Chapter 16

LINEAR EQUATIONS AND INEQUATIONS

In the previous classes, you have studied mathematical open sentences like $x + 3 = 5$ or $2x - 1 = 9$ which may or may not be true depending upon the value of x. You have learnt how to find **truth set** or **solution set** of such equations. You also studied **word problems** where you first have to write equations corresponding to the given problem stated in words and then find the solution set. In this chapter, we shall strengthen these concepts and introduce the idea of *inequations*.

EQUATIONS

An (algebraic) equation is a statement that two expressions are equal. It may involve one or more than one variables (literals). Thus, $2x - 1 = 9$, $x^2 - 3x + 2 = 0$ are equations in one variable and $5x - 7y = 3$, $x^2 + 3xy + 5y^2 = 4$ are equations in two variables.

Linear equation

An equation of the type $ax + b = 0$, where $a \neq 0$, is called a ***linear equation*** *in the variable x.*

Thus, $2x - 1 = 9$ is a linear equation in the variable x.

Solution or root

Any value (or values) of the variable (or variables) which when substituted in an equation makes its both sides equal is called a ***solution*** *(or* ***root****) of the equation.*

Thus, a number α is a root (or solution) of the equation $ax + b = 0$ if and only if $a\alpha + b = 0$.

For example, when we substitute $x = 5$ in the equation $2x - 1 = 9$, we get $2 \times 5 - 1 = 9$ *i.e.* $9 = 9$, which is true. Therefore, 5 is a solution (or root) of the equation $2x - 1 = 9$.

Solving an equation

To solve an equation is to find all its solutions (or roots), and the process of finding all the solutions is called **solving the equation**.

Permissible rules for solving equations

* *We can add the same number or expression to both sides of an equation.*
* *We can subtract the same number or expression from both sides of an equation.*
* *We can multiply both sides of an equation by the same non-zero number or expression.*
* *We can divide both sides of an equation by the same non-zero number or expression.*
* *A term may be transposed from one side of the equation to the other side, but its sign will change.*

Solving linear equations in one variable

For solving a linear equation in one variable, proceed as under :

(*i*) *Simplify both sides by removing group symbols and collecting like terms.*

(*ii*) *Remove fractions (or decimals) by multiplying both sides by an appropriate factor (L.C.M. of fractions or a power of 10 in case of decimals).*

(*iii*) *Isolate all variable terms on one side and all constants on the other side. Collect like terms.*

(*iv*) *Make the coefficient of the variable 1.*

Remark

The solution may be checked (verified) by substituting in the original equation.

Example 1. Solve the following equations :

(*i*) $3 - 4x = 2x + 25$ (*ii*) $5 - 3(5x + 2) = 4(7 - 3x) + 1$.

Solution. (*i*) Given $3 - 4x = 2x + 25$

$\Rightarrow\ -4x - 2x = 25 - 3$

$\Rightarrow\ -6x = 22 \Rightarrow x = -\frac{22}{6} = -\frac{11}{3}$.

(*ii*) Given $5 - 3(5x + 2) = 4(7 - 3x) + 1$

$\Rightarrow\ 5 - 15x - 6 = 28 - 12x + 1$

$\Rightarrow\ -15x - 1 = -12x + 29$

$\Rightarrow\ -15x + 12x = 29 + 1$

$\Rightarrow\ -3x = 30 \Rightarrow x = -10$.

Example 2. Solve the following equations :

(*i*) $\frac{3x}{5} - \frac{x}{3} = \frac{x}{6} + 1\frac{1}{2}$ (*ii*) $\frac{3y-1}{5} - \frac{1+y}{2} = 3 - \frac{y-1}{4}$.

Solution. (*i*) Given $\frac{3x}{5} - \frac{x}{3} = \frac{x}{6} + \frac{3}{2}$

Multiplying both sides by 30, L.C.M. of 5, 3, 6 and 2, we get

$18x - 10x = 5x + 45$

$\Rightarrow\ 18x - 10x - 5x = 45$

$\Rightarrow\ 3x = 45 \Rightarrow x = 15$.

(*ii*) Given $\frac{3y-1}{5} - \frac{1+y}{2} = 3 - \frac{y-1}{4}$

Multiplying both sides by 20, L.C.M. of 5, 2 and 4, we get

$4(3y - 1) - 10(1 + y) = 20 \times 3 - 5(y - 1)$

$\Rightarrow\ 12y - 4 - 10 - 10y = 60 - 5y + 5$

$\Rightarrow\ 2y - 14 = -5y + 65$

$\Rightarrow\ 2y + 5y = 65 + 14$

$\Rightarrow\ 7y = 79 \Rightarrow y = \frac{79}{7} = 11\frac{2}{7}$.

Cross-multiplication

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ or $bc = ad$.

This is known as ***cross-multiplication*** $\frac{a}{b} \times \frac{c}{d}$

Consider the equation

$\frac{ax+b}{cx+d} = \frac{m}{n} \Rightarrow (ax+b) \times n = (cx+d) \times m$ (By cross-multiplication)

Example 3. Solve the following equations :

(i) $\frac{3}{x+8} = \frac{4}{6-x}$ (ii) $\frac{x+1}{x-1} = \frac{2x+3}{2x-5}$.

Solution. (i) Given $\frac{3}{x+8} = \frac{4}{6-x}$

$\Rightarrow 4(x+8) = 3(6-x)$ (By cross-multiplication)

$\Rightarrow 4x + 32 = 18 - 3x$

$\Rightarrow 4x + 3x = 18 - 32$

$\Rightarrow 7x = -14 \Rightarrow x = -2.$

(ii) Given $\frac{x+1}{x-1} = \frac{2x+3}{2x-5}$

$\Rightarrow (x-1)(2x+3) = (x+1)(2x-5)$ (By cross-multiplication)

$\Rightarrow 2x^2 + 3x - 2x - 3 = 2x^2 - 5x + 2x - 5$

$\Rightarrow 2x^2 + x - 3 = 2x^2 - 3x - 5$

$\Rightarrow 2x^2 + x - 2x^2 + 3x = -5 + 3$

$\Rightarrow 4x = -2 \Rightarrow x = -\frac{1}{2}.$

Example 4. If $p = x + 1$ and $\frac{4p-3}{2} - \frac{3x+2}{5} = \frac{3}{2}$, find x.

