

## Chapter 17

# SIMULTANEOUS LINEAR EQUATIONS

## LINEAR EQUATION IN TWO VARIABLES

An equation of the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are real numbers and  $a$  and  $b$  both are non-zero, is called a **linear equation** in two variables  $x$  and  $y$ .

**For example :**

$x + y - 5 = 0$  is a linear equation in the two variables (unknown)  $x$  and  $y$ . Note that  $x = 2$  and  $y = 3$  satisfies this linear equation.

## SIMULTANEOUS LINEAR EQUATIONS

Let us consider two linear equations in two variables.

$$x + y - 5 = 0$$

$$2x - y - 1 = 0$$

These two equations are said to form a **pair of simultaneous linear equations** in two variables  $x$  and  $y$ .

A **solution** to a pair of simultaneous linear equations in two variables is an ordered pair of real numbers which satisfy both the equations.

In the above example,  $x = 2$ ,  $y = 3$  is a solution to the pair of simultaneous linear equations. We can check this by substituting  $x = 2$ ,  $y = 3$  in each of these two equations.

The process of finding an ordered pair of real numbers which satisfy both the equations is called **solving a pair of simultaneous equations**.

There are two methods to solve a pair of simultaneous linear equations:

- (i) Substitution method      (ii) Addition-subtraction (or Elimination) method.

### Substitution Method

**Procedure to solve a pair of simultaneous linear equations in two variables :**

- (i) Solve one of the given equations for one of the variables.
- (ii) Substitute that value of the variable in the other equation.
- (iii) Solve the resulting single variable equation and substitute this value into either of the two original equations and solve it to find the value of the other variable.



### Remark

The solution may be checked by substituting in both the original equations.

**Example 1.** Solve the following pair of simultaneous linear equations :

$$2x + 7y = 30, \quad x - 3y = 2.$$

**Solution.**

Given equations are :

$$2x + 7y = 30 \quad \dots(i)$$

$$x - 3y = 2 \quad \dots(ii)$$

We can use any equation to express one variable in terms of another. To avoid fractions, we can express  $x$  in terms of  $y$  by using equation (ii).

$$x - 3y = 2 \quad \Rightarrow \quad x = 3y + 2$$

Substituting this value of  $x$  in equation (i), we get

$$2(3y + 2) + 7y = 30$$

$$\Rightarrow \quad 6y + 4 + 7y = 30$$

$$\Rightarrow \quad 13y = 30 - 4 \Rightarrow 13y = 26 \Rightarrow y = 2.$$

Substituting this value of  $y$  in equation (ii), we get

$$x - 3 \times 2 = 2 \quad \Rightarrow \quad x - 6 = 2 \quad \Rightarrow \quad x = 8.$$

Hence, the solution is  $x = 8, y = 2$ .

**Example 2.** Solve  $4x + 3y = 14, 9x - 5y = 55$  by substitution method.

**Solution.**

Given equations are :

$$4x + 3y = 14 \quad \dots(i)$$

$$9x - 5y = 55 \quad \dots(ii)$$

Using equation (i), we get

$$4x + 3y = 14 \quad \Rightarrow \quad x = \frac{14 - 3y}{4}.$$

Substituting this value of  $x$  in equation (ii), we get

$$9 \times \frac{14 - 3y}{4} - 5y = 55$$

$$\Rightarrow \quad 9(14 - 3y) - 20y = 220 \quad \text{(Multiplying by 4)}$$

$$\Rightarrow \quad 126 - 27y - 20y = 220$$

$$\Rightarrow \quad -47y = 220 - 126 \quad \Rightarrow \quad -47y = 94$$

$$\Rightarrow \quad y = -2.$$

Substituting this value of  $y$  in equation (i), we get

$$4x + 3 \times (-2) = 14 \quad \Rightarrow \quad 4x - 6 = 14$$

$$\Rightarrow \quad 4x = 20 \quad \Rightarrow \quad x = 5$$

Hence, the solution is  $x = 5, y = -2$ .



