CHAPTER 15

FACTORISATION

15.1 REVIEW

Factors

Each of the numbers (constant or variable), which form a product is called a factor of the product.

(i) 5 and x are factors of the product 5x.

(ii) $(2x - 5)$ and $(3x + 2)$ are the factors of $(2x - 5)(3x + 2)$.

Since, $(2x - 5)(3x + 2) = 2x(3x + 2) - 5(3x + 2)$

$= 6x^2 + 4x - 15x - 10$

$= 6x^2 - 11x - 10$

$\therefore$ $2x - 5$ and $3x + 2$ are the factors of $6x^2 - 11x - 10$.

TEST YOURSELF

1. Factors of xy are ..... and .....
2. Factors of xy $(4x + 7)$ are ....., ... and .....
3. Factors of $xy(4x + 7)(x - 8)$ are ....., ..., ... and .....
4. Since, $4x^2 - 9 = (2x)^2 - (3)^2 = (2x + 3)(2x - 3)$;
   factors of $4x^2 - 9$ are ..... and .....
5. Since, $x^2 - 5xy + 6y^2 = (x - 3y)(x - 2y)$;
   factors of $x^2 - 5xy + 6y^2$ are .....

15.2 FACTORISATION

Factorisation means to find two or more expressions whose product is equal to the given expression.

15.3 FACTORISATION BY TAKING OUT COMMON FACTORS

Steps : 1. Find by inspection, the largest monomial that will divide each term of the given polynomial completely.

2. Divide each term of the given polynomial by this monomial (factor) and enclose the quotient within brackets keeping this common monomial outside the bracket.

Example 1 :

Factorise :

(i) $5x^2 - 10x$

(ii) $3x^2y - 6xy^2 + 9xy$

Solution :

(i) By inspection, we find that the largest monomial which divides each term of the given polynomial $5x^2 - 10x$ is $5x$. [Step 1]

$\therefore$ $$5x^2 - 10x = 5x\left(\frac{5x^2}{5x} - \frac{10x}{5x}\right)$$ [Step 2]

$$= 5x\,(x - 2)$$ **(Ans.)**

(ii) $$3x^2y - 6xy^2 + 9xy = 3xy\left(\frac{3x^2y}{3xy} - \frac{6xy^2}{3xy} + \frac{9xy}{3xy}\right)$$

$$= 3xy\,(x - 2y + 3)$$ **(Ans.)**

Example 2 :

Factorise :

(i) $-10a^4x^2 - 15a^6x^4 + 20a^7x^5$ (ii) $2x(a + b) - 3y(a + b)$

Solution :

(i) $-10a^4x^2 - 15a^6x^4 + 20a^7x^5 = -5a^4x^2\left(\frac{-10a^4x^2}{-5a^4x^2} - \frac{15a^6x^4}{-5a^4x^2} + \frac{20a^7x^5}{-5a^4x^2}\right)$

$= -5a^4x^2(2 + 3a^2x^2 - 4a^3x^3)$ **(Ans.)**

(ii) $2x(a + b) - 3y(a + b) = (a + b)\left[\frac{2x(a+b)}{a+b} - \frac{3y(a+b)}{a+b}\right]$

$= (a + b)(2x - 3y)$ **(Ans.)**

EXERCISE 15 (A)

Factorise :

1. $15x + 5$
2. $a^3 - a^2 + a$
3. $3x^2 + 6x^3$
4. $4a^2 - 8ab$
5. $2x^3b^2 - 4x^5b^4$
6. $15x^4y^3 - 20x^3y$
7. $a^3b - a^2b^2 - b^3$
8. $6x^2y + 9xy^2 + 4y^3$
9. $17a^6b^8 - 34a^4b^6 + 51a^2b^4$
10. $3x^5y - 27x^4y^2 + 12x^3y^3$
11. $x^2(a - b) - y^2(a - b) + z^2(a - b)$
12. $(x + y)(a + b) + (x - y)(a + b)$
13. $2b(2a + b) - 3c(2a + b)$
14. $12abc - 6a^2b^2c^2 + 3a^3b^3c^3$
15. $4x(3x - 2y) - 2y(3x - 2y)$
16. $(a + 2b)(3a + b) - (a + b)(a + 2b) + (a + 2b)^2$
17. $6xy(a^2 + b^2) + 8yz(a^2 + b^2) - 10xz(a^2 + b^2)$

15.4 FACTORISATION BY GROUPING

A given algebraic expression, containing an even number of terms may be resolved into factors, if its terms can be arranged in groups such that each group has a common factor.