Solution. Given $p = x + 1$ ...(i)

and $\frac{4p-3}{2} - \frac{3x+2}{5} = \frac{3}{2}$ ...(ii)

Substituting the value of p from (i) in (ii), we get

$\frac{4(x+1)-3}{2} - \frac{3x+2}{5} = \frac{3}{2} \Rightarrow \frac{4x+1}{2} - \frac{3x+2}{5} = \frac{3}{2}$

Multiplying both sides by 10, L.C.M. of 2, 5 and 2, we get

$5(4x+1) - 2(3x+2) = 15$

$\Rightarrow 20x + 5 - 6x - 4 = 15$

$\Rightarrow 14x + 1 = 15 \Rightarrow 14x = 15 - 1$

$\Rightarrow 14x = 14 \Rightarrow x = 1.$

Exercise 16.1

Solve the following (1 to 9) equations :

1. (i) $4x - 8 = 7 - x$ (ii) $3x - 7 = 3(5 - x)$
2. (i) $3(2x - 1) = 5 - (3x - 2)$ (ii) $5x - 2[x - 3(x - 5)] = 6$
3. (i) $\frac{x-1}{3} = \frac{x+2}{6} + 3$ (ii) $\frac{x+7}{3} = 1 + \frac{3x-2}{5}$
4. (i) $\frac{y+1}{3} - \frac{y-1}{2} = \frac{1+2y}{3}$ (ii) $\frac{p}{3} + \frac{p}{4} = 55 - \frac{p+40}{5}$
5. (i) $0{\cdot}3(6 - x) = 0{\cdot}4(x + 8)$ (ii) $\frac{1}{2}(x + 1{\cdot}7) - \frac{1}{3}(x - 2{\cdot}3) = 1$

6. (i) $\frac{3x+2}{x-6} = 5$ (ii) $\frac{1}{x-3} + \frac{3}{7} = \frac{1}{2}$

7. (i) $\frac{5}{x} = \frac{7}{x-4}$ (ii) $\frac{2x-3}{2x-1} = \frac{3x-1}{3x+1}$

8. (i) $\frac{2x+5}{2} - \frac{5x}{x-1} = x$ (ii) $\frac{1}{5}\left(\frac{1}{3x} - 5\right) = \frac{1}{3}\left(3 - \frac{1}{x}\right)$

9. (i) $\frac{4}{x-3} + \frac{2}{x-2} = \frac{6}{x}$ (ii) $\frac{3}{2x-1} + \frac{4}{2x+1} = \frac{7}{2x}$.

10. If $x = p + 1$, find the value of p from the equation $\frac{1}{2}(5x - 30) - \frac{1}{3}(1 + 7p) = \frac{1}{4}$.

11. Solve $\frac{x+3}{3} - \frac{x-2}{2} = 1$. Hence find p if $\frac{1}{x} + p = 1$.

12. Solve $\frac{2x+1}{10} - \frac{3-2x}{15} = \frac{x-2}{6}$. Hence find the value of y if $\frac{2}{x} + \frac{5}{y} = 5$.

PROBLEMS BASED ON LINEAR EQUATIONS

Problems stated in words are called ***word*** *or* ***applied problems****.*

Success with word problems comes with practice. Solving word problems involves two steps; translating the words of the problem into an algebraic equation and then solving the resulting equation.

Solving word problems

Due to the wide variety of word (or applied) problems, there is no single technique that works in all cases. However, the following general suggestions should prove helpful :

* Read and reread the statement of the problem carefully, and determine what quantity must be found.
* Represent the unknown quantity by a letter.
* Determine which expressions are equal and write an equation.
* Solve the resulting equation.

Example 1. The sum of two numbers is 90 and the greater number exceeds thrice the smaller by 14. Find the numbers.

Solution. Let the smaller number be x.

As the sum of two numbers is 90, the greater number is $90 - x$.

Thrice the smaller number $= 3x$

According to the problem,

$$90 - x = 3x + 14$$

$$\Rightarrow \quad -x - 3x = 14 - 90$$

$$\Rightarrow \quad -4x = -76 \quad \Rightarrow \quad x = 19$$

Then $90 - x = 90 - 19 = 71$

Hence, the two numbers are 19 and 71.

Example 2. Find two consecutive even integers such that two-fifth of the smaller exceeds two-eleventh of the larger by 4.

Solution. Let the smaller even integer be x, then the other consecutive even integer is $x + 2$.

According to the question,

$$\frac{2}{5}x = \frac{2}{11}(x + 2) + 4$$

Multiplying both sides by 55, L.C.M. of 5 and 11, we get

$$22x = 10(x + 2) + 220$$

$$\Rightarrow \quad 22x = 10x + 20 + 220$$

$$\Rightarrow 22x - 10x = 240$$

$$\Rightarrow \quad 12x = 240 \quad \Rightarrow \quad x = 20$$

Then $\quad x + 2 = 20 + 2 = 22$

Hence, the required consecutive even integers are 20 and 22.

Example 3. The denominator of a fraction exceeds its numerator by 4. If the numerator and denominator are both increased by 3, the new fraction becomes $\frac{4}{5}$. Find the original fraction.

Solution. Let the numerator of the original fraction be x.

Then, its denominator = $x + 4$

$\therefore$ The original fraction = $\frac{x}{x+4}$

According to the problem,

$$\frac{x+3}{(x+4)+3} = \frac{4}{5} \quad \Rightarrow \quad \frac{x+3}{x+7} = \frac{4}{5}$$

$\Rightarrow \quad 5(x + 3) = 4(x + 7)$ (By cross-multiplication)

$$\Rightarrow \quad 5x + 15 = 4x + 28$$

$$\Rightarrow \quad 5x - 4x = 28 - 15 \quad \Rightarrow \quad x = 13$$

$\therefore$ The original fraction = $\frac{13}{13+4} = \frac{13}{17}$.

Example 4. What number should be added to each of the numbers 3, 5, 13 and 19 so that the resulting numbers may be in proportion?