### Remark

An alternative method of solving a pair of simultaneous linear equations that is perhaps superior to substitution for systems such as

$$4x + 3y = 14, 9x - 5y = 55$$

is known as **addition-subtraction method** or **elimination method**. In example 2, we noticed that solving these equations by using substitution method involves fractions regardless of our choice of variable and choice of equation. The addition subtraction method enables us to avoid fractions.

## Addition–Subtraction Method

### Procedure :

- \* Multiply one or both equations (if necessary) to transform them so that addition or subtraction will eliminate one variable.
- \* Solve the resulting single variable equation and substitute this value into either of the two original equations, and solve to find the value of the second variable.

**Example 3.** Solve  $4x + 3y = 14$ ,  $9x - 5y = 55$  by addition-subtraction method.

#### Solution.

Given equations are :

$$4x + 3y = 14 \quad \dots(i)$$

$$9x - 5y = 55 \quad \dots(ii)$$

To make coefficients of  $y$  equal (numerically), multiplying equation (i) by 5 and equation (ii) by 3, we get

$$20x + 15y = 70 \quad \dots(iii)$$

$$27x - 15y = 165 \quad \dots(iv)$$

Adding (iii) and (iv), we get

$$47x = 235 \quad \Rightarrow \quad x = 5.$$

Substituting  $x = 5$  in equation (i), we get

$$4 \times 5 + 3y = 14 \quad \Rightarrow \quad 20 + 3y = 14$$

$$\Rightarrow \quad 3y = -6 \quad \Rightarrow \quad y = -2.$$

Hence, the solution is  $x = 5$ ,  $y = -2$ .

**Example 4.** Solve :  $14x - 3y = 54$ ,  $21x - 8y = 95$ .

#### Solution.

Given equations are :

$$14x - 3y = 54 \quad \dots(i)$$

$$21x - 8y = 95 \quad \dots(ii)$$

L.C.M. of coefficients of  $x$  (14 and 21) is 42. So, to make both coefficients equal to 42, multiply equation (i) by 3 and equation (ii) by 2 to get

$$42x - 9y = 162 \quad \dots(iii)$$

$$42x - 16y = 190 \quad \dots(iv)$$

Subtracting (iv) from (iii), we get

$$7y = -28 \quad \Rightarrow \quad y = -4.$$

Substituting  $y = -4$  in equation (i), we get

$$14x - 3 \times (-4) = 54 \quad \Rightarrow \quad 14x + 12 = 54$$

$$\Rightarrow \quad 14x = 42 \quad \Rightarrow \quad x = 3.$$

Hence, the solution is  $x = 3$ ,  $y = -4$ .

**Example 5.** Solve :

$$3 - 2(3x + 4y) = x, \quad \frac{x-3}{4} - \frac{y-4}{5} = 2\frac{1}{10}.$$

#### Solution.

The given pair of simultaneous linear equations is

$$3 - 2(3x + 4y) = x \quad \dots (i)$$

$$\text{and } \frac{x-3}{4} - \frac{y-4}{5} = 2\frac{1}{10} \quad \dots (ii)$$

From (i), we get

$$3 - 6x - 8y - x = 0 \Rightarrow -7x - 8y + 3 = 0$$

$$\Rightarrow 7x + 8y - 3 = 0 \quad \dots (iii)$$

Multiplying (ii) by 20 (L.C.M. of 4 and 5), we get

$$5(x - 3) - 4(y - 4) = 42$$

$$\Rightarrow 5x - 15 - 4y + 16 = 42$$

$$\Rightarrow 5x - 4y - 41 = 0 \quad \dots (iv)$$

Multiplying (iv) by 2, we get

$$10x - 8y - 82 = 0 \quad \dots (v)$$

Adding (iii) and (v), we get

$$17x - 85 = 0 \Rightarrow 17x = 85 \Rightarrow x = 5.$$

Putting  $x = 5$  in (iii), we get

$$7 \times 5 + 8y - 3 = 0 \Rightarrow 35 + 8y - 3 = 0$$

$$\Rightarrow 8y + 32 = 0 \Rightarrow 8y = -32 \Rightarrow y = -4.$$

Hence, the solution is  $x = 5, y = -4$ .

