Chapter 19

COORDINATE SYSTEM AND GRAPHS

In previous class, you have learnt the basic terminology of coordinate geometry, plotting of points and graphs of linear equations in two variables x and y. In this chapter, we shall strengthen these concepts and introduce how to solve graphically a pair of simultaneous linear equations in two variables.

COORDINATE SYSTEM

Draw two number lines X′OX and Y′OY perpendicular to each other (horizontal and vertical) on a graph paper to intersect each other at the point O. Then

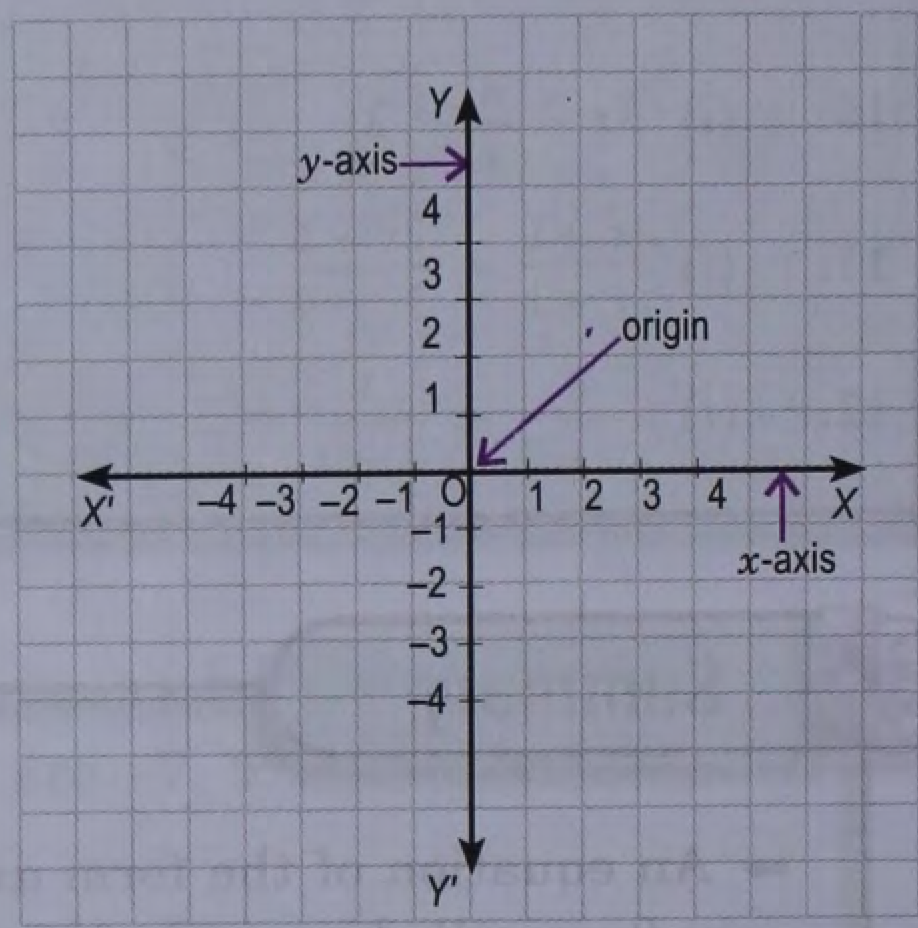
(i) the horizontal line X′OX is called ***x*-axis**.

(ii) the vertical line Y′OY is called ***y*-axis**.

(iii) X′OX and Y′OY taken together are called **coordinate axes**.

(iv) the point O is called the **origin**.

The configuration so formed is called a **coordinate system** or **coordinate plane**.