Solution. Let the required number to be added be x.

On adding x to each of the given numbers, the numbers become $3 + x$, $5 + x$, $13 + x$ and $19 + x$.

As the resulting numbers are in proportion, we get

$$\frac{3+x}{5+x} = \frac{13+x}{19+x}$$

$\Rightarrow \quad (3 + x)(19 + x) = (5 + x)(13 + x)$ (By cross-multiplication)

$$\Rightarrow \quad 57 + 3x + 19x + x^2 = 65 + 5x + 13x + x^2$$

$$\Rightarrow \quad x^2 + 22x + 57 = x^2 + 18x + 65$$

$$\Rightarrow x^2 + 22x - x^2 - 18x = 65 - 57$$

$$\Rightarrow 4x = 8 \Rightarrow x = 2$$

Hence, the required number to be added is 2.

Example 5. The sum of the digits of a two digit number is 12. If 18 is added to it, the digits are reversed. Find the number.

Solution. Let the unit's digit be x.

As the sum of both digits is 12, the ten's digit is $12 - x$.

$\therefore$ The number = $(12 - x) \times 10 + x$

On reversing the digits, we have x at ten's place and $(12 - x)$ at unit's place.

$\therefore$ New number = $x \times 10 + (12 - x)$

According to given information,

$$x \times 10 + (12 - x) = (12 - x) \times 10 + x + 18$$
$$\Rightarrow \quad 10x + 12 - x = 120 - 10x + x + 18$$
$$\Rightarrow \quad 9x + 12 = -9x + 138$$
$$\Rightarrow \quad 9x + 9x = 138 - 12$$
$$\Rightarrow \quad 18x = 126 \Rightarrow x = 7$$

$\therefore$ Unit's digit = 7 and ten's digit = 12 – 7 = 5

$\therefore$ The original number = 57.

Example 6. Michael is 6 times as old as his grand daughter Laura.

(*i*) If Laura's present age is x years, write down in terms of x the age of Michael in 15 years time.

(*ii*) In 15 years time, Michael will be 3 times as old as Laura. Write an equation in x and hence find their present ages.

Solution. (*i*) Given, Laura's present age is x years.

$\therefore$ Michael's present age = $6x$ years

In 15 years time, Michael's age will be $(6x + 15)$ years.

(*ii*) In 15 years time, Laura's age will be $(x + 15)$ years.

According to the given information,

$$6x + 15 = 3(x + 15)$$
$$\Rightarrow \quad 6x + 15 = 3x + 45$$
$$\Rightarrow \quad 6x - 3x = 45 - 15$$
$$\Rightarrow \quad 3x = 30 \Rightarrow x = 10$$

$\therefore$ Laura's present age = 10 years

and Michael's present age = (6×10) years = 60 years.

Example 7. A bag contains 10 paise, 25 paise and 50 paise coins. The number of 25 paise coins is three times the number of 10 paise coins. If 50 paise coins are 5 more than 25 paise coins and the total value of the money in the bag is ₹ 120, find the number of 10 paise coins.

Solution. Let the number of 10 paise coins in the bag be x.

Then, the number of 25 paise coins = $3x$

and the number of 50 paise coins = $3x + 5$

Value of 10 paise coins = ₹ $\frac{x}{10}$,

value of 25 paise coins = ₹ $\frac{3x}{4}$

and the value of 50 paise coins = ₹ $\frac{1}{2}(3x + 5)$

According to the question,

$$\frac{x}{10} + \frac{3x}{4} + \frac{1}{2}(3x + 5) = 120$$

Multiplying both sides by 20, L.C.M. of 10, 4 and 2, we get

$$2x + 15x + 10(3x + 5) = 120 \times 20$$

$\Rightarrow \quad 17x + 30x + 50 = 2400$

$\Rightarrow \quad 47x = 2400 - 50 = 2350$

$\Rightarrow \quad x = 50$

Hence, the number of 10 paise coins in the bag = 50.

Example 8. A man covers a distance of 24 km in $3\frac{1}{2}$ hours partly on foot at the speed of 4·5 km/hr and partly on bicycle at the speed of 10 km/hr. Find the distance covered on foot.

Solution. Let the distance covered on foot be d km.

Then, the distance covered on bicycle = $(24 - d)$ km

$\therefore$ Time taken to cover d km on foot = $\frac{\text{distance}}{\text{speed}} = \frac{d}{4 \cdot 5}$ hours.

Time taken to cover $(24 - d)$ km on bicycle = $\frac{24 - d}{10}$ hours.

According to the problem,

$$\frac{d}{4 \cdot 5} + \frac{24 - d}{10} = 3\frac{1}{2}$$

$\Rightarrow \quad \frac{2d}{9} + \frac{24 - d}{10} = \frac{7}{2}$

$\Rightarrow \quad 20d + 9(24 - d) = 45 \times 7$ (Multiplying both side by 90)

$\Rightarrow \quad 20d + 216 - 9d = 315$

$\Rightarrow \quad 11d = 315 - 216 = 99 \Rightarrow d = 9$

$\therefore$ The distance covered on foot = 9 km.

Example 9. A boat covers a certain distance downstream in 3 hours and it covers the same distance upstream in 5 hours. If the speed of the boat in still water is 8 km/hr, find the speed of the stream.

Solution. Let the speed of the stream be x km/hr.

Since the speed of the boat in still water is 8 km/hr,

the speed of the boat downstream = $(8 + x)$ km/hr

and the speed of the boat upstream = $(8 - x)$ km/hr

Then the distance covered by the boat downstream in 3 hours

$= 3(8 + x)$ km

The distance covered by the boat upstream in 5 hours

$= 5(8 - x)$ km

As the distances covered by the boat downstream and upstream are equal, we get

$$3(8 + x) = 5(8 - x)$$

$\Rightarrow \quad 24 + 3x = 40 - 5x$

$\Rightarrow \quad 3x + 5x = 40 - 24$

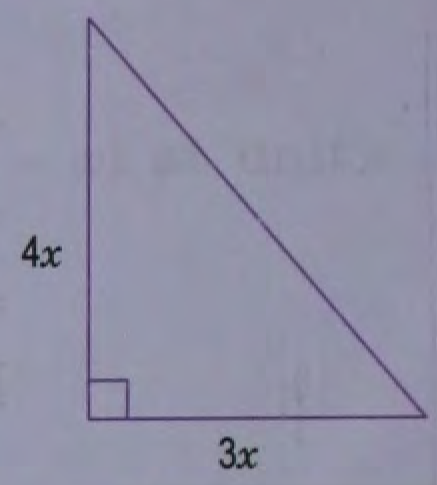
$\Rightarrow \quad 8x = 16 \Rightarrow x = 2$

Hence, the speed of the stream = 2 km/hr.

