8. $x = -y - 1$ and $2y = 1 - x$

9. $2x - 3y = -6$ and $x - \frac{y}{2} = 1$

10. $4x + 3y = 1$ and $2x - y = 3$

11. Draw the graphs of $3x - 2y = 6$ and $3x - 2y = 9$ on the same graph paper. What do you observe ?

12. Draw the graphs of $2x - 3y = 6$ and $3x + 2y = 6$ on the same graph paper. What do you observe ?

ANSWERS

TEST YOURSELF

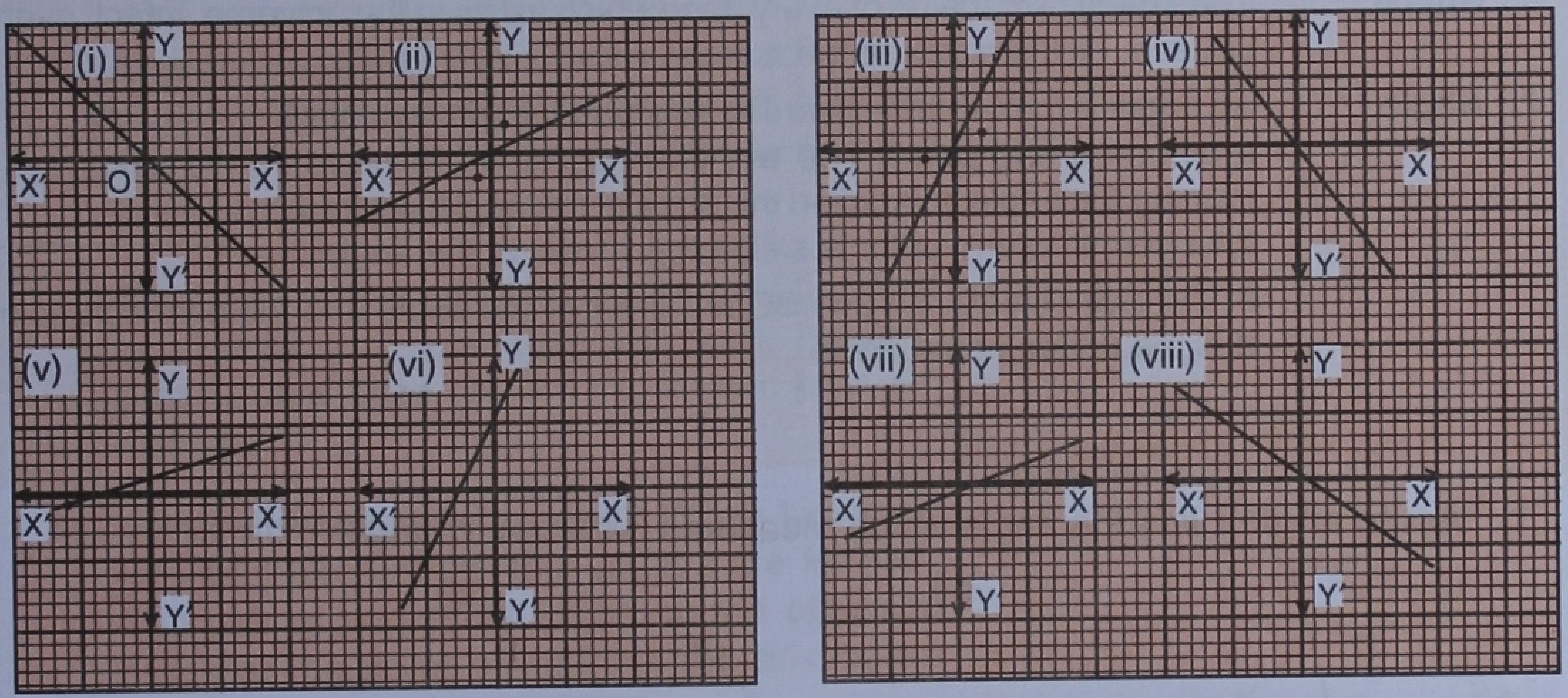
1. perpendicular ; line ; zero **2.** horizontal ; x-axis **3.** vertical ; y-axis **4.** origin **5.** (i) ture (ii) true (iii) false (iv) true (v) false (vi) true **6.** (i) (3, 5), (7, 8), (8, 3); (6, 4) (ii) (–3, 5), (–6, 2); (–2, 3) (iii) (–3, –5), (–4, – 4), (–6, –5) (iv) (3, –5), (5, –4), (5, –5), (5, –3) **7.** origin **8.** y-axis ; x-axis **9.** y-axis ; x-axis **10.** 0 ; 0

EXERCISE 22(A)