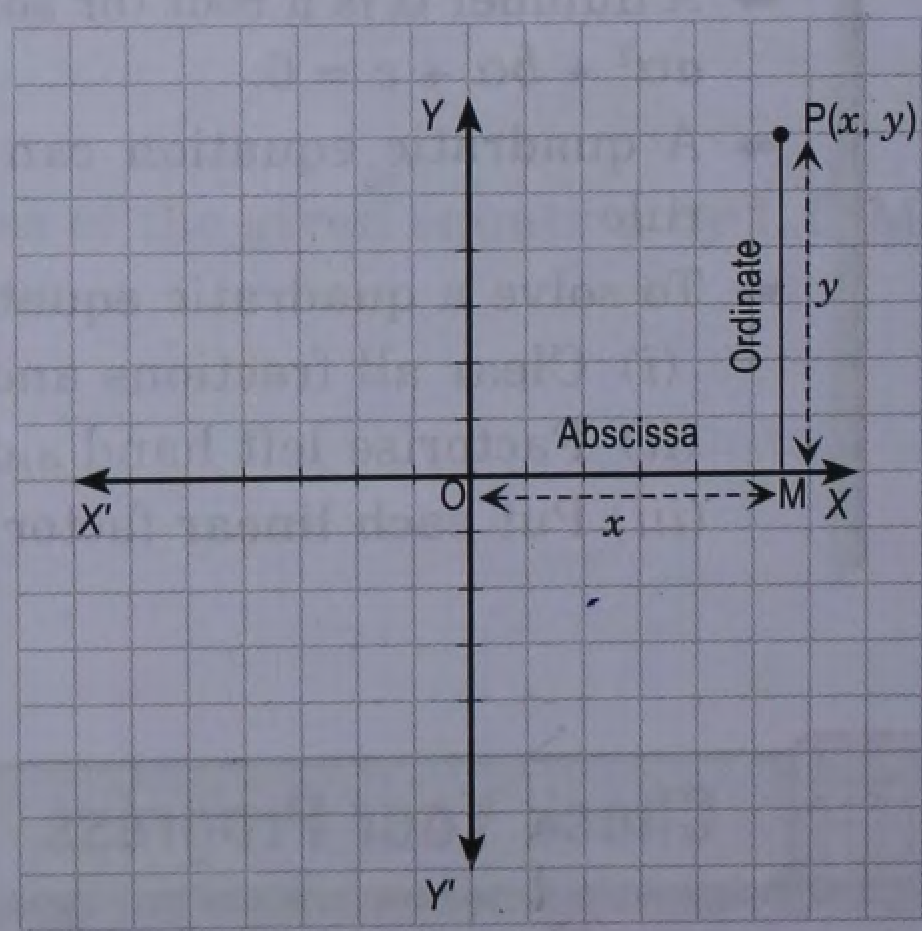
Coordinates of a point

Let P be any point in the coordinate plane. From P, draw PM perpendicular to X′OX. Then

(i) OM is called ***x*-coordinate** or **abscissa** of P and is usually denoted by x.

(ii) MP is called ***y*-coordinate** or **ordinate** of P and is usually denoted by y.

(iii) x and y taken together are called coordinates of P. It is written as (x, y) or $P(x, y)$.



Thus, corresponding to a point P in the coordinate plane, we get an ordered pair (x, y) of real numbers; conversely, corresponding to every ordered pair (x, y) of real numbers, we get a point P in the coordinate plane whose abscissa = x and ordinate = y.

For example, if the abscissa of a point P in the coordinate plane is 2 and ordinate is 3, then coordinates of P are (2, 3). So we get the ordered pair (2, 3); conversely, corresponding to the ordered pair (2, 3) we get a point in the coordinate plane whose abscissa is 2 and ordinate is 3.

Convention for signs of coordinates

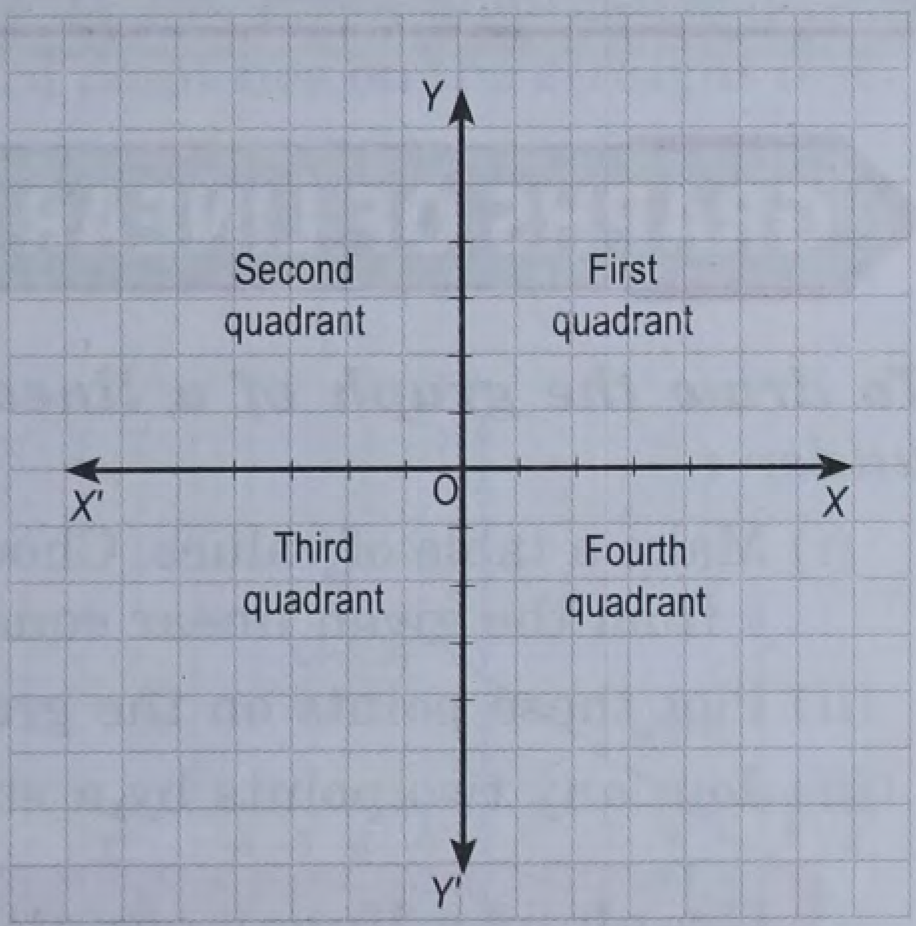
(*i*) The x-coordinate (abscissa) of a point is **positive** if it is measured to the right of origin and **negative** if it is measured to the left of origin.

