CHAPTER 21

LINEAR INEQUATIONS : NUMBER LINE

21.1 INTRODUCTION

1. Equation	A *statement*, which says that *one thing is equal to another*, is called an *equation. e.g.* (i) $x = 5$ (ii) $3x = 7$ (iii) $2x - 5 = 10$, etc.
2. Inequation	A *statement*, which says that *one thing is not equal to another* (*i.e.*, either it is greater or lesser), is called an *inequation.* *e.g.* (i) $x < 7$ (read as x *is less than* 7) (ii) $x > 5$ (read as x *is greater than* 5)
3. Connecting-verbs	The symbols =, ≠, <, >, etc. are called *connecting verbs.* (i) '<' means; 'is less than' , (ii) '>' means; 'is greater than' , (iii) '≤' means; 'is less than or equal to' , (iv) '≥' means; 'is greater than or equal to'.

21.2 REPLACEMENT SET AND SOLUTION SET

For any linear inequation in x, the set from which the value(s) of variable x is chosen, is called the **replacement set** or the **universal set.**

The set of elements of the replacement set (universal set), which satisfy the given inequation, is called **the solution set** or **the truth set.**

e.g. Consider the inequation (statement) $x > 6$;

(i) if replacement set = {2, 4, 6, 8, 10}
then, the solution set = {8, 10}

(ii) if replacement set = {1, 3, 5, 7, 9, 11}
then, the solution set = {7, 9, 11}

TEST YOURSELF

1. (i) If $x \in$ N (Natural numbers) and $x < 5$; then x = ..., ..., ... or ....
   (ii) If $x \in$ W (Whole numbers) and $x < 5$; then x = ..., ..., ..., ... or ....
   (iii) If x = Z (integers) and $-2 \leq x < 3$; then x = ..., ..., ..., ... or ....
2. If the replacement set = {–4, –3, –2, –1, 0, 1, 2, 3}, write the solution set for each of the following :
   (i) $\{x : x > 1\}$ = ...
   (ii) $\{x : x < 1\}$ = ...
   (iii) $\{x : -3 < x \leq 2\}$ = ...
   (iv) $\{x : -2 \leq x < 2\}$ = ...

21.3 PROPERTIES

1. Adding the same number to each side of an inequation, does not change the sign of inequality.
   i.e. if $a > b$, then $a + c > b + c$
   and, if $a < b$, then $a + c < b + c$.

2. Subtracting the same number from each side of an inequation, does not change the sign of inequality.
   i.e. if $a > b$, then $a - c > b - c$
   and, if $a < b$, then $a - c < b - c$.
3. Multiplying each side of an inequation by a positive number, does not change the sign of inequality.
   i.e. if $a > b$ and c is positive (*i.e.* $c > 0$) then, $a.c > b.c$
   also, if $a < b$ and $c > 0$; then $a \cdot c < b \cdot c$.
4. Multiplying each side of an inequation by a negative number, reverses the sign of inequality.
   i.e. if $a > b$ and c is negative (*i.e.* $c < 0$), then $a \cdot c < b \cdot c$;
   also, if $a < b$ and $c < 0$; then $a \cdot c > b \cdot c$.
5. Dividing each side of an inequation by a positive number, does not change the sign of inequality.
   i.e. if $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$
   also, if $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.
6. Dividing each side of an inequation by a negative number, reverses the sign of inequality.
   i.e. if $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$
   also, if $a < b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.

TEST YOURSELF

3. $16 > 15 \Rightarrow 16 + 8 >$ .... $\Rightarrow$ .... > ....
4. $8 < 10 \Rightarrow 8 - 4$ .... $\Rightarrow$ ....
5. $3 < 4 \Rightarrow -5 \times 3$ .... $-5 \times 4 \Rightarrow$ .... > ....
6. $6 > -5 \Rightarrow 6 \times -4$ .... $-5 \times -4 \Rightarrow$ ....
7. $20 > -8 \Rightarrow \frac{20}{4}$ .... $\Rightarrow$ ....
8. $15 < 21 \Rightarrow \frac{15}{-3}$ .... $\Rightarrow$ ....

Example 1 :

Find the solution set of the inequation :

(i) $12 + 6x > 0$; where x is a negative integer.

(ii) $30 - 4(2x - 1) < 30$; where x is a positive integer.