**Note.** In particular, if the coefficient of  $x$  in the first equation is numerically equal to the coefficient of  $y$  in the second equation and the coefficient of  $y$  in the first equation is numerically equal to the coefficient of  $x$  in the second equation, then it is better to proceed as in example 6 (given below).

**Example 6.** Solve :  $15x - 14y = 117, 14x - 15y = 115$ .

**Solution.**

Given equations are :

$$15x - 14y = 117 \quad \dots(i)$$

$$14x - 15y = 115 \quad \dots(ii)$$

Adding (i) and (ii), we get

$$29x - 29y = 232$$

$$\Rightarrow x - y = 8 \quad \dots(iii)$$

Subtracting (ii) from (i), we get

$$x + y = 2 \quad \dots(iv)$$

On adding (iii) and (iv), we get

$$2x = 10 \Rightarrow x = 5.$$

Substituting  $x = 5$  in (iv), we get

$$5 + y = 2 \Rightarrow y = -3.$$

Hence, the solution is  $x = 5, y = -3$ .

**Example 7.** Solve :  $y - \frac{3}{x} = 8, 2y + \frac{7}{x} = 3$ .

**Solution.**

Let  $\frac{1}{x} = z$ , then the given equations become

$$y - 3z = 8 \quad \dots(i)$$

$$2y + 7z = 3 \quad \dots(ii)$$

Multiplying equation (i) by 2, we get

$$2y - 6z = 16 \quad \dots(iii)$$

Subtracting (iii) from (ii), we get

$$13z = -13 \Rightarrow z = -1$$

Substituting  $z = -1$  in (i), we get

$$y - 3 \times (-1) = 8 \Rightarrow y + 3 = 8 \Rightarrow y = 5.$$

$$\text{Now, } z = -1 \Rightarrow \frac{1}{x} = -1 \Rightarrow x = -1.$$

Hence, the solution is  $x = -1, y = 5$ .

**Example 8.**

Solve :  $\frac{6}{x} + \frac{7}{y} = 4, \frac{5}{x} - \frac{4}{y} = 23$ .

**Solution.**

Let  $\frac{1}{x} = a$  and  $\frac{1}{y} = b$ , then the given equations become

$$6a + 7b = 4 \quad \dots(i)$$

$$5a - 4b = 23 \quad \dots(ii)$$

Multiplying (i) by 4 and (ii) by 7, we get

$$24a + 28b = 16 \quad \dots(iii)$$

$$35a - 28b = 161 \quad \dots(iv)$$

On adding (iii) and (iv), we get

$$59a = 177 \Rightarrow a = 3$$

Substituting  $a = 3$  in (i), we get

$$6 \times 3 + 7b = 4 \Rightarrow 18 + 7b = 4$$

$$\Rightarrow 7b = -14 \Rightarrow b = -2$$

$$\text{Now, } a = 3 \Rightarrow \frac{1}{x} = 3 \Rightarrow x = \frac{1}{3}.$$

$$\text{and } b = -2 \Rightarrow \frac{1}{y} = -2 \Rightarrow y = -\frac{1}{2}.$$

Hence, the solution is  $x = \frac{1}{3}, y = -\frac{1}{2}$ .

**Exercise 17.1**

Solve the following (1 to 3) simultaneous linear equations by using substitution method :

1. (i)  $4x - 3y = 8$

$$x - 2y = -3$$

(ii)  $5x + 3y = 24$

$$3x - y = 20$$

2. (i)  $x + y - 5 = 0$

$$y - 2 = 2x$$

(ii)  $x + 2y + 9 = 0$

$$3x + 4y + 17 = 0$$

3. (i)  $2x - 5y = 4$

$$3x - 2y = -16$$

(ii)  $5x - 7y = -9$

$$2x + 5y = 12$$

Solve the following (4 to 5) pairs of simultaneous linear equations by using addition-subtraction method :