(*ii*) The y-coordinate (ordinate) of a point is **positive** if it is measured above the origin and **negative** if it is measured below the origin.

Remarks

- The coordinates of the origin are (0, 0).
- For any point on x-axis, its ordinate is zero so the coordinates of any point on x-axis are $(x, 0)$. Thus, each of the points (3, 0), (−7, 0), (0, 0) lies on x-axis.
- For any point on y-axis, its abscissa is zero so the coordinates of any point on y-axis are $(0, y)$. Thus, each of the points (0, 3), (0, −7), (0, 0) lies on y-axis.

Quadrants



The two axes divide the plane into four parts called **quadrants**.

(*i*) XOY is called first quadrant.

Here both x and y are positive.

(*ii*) X′OY is called second quadrant.

Here x is negative and y is positive.

(*iii*) X′OY′ is called third quadrant.

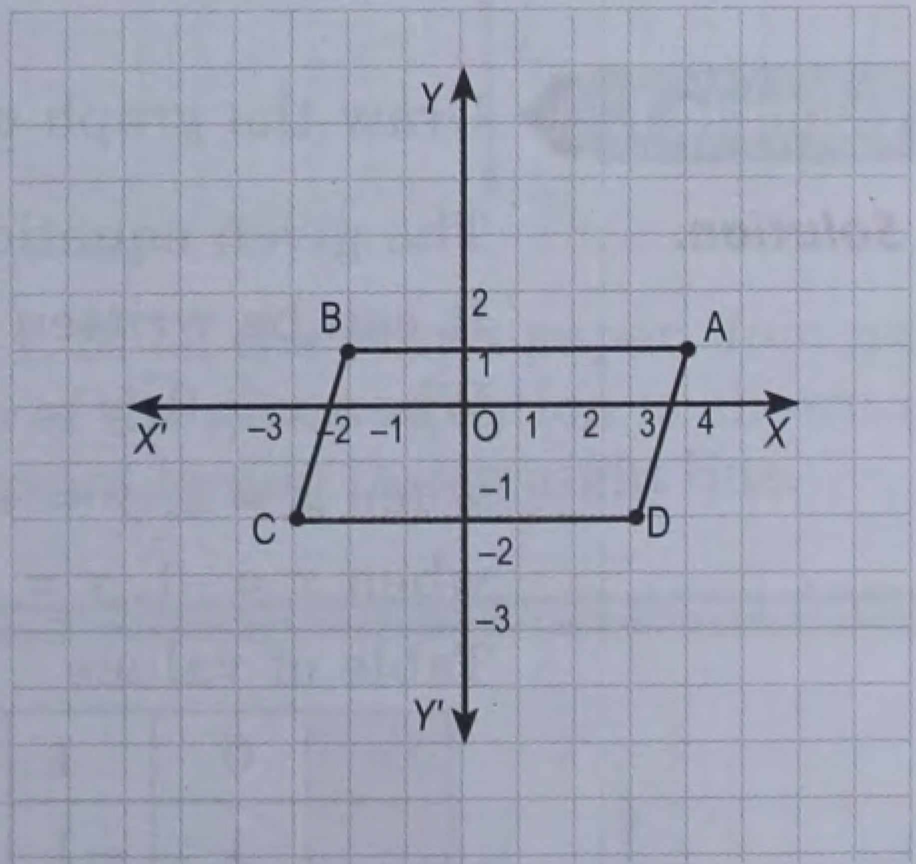
Here both x and y are negative.

(*iv*) Y′OX is called fourth quadrant.

Here x is positive and y is negative.

Example. Plot the points A (4, 1), B (−2, 1), C (− 3, −2) and D (3, −2). Name the figure ABCD. Find its area.

Solution. Points A, B, C, D are marked in the adjoining figure. It is easy to see that it is a *parallelogram*.



Area = base × height

= 6 units × 3 units

= 18 square units.

Exercise 19.1

1. State whether true or false :

(*i*) The point (4·5, 0) lies on x-axis.

(*ii*) The point (0, −4) lies on x-axis.

(*iii*) The point (0, –3·5) lies on *y*-axis.

(*iv*) If the point (*x*, *y*) lies on *x*-axis, then its abscissa is zero.

(*v*) If the point (*x*, *y*) lies on *y*-axis, then its ordinate is zero.

(*vi*) The point (*x*, *y*) lies on *y*-axis if $x = 0$.

(*vii*) The point (–3, –2) lies in the fourth quadrant.

(*viii*) The point (5, –3) lies in the second quadrant.

