LINEAR EQUATIONS IN ONE VARIABLE

(With problems based on linear equations)

18.1 REVIEW

1. Equation	A statement which states that the two algebraic *expressions are equal* is called an *equation*. *e.g.* Each of the following algebraic statement is an equation. (i) $7x^2 + 8 = x - 3$, (ii) $3x - 4y = 8$, (iii) $x - 3 = 3x + 8$, etc.
2. Linear Equation	The equation involving only one variable (unknown) in first order *(i.e. with* highest power equal to one) is called a linear equation. *e.g.* $3x - 5 = 0$, $8 - y = 15$, $7 + 3z = 10$, etc.

1. To solve an equation means to find the value of its variable (*i.e.* x, y or z, etc).
2. A **linear equation** has **only one solution,** which is called its **root.**

An equation remains unaltered (unchanged) on :
- (i) *adding the same number to both sides of it;*
- (ii) *subtracting the same number from both sides of it;*
- (iii) *multiplying both sides of it by the same number;* and
- (iv) *dividing both sides of it by the same number.*

TEST YOURSELF

1. $x + 3 = 5 \Rightarrow x + 3 - 3 =$ .... $\Rightarrow x =$ ....
2. $y - 2 = 4 \Rightarrow y - 2 + 2 =$ .... $\Rightarrow y =$ ....
3. $3z = 15 \Rightarrow \frac{3z}{3} =$ .... $\Rightarrow z =$ ....
4. $\frac{x}{5} = -2 \Rightarrow \frac{x}{5} \times 5 =$ .... $\Rightarrow x =$ ....
5. $3y - 2 = 10 \Rightarrow 3y =$ .... $\Rightarrow y =$ ....
6. $\frac{m}{5} + 7 = 9 \Rightarrow \frac{m}{5} =$ .... $\Rightarrow m =$ ....
7. $4x - 13 = 7 - x \Rightarrow 4x + x =$ .... $\Rightarrow 5x =$ .... and $x =$ ....
8. $2(7z - 3) = 3(4z - 2) \Rightarrow$ .... = ...

   $\Rightarrow 14z - 12z =$ .... $\Rightarrow 2z =$ .... and $z =$ ....

Example 1 :

Solve : $21 - 3(a - 7) = a + 20$

Solution :

$$21 - 3a + 21 = a + 20$$

$$\Rightarrow 42 - 20 = a + 3a \Rightarrow 22 = 4a \quad \therefore a = \frac{22}{4} = 5\frac{1}{2}$$ **(Ans.)**

Example 2 :

Solve : $\frac{y+2}{4} - \frac{y-3}{3} = \frac{1}{2}$

Solution :

Since, L.C.M. of denominators 4, 3 and 2 = 12

$$\therefore \quad 12 \times \frac{y+2}{4} - 12 \times \frac{y-3}{3} = 12 \times \frac{1}{2} \quad \text{[Multiplying each term by 12]}$$

$$\Rightarrow \quad 3(y+2) - 4(y-3) = 6$$

$$\Rightarrow \quad 3y + 6 - 4y + 12 = 6$$

$$\Rightarrow \quad -y = -12 \Rightarrow y = 12 \quad \text{(Ans.)}$$

Example 3 :

Solve : (i) $\frac{5}{x} = \frac{7}{x-4}$ (ii) $\frac{a-2}{a+4} = \frac{a-3}{a+1}$

Solution :

On cross-multiplying; we get :

(i)

$$7x = 5(x-4)$$

$$\Rightarrow \quad 7x = 5x - 20$$

$$\Rightarrow \quad 7x - 5x = -20$$

$$\Rightarrow \quad 2x = -20$$

$$\Rightarrow \quad x = -10 \quad \text{(Ans.)}$$

(ii)

$$(a-2)(a+1) = (a-3)(a+4)$$

$$\Rightarrow \quad a^2 - 2a + a - 2 = a^2 - 3a + 4a - 12$$

$$\Rightarrow \quad a^2 - a - 2 = a^2 + a - 12$$

$$\Rightarrow \quad a^2 - a - a^2 - a = -12 + 2$$

$$\Rightarrow \quad -2a = -10$$

$$\Rightarrow \quad a = 5 \quad \text{(Ans.)}$$

Example 4 :

Solve : $\frac{2x+1}{10} - \frac{3-2x}{15} = \frac{x-2}{6}$. Hence, find the value of y, if $\frac{1}{x} + \frac{1}{y} = 3$.