Example 10. The two perpendicular sides of a right angled triangle are in ratio 3 : 4 and its perimeter is 96 cm. Find the three sides.

Solution. As the two perpendicular sides are in ratio 3 : 4, let these be $3x$ and $4x$.

By Pythagoras theorem,



$$\text{hypotenuse} = \sqrt{(3x)^2 + (4x)^2} = \sqrt{9x^2 + 16x^2}$$
$$= \sqrt{25x^2} = 5x$$

$\therefore$ Perimeter = $3x + 4x + 5x = 12x$

By given condition, $12x = 96$ cm

$\Rightarrow \quad x = \frac{96}{12}$ cm = 8 cm

Hence, the three sides are $3x$, $4x$ and $5x$ *i.e.* 24 cm, 32 cm and 40 cm.

Example 11. The perimeter of a rectangle is 240 cm. If its length is decreased by 10% and breadth is increased by 20%, we get the same perimeter. Find the length and the breadth of the rectangle.

Solution. Given, perimeter of rectangle = 240 cm

$\Rightarrow$ 2(length + breadth) = 240 cm $\Rightarrow$ length + breadth = 120 cm.

Let the length of the rectangle be x cm, then its breadth = $(120 - x)$ cm.

Length is decreased by 10%,

$$\text{new length} = \left(1 - \frac{10}{100}\right) \text{ of } x \text{ cm} = \left(1 - \frac{1}{10}\right)x \text{ cm} = \frac{9}{10}x \text{ cm}$$

Breadth is increased by 20%,

$$\text{new breadth} = \left(1 + \frac{20}{100}\right) \text{ of } (120 - x) \text{ cm}$$
$$= \left(1 + \frac{1}{5}\right)(120 - x) \text{ cm} = \frac{6(120 - x)}{5} \text{ cm}$$

According to given information, perimeter of the rectangle remains same

$\Rightarrow \quad 2\left(\frac{9}{10}x + \frac{6(120 - x)}{5}\right) = 240$

$\Rightarrow \quad \frac{9}{10}x + \frac{6(120 - x)}{5} = 120$

$\Rightarrow \quad 9x + 12(120 - x) = 120 \times 10$ (Multiplying both sides by 10)

$\Rightarrow \quad 9x + 1440 - 12x = 1200$

$\Rightarrow \quad -3x = 1200 - 1440$

$\Rightarrow \quad -3x = -240 \quad \Rightarrow \quad x = 80.$

$\therefore$ Length of original rectangle = 80 cm and its breadth = (120 – 80) cm = 40 cm.

Exercise 16.2

1. Three more than twice a number is equal to four less than the number. Find the number.
2. Three times of a number increased by 2 is equal to one less than four times the number. Find the number.
3. When four consecutive integers are added, the sum is 46. Find the integers.

4. When three consecutive even integers are added, the sum is zero. Find the integers.
5. Find three consecutive integers such that third is one and a half times the first.
6. Find two consecutive odd integers such that two-fifth of the smaller exceeds two-ninth of the greater by 4.
7. The denominator of a fraction is 1 more than twice its numerator. If the numerator and denominator are both increased by 5, it becomes $\frac{3}{5}$. Find the original fraction.
8. Find two positive numbers in the ratio 2 : 5 such that their difference is 15.
9. What number should be added to each of the numbers 12, 22, 42 and 72 so that the resulting numbers may be in proportion?
10. The ten's digit of a two digit number is three times the unit's digit. When the digits are reversed, we get 36 less than the number. Find the original number.
11. The sum of the digits of a two digit number is 9. If the digits are reversed, the number is increased by 45. Find the original number.
12. A student purchased two books at a total cost of ₹ 66. If one book costs ₹ 6 more than the other, find the cost of each book.
13. In an election in a class, 62 students voted for Arnav or Tushar. If Arnav won by a margin of 8 votes, find the number of votes polled by Tushar.
14. Ritu is now four times as old as his brother Raju. In 4 years time, her age will be twice of Raju's age. What are their present ages?
15. A father is 7 times as old as his son. Two years ago, the father was 13 times as old as his son. How old are they now?
16. The ages of Sona and Sonali are in the ratio 5 : 3. Five years hence, the ratio of their ages will be 10 : 7. Find their present ages.
17. Deepa has 7 more ₹ 2 coins than 50 paise coins. If the value of all the coins is ₹ 76·50, find the number of each type of coins.
18. A bag contains ₹ 155 in the form of 1 rupee, 50 paise and 10 paise coins in the ratio of 3 : 5 : 7. Find the number of each type of coins.
19. A local bus is carrying 40 passengers, some with 50 paise tickets and the remaining with ₹ 1·50 tickets. If the total receipts from these passengers is ₹ 32, find the number of passengers with 50 paise tickets.
20. On a school picnic, a group of students agree to pay equally for the use of a full boat and pay ₹ 10 each. If there had been 3 more students in the group, each would have paid ₹ 2 less. How many students were there in the group?
21. Two supplementary angles differ by 50°. Find the measure of each angle.
22. If the angles of a triangle are in the ratio 5 : 6 : 7, find the angles.
23. Two equal sides of an isosceles triangle are $3x - 1$ and $2x + 2$ units. The third side is $2x$ units. Find x and the perimeter of the triangle.
24. If each side of a triangle is increased by 4 cm, the ratio of the perimeters of the new triangle and the given triangle is 7 : 5. Find the perimeter of the given triangle.
    [**Hint.** Let the perimeter of given triangle be x cm. As each side is increased by 4 cm, so the perimeter is increased by (3×4) cm *i.e.* 12 cm. According to given information, $\frac{x+12}{x} = \frac{7}{5}$.]