4. (i)  $x + 3y = 5$

$$7x - 8y = 6$$

(ii)  $3x - 7y = 35$

$$2x + 5y = 4$$

5. (i)  $4x + 7y = 9$

$$10x + 9y = 31$$

(ii)  $5x + 4y = 43$

$$7x - 6y = 37$$

Solve the following (6 to 12) pairs of simultaneous linear equations :

$$6. \quad (i) \quad \begin{aligned} 3x - 2y &= 5 \\ 5x - 3y &= 13 \end{aligned}$$

$$(ii) \quad \begin{aligned} 4x &= 3y + 5 \\ 8x &= 7y + 1 \end{aligned}$$

$$7. \quad (i) \quad \begin{aligned} \frac{p}{2} - q &= 22 \\ \frac{p}{4} - \frac{q}{6} &= 7 \end{aligned}$$

$$(ii) \quad \begin{aligned} \frac{3}{4}a - \frac{2}{3}b &= 1 \\ \frac{3}{8}a - \frac{1}{6}b &= 1 \end{aligned}$$

$$8. \quad (i) \quad \begin{aligned} 3x + 2(y - 8) &= 0 \\ 5(x - 5) + 3y &= 0 \end{aligned}$$

$$(ii) \quad \begin{aligned} 2(x - 3) + 3(y - 5) &= 0 \\ 5(x - 1) + 4(y - 4) &= 0 \end{aligned}$$

$$9. \quad (i) \quad \begin{aligned} \frac{2x+1}{7} + \frac{5y-3}{3} &= 12 \\ \frac{3x-2}{2} - \frac{4y+3}{9} &= 13 \end{aligned}$$

$$(ii) \quad \begin{aligned} \frac{x+7}{5} - \frac{2x-y}{4} &= 3y - 5 \\ \frac{4x-32}{6} + \frac{5y-7}{2} &= 18 - 5x \end{aligned}$$

$$10. \quad (i) \quad \begin{aligned} 41x + 53y &= 135 \\ 53x + 41y &= 147 \end{aligned}$$

$$(ii) \quad \begin{aligned} 78x - 97y &= 116 \\ 97x - 78y &= 59 \end{aligned}$$

$$11. \quad (i) \quad \begin{aligned} 3x - \frac{2}{y} &= 5 \\ 4x + \frac{3}{y} &= 1 \end{aligned}$$

$$(ii) \quad \begin{aligned} \frac{4}{x} - y &= 1 \\ \frac{6}{x} - 4y &= 9 \end{aligned}$$

$$12. \quad (i) \quad \begin{aligned} \frac{8}{x} - \frac{5}{y} &= 34 \\ \frac{3}{x} - \frac{2}{y} &= 13 \end{aligned}$$

$$(ii) \quad \begin{aligned} \frac{8}{x} - \frac{15}{y} &= 1 \\ \frac{2}{x} + \frac{10}{y} &= 3 \end{aligned}$$

## PROBLEMS ON SIMULTANEOUS LINEAR EQUATIONS

A wide variety of word problems involving two unknown quantities can be solved by assuming two unknown quantities as  $x$  and  $y$  and then writing linear equations corresponding to given conditions. Then solve the pair of linear equations simultaneously to get the values of  $x$  and  $y$ .

**Example 1.** Twice one number minus three times a second number is equal to 1, and the sum of these numbers is 13. Find the numbers.

**Solution.** Let the two numbers be  $x$  and  $y$ .

According to the problem,

$$2x - 3y = 1 \quad \dots(i)$$

$$\text{and} \quad x + y = 13 \quad \dots(ii)$$

Multiplying equation (ii) by 3, we get

$$3x + 3y = 39 \quad \dots(iii)$$

On adding (i) and (iii), we get

$$5x = 40 \quad \Rightarrow \quad x = 8.$$

Substituting  $x = 8$  in (ii), we get

$$8 + y = 13 \quad \Rightarrow \quad y = 5.$$

Hence, the required numbers are 8 and 5.

**Example 2.**

If 1 is added to the numerator and 3 to the denominator of a fraction, it becomes  $\frac{1}{2}$ . If 1 is subtracted from the numerator and 2 is added to the denominator, it becomes  $\frac{2}{5}$ . Find the fraction.

**Solution.**

Let the required fraction be  $\frac{x}{y}$ .