Steps :
1. Arrange the terms of the given expression in suitable groups such that each group has a common factor.
2. Factorise each group.
3. Take out the factor which is common to each group.

Example 3 :

Factorise : $ax - bx + ay - by$

Solution :

$ax - bx + ay - by$

$= (ax - bx) + (ay - by)$ [Step 1]

$= x(a - b) + y(a - b)$ [Step 2]

$= (a - b)(x + y)$ **(Ans.)** [Step 3]

Or $ax - bx + ay - by$

$= ax + ay - bx - by$ [Step 1]

$= a(x + y) - b(x + y)$ [Step 2]

$= (x + y)(a - b)$ **(Ans.)** [Step 3]

Example 4 :

Factorise :

(i) $y^3 - 3y^2 + 2y - 6 - xy + 3x$ (ii) $a^2 - (b + 5)\,a + 5b$

Solution :

(i) $y^3 - 3y^2 + 2y - 6 - xy + 3x = (y^3 - 3y^2) + (2y - 6) - (xy - 3x)$ [Step 1]

$= y^2(y - 3) + 2(y - 3) - x(y - 3)$ [Step 2]

$= (y - 3)(y^2 + 2 - x)$ **(Ans.)** [Step 3]

(ii) $a^2 - (b + 5)\,a + 5b = a^2 - ab - 5a + 5b$ [Removing the bracket]

$= (a^2 - ab) - (5a - 5b)$ [Step 1]

$= a(a - b) - 5(a - b)$ [Step 2]

$= (a - b)\,(a - 5)$ **(Ans.)** [Step 3]

EXERCISE 15 (B)

Factorise :

1. $a^2 + ax + ab + bx$
2. $a^2 - ab - ca + bc$
3. $ab - 2b + a^2 - 2a$
4. $a^3 - a^2 + a - 1$
5. $2a - 4b - xa + 2bx$
6. $xy - ay - ax + a^2 + bx - ab$
7. $3x^5 - 6x^4 - 2x^3 + 4x^2 + x - 2$
8. $-x^2y - x + 3xy + 3$
9. $6a^2 - 3a^2b - bc^2 + 2c^2$
10. $3a^2b - 12a^2 - 9b + 36$
11. $x^2 - (a - 3)x - 3a$
12. $x^2 - (b - 2)x - 2b$
13. $a(b - c) - d(c - b)$
14. $ab^2 - (a - c)\,b - c$
15. $(a^2 - b^2)\,c + (b^2 - c^2)a$
16. $a^3 - a^2 - ab + a + b - 1$
17. $ab(c^2 + d^2) - a^2cd - b^2cd$
18. $2ab^2 - aby + 2cby - cy^2$
19. $ax + 2bx + 3cx - 3a - 6b - 9c$
20. $2ab^2c - 2a + 3b^3c - 3b - 4b^2c^2 + 4c$

15.5 FACTORISATION OF DIFFERENCE OF TWO SQUARES

Since, the product of $(x + y)$ and $(x - y)$ is $x^2 - y^2$

$\therefore$ Factors of $x^2 - y^2 = (x + y)(x - y)$

Difference of squares of two terms = Sum of the two terms × their difference.