25. The length of a rectangle is 5 cm less than twice its breadth. If the length is decreased by 3 cm and breadth increased by 2 cm, the perimeter of the resulting rectangle is 72 cm. Find the area of the original rectangle.

26. A rectangle is 10 cm long and 8 cm wide. When each side of the rectangle is increased by x cm, its perimeter is doubled. Find the equation in x and hence find the area of the new rectangle.

27. Two boats are 110 km apart. They travel towards each other, one travelling 5 km/hr slower than the other. If they meet in 2 hours, find the speed of each boat.

28. A passenger train travelling at a constant speed covers a distance in 3 hours while an express train travelling 30 km/hr faster covers the same distance in 2 hours. Find the speeds of the two trains.

29. A man walks from his house to his daughter's school at a speed of 3 km/hr and returns at a speed of 4 km/hr. If he takes 21 minutes for the total journey, find the distance between his house and the school.

30. A steamer travels 90 km downstream in the same time as it takes to travel 60 km upstream. If the speed of the stream is 5 km/hr, find the speed of the steamer in still water.

[**Hint.** Let the speed of the steamer in still water be x km/hr, then the speed downstream $= (x + 5)$ km/hr and the speed upstream $= (x - 5)$ km/hr.

According to the problem, $\frac{90}{x+5} = \frac{60}{x-5}$.]

LINEAR INEQUATIONS

Statements such as

$$x < 3,\ x + 5 \le 9,\ 2x - 1 > 5,\ 3x + 5 \ge 2,\ \frac{x-1}{2} < 3x + 5$$

are called **linear inequations**.

In general, a linear inequation in the variable x can be written in any one of the forms:

(*i*) $ax + b < 0$

(*ii*) $ax + b \le 0$

(*iii*) $ax + b > 0$

(*iv*) $ax + b \ge 0$

where a and b are real numbers, $a \ne 0$.

Replacement set

The set from which the values of the variable (involved in the inequation) are chosen is called the ***replacement set*** *or* ***universal set****.*

Solution set

A ***solution*** *of an inequation is a number (chosen from the replacement set) which, when substituted for the variable, makes the inequation true. The set of all the solutions of an inequation is called the* ***solution set*** *or* ***truth set*** *of the inequation.*

For example, consider the inequation $x < 4$.

Replacement set	*Solution set*
(*i*) {1, 2, 3, 4, 5, 6, 7, 8}	{1, 2, 3}
(*ii*) {–1, 0, 1, 2, 5, 9}	{–1, 0, 1, 2}
(*iii*) {–5, 5, 10}	{–5}
(*iv*) {5, 6, 7, 8, 9, 10}	ϕ
(*v*) **N**	{1, 2, 3}
(*vi*) **W**	{0, 1, 2, 3}

Note that the solution set depends upon the replacement set.

Representing solution of an inequation on a number line

Solution of every inequation can be represented on a number line. See the following examples:

Example 1. Represent the solution of $x < 5$, $x \in$ **N**, on a number line.

Solution. Given $x < 5$, $x \in$ **N**

$\therefore$ The solution set = {1, 2, 3, 4}

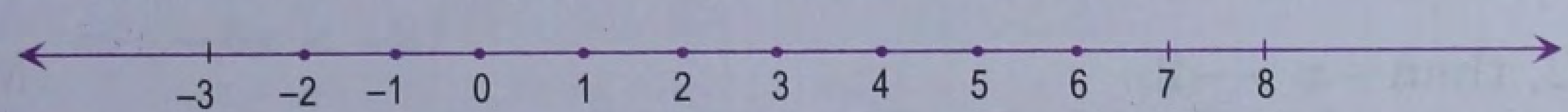
The solution set is shown by thick dots on the number line.

Example 2. Represent the solution of $-2 \le x < 7$, $x \in$ **I**, graphically.

Solution. Given $-2 \le x < 7$, $x \in$ **I**

$\therefore$ The solution set = {–2, –1, 0, 1, 2, 3, 4, 5, 6}

The solution set is shown by thick dots on the number line.

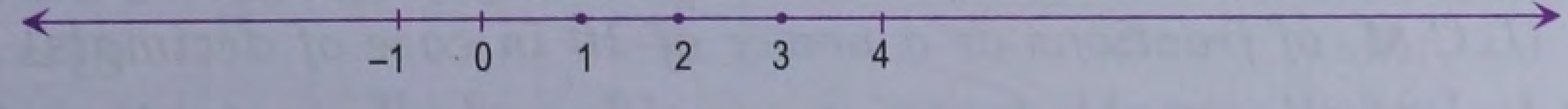


Example 3. Graph the solution set of $x \le 3$ on a number line if the replacement set is:

(*i*) **N** (*ii*) **W** (*iii*) **I** (*iv*) **R**

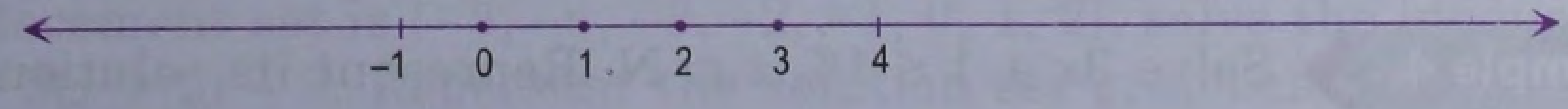
Solution. (*i*) Given $x \le 3$, replacement set = **N**

$\therefore$ The solution set is {1, 2, 3}, it is shown by thick dots on the number line.



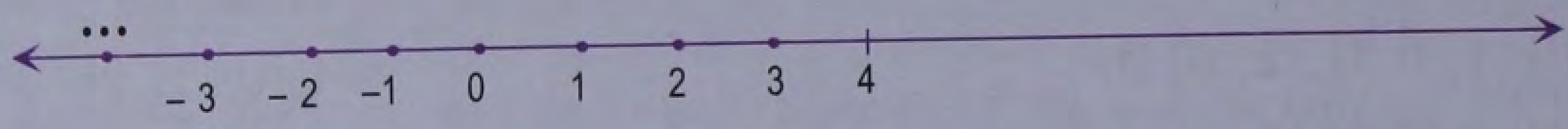
(*ii*) Given $x \le 3$, replacement set = **W**

$\therefore$ The solution set = {0, 1, 2, 3}, it is shown by thick dots on the number line.