Solution :

Since, L.C.M. of denominators 10, 15 and 6 = 30, multiply each fraction by 30 to get :

$$30 \times \frac{2x+1}{10} - 30 \times \frac{3-2x}{15} = 30 \times \frac{x-2}{6}$$

$$\Rightarrow \quad 3(2x+1) - 2(3-2x) = 5(x-2)$$

$$\Rightarrow \quad 6x + 3 - 6 + 4x = 5x - 10$$

$$\Rightarrow \quad 10x - 5x = -10 + 3$$

$$\Rightarrow \quad 5x = -7 \text{ and } x = -\frac{7}{5} \quad \text{(Ans.)}$$

Now, $\frac{1}{x} + \frac{1}{y} = 3 \Rightarrow -\frac{5}{7} + \frac{1}{y} = 3$ $\quad \left[x = -\frac{7}{5} \Rightarrow \frac{1}{x} = -\frac{5}{7}\right]$

$$\Rightarrow \quad \frac{1}{y} = 3 + \frac{5}{7} = \frac{26}{7}$$

$$\therefore \quad y = \frac{7}{26} \quad \text{(Ans.)}$$

EXERCISE 18 (A)

Solve the following equations :

1. $20 = 6 + 2x$
2. $15 + x = 5x + 3$
3. $\frac{3x+2}{x-6} = -7$
4. $3a - 4 = 2(4 - a)$
5. $3(b - 4) = 2(4 - b)$
6. $\frac{x+2}{9} = \frac{x+4}{11}$
7. $\frac{x-8}{5} = \frac{x-12}{9}$
8. $5(8x + 3) = 9(4x + 7)$

9. $3(x + 1) = 12 + 4(x - 1)$

10. $\frac{3x}{4} - \frac{1}{4}(x - 20) = \frac{x}{4} + 32$

11. $3a - \frac{1}{5} = \frac{a}{5} + 5\frac{2}{5}$

12. $\frac{x}{3} - 2\frac{1}{2} = \frac{4x}{9} - \frac{2x}{3}$

13. $\frac{4(y+2)}{5} = 7 + \frac{5y}{13}$

14. $\frac{a+5}{6} - \frac{a+1}{9} = \frac{a+3}{4}$

15. $\frac{2x-13}{5} - \frac{x-3}{11} = \frac{x-9}{5} + 1$

16. $6(6x - 5) - 5(7x - 8) = 12(4 - x) + 1$

17. $(x - 5)(x + 3) = (x - 7)(x + 4)$

18. $(x - 5)^2 - (x + 2)^2 = -2$

19. $(x - 1)(x + 6) - (x - 2)(x - 3) = 3$

20. $\frac{3x}{x+6} - \frac{x}{x+5} = 2$

21. $\frac{1}{x-1} + \frac{2}{x-2} = \frac{3}{x-3}$

22. $\frac{x-1}{7x-14} = \frac{x-3}{7x-26}$

23. $\frac{1}{x-1} - \frac{1}{x} = \frac{1}{x+3} - \frac{1}{x+4}$

24. Solve : $\frac{2x}{3} - \frac{x-1}{6} + \frac{7x-1}{4} = 2\frac{1}{6}$.

Hence, find the value of 'a', if $\frac{1}{a} + 5x = 8$.

25. Solve : $\frac{4-3x}{5} + \frac{7-x}{3} + 4\frac{1}{3} = 0$.

Hence, find the value of 'p', if $3p - 2x + 1 = 0$.

18.2 TO SOLVE PROBLEMS BASED ON LINEAR EQUATIONS

Steps :

1. Read the problem carefully to know what is given and what is to be found.
2. Represent the unknown quantity as x or some other letter such as a, b, y, z, etc.
3. According to the conditions, given in the problem, write the relation between the given quantity and quantity to be found.
4. Solve the equation to obtain the value of the unknown.

Example 5 :

Find a number such that one-fifth of it is less than its one-fourth by 3.

Solution :

Let the required number be x.

Since, one-fifth of x = $\frac{x}{5}$ and one-fourth of it = $\frac{x}{4}$; then according to the given statement :

$$\frac{x}{4} - \frac{x}{5} = 3 \Rightarrow \frac{5x - 4x}{20} = 3 \qquad \text{[L.C.M. of 4 and 5 = 20]}$$

$$\Rightarrow \quad x = 3 \times 20 = 60 \qquad \textbf{(Ans.)}$$

Example 6 :

The difference of the squares of two consecutive even natural numbers is 92. Taking x as the smaller of the two numbers, form an equation in x and hence find the larger of the two.

Solution :

Since, the consecutive even natural numbers differ by 2 and it is given that the smaller of the two numbers is x; therefore, the next (larger) even number is x + 2.