Solution :

(i) $12 + 6x > 0 \Rightarrow 6x > -12$

$\Rightarrow x > -2$ [Dividing by 6]

$\because$ x is a negative integer $\therefore$ **Solution set = {– 1}** **(Ans.)**

(ii) $30 - 4(2x - 1) < 30 \Rightarrow 30 - 8x + 4 < 30$

$\Rightarrow 34 - 8x < 30$

$\Rightarrow -8x < 30 - 34$

$$\Rightarrow \quad -8x < -4$$

$$\Rightarrow \quad \frac{-8x}{-8} > \frac{-4}{-8} \qquad \text{[Dividing by −8]}$$

$$\Rightarrow \quad x > \frac{1}{2}$$

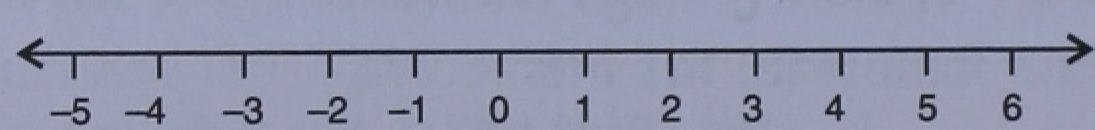
$\because$ x is a positive integer $\therefore$ **Solution set = {1, 2, 3, 4, 5, ...}** **(Ans.)**

EXERCISE 21 (A)

1. If the replacement set is the set of natural numbers, solve :
   (i) $x - 5 < 0$ (ii) $x + 1 \le 7$
   (iii) $3x - 4 > 6$ (iv) $4x + 1 \ge 17$
2. If the replacement set = {– 6, – 3, 0, 3, 6, 9}; find the truth set of the following :
   (i) $2x - 1 > 9$ (ii) $3x + 7 \le 1$
3. Solve : $7 > 3x - 8$; $x \in N$.
4. Solve : $-17 < 9y - 8$; $y \in Z$.
5. Solve : $9x - 7 \le 28 + 4x$; $x \in W$.
6. Solve : $\frac{2}{3}x + 8 < 12$; $x \in W$.
7. Solve : $-5(x + 4) > 30$; $x \in Z$.
8. Solve the inequation $8 - 2x \ge x - 5$; $x \in N$.
9. Solve the inequality $18 - 3(2x - 5) > 12$; $x \in W$.
10. Solve : $\frac{2x+1}{3} + 15 \le 17$; $x \in W$.

21.4 NUMBER LINE

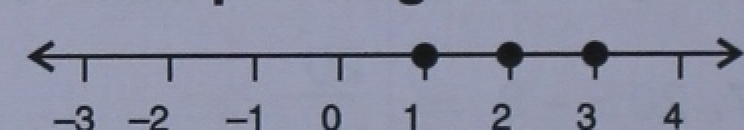
A number line is a graph (straight line) on which real numbers are marked as shown below :



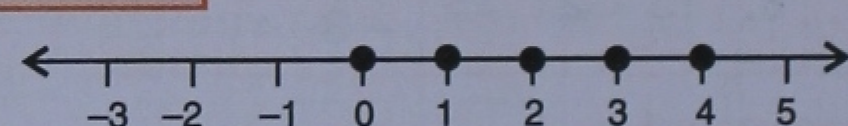
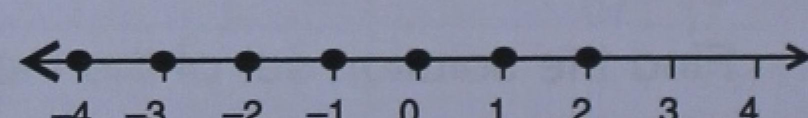
(A number line)

The solution of every inequation can be represented on a number line.

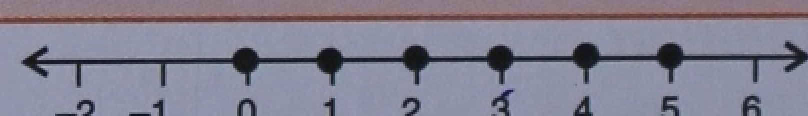
For example :

	Inequation	**Solution set**	**Corresponding number line**
1.	$x < 4$ and $x \in N$	{1, 2, 3}	

Thick dots on the number line represent the solution.

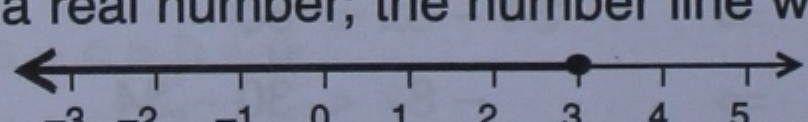
2.	$x < 5$; $x \in W$	{0, 1, 2, 3, 4}	
3.	$x < 3$; $x \in Z$	{......, –3, –2, –1, 0, 1, 2}	

The dark arrow on the left side shows that the solution set continues towards left side.