By the given conditions, we have

$$\frac{x+1}{y+3} = \frac{1}{2} \quad \text{and} \quad \frac{x-1}{y+2} = \frac{2}{5}$$

$$\Rightarrow 2x + 2 = y + 3 \quad \text{and} \quad 5x - 5 = 2y + 4$$

$$\Rightarrow 2x - y = 1 \quad \dots(i)$$

$$\text{and } 5x - 2y = 9 \quad \dots(ii)$$

Multiplying equation (i) by 2, we get

$$4x - 2y = 2 \quad \dots(iii)$$

Subtracting (iii) from (ii), we get

$$x = 7.$$

Substituting  $x = 7$  in (i), we get

$$2 \times 7 - y = 1 \quad \Rightarrow \quad 14 - y = 1$$

$$\Rightarrow -y = -13 \quad \Rightarrow \quad y = 13.$$

Hence, the required fraction =  $\frac{7}{13}$ .

**Example 3.**

A two digit number is four times the sum of its digits. If 27 is added to the number, its digits are reversed. Find the number.

**Solution.**

Let  $x$  be the digit at ten's place and  $y$  be the digit at unit's place. Then the number is  $10x + y$ .

According to the first condition of the given problem,

$$10x + y = 4(x + y)$$

$$\Rightarrow 10x + y = 4x + 4y$$

$$\Rightarrow 10x - 4x = 4y - y$$

$$\Rightarrow 6x = 3y \quad \Rightarrow \quad 2x = y. \quad \dots(i)$$

The number formed by reversing the digits is  $10y + x$ .

According to the second condition of the given problem,

$$10y + x = 10x + y + 27$$

$$\Rightarrow 10y - y = 10x - x + 27$$

$$\Rightarrow 9y = 9x + 27 \quad \Rightarrow \quad y = x + 3 \quad \dots(ii)$$

Substituting  $y = 2x$  in (ii), we get

$$2x = x + 3 \quad \Rightarrow \quad x = 3.$$

$\therefore$  From (i),  $y = 2 \times 3 = 6$ .

Hence, the required number is 36.

**Example 4.**

The cost of 7 kg of sugar and 5 kg of rice is ₹ 354, and the cost of 6 kg of sugar and 7 kg of rice is ₹ 393. Find the cost of sugar and rice per kg.

**Solution.**

Let the cost of sugar per kg be ₹  $x$  and that of the rice per kg be ₹  $y$ .

According to the problem, we have

$$7x + 5y = 354 \quad \dots(i)$$

$$6x + 7y = 393 \quad \dots(ii)$$

Multiplying (i) by 7 and (ii) by 5, we get

$$49x + 35y = 2478 \quad \dots(iii)$$

$$30x + 35y = 1965 \quad \dots(iv)$$

Subtracting (iv) from (iii), we get

$$19x = 513 \quad \Rightarrow \quad x = 27.$$

Substituting  $x = 27$  in (i), we get

$$7 \times 27 + 5y = 354 \quad \Rightarrow \quad 189 + 5y = 354$$

$$\Rightarrow \quad 5y = 165 \quad \Rightarrow \quad y = 33.$$

Hence, sugar costs ₹ 27 per kg and rice costs ₹ 33 per kg.

### Example 5.

The weekly incomes of A and B are in the ratio 3 : 4 and their weekly expenditures are in the ratio 1 : 2. If each saves ₹ 1000 per week, find their weekly incomes.

### Solution.

As the weekly incomes of A and B are in the ratio 3 : 4, let their weekly incomes be ₹  $3x$  and ₹  $4x$  respectively.

As their weekly expenditures are in the ratio 1 : 2, let their weekly expenditures be ₹  $y$  and ₹  $2y$  respectively.

Then, weekly savings of A = ₹  $(3x - y)$

and weekly savings of B = ₹  $(4x - 2y)$ .

Since each saves ₹ 1000 per week, we have

$$3x - y = 1000 \quad \dots(i)$$

$$4x - 2y = 1000 \quad \Rightarrow \quad 2x - y = 500 \quad \dots(ii)$$

Subtracting (ii) from (i), we get  $x = 500$

$$\therefore 3x = 3 \times 500 = 1500 \text{ and } 4x = 4 \times 500 = 2000.$$

Hence, A's weekly income is ₹ 1500 and that of B is ₹ 2000.