According to the given statement :

$$(x + 2)^2 - x^2 = 92 \qquad \text{[Difference of the squares]}$$

$$\Rightarrow \quad x^2 + 4x + 4 - x^2 = 92$$

$\Rightarrow$ $4x = 92 - 4 = 88$

$\Rightarrow$ $x = 22$

$\therefore$ **Larger even number** $= x + 2 = 22 + 2 = 24$ **(Ans.)**

In case of integers, natural numbers and whole numbers :

1. Consecutive numbers are taken as : x + 1, x + 2, ...
2. Consecutive even numbers are taken as : x, x + 2, x + 4, ...; where x is an ....
3. Consecutive odd numbers are also taken as : x, x + 2, x + 4, ...; where x is an ....
4. Consecutive multiples of 3 are taken as : x, x + 3, x + 6, ...; ....

Example 7 :

A rectangle is 8 cm long and 5 cm wide. Its perimeter is doubled when each of its sides is increased by x cm. Form an equation in x and find the new length of the rectangle.

Solution :

Since, length of the rectangle = 8 cm and its width = 5 cm

$\therefore$ Its perimeter = 2(length + width)

= 2(8 + 5)cm = 26 cm

On increasing each of its sides by x cm,

its new length = (8 + x) cm

and, new width = (5 + x) cm

$\therefore$ Its new perimeter = 2(8 + x + 5 + x) cm

= (26 + 4x) cm

Given : new perimeter = 2 times the original perimeter

$\Rightarrow$ $26 + 4x = 2 \times 26$

$\Rightarrow$ $4x = 52 - 26 = 26$

$\Rightarrow$ $x = \frac{26}{4} = 6.5$ cm

And, **the new length of the rectangle** = (8 + x) cm

= (8 + 6.5) cm = **14.5 cm.** **(Ans.)**

Example 8 :

A man is 24 years older than his son. In 2 years, his age will be twice the age of his son. Find their present ages.

Solution :

Let the present age of the son be x years

$\therefore$ Present age of the father = (x + 24) years

In 2 years :

The man's age will be (x + 24) + 2 = (x + 26) years

and son's age will be x + 2 years

According to the problem : $x + 26 = 2(x + 2)$

On solving we get : $x = 22$

$\therefore$ **Present age of the man** = x + 24 = 22 + 24 = **46 years**

and, **Present age of the son** = x = **22 years.** **(Ans.)**

Example 9 :

One day a boy walked from his house to his school at the speed of 4 km/hr and he reached ten minutes late to the school. Next day, he ran at the speed of 8 km/hr and was 5 minutes early to the school. Find the distance between his house and school.

Solution :

Let the distance between his house and school be x km.

Since, time $= \frac{\text{distance}}{\text{speed}}$

∴ To reach the school, first day he takes $\frac{x}{4}$ hrs and next day he takes $\frac{x}{8}$ hrs .

Since, the difference of two timings = 10 minutes + 5 minutes = 15 minutes = $\frac{1}{4}$ hrs

$$\therefore \quad \frac{x}{4} - \frac{x}{8} = \frac{1}{4}$$

On solving, we get : **x = 2 km.** **(Ans.)**

Example 10 :

Two consecutive even numbers are such that half of the larger exceeds one-fourth of the smaller by 5. Find the numbers.

Solution :

Let the required even numbers be x and x + 2.

Given : $\frac{1}{2}(x+2) - \frac{1}{4}x = 5$

$\Rightarrow \quad \frac{2x+4-x}{4} = 5$

$\Rightarrow \quad x + 4 = 20$ *i.e.* $x = 20 - 4 = 16$

∴ **Required numbers** = x and x + 2

= 16 and 16 + 2 = **16 and 18** **(Ans.)**

Example 11 :

A person is paid ₹ 150 for each day he works and is fined ₹ 30 for each day he remains absent. If in 40 days, he earned ₹ 3,300; find for how many days did he work ?

Solution :

Let the man works for x days

∴ He remains absent for (40 – x) days.

Since, the man gets ₹ 150 for each day he worked and is fined ₹ 30 for each day he remains absent.