Example 5 :

Factorise : $25a^2 - 36b^2$

Solution :

$25a^2 - 36b^2 = (5a)^2 - (6b)^2 = (5a + 6b)(5a - 6b)$ **(Ans.)**

Example 6 :

Factorise :

(i) $1 - 4(a - 2b)^2$ (ii) $9(x + y)^2 - 16(x - 3y)^2$

Solution :

(i) $1 - 4(a - 2b)^2 = 1 - 2^2\,(a - 2b)^2$

$= 1 - [2(a - 2b)]^2$

$= 1^2 - (2a - 4b)^2$

$= (1 + \overline{2a - 4b})\,(1 - \overline{2a - 4b})$

$= (1 + 2a - 4b)(1 - 2a + 4b)$ **(Ans.)**

(ii) $9(x + y)^2 - 16(x - 3y)^2 = [3(x + y)]^2 - [4(x - 3y)]^2$

$= (3x + 3y)^2 - (4x - 12y)^2$

$= (\overline{3x + 3y} + \overline{4x - 12y})(\overline{3x + 3y} - \overline{4x - 12y})$

$= (3x + 3y + 4x - 12y)(3x + 3y - 4x + 12y)$

$= (7x - 9y)(15y - x)$ **(Ans.)**

EXERCISE 15 (C)

Factorise :

1. $16 - 9x^2$
2. $1 - 100a^2$
3. $4x^2 - 81y^2$
4. $\frac{4}{25} - 25b^2$
5. $(a + 2b)^2 - a^2$
6. $(5a - 3b)^2 - 16b^2$
7. $a^4 - (a^2 - 3b^2)^2$
8. $(5a - 2b)^2 - (2a - b)^2$
9. $1 - 25\,(a + b)^2$
10. $4(2a + b)^2 - (a - b)^2$
11. $25(2x + y)^2 - 16(x - y)^2$
12. $49(x - y)^2 - 9(2x + y)^2$
13. $\left(6\frac{2}{3}\right)^2 - \left(2\frac{1}{3}\right)^2$
14. $\left(7\frac{3}{10}\right)^2 - \left(2\frac{1}{10}\right)^2$
15. $(0{\cdot}7)^2 - (0{\cdot}3)^2$
16. $(4{\cdot}5)^2 - (1{\cdot}5)^2$
17. $75(x + y)^2 - 48(x - y)^2$
18. $a^2 + 4a + 4 - b^2$
19. $a^2 - b^2 - 2b - 1$
20. $x^2 + 6x + 9 - 4y^2$

15.6 FACTORISATION OF TRINOMIALS

Since, the product of two binomials $(2a + b)$ and $(3a - 5b)$

$= (2a + b)(3a - 5b)$

$= 6a^2 - 7ab - 5b^2$; **which is a trinomial.**

$\therefore$ The factors of a trinomial $6a^2 - 7ab - 5b^2$ are the binomials $(2a + b)$ and $(3a - 5b)$.

Before learning the factorisation of a trinomial, it is essential to know how to find out the two numbers whose product and sum are given.

Example 7 :

Find the numbers whose :

(i) product = 6 and sum = 5 (ii) product = 6 and sum = – 5

(iii) product = – 6 and sum = 5 (iv) product = – 6 and sum = – 5

Solution :

(i) Since, product = 6 and sum = 5. The *product* and the *sum* of two numbers are *positive* only when *both the numbers are positive.*

By trial, we find that **the required two numbers are 3 and 2.** **(Ans.)**

Product of 3 and 2 = $3 \times 2 = 6$ and their sum = $3 + 2 = 5$

(ii) Since, product = 6 and sum = – 5

The *product* of two numbers is *positive* and their *sum* is *negative* only when *both the numbers are negative.*

$\therefore$ **Required numbers are – 3 and – 2.** **(Ans.)**

(iii) Since, product = – 6 and sum = 5.

The *product* of two numbers is *negative* and their *sum* is *positive* only when the *larger* of the two numbers is *positive* and the *smaller* is *negative.*

By trial, we find **that the required two numbers are 6 and – 1.** **(Ans.)**

(iv) Since, product = – 6 and sum = – 5. The *product* of two numbers is *negative* and their *sum* is also *negative* only when the *larger* is *negative* and the *smaller* is *positive.*

∴ By trial, we find that **the required two numbers are – 6 and 1.** **(Ans.)**

TEST YOURSELF

6. The two numbers whose :

(a) product = 18 and sum = 11 are ..... and .....

(b) product = 32 and sum = – 12 are ..... and .....

(c) product = – 24 and sum = 2 are ..... and .....