2. Plot the following points on the same graph paper :

(*i*) (–3, 5) (*ii*) (4, 2·5) (*iii*) (–1, –4) (*iv*) (0, –4)

(*v*) (3, – 5) (*vi*) (–4·5, 0) (*vii*) (–4, –1) (*viii*) (–2, 6)

3. Plot the points A (1, 2), B (–4, 2), C (–4, –1) and D (1, –1). What kind of quadrilateral is ABCD? Find its area.
4. Plot the points A (2, 0), B (0, 5) and C (–2, 0). What kind of triangle is ABC? Find its area.
5. Plot a rectangle which lies in first quadrant, has origin as one vertex, is 6 units long along *x*-axis and 4 units long along *y*-axis. Give the coordinates of its vertices.

GRAPHS OF LINEAR EQUATIONS

To draw the graph of a linear equation in two variables x and y, proceed as under :

(*i*) Make a table of values. Choose three values of *x* and find the corresponding values of *y* from the given linear equation. As far as possible, take the integral values of *x*.

(*ii*) Plot these points on the graph paper (coordinate plane).

(*iii*) Join any two points by a straight line and check that the third point lies on it.

Graph of a linear equation in two variables is always a straight line

Example 1. Draw the graph of the equation $2x + y - 1 = 0$.

Solution. The given equation is $2x + y - 1 = 0$

It can be written as $y = -2x + 1$

When $x = 0$, $y = -2 \times 0 + 1 = 1$;

when $x = 1$, $y = -2 \times 1 + 1 = -1$;

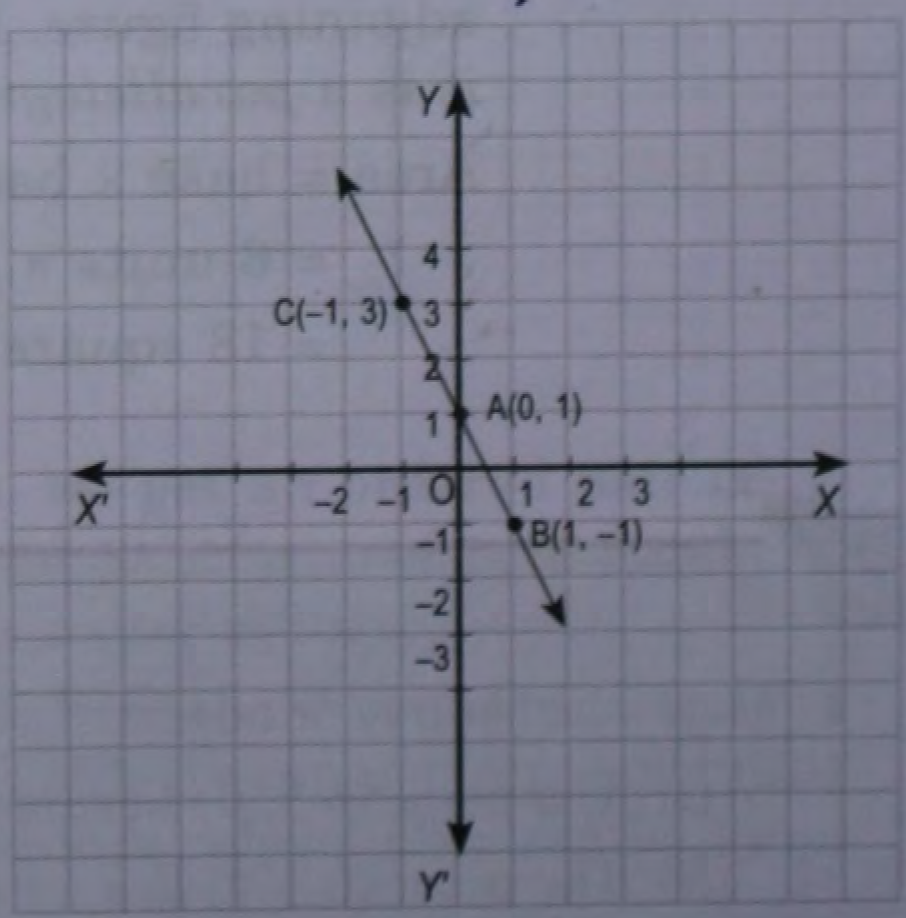
when $x = -1$, $y = -2 \times (-1) + 1 = 3$

Table of values

x	0	1	–1
y	1	–1	3

Plot the points A(0, 1), B(1, –1) and C(–1, 3) on the graph paper. Join any two points by a straight line. The graph of the given equation is shown in the adjoining figure.

Observe that the third point lies on the straight line.



Example 2. Draw the graph of the equation $3x - 2y = 5$.