(*iii*) Given $x \leq 3$, replacement set = **I**

$\therefore$ The solution set is {..., –2, –1, 0, 1, 2, 3}, it is represented on the number line as shown below :

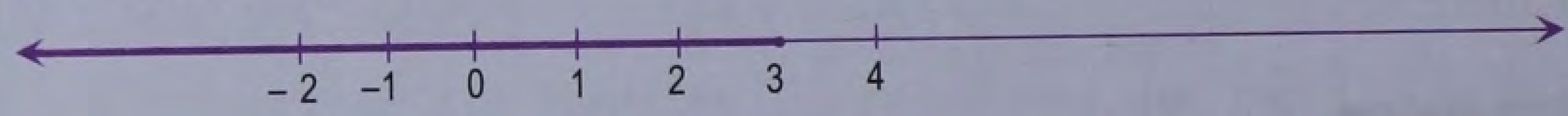


Three dots marked above the line on left hand side indicate that the marking continues on the left side.

(*iv*) Given $x \leq 3$, replacement set = **R**.

$\therefore$ The solution set = $\{x; x \in \mathbf{R}, x \leq 3\}$

The graph of the solution set is shown by thick portion of the number line. The thick dot at 3 indicates that the number 3 is included in the solution set. The dark line and arrow on the left side indicate that the solution set contains all real numbers whose value is less than or equal to 3.



Solving linear inequations in one variable

The rules for solving inequations are similar to those for solving equations except for multiplying or dividing by a negative number.

You can do any of the following to an inequation :

* add the same number or expression to both sides.
* subtract the same number or expression from both sides.
* multiply both sides by the same **positive** number.
* divide both sides by the same **positive** number.

However, when you multiply or divide by the same **negative** number, the **symbol of inequation is reversed**.

For example :

(*i*) If $x < 2$, then $-x > -2$ (Multiplying by –1)

(*ii*) $3x - 1 \geq 5$, then $-4(3x - 1) \leq -20$ (Multiplying by –4)

(*iii*) $-6x \leq 12$, then $x \geq -2$ (Dividing by –6)

> ***Thus, always reverse the symbol of an inequation when multiplying or dividing by a negative number.***

Procedure to solve a linear inequation in one variable :

(*i*) *Simplify both sides by removing group symbols and collecting like terms.*

(*ii*) *Remove fractions (or decimals) by multiplying both sides by an appropriate number (L.C.M. of fractions or a power of 10 in case of decimals).*

(*iii*) *Isolate all variable terms on one side and all constants on the other side. Collect like terms.*

(*iv*) *Make the coefficient of the variable 1.*

(*v*) *Choose the solution set from the replacement set.*

Example 4. Solve $3x + 1 \leq 16$, $x \in$ **N**. Represent its solution on the number line.

Solution. Given $3x + 1 \leq 16$

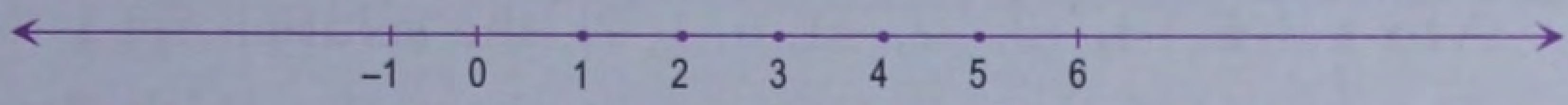
$\Rightarrow \quad 3x + 1 - 1 \leq 16 - 1$ (Subtracting 1 from both sides)

$\Rightarrow \quad 3x \leq 15$

$\Rightarrow \quad x \leq 5$ (Dividing both sides by 3)

As $x \in \mathbf{N}$, the solution set = {1, 2, 3, 4, 5}

The solution set is shown by thick dots on the number line.



Example 5. Solve the inequation $\frac{2x-1}{3} \leq 4$, $x \in \mathbf{W}$.

Solution. Given $\frac{2x-1}{3} \leq 4$

$\Rightarrow \quad 2x - 1 \leq 12$ (Multiplying both sides by 3)

$\Rightarrow \quad 2x - 1 + 1 \leq 12 + 1$ (Adding 1 to both sides)

$\Rightarrow \quad 2x \leq 13$

$\Rightarrow \quad x \leq 6{\cdot}5$ (Dividing both sides by 2)

As $x \in \mathbf{W}$, the solution set is {0, 1, 2, 3, 4, 5, 6}.

Example 6. Solve the following inequations :

(*i*) $11 + 2x > 5$, where x is a negative integer.

(*ii*) $25 - 3\,(2x - 5) < 19$, where x is a positive integer.

Solution. (*i*) Given $11 + 2x > 5$

$\Rightarrow \quad 11 + 2x - 11 > 5 - 11$ (Subtract 11)

$\Rightarrow \quad 2x > -6$

$\Rightarrow \quad x > -3$ (Divide by 2)

As x is a negative integer, the solution set is {–2, – 1}.

(*ii*) Given $25 - 3(2x - 5) < 19$

$\Rightarrow \quad 25 - 6x + 15 < 19$ (Removing group symbol)

$\Rightarrow \quad 40 - 6x < 19$

$\Rightarrow \quad 40 - 6x - 40 < 19 - 40$ (Subtract 40)

$\Rightarrow \quad -6x < -21$

$\Rightarrow \quad x > \frac{7}{2}$ (Divide by –6 and reverse the symbol)

As x is a positive integer, the solution set is {4, 5, 6, ...}.

Example 7. Solve the inequality $3 - 2x \geq x - 10$, $x \in \mathbf{N}$.

Solution. Given $3 - 2x \geq x - 10$

$\Rightarrow \quad 3 - 2x - 3 \geq x - 10 - 3$ (Subtract 3)

$\Rightarrow \quad -2x \geq x - 13$

$\Rightarrow \quad -2x - x \geq x - 13 - x$ (Subtract x)

$\Rightarrow \quad -3x \geq -13$

$\Rightarrow \quad x \leq \frac{13}{3}$ (Divide by –3 and reverse the symbol)

As $x \in \mathbf{N}$, the solution set is {1, 2, 3, 4}.