4.	$-3 \le x < 6$; $x \in W$	{0, 1, 2, 3, 4, 5}	
5.	$-3 \le x < 6$; $x \in Z$	{–3, –2, –1, 0, 1, 2, 3, 4, 5}	

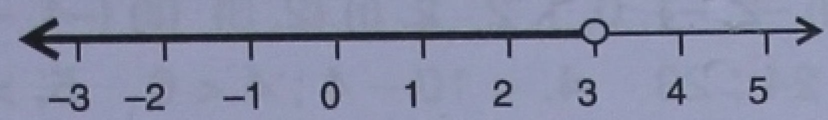
IMPORTANT

1. For $x \le 3$ where x is a real number; the number line will be as shown below :



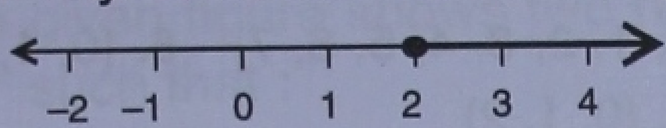
The dark circle around 3, shows 3 is included in the solution and the dark line with dark arrow on the left of number 3 shows that every number less than 3 is also included in the solution.

2. For $x < 3$ where $x \in R$; the number line will be as shown below :

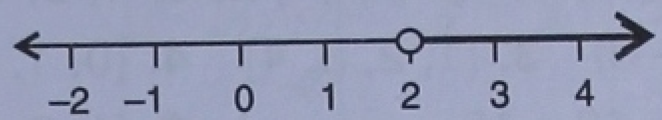


The hollow circle around 3, shows 3 is not included in the solution and the dark line with dark arrow on the left of number 3 shows that every number less than 3 is included in the solution.

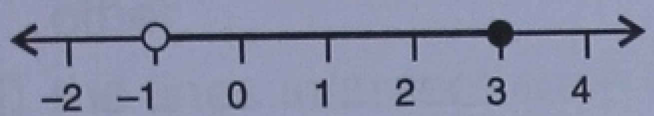
Similarly consider the following number lines :

1. 

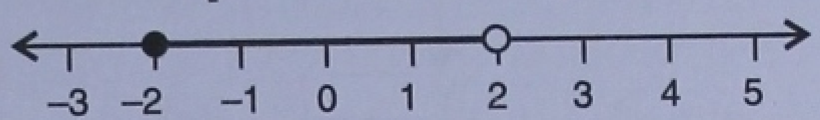
$[x \geq 2 \text{ and } x \in R]$

2. 

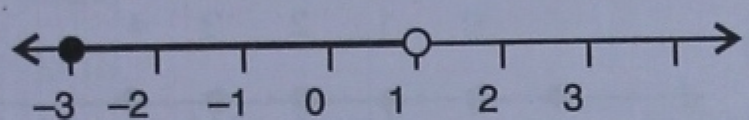
$[x > 2 \text{ and } x \in R]$

3. 

$[-1 < x \leq 3 \text{ and } x \in R]$

4. 

$[-2 \leq x < 2 \text{ and } x \in R]$

5. 

$[-3 \leq x < 1 \text{ and } x \in R]$

Example 2 :

Graph the solution set on a number line if $-2x + 14 < 6$; where x is a real number.

Solution :

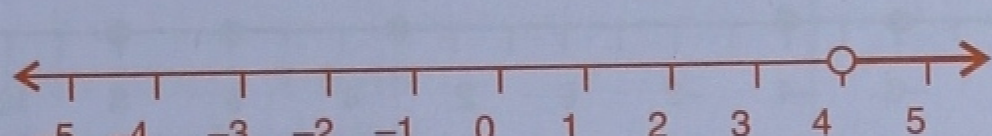
$-2x + 14 < 6 \Rightarrow -2x < 6 - 14$

$\Rightarrow -2x < -8$

$\Rightarrow \frac{-2x}{-2} > \frac{-8}{-2}$ [Division by a negative number, reverses the sign of inequality]

$\Rightarrow x > 4$ **(Ans.)**

$\therefore$ The required graph is :



EXERCISE 21 (B)

Solve and graph the solution set on a number line :