Solution. The given equation is $3x - 2y = 5$

It can be written as

$$3x - 5 = 2y \text{ or } y = \frac{3x - 5}{2}$$

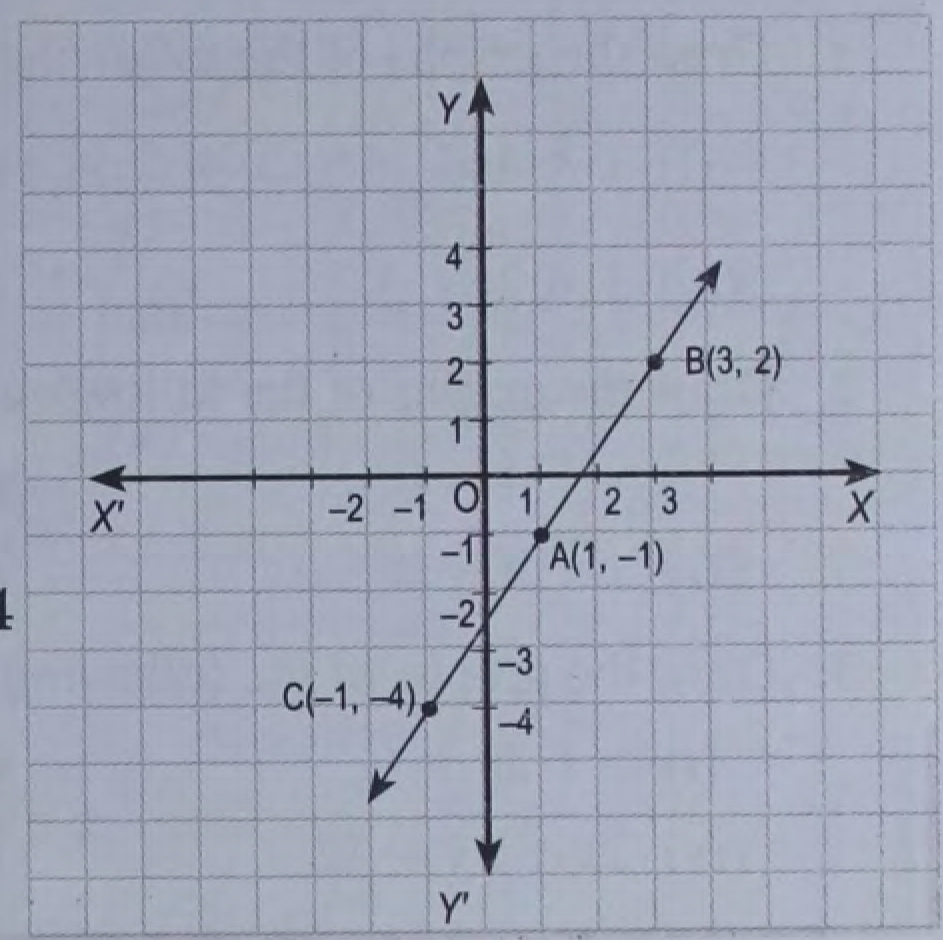
When $x = 1$, $y = \frac{3 \times 1 - 5}{2} = \frac{-2}{2} = -1$;

when $x = 3$, $y = \frac{3 \times 3 - 5}{2} = \frac{4}{2} = 2$;

when $x = -1$, $y = \frac{3 \times (-1) - 5}{2} = \frac{-8}{2} = -4$

Table of values

x	1	3	–1
y	–1	2	–4



Plot the points A(1, –1), B(3, 2) and C(–1, –4) on the graph paper. Join any two points by a straight line. The graph of the given equation is shown in the above figure. Observe that the third point lies on the straight line.

Example 3. Draw the graph of the equation $2x + 3y = 7$.

Solution. The given equation is $2x + 3y = 7$

It can be written as

$$3y = 7 - 2x \text{ or } y = \frac{7 - 2x}{3}$$

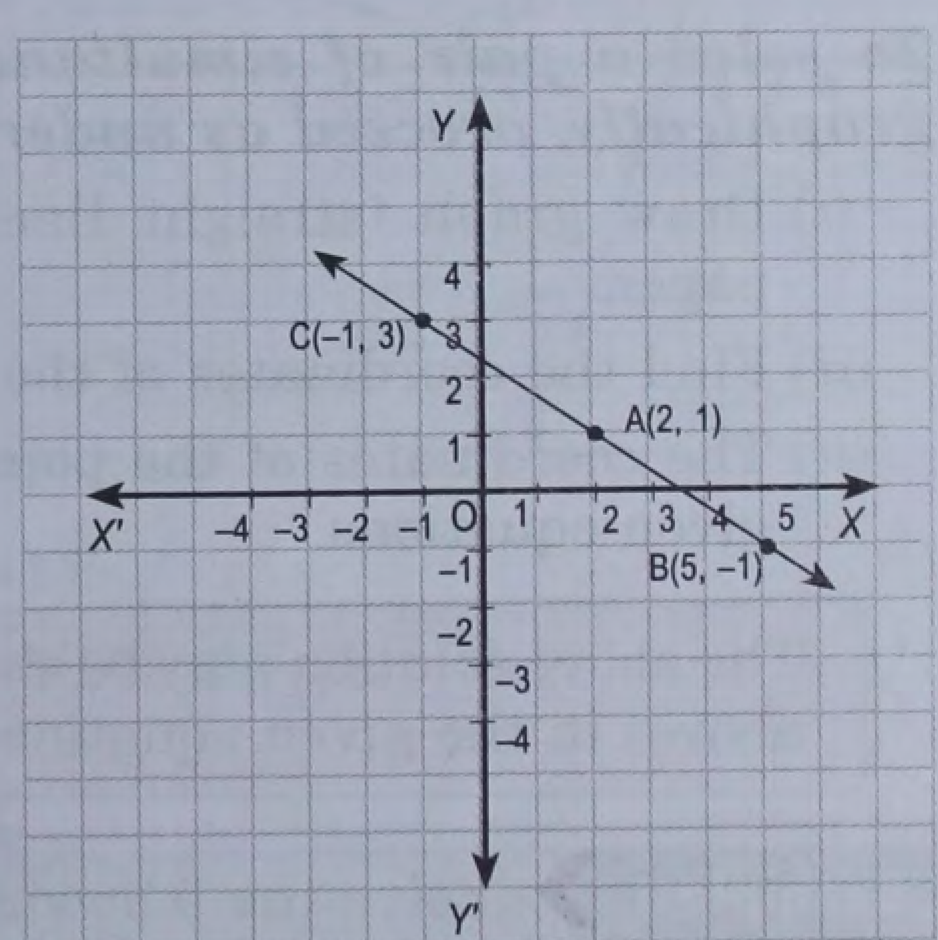
When $x = 2$, $y = \frac{7 - 2 \times 2}{3} = \frac{3}{3} = 1$;