$\therefore \quad 150x - 30(40 - x) = 3{,}300$

$\Rightarrow \quad 150x - 1200 + 30x = 3{,}300$

$\Rightarrow \quad 180x = 3{,}300 + 1{,}200 = 4{,}500$

and, $\quad x = \frac{4{,}500}{180} = 25$

∴ **The man worked for 25 days** **(Ans.)**

TEST YOURSELF

9. Two numbers differ by 5; if one number is x; the other number = ... or ....
10. (i) Consecutive multiples of 8, differ by ....
    (ii) Consecutive multiple of 15, differ by ....
11. If x years and (2x – 7) years are the present ages of A and B respectively, their ages :
    (a) **6 years ago** were, A = ... and B = ... = ...
    (b) **10 years hence**, will be, A = ... and B = ... = ...

EXERCISE 18 (B)

1. Fifteen less than 4 times a number is 9. Find the number.
2. If Megha's age is increased by three times her age, the result is 60 years. Find her age.
3. 28 is 12 less than 4 times a number. Find the number.
4. Five less than 3 times a number is – 20. Find the number.
5. Fifteen more than 3 times Neetu's age is the same as 4 times her age. How old is she ?
6. A number decreased by 30 is the same as 14 decreased by 3 times the number. Find the number.
7. A's salary is same as 4 times B's salary. If together they earn ₹ 3,750 a month, find the salary of each.
8. Separate 178 into two parts so that the first part is 8 less than twice the second part.
9. Six more than one-fourth of a number is two-fifth of the number. Find the number.
10. The length of a rectangle is twice its width. If its perimeter is 54 cm, find its length.
11. A rectangle's length is 5 cm less than twice its width. If the length is decreased by 5 cm and width is increased by 2 cm; the perimeter of the resulting rectangle will be 74 cm. Find the length and the width of the original rectangle.
12. The sum of three consecutive odd numbers is 57. Find the numbers.
13. A man's age is three times that of his son and in twelve years he will be twice as old as his son would be. What are their present ages ?
14. A man is 42 years old and his son is 12 years old. In how many years will the age of the son be half the age of the man at that time ?
15. A man completed a trip of 136 km in 8 hours. Some parts of the trip was covered at 15 km/hr and the remaining at 18 km/hr. Find the part of the trip covered at 18 km/hr.
16. The difference of two numbers is 3 and the difference of their square is 69. Find the numbers.
17. Two consecutive natural numbers are such that one-fourth of the smaller exceeds one-fifth of the greater by 1. Find the numbers.
18. Three consecutive whole numbers are such that if they be divided by 5, 3 and 4 respectively; the sum of the quotients is 40. Find the numbers.
19. If the same number be added to the numbers 5, 11, 15 and 31, the resulting numbers are in proportion. Find the number.

> a, b, c and d are in proportion $\Rightarrow \frac{a}{b} = \frac{c}{d}$.

20. The present age of a man is twice that of his son. Eight years hence, their ages will be in the ratio 7 : 4. Find their present ages.

ANSWERS

TEST YOURSELF

1. 5 – 3; 2 **2.** 4 + 2 ; 6 **3.** $\frac{15}{3}$; 5 **4.** –2 × 5 ; – 10 **5.** 12 ; 4 **6.** 2 ; 10 **7.** 7 + 13 ; 20 ; 4 **8.** 14z – 6; 12z – 6; 6 – 6 ; 0 ; 0 **9.** x + 5 or x – 5 **10.** (i) 8 (ii) 15 **11.** (a) (x – 6) years ; (2x – 7 – 6) years; (2x – 13) years ; (b) (x + 10) years ; (2x – 7 + 10) years ; (2x + 3) years

EXERCISE 18(A)

1. 7 **2.** 3 **3.** 4 **4.** 2·4 **5.** 4 **6.** 7 **7.** 3 **8.** 12 **9.** – 5 **10.** 108 **11.** 2 **12.** 4·5 **13.** 13 **14.** $-\frac{1}{7}$ **15.** 14 **16.** 3 **17.** – 13 **18.** $x = 1\frac{9}{14}$ **19.** $1\frac{1}{2}$ **20.** $-4\frac{8}{13}$ **21.** $1\frac{1}{2}$ **22.** 8 **23.** $-1\frac{1}{2}$ **24.** x = 1 and $a = \frac{1}{3}$ **25.** x = 8 and p = 5

EXERCISE 18(B)

1. 6 **2.** 15 years **3.** 10 **4.** –5 **5.** 15 years **6.** 11 **7.** A's = ₹ 3000 ; B's = ₹ 750 **8.** 116 and 62 **9.** 40 **10.** 18 cm **11.** 25 cm and 15 cm **12.** 17, 19 and 21 **13.** 36 years and 12 years **14.** 18 years **15.** 96 km **16.** 13 and 10 **17.** 24 and 25 **18.** 50, 51 and 52 **19.** 1 **20.** 48 years and 24 years