(d) product = – 12 and sum = 11 are ..... and .....

(e) product = – 6 and sum = – 5 are ..... and .....

(f) product = – 44 and sum = – 7 are ..... and .....

Note : The standard forms of a trinomial are :

(i) $6x^2 + 11x + 3$ *i.e.* descending order of the powers of its literal coefficients.

(ii) $3 + 11x + 6x^2$ *i.e.* ascending order of the powers of its literal coefficients.

To factorise a given trinomial, the following steps should be adopted :

1. Find the product of the first and the last terms of the trinomial with their signs. In case of trinomial $6x^2 + 11x + 3$, the product of its first and last terms = $6x^2 \times 3 = 18x^2$.
2. Split the middle term of the given trinomial (*i.e.* + 11x) such that the sum of these two terms is equal to the middle term and their product is equal to the product obtained in step 1 (*i.e.* $18x^2$)

   By trial, we find that the two such terms are + 9x and + 2x.
3. Now by forming the suitable groups, factorise the given trinomial.

   i.e. $\mathbf{6x^2 + 11x + 3} = 6x^2 + 9x + 2x + 3$

   $= 3x(2x + 3) + 1(2x + 3)$

   $= \mathbf{(2x + 3)(3x + 1)}$ **(Ans.)**

Example 8 :

Factorise :

(i) $x^2 - 9x + 20$ (ii) $y^2 + 5y - 24$ (iii) $1 - 3a - 28a^2$

Solution :

(i) Given trinomial = $x^2 - 9x + 20$

The product of its first and the last terms = $x^2 \times 20 = 20x^2$

Splitting the middle term (*i.e.* –9x) into two terms so that their product is $20x^2$ and sum is –9x; we get : –5x and –4x.

∴ $\mathbf{x^2 - 9x + 20} = x^2 - 5x - 4x + 20$

$= x(x - 5) - 4(x - 5)$

$= \mathbf{(x - 5)(x - 4)}$ **(Ans.)**

(ii) Given trinomial is $y^2 + 5y - 24$

The product of its first and the last terms = $y^2 \times -24 = -24y^2$

and, the middle term = $+5y$.

Now find two terms whose product should be $-24y^2$ and sum should be $+5y$. By trial, we find that the required two terms are $+8y$ and $-3y$.

$\therefore$ $\mathbf{y^2 + 5y - 24} = y^2 + 8y - 3y - 24$

$= y(y + 8) - 3(y + 8) = \mathbf{(y + 8)(y - 3)}$ **(Ans.)**

(iii) Given trinomial is $1 - 3a - 28a^2$

Product of the first and the last terms = $1 \times -28a^2 = -28a^2$

and, the middle term = $-3a$

By trial, we find that two terms whose product is $-28a^2$ and sum is $-3a$ are $-7a$ and $+4a$.

$\therefore$ $\mathbf{1 - 3a - 28a^2} = 1 - 7a + 4a - 28a^2$

$= 1(1 - 7a) + 4a(1 - 7a) = \mathbf{(1 - 7a)(1 + 4a)}$ **(Ans.)**

Example 9 :

Factorise : (i) $(a + b)^2 - 11(a + b) - 42$ (ii) $7 + 10(x - y) - 8(x - y)^2$

Solution :

(i) $\mathbf{(a + b)^2 - 11(a + b) - 42} = x^2 - 11x - 42$ [Taking $a + b = x$]

$= x^2 - 14x + 3x - 42$ [Splitting the middle term]

$= x(x - 14) + 3(x - 14)$

$= (x - 14)(x + 3)$

$= \mathbf{(a + b - 14)(a + b + 3)}$ **(Ans.)** [Substituting $x = a + b$]

(ii) $\mathbf{7 + 10(x - y) - 8(x - y)^2} = 7 + 10a - 8a^2$ [Taking $x - y = a$]

$= 7 + 14a - 4a - 8a^2$ [Splitting the middle term]

$= 7(1 + 2a) - 4a(1 + 2a)$

$= (1 + 2a)(7 - 4a)$

$= [1 + 2(x - y)][7 - 4(x - y)]$ [Substituting $a = x - y$]