when $x = 5$, $y = \frac{7 - 2 \times 5}{3} = \frac{-3}{3} = -1$;

when $x = -1$, $y = \frac{7 - 2 \times (-1)}{3} = \frac{9}{3} = 3$

Table of values

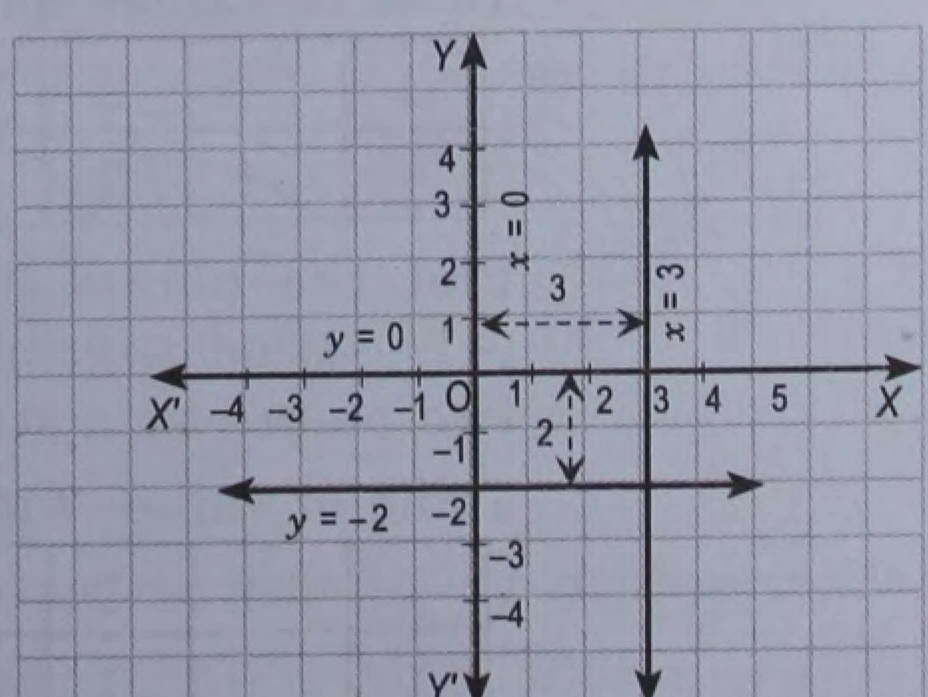
x	2	5	–1
y	1	–1	3



Plot the points A(2, 1), B(5, –1) and C(–1, 3) on the graph paper. Join any two points by a straight line. The graph of the given equation is shown in the above figure. Observe that the third point lies on the straight line.

Remarks

- The graph of the equation $y = 0$ is x-axis.
- The graph of the equation $x = 0$ is y-axis.
- The graph of the equation $y = -2$ is a straight line parallel to x-axis situated at a distance 2 units below it.
- The graph of the equation $x = 3$ is a straight line parallel to y-axis situated at a distance 3 units to the right of y-axis.



Exercise 19.2

1. Draw the graphs of the following equations :

   (i) $y = 4x$ (ii) $y = -3x$ (iii) $y = \frac{3}{2}x$

   (iv) $y = 2x - 1$ (v) $y = 3x + 1$ (vi) $y = 5 - 3x$

2. Draw the graphs of the following equations :

   (i) $y + 4 = 0$ (ii) $2y - 5 = 0$ (iii) $2y + 5 = 0$

   (iv) $x + 4 = 0$ (v) $2x - 3 = 0$ (vi) $2x + 7 = 0$

3. Draw the graphs of the following equations :

   (i) $x + 2y = 0$ (ii) $3x - 2y = 9$ (iii) $3x - 2y = 11$

   (iv) $2x + 3y = 12$ (v) $x + 2y + 1 = 0$ (vi) $2x - 5y = 2$

SOLUTION OF A PAIR OF SIMULTANEOUS LINEAR EQUATIONS GRAPHICALLY

To solve a pair of simultaneous linear equations in two variables x and y graphically, proceed as under :

(i) Draw graph (straight line) for each of the given equation on the same graph paper.