1. $x - 5 < -2;\ x \in N$
2. $3x - 1 > 5;\ x \in W$
3. $-3x + 12 < -15;\ x \in R$
4. $7 \geq 3x - 8;\ x \in W$
5. $8x - 8 \leq -24;\ x \in Z$
6. $8x - 9 \geq 35 - 3x;\ x \in N$
7. $5x + 4 > 8x - 11;\ x \in Z$
8. $\frac{2x}{5} + 1 < -3;\ x \in R$
9. $\frac{x}{2} > -1 + \frac{3x}{4};\ x \in N$
10. $\frac{2}{3}x + 5 \leq \frac{1}{2}x + 6;\ x \in W$
11. Solve the inequation $5(x - 2) > 4(x + 3) - 24$ and represent its solution on a number line. Given the replacement set is {– 4, – 3, – 2, – 1, 0, 1, 2, 3, 4}.
12. Solve $\frac{2}{3}(x - 1) + 4 < 10$ and represent its solution on a number line. Given replacement set is {– 8, – 6, – 4, 3, 6, 8, 12}.
13. For each inequation, given below, represent the solution on a number line :

(i) $\frac{5}{2} - 2x \geq \frac{1}{2},\ x \in W$

(ii) $3(2x - 1) \geq 2(2x + 3),\ x \in Z$

(iii) $2(4 - 3x) \leq 4(x - 5),\ x \in W$

(iv) $4(3x + 1) > 2(4x - 1)$, x is a negative integer

(v) $\frac{4 - x}{2} < 3,\ x \in R$

(vi) $-2(x + 8) \leq 8,\ x \in R$

ANSWERS

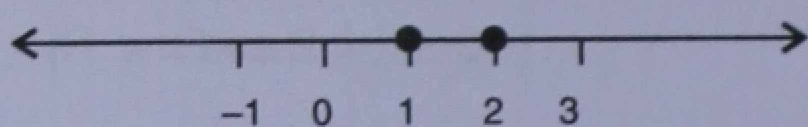
TEST YOURSELF

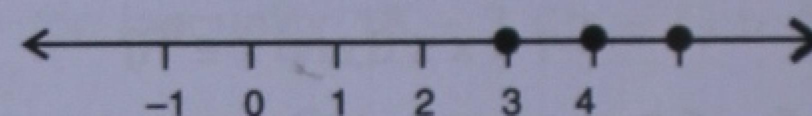
1. (i) 1, 2, 3, 4 (ii) 0, 1, 2, 3, 4 (iii) –2, –1, 0, 1, 2 **2.** (i) {2, 3} (ii) {–4, –3, –2, –1, 0} (iii) {–2, –1, 0, 1, 2} (iv) {–2, –1, 0, 1} **3.** 15 + 8 ; 24 ; 23 **4.** < 10 – 4 ; 4 < 6 **5.** > ; –15 > –16 **6.** < ; –24 < 20 **7.** > $\frac{-8}{4}$; 5 > – 2 **8.** > $\frac{21}{-3}$; –5 > –7

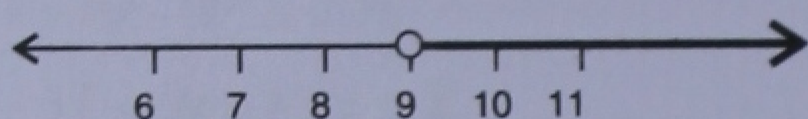
EXERCISE 21(A)

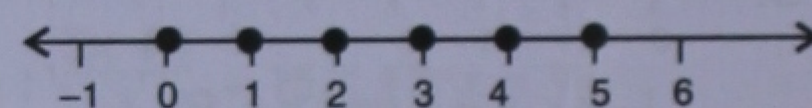
1. (i) {1, 2, 3, 4} (ii) {1, 2, 3, 4, 5, 6} (iii) {4, 5, 6, ...} (iv) {4, 5, 6, ...} **2.** (i) {6, 9} (ii) {–6, –3} **3.** {1, 2, 3, 4} **4.** {0, 1, 2, 3, ...} **5.** {0, 1, 2, 3, 4, 5, 6, 7} **6.** {0, 1, 2, 3, 4, 5} **7.** {–11, –12, –13, ...} **8.** {1, 2, 3, 4} **9.** {0, 1, 2, 3} **10.** {0, 1, 2}

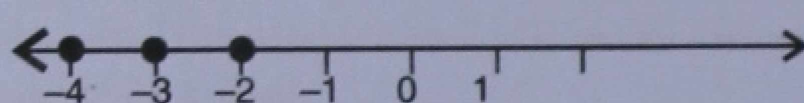
EXERCISE 21(B)