$= \mathbf{(1 + 2x - 2y)(7 - 4x + 4y)}$ **(Ans.)**

EXERCISE 15 (D)

Factorise :

1. $x^2 + 6x + 8$
2. $x^2 + 4x + 3$
3. $a^2 + 5a + 6$
4. $a^2 - 5a + 6$
5. $a^2 + 5a - 6$
6. $x^2 + 5xy + 4y^2$
7. $a^2 - 3a - 40$
8. $x^2 - x - 72$
9. $x^2 - 10xy + 24y^2$
10. $2a^2 + 7a + 6$
11. $3a^2 - 5a + 2$
12. $7b^2 - 8b + 1$
13. $2a^2 - 17ab + 26b^2$
14. $2x^2 + xy - 6y^2$
15. $4c^2 + 3c - 10$
16. $14x^2 + x - 3$
17. $6 + 7b - 3b^2$
18. $5 + 7x - 6x^2$
19. $4 + y - 14y^2$
20. $5 + 3a - 14a^2$
21. $(2a + b)^2 + 5(2a + b) + 6$
22. $1 - (2x + 3y) - 6(2x + 3y)^2$
23. $(x - 2y)^2 - 12(x - 2y) + 32$
24. $8 + 6(a + b) - 5(a + b)^2$
25. $2(x + 2y)^2 - 5(x + 2y) + 2$

15.7 FACTORISING A PERFECT SQUARE TRINOMIAL

Square of a binomial is called a perfect square trinomial.

Since, $(a + b)^2 = a^2 + 2ab + b^2$

and, $(a - b)^2 = a^2 - 2ab + b^2$

$\therefore$ $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$ are perfect square trinomials.

Any trinomial which can be expressed as $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$ is perfect square trinomial.

Example 10 :

(i) Is $4x^2 + 12xy + 9y^2$ a perfect square trinomial ?

(ii) Is $x^2 - 6xy + 36y^2$ a perfect square trinomial ?

Solution :

(i) $4x^2 + 12xy + 9y^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2$

$= a^2 + 2ab + b^2$ [Taking $2x = a$ and $3y = b$]

$= (a + b)^2$

$= (2x + 3y)^2$ **(Ans.)**

$\therefore$ **The given trinomial $4x^2 + 12xy + 9y^2$ is a perfect square trinomial.**

(ii) $x^2 - 6xy + 36y^2 = (x)^2 - x \times 6y + (6y)^2$

$= a^2 - ab + b^2$ [Taking $x = a$ and $6y = b$]

Since, the given trinomial cannot be expressed as $a^2 - 2ab + b^2$; **it is not a perfect square trinomial.** **(Ans.)**

15.8 FACTORISING COMPLETELY

Example 11 :

Factorise completely : (i) $8x^3 - 18xy^2$ (ii) $3x^2 + 12x - 36$

Solution :

(i) $8x^3 - 18xy^2 = 2x(4x^2 - 9y^2)$ [Taking out the common]

$= 2x[(2x)^2 - (3y)^2]$ [Converting in the form $a^2 - b^2$]

$= 2x(2x + 3y)(2x - 3y)$ **(Ans.)**

(ii) $3x^2 + 12x - 36 = 3(x^2 + 4x - 12)$ [Taking out the common factor]

$= 3(x^2 + 6x - 2x - 12)$ [Factorising the trinomial]

$= 3[x(x + 6) - 2(x + 6)]$

$= 3(x + 6)(x - 2)$ **(Ans.)**

Example 12 :

Factorise completely : (i) $x^2 + 4xy + 4y^2 - 9z^2$ (ii) $16x^4 - y^4$

Solution :

(i) In the given expression $x^2 + 4xy + 4y^2$ is a perfect square trinomial as :

$x^2 + 4xy + 4y^2 = x^2 + 2 \times x \times 2y + (2y)^2$

$= a^2 + 2ab + b^2$ [Taking $x = a$ and $2y = b$]

$= (a + b)^2$

$= (x + 2y)^2$ [Substituting]