(ii) Find the coordinates of the point of intersection of the two lines drawn.

(iii) The coordinates of the point of intersection of the two lines is the solution of the given equations.

> The above solution may be checked by substituting the values of x and y (obtained above) in the given equations.

Example 1. Solve the following pair of simultaneous linear equations graphically :
$2x - y - 1 = 0$ and $x - 2y + 1 = 0$.

Solution. The given equations can be written as

$$y = 2x - 1 \quad ...(i)$$

and $$y = \frac{x+1}{2} \quad ...(ii)$$

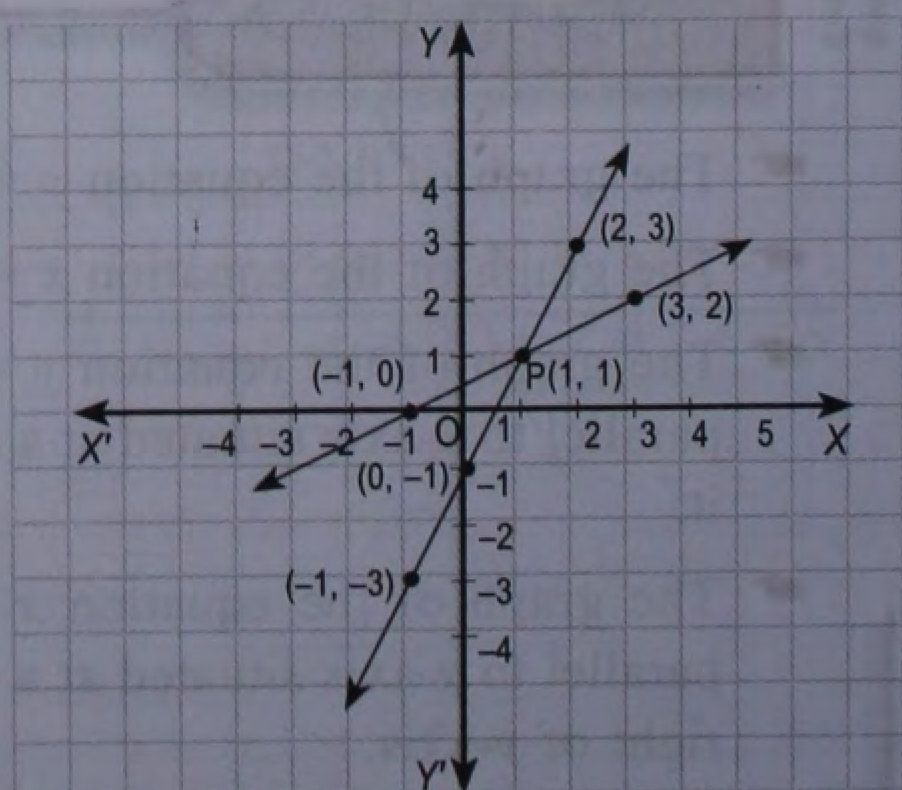
Table of values for equation (i)

x	0	2	–1
y	–1	3	–3

Plot the points (0, –1), (2, 3) and (–1, –3) on a graph paper. Join any two points by a straight line.

Table of values for equation (ii)

x	–1	1	3
y	0	1	2



Plot the points (–1, 0), (1, 1) and (3, 2) on the same graph paper. Join any two points by a straight line. The graphs of both the straight lines are shown in the above figure.

The lines intersect at the point P(1, 1).

∴ The solution of the given equations is $x = 1, y = 1$.

Example 2. Solve the following pair of simultaneous linear equations graphically :

$$3x - y = 7 \quad \text{and} \quad 2x + 5y + 1 = 0.$$

Solution. The given equations can be written as

$$y = 3x - 7 \qquad ...(i)$$

and $$y = -\frac{2x+1}{5} \qquad ...(ii)$$

Table of values for equation (*i*)