2. A = (2, 1) ; B = (–2, 2) ; C = (–3, 3) ; D = (–3, –3) ; E = (3, 0) ; F = (0, 3) ; G = (–2, 0); H = (0, –1) ; I = (3, –2) ; J = (2, –3) **4.** (i) Parallelogram (ii) Rectangle (iii) Square (iv) Isosceles triangle **5.** D = (5, –1) **6.** C = (–2, –3) **7.** Rectangle ; 35 sq. units **8.** Isosceles triangle; 10 sq. units **9.** A = (4, 0), B = (4, 5) and C = (0, 5) **10.** A = (6, 0), B = (6, 6) and C = (0, 6)

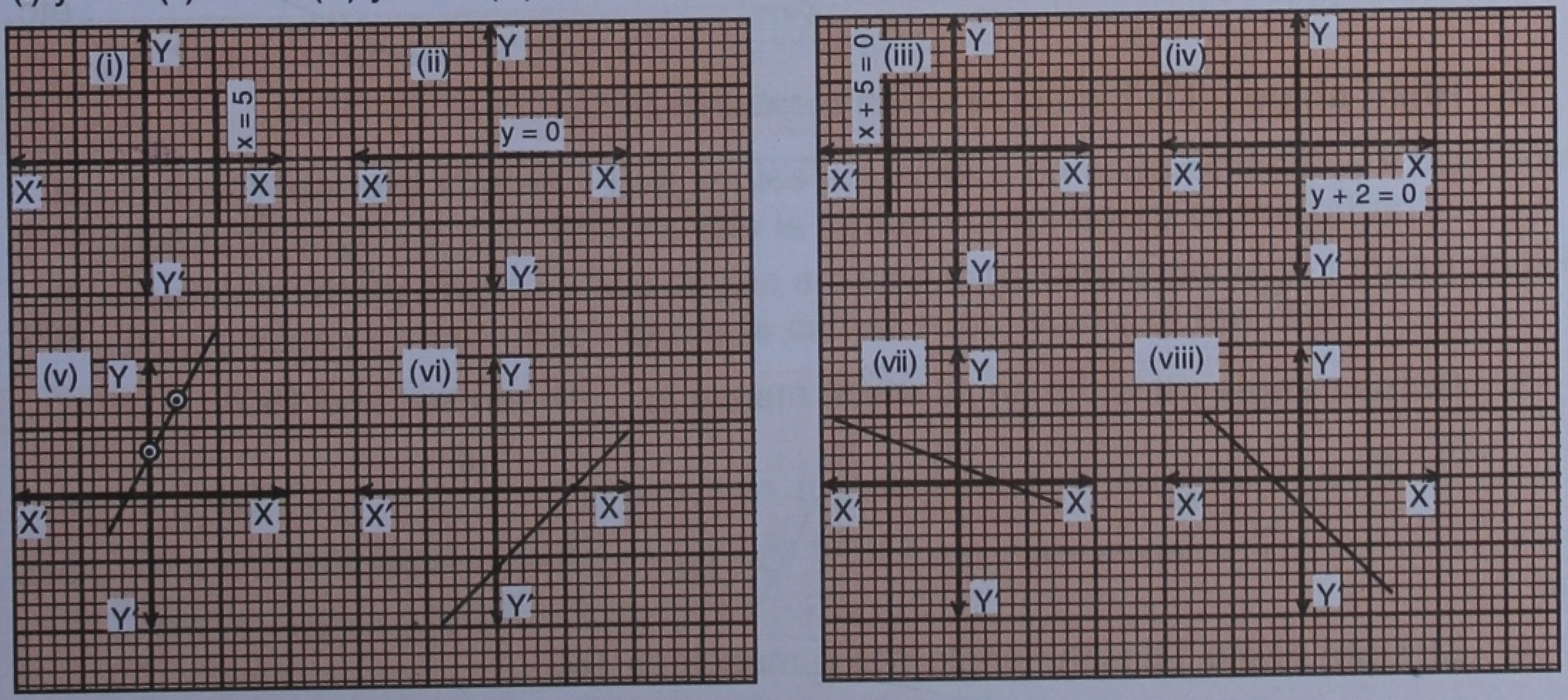
EXERCISE 22(B)

1.



2. (i) y = 0 (ii) x = 0 (iii) y-axis (iv) x-axis (v) y-axis (vi) x-axis

3.



EXERCISE 22(C)

1. (0, 2) **2.** (4, 3) **3.** (–5, 5) **4.** (–2, –3) **5.** (5, 2) **6.** (3, 2) **7.** (4, –3) **8.** (–3, 2) **9.** (3, 4) **10.** (1, –1) **11.** The lines are parallel to each other. **12.** The lines are perpendicular to each other.