$\therefore\ x^2 + 4xy + 4y^2 - 9z^2 = (x + 2y)^2 - (3z)^2$

$= (x + 2y + 3z)(x + 2y - 3z)$ **(Ans.)**

(ii) $16x^4 - y^4 = (4x^2)^2 - (y^2)^2$

$= (4x^2 + y^2)(4x^2 - y^2)$

$= (4x^2 + y^2)[(2x)^2 - (y)^2]$

$= (4x^2 + y^2)(2x + y)(2x - y)$ **(Ans.)**

EXERCISE 15 (E)

1. In each case, find whether the trinomial is a perfect square or not :
   (i) $x^2 + 14x + 49$ (ii) $a^2 - 10a + 25$
   (iii) $4x^2 + 4x + 1$ (iv) $9b^2 + 12b + 16$
   (v) $16x^2 - 16xy + y^2$ (vi) $x^2 - 4x + 16$

Factorise completely :

2. $2 - 8x^2$
3. $8x^2y - 18y^3$
4. $ax^2 - ay^2$
5. $25x^3 - x$
6. $a^4 - b^4$
7. $16x^4 - 81y^4$
8. $625 - x^4$
9. $x^2 - y^2 - 3x - 3y$
10. $x^2 - y^2 - 2x + 2y$
11. $3x^2 + 15x - 72$
12. $2a^2 - 8a - 64$
13. $5b^2 + 45b + 90$
14. $3x^2y + 11xy + 6y$
15. $5ap^2 + 11ap + 2a$
16. $a^2 + 2ab + b^2 - c^2$
17. $x^2 + 6xy + 9y^2 + x + 3y$
18. $4a^2 - 12ab + 9b^2 + 4a - 6b$
19. $2a^2b^2 - 98b^4$
20. $a^2 - 16b^2 - 2a - 8b$

TEST YOURSELF

7. $5x + 10xy - 7ax - 14axy = 5x(\ldots\ldots) - 7ax\ (\ldots\ldots) = \ldots\ldots$
8. $(3x + 9y)\ (6x - 18y) = 3(\ldots\ldots) \times 6(\ldots\ldots) = 18(\ldots\ldots)$
9. $8x^2 - \frac{8}{x^2} = \ldots\ldots = \ldots\ldots$
10. $8x^2 - 20xy + 8y^2 = \ldots\ldots = \ldots\ldots\ \ldots\ldots$
    $= \ldots\ldots\ \ldots\ldots$

EXERCISE 15 (F)

1. Factorise :
   (i) $6x^3 - 8x^2$
   (ii) $35a^3b^2c + 42ab^2c^2$
   (iii) $36x^2y^2 - 30x^3y^3 + 48x^3y^2$
   (iv) $8(2a + 3b)^3 - 12(2a + 3b)^2$
   (v) $9a(x - 2y)^4 - 12a\ (x - 2y)^3$
2. Factorise :
   (i) $a^2 - ab - 3a + 3b$
   (ii) $x^2y - xy^2 + 5x - 5y$
   (iii) $a^2 - ab\ (1 - b) - b^3$
   (iv) $xy^2 + (x - 1)y - 1$
   (v) $(ax + by)^2 + (bx - ay)^2$
   (vi) $ab(x^2 + y^2)\ -xy(a^2 + b^2)$
   (vii) $m - 1 - (m - 1)^2 + am - a$
3. Factorise :
   (i) $a^2 - (b - c)^2$
   (ii) $25(2x - y)^2 - 16(x - 2y)^2$
   (iii) $16(5x + 4)^2 - 9(3x - 2)^2$
   (iv) $9x^2 - \frac{1}{16}$
   (v) $25(x - 2y)^2 - 4$
4. Factorise :
   (i) $a^2 - 23a + 42.$ (ii) $a^2 - 23a - 108$
   (iii) $1 - 18x - 63x^2$ (iv) $5x^2 - 4xy - 12y^2$
   (v) $x(3x + 14) + 8$ (vi) $5 - 4x(1 + 3x)$
   (vii) $x^2y^2 - 3xy - 40$
   (viii) $(3x - 2y)^2 - 5(3x - 2y) - 24$
   (ix) $12(a + b)^2 - (a + b) - 35$
5. Factorise :
   (i) $15(5x - 4)^2 - 10(5x - 4)$
   (ii) $3a^2x - bx + 3a^2 - b$
   (iii) $b(c - d)^2 + a(d - c) + 3(c - d)$
   (iv) $ax^2 + b^2y - ab^2 - x^2y$
   (v) $1 - 3x - 3y - 4(x + y)^2$
6. Factorise :
   (i) $2a^3 - 50a$ (ii) $54a^2b^2 - 6$
   (iii) $64a^2b - 144b^3$ (iv) $(2x - y)^3 - (2x - y)$
   (v) $x^2 - 2xy + y^2 - z^2$
   (vi) $x^2 - y^2 - 2yz - z^2$
   (vii) $7a^5 - 567a$ (viii) $5x^2 - \frac{20x^4}{9}$
7. Factorise $xy^2 - xz^2$, Hence, find the value of :
   (i) $9 \times 8^2 - 9 \times 2^2$
   (ii) $40 \times 5.5^2 - 40 \times 4.5^2$
8. Factorise :
   (i) $(a - 3b)^2 - 36\ b^2$
   (ii) $25(a - 5b)^2 - 4(a - 3b)^2$
   (iii) $a^2 - 0{\cdot}36\ b^2$
   (iv) $a^4 - 625$
   (v) $x^4 - 5x^2 - 36$
   (vi) $15(2x - y)^2 - 16(2x - y) - 15$
9. Factorise $a^2b - b^3$. Using this result, find the value of $101^2 \times 100 - 100^3$
10. Evaluate (using factors) : $301^2 \times 300 - 300^3$.