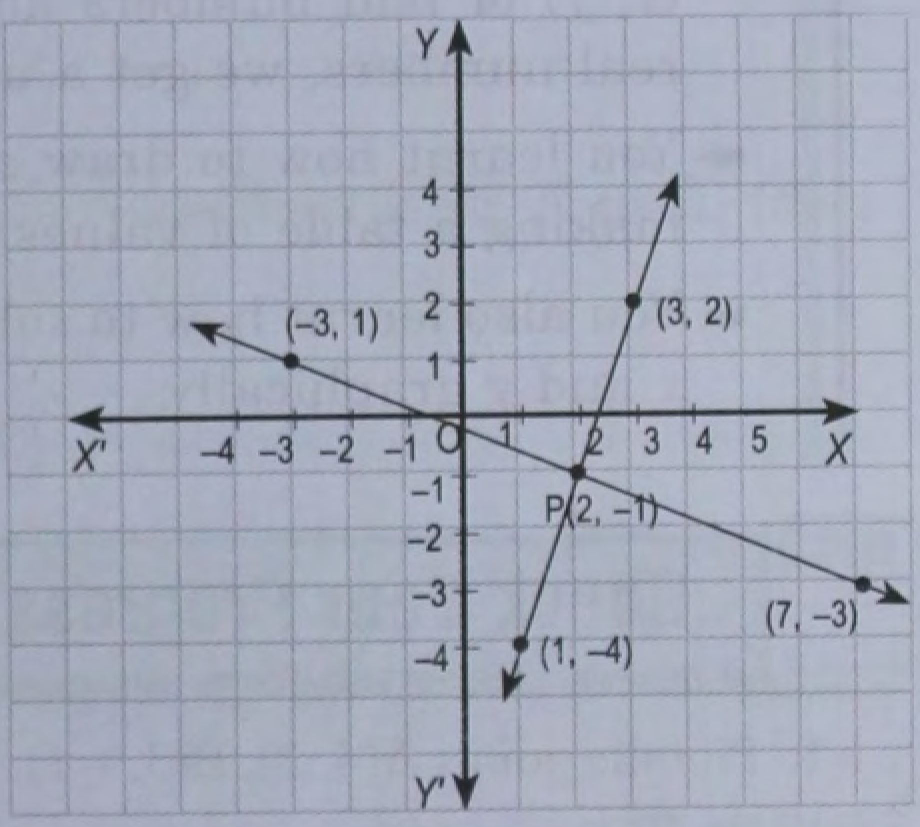
x	1	2	3
y	–4	–1	2

Plot the points (1, –4), (2, –1) and (3, 2) on a graph paper. Join any two points by a straight line.

Table of values for equation (*ii*)

x	2	7	–3
y	–1	–3	1

Plot the points (2, –1), (7, –3) and (–3, 1) on the same graph paper. Join any two points by a straight line. The graphs of both the straight lines are shown in the adjoining figure.



The lines intersect at the point P(2, –1).

∴ The solution of the given equations is $x = 2, y = -1$.

Exercise 19.3

Solve the following (1 to 8) pair of simultaneous linear equations graphically :

1. $x + y = 0$ and $x - y = 4$
2. $y = 2x - 3$ and $x + 3y = 5$
3. $y = 2x + 1$ and $x + 2y + 3 = 0$
4. $x + 3y - 4 = 0$ and $3x - y - 2 = 0$
5. $2x + y - 3 = 0$ and $3x + 2y - 4 = 0$
6. $2x - 3y + 6 = 0$ and $2x - y - 2 = 0$
7. $2x = y + 3$ and $4x + 3y = 1$
8. $x + y + 2 = 0$ and $3x - 4y = 15$
9. Draw the graphs of the linear equations $x = -2, x = 5, y = 0$ and $y = 4$ on the same graph paper. Hence find the area of the quadrilateral enclosed by these lines.

Summary

- Two number lines X′OX and Y′OY drawn horizontal and vertical respectively on a graph paper form coordinate system. The point O is called origin. The horizontal line X′OX is called x-axis and vertical line Y′OY is called y-axis. The lines X′OX and Y′OY taken together are called coordinate axes.
- From any point P in the coordinate plane, if we draw PM perpendicular to X′OX, then
  (*i*) OM $(= x)$ is called x-coordinate or abscissa of P.
  (*ii*) MP $(= y)$ is called y-coordinate or ordinate of P.
  (*iii*) Coordinates of P are written as (x, y) or $P(x, y)$.
- The x-coordinate is taken positive to the right of origin and negative to the left of origin. The y-coordinate is taken positive above the origin and negative below the origin.
- Corresponding to every point in the coordinate plane, we get a unique ordered pair (x, y) of real numbers and converely, corresponding to every ordered pair (x, y) of real numbers, we get a unique point in the coordinate plane.
- You learnt how to draw a straight line corresponding to a given linear equation by making a table of values and then plotting the points.
- You also learnt how to solve a pair of simultaneous linear equations in two variables x and y graphically.

Check Your Progress

1. Plot the points A(4, 3), B(3, – 1), C(–3, – 1) and D(–2, 3). What kind of quadrilateral is ABCD? Find its area.

2. Three vertices of a square are A(2, 3), B(–3, 3) and C(–3, –2). Plot these points on a graph paper and using these points find the coordinates of the fourth vertex. Also find the area of the square.

3. Draw the graphs of the following equations :

(*i*) $2x + 3y = 12$ (*ii*) $3x - 2y = 11$ (*iii*) $2x + 5y = 3$

4. Draw the graphs of $2x - 3y = 6$ and $2x - 3y = 3$ on the same graph paper. What do you observe?

5. Solve the following simultaneous linear equations graphically :

(*i*) $2x - 3y = 4$ and $3y - x = 1$

(*ii*) $2x - y - 3 = 0$ and $x + 2y - 14 = 0$