Example 8. If the replacement set is {–5, –4, –3, –2, –1, 0, 1, 2}, solve the inequation $3(x - 1) > 2(x + 2) - 9$. Represent the solution set graphically.

Solution. Given $3(x - 1) > 2(x + 2) - 9$

$\Rightarrow \quad 3x - 3 > 2x + 4 - 9$

$\Rightarrow$ $3x - 3 + 3 > 2x + 4 - 9 + 3$ (Add 3)

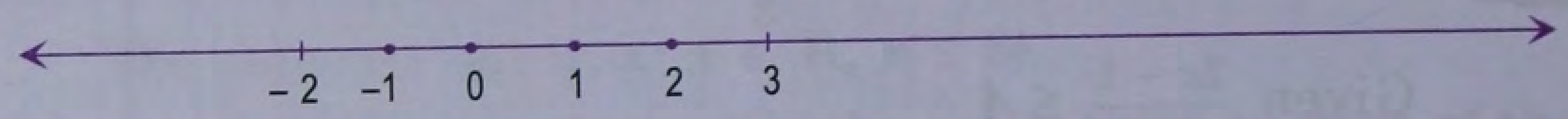
$\Rightarrow$ $3x > 2x - 2$ (Subtract $2x$)

$\Rightarrow$ $3x - 2x > 2x - 2 - 2x$

$\Rightarrow$ $x > -2$

As $x \in \{-5, -4, -3, -2, -1, 0, 1, 2\}$, the solution set is $\{-1, 0, 1, 2\}$.

The graph of the solution set is shown by thick dots on the number line.



Example 9. Solve $1 \leq 3(x - 2) + 4 < 10$, $x \in \mathbf{W}$. Also represent its solution on the number line.

Solution. Given $1 \leq 3(x - 2) + 4 < 10$

$\Rightarrow$ $1 \leq 3x - 6 + 4 < 10$

$\Rightarrow$ $1 \leq 3x - 2 < 10$

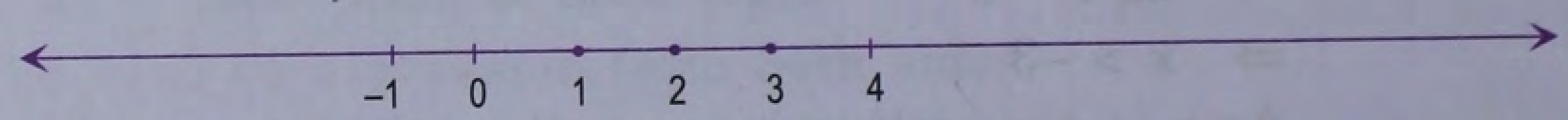
$\Rightarrow$ $1 + 2 \leq 3x - 2 + 2 < 10 + 2$ (Add 2)

$\Rightarrow$ $3 \leq 3x < 12$

$\Rightarrow$ $1 \leq x < 4$ (Divide by 3)

As $x \in \mathbf{W}$, the solution set is $\{1, 2, 3\}$.

The solution set is shown by thick dots on the number line.



Example 10. Find the solution set of $-\frac{1}{5} < \frac{3x}{10} + 1 \leq \frac{2}{5}$, $x \in \mathbf{R}$. Graph the solution set on the number line.

Solution. Given $-\frac{1}{5} < \frac{3x}{10} + 1 \leq \frac{2}{5}$

Multiplying throughout by 10, L.C.M. of fractions, we get

$-2 < 3x + 10 \leq 4$

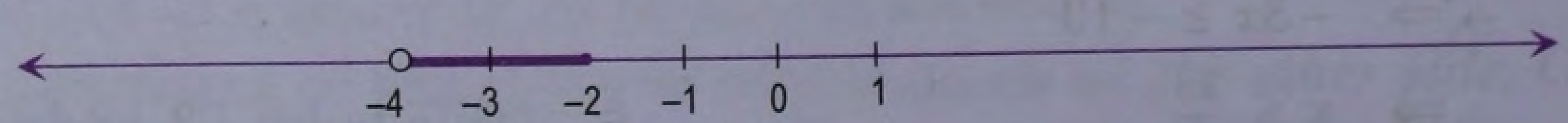
$\Rightarrow$ $-2 - 10 < 3x + 10 - 10 \leq 4 - 10$ (Subtract 10)

$\Rightarrow$ $-12 < 3x \leq -6$

$\Rightarrow$ $-4 < x \leq -2$ (Divide by 3)

As $x \in \mathbf{R}$, the solution set is $\{x : x \in \mathbf{R}, -4 < x \leq -2\}$.

The graph of the solution set is shown by the thick portion of the number line. The thick dot at -2 indicates that the number -2 is included in the solution and the open circle at -4 indicates that the number -4 is not included in the solution set.



Exercise 16.3

1. If the replacement set = $\{-7, -5, -3, -1, 0, 1, 3\}$, find the solution set of :

   (*i*) $x > -2$ (*ii*) $x < -2$ (*iii*) $x > 2$

   (*iv*) $-5 < x \leq 5$ (*v*) $-8 < x < 1$ (*vi*) $0 \leq x \leq 4$.

2. Represent the solution of the following inequations graphically :

(*i*) $x \le 4, x \in \mathbf{N}$ (*ii*) $x < 5, x \in \mathbf{W}$ (*iii*) $-3 \le x < 3, x \in \mathbf{I}$

(*iv*) $-2 \le x < 3, x \in \mathbf{R}$ (*v*) $x \ge -2, x \in \mathbf{R}$ (*vi*) $x < 4, x \in \mathbf{R}$.

3. If the replacement set is $\{-6, -4, -2, 0, 2, 4, 6\}$, then represent the solution set of the inequation $-4 \le x < 4$ graphically.

4. Find the solution set of the inequation $x < 4$ if the replacement set is

(*i*) $\{1, 2, 3, ..., 10\}$ (*ii*) $\{-1, 0, 1, 2, 5, 8\}$

(*iii*) $\{-5, 10\}$ (*iv*) $\{5, 6, 7, 8, 9, 10\}$.