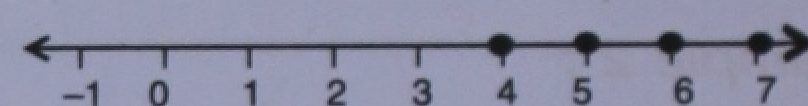
1. x < 3;


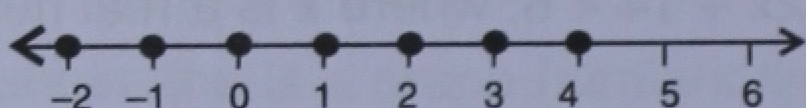
2. x > 2;


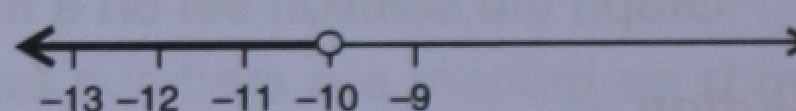
3. x > 9;


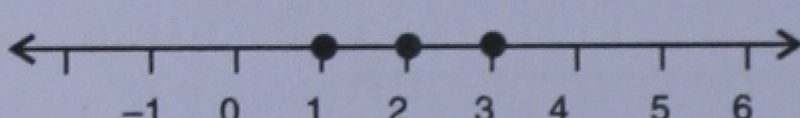
4. x ≤ 5;


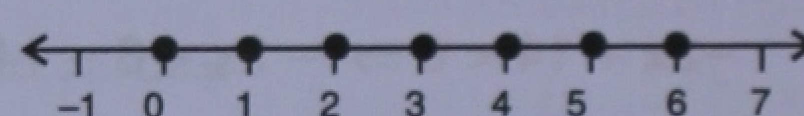
5. x ≤ – 2;


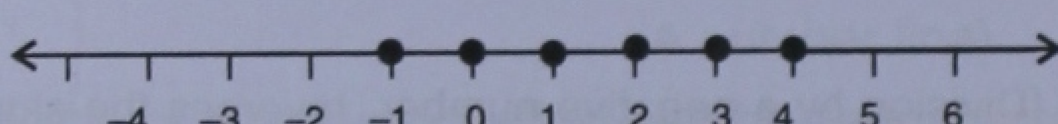
6. x ≥ 4;


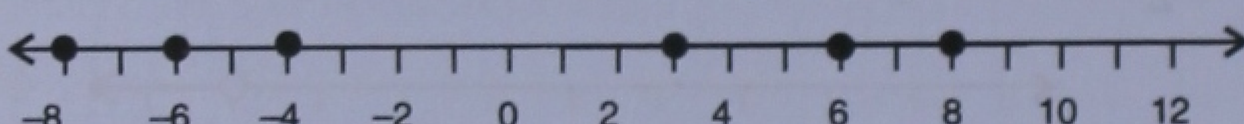
7. x < 5;


8. x < –10;


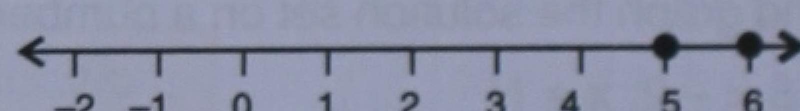
9. x < 4;


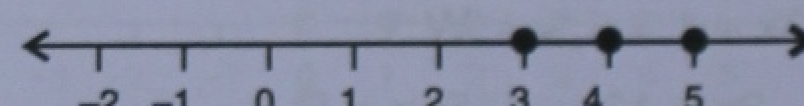
10. x ≤ 6;


11. x > – 2;


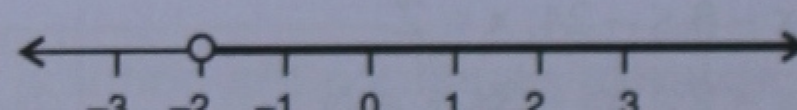
12. x < 10


13. (i) x ≤ 1, x ∈ W ⇒ Solution = {0, 1} ⇒


(ii) x ≥ $4\frac{1}{2}$, x ∈ Z ⇒ Solution = {5, 6, ...} ⇒


(iii) x ≥ 2·8, x ∈ W ⇒ Solution = {3, 4, 5, ...} ⇒


(iv) x > –1·5, x ∈ {negative integers} ⇒ Solution = {–1} ⇒


(v) x > –2, x ∈ R ⇒ Solution = {x > –2, x ∈ R} ⇒


(vi) Solution : {x ≥ –12, x ∈ R} ⇒