### Example 6.

Four years ago, Marina was three times old as her daughter. Six years from now, the mother will be twice old as her daughter. Find their present ages.

### Solution.

Let the present age of Marina be  $x$  years and that of her daughter be  $y$  years.

Four years ago,

Marina's age =  $(x - 4)$  years

and her daughter's age =  $(y - 4)$  years

According to the first condition of the problem,

$$x - 4 = 3(y - 4)$$

$$\Rightarrow \quad x - 3y = -8 \quad \dots(i)$$

Six years from now,

Marina's age =  $(x + 6)$  years

and her daughter's age =  $(y + 6)$  years



According to the second condition of the problem,

$$x + 6 = 2(y + 6)$$

$$\Rightarrow x - 2y = 6 \quad \dots(ii)$$

Subtracting (i) from (ii), we get  $y = 14$ .

Substituting  $y = 14$  in (ii), we get

$$x - 2 \times 14 = 6 \quad \Rightarrow \quad x - 28 = 6 \quad \Rightarrow \quad x = 34.$$

Hence, Marina's present age is 34 years and that of her daughter is 14 years.

### Example 7.

At a certain time in a deer park, there were 45 heads and 146 legs counting both the deer and the human visitors. How many deer and how many visitors were there?

### Solution.

Let the number of deer in the park be  $x$  and that of human visitors be  $y$ .

As each deer and each human has one head, we have

$$x + y = 45 \quad \dots(i)$$

As each deer has four legs and each human has 2 legs, we have

$$4x + 2y = 146$$

$$\Rightarrow 2x + y = 73 \quad \dots(ii)$$

Subtracting (i) from (ii), we get  $x = 28$ .

Substituting  $x = 28$  in (i), we get

$$28 + y = 45 \quad \Rightarrow \quad y = 17.$$

Hence, there are 28 deer and 17 human visitors in the park.

### Example 8.

A boat goes 25 km downstream in 2.5 hours and 12 km upstream in 3 hours. Find the speed of the boat in still water and the speed of the stream.

### Solution.

Let the speed of the boat in still water be  $x$  km/hr

and the speed of the stream be  $y$  km/hr.

Then the speed of the boat downstream =  $(x + y)$  km/hr,

and the speed of the boat upstream =  $(x - y)$  km/hr.

Since  $\text{speed} = \frac{\text{distance}}{\text{time}}$ ,

$$\therefore x + y = \frac{25}{2.5} \quad \Rightarrow \quad x + y = 10 \quad \dots(i)$$

$$\text{and } x - y = \frac{12}{3} \quad \Rightarrow \quad x - y = 4 \quad \dots(ii)$$

On adding (i) and (ii), we get

$$2x = 14 \quad \Rightarrow \quad x = 7.$$

Substituting  $x = 7$  in (i), we get

$$7 + y = 10 \quad \Rightarrow \quad y = 3.$$

Hence, the speed of the boat in still water is 7 km/hr and the speed of the stream is 3 km/hr.