5. If the replacement set $= \{-6, -3, 0, 3, 6, 9, 12\}$, find the truth set of the following :

(*i*) $2x - 3 > 7$ (*ii*) $3x + 8 \le 2$ (*iii*) $-3 < 1 - 2x$.

6. Solve the following inequations :

(*i*) $4x + 1 < 17, x \in \mathbf{N}$ (*ii*) $4x + 1 \le 17, x \in \mathbf{W}$

(*iii*) $4 > 3x - 11, x \in \mathbf{N}$ (*iv*) $-17 \le 9x - 8, x \in \mathbf{Z}$.

7. Solve the following inequations :

(*i*) $\frac{2y-1}{5} \le 2, y \in \mathbf{N}$ (*ii*) $\frac{2y+1}{3} + 1 \le 3, y \in \mathbf{W}$

(*iii*) $\frac{2}{3}p + 5 < 9, p \in \mathbf{W}$ (*iv*) $-2(p + 3) > 5, p \in \mathbf{I}$.

8. Solve the following inequations :

(*i*) $2x - 3 < x + 2, x \in \mathbf{N}$ (*ii*) $3 - x \le 5 - 3x, x \in \mathbf{W}$

(*iii*) $3(x - 2) < 2(x - 1), x \in \mathbf{W}$ (*iv*) $\frac{3}{2} - \frac{x}{2} > -1, x \in \mathbf{N}$.

9. If the replacement set is $\{-3, -2, -1, 0, 1, 2, 3\}$, solve the inequation $\frac{3x-1}{2} < 2$. Also represent its solution on the number line.

10. Solve $\frac{x}{3} + \frac{1}{4} < \frac{x}{6} + \frac{1}{2}, x \in \mathbf{W}$. Also represent its solution on the number line.

11. Solve the following inequations and graph their solutions on a number line :

(*i*) $-4 \le 4x < 14, x \in \mathbf{N}$ (*ii*) $-1 < \frac{x}{2} + 1 \le 3, x \in \mathbf{I}$

(*iii*) $-3 < \frac{2x}{3} - 1 \le 3, x \in \mathbf{R}$ (*iv*) $-\frac{2}{3} < -\frac{x}{3} + 1 \le \frac{2}{3}, x \in \mathbf{R}$.

Summary

- An equation of the type $ax + b = 0$, where $a \ne 0$, is called a linear equation in the variable x.
- A number α is a root (or solution) of the equation $ax + b = 0$ if and only if $a\alpha + b = 0$.
- To solve an equation, you can do any of the following :
  - add the same number or expression to both sides.
  - subtract the same number or expression from both sides.
  - multiply both sides by the same non-zero number or expression.
  - divide both sides by the same non-zero number or expression.
  - transpose a term from one side to the other side by changing its sign.

- While solving word (or applied) problems involving one unknown, we first have to write an equation corresponding to the given statement and then solve this equation to find the value of unknown.
- A linear inequation in the variable x can be written in any one of the forms :
  (*i*) $ax + b < 0$ (*ii*) $ax + b \leq 0$ (*iii*) $ax + b > 0$ (*iv*) $ax + b \geq 0$
  where a and b are real numbers, $a \neq 0$.
- The set from which the values of the variable are chosen is called the **replacement set** or **universal set**.
- A **solution** of an inequation is a number (chosen from the replacement set) which, when substituted for the variable, makes the inequation true.
- The set of all solutions of an inequation is called its **solution** (or **truth**) **set**.
- Rules for solving inequations are similar to those for solving equations. However, when you multiply or divide both sides of an inequation by a negative number, then the symbol of the inequation is reversed.
- Solution set of an inequation can be represented on a number line.

Check Your Progress

1. Solve the following equations :

   (*i*) $\frac{2x+5}{2} - \frac{5x}{x-1} = x$ (*ii*) $2 + \frac{2x-3}{2x+3} = \frac{3x+4}{x+2}$

   (*iii*) $\frac{1}{x-1} - \frac{2}{x+1} = \frac{3}{2(x^2-1)}$ (*iv*) $0{\cdot}5\,(4x+1) = 0{\cdot}3\,(2x+1) + 1{\cdot}6$

2. The difference between squares of two consecutive even integers is 92. Find the integers.
3. One-half of a number is equal to one-third of its succeeding number. Find the first number.
4. Find a number such that if you add 8 and divide the sum by 5 you will get the same answer as if you had subtracted 2 and then divided by 3.
5. The ten's digit of a two digit number exceeds its unit's digit by 5. When digits are reversed, the new number added to the original number becomes 99. Find the original number.
6. Ankita is 2 years younger than Anu. Four years hence Anu will be twice old as what Ankita was 3 years ago. Find their present ages.
7. Grandpa's age in years is equal to his dog's age in months. If the difference in their ages is 55 years, find how old the dog is.
8. In a shooting competition, a person receives ₹ 5 if he hits the mark and pays ₹ 2 if he misses it. He tried 60 shots and was paid ₹ 13. How many times did he hit the mark?
9. Vandana has ₹ 103 in five rupee and two rupee notes. She has 26 notes in all. How many five rupee notes does she have?
10. A certain number of students of a school appeared for an examination in which one-fifth of the whole plus 10 got first division, one-fourth plus 17 got second division and one-sixth minus 3 got third division. If the remaining 68 students failed, find the total number of students who appeared for the exam.
11. Ajay covers a distance of 240 km in $4\frac{1}{4}$ hours. Some part of the journey was covered at the speed of 45 km/hr and the remaining at 60 km/hr. Find the distance covered by him at the rate of 60 km/hr.
12. If $x \in$ {even integers}, represent the solution set of the inequation $-5 \leq x < 5$ on a number line.
13. Solve the following inequations and graph their solution on a number line :

    (*i*) $3 - \frac{x}{2} > 2 - \frac{x}{3}, x \in \mathbf{W}$ (*ii*) $-\frac{1}{4} \leq \frac{1}{2} - \frac{x}{3} < 2, x \in \mathbf{I}$.