 Exercise 17.2

1. The sum of two numbers is 40 and their difference is 8. Find the numbers.
2. The sum of two numbers is 2 and their difference is 8. Find the numbers.
3. The sum of two numbers is 2 and their difference is 1. Find the numbers.
4. Twice one number minus thrice a second number is equal to 2, and the sum of these numbers is 11. Find the numbers.
5. The sum of two numbers is 35. If the larger is doubled and the smaller is tripled, the difference is 5. Find the numbers.
6. Two numbers are in the ratio 4 : 7. If thrice the larger is added to twice the smaller, we get 87. Find the numbers.
7. Two numbers are in the ratio 2 : 3. If 20 is added to each, the ratio becomes 6 : 7. Find the numbers.
8. If 4 is added to the numerator of a certain fraction, it becomes  $\frac{7}{10}$ . If 4 is added to the denominator, it becomes  $\frac{1}{2}$ . Find the fraction.
9. The sum of the digits of a two digit number is 12. If the digits are reversed, the new number is 12 less than double the original number. Find the number.
10. A number of two digits is 4 more than 3 times the sum of the digits; and if 27 is added to it, the digits are reversed. Find the number.
11. Four erasers and six pens cost ₹ 36 while three erasers and nine pens cost ₹ 45. Find the cost of one eraser and one pen.
12. The cost of 5 kg of sugar and 7 kg of rice is ₹ 153 and the cost of 7 kg of sugar and 5 kg of rice is ₹ 147. Find the cost of 6 kg of sugar and 10 kg of rice.
13. Sanjay paid ₹ 27 for 5 mangoes and 8 oranges. If a mango costs ₹ 1.50 more than an orange, what is the cost of each mango?
14. Drawing pencils cost 80 paise each and coloured pencils cost ₹ 1.10 each. Two dozen assorted pencils cost ₹ 21.60. How many coloured pencils are there?
15. Two years ago, Sumitra was five times as old as her daughter, and eight years hence, the mother will be thrice old as her daughter. Find their present ages.
16. Six years ago, the ages of Sujata and Sushma were in the ratio 4 : 3. Nine years from now, their ages will be in the ratio 7 : 6. Find their present ages.
17. The monthly incomes of A and B are in the ratio 3 : 4 and their monthly expenditures are in the ratio 7 : 11. If each saves ₹ 2500 per month, find the monthly income of each.
18. A man travels for  $x$  hours at 4 km/hr and then for  $y$  hours at 6 km/hr. If he covers a total distance of 23 km in 5 hours, find  $x$  and  $y$ .
19. A boat can go 70 km downstream in 5 hours and 42 km upstream in 7 hours. Find the speed
  - (i) of the boat in still water
  - (ii) of the current.

## Summary

- ➔ An equation of the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are real numbers and  $a$  and  $b$  both are non-zero, is called a linear equation in two variables  $x$  and  $y$ .
- ➔ Two linear equations in two variable form a system of simultaneous linear equations.
- ➔ A solution to a system of two simultaneous linear equations in two variables is an ordered pair of real numbers which satisfy both the equations.
- ➔ There are two methods to solve a pair of simultaneous linear equations :
  - (i) Substitution method
  - (ii) Addition-Subtraction (or elimination) method.
- ➔ A wide variety of word problems involving two unknown quantities can be solved by assuming two unknowns as  $x$  and  $y$  and then writing linear equations corresponding to given conditions.

## Check Your Progress

Solve the following (1 to 3) pairs of simultaneous linear equations :

1. (i)  $x + 2y = 11$   
 $2x - y = 2$

(ii)  $3x + 4y + 11 = 0$   
 $5x + 6y + 7 = 0$

2. (i)  $2(x - 2y) = 3y$

(ii)  $x + \frac{y}{2} = 13$

$3y - 2x = 1$

$\frac{x}{3} - y = 2$

3. (i)  $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$

(ii)  $\frac{3}{2x} + \frac{2}{3y} = -\frac{1}{3}$

$\frac{3}{x} + \frac{2}{y} = 0$

$\frac{3}{4x} + \frac{1}{2y} = -\frac{1}{8}$

4. Find two numbers such that three times the smaller exceeds twice the larger by 1 and eight times the smaller exceeds five times the larger by 6.
5. I think of a pair of numbers. If I add 5 to the first number, I obtain twice the second number; and if I add 14 to the second, I obtain twice the first number. What are the numbers?
6. If 1 is added to the numerator of a fraction, it becomes  $\frac{1}{5}$ ; if 1 is taken away from the denominator, it becomes  $\frac{1}{7}$ . Find the fraction.
7. The sum of the digits of a two digit number is 5. The digit obtained by increasing the digit in ten's place by unity is one-eighth of the number. Find the number.
8. A number of two digits is 3 more than twice the sum of digits; and the number formed by reversing the digits is 3 more than 8 times the sum of digits. Find the number.
9. Six years hence a man's age will be three times his son's age and three years ago he was nine times as old as his son. Find their present ages.
10. A man has some notes of denominations ₹ 20 and ₹ 5 which amount to ₹ 380. If the number of notes of each kind are interchanged, they amount to ₹ 60 less as before. Find the number of notes of each denomination.
11. Divide 700 into two parts such that 40% of one part exceeds 60% of the other by 80.