ANSWERS

TEST YOURSELF

1. x, y **2.** x, y ; 4x + 7 **3.** x, y, (4x + 7), (x – 8) **4.** (2x + 3), (2x – 3) **5.** (x – 3y) and (x – 2y)
6. (a) 9 ; 2 (b) –4 ; –8, (c) 6 ; –4 (d) 12 ; –1 (e) –6 ; 1 (f) –11 ; 4 **7.** 1 + 2y, 1 + 2y, (1 + 2y)
(5x – 7ax) = x(1 + 2y) (5 – 7a) **8.** x + 3y ; x – 3y ; $x^2 - 9y^2$ **9.** $8(x^2 - \frac{1}{x^2})$; $8(x + \frac{1}{x})(x - \frac{1}{x})$
10. $4(2x^2 - 5xy + 2y^2) = 4(2x^2 - 4xy - xy + 2y^2) = 4[2x(x - 2y) - y(x - 2y)] = 4(x - 2y)(2x - y)$

EXERCISE 15(A)

1. 5(3x + 1) **2.** $a(a^2 - a + 1)$ **3.** $3x^2(1 + 2x)$ **4.** 4a(a – 2b) **5.** $2x^3b^2(1 - 2x^2b^2)$ **6.** $5x^3y(3xy^2 - 4)$
7. $b(a^3 - a^2b - b^2)$ **8.** $y(6x^2 + 9xy + 4y^2)$ **9.** $17a^2b^4(a^4b^4 - 2a^2b^2 + 3)$ **10.** $3x^3y(x^2 - 9xy + 4y^2)$
11. $(a - b)(x^2 - y^2 + z^2)$ **12.** 2x(a + b) **13.** (2a + b) (2b – 3c) **14.** $3abc(4 - 2abc + a^2b^2c^2)$
15. 2(3x – 2y) (2x – y) **16.** (a + 2b) (3a + 2b) **17.** $2(a^2 + b^2)(3xy + 4yz - 5xz)$

EXERCISE 15(B)

1. (a + x) (a + b) **2.** (a – b) (a – c) **3.** (a – 2) (b + a) **4.** $(a - 1)(a^2 + 1)$ **5.** (a – 2b) (2 – x)
6. (x – a) (y – a + b) **7.** $(x - 2)(3x^4 - 2x^2 + 1)$ **8.** (xy + 1) (3 – x) **9.** $(2 - b)(3a^2 + c^2)$ **10.** $3(b - 4)(a^2 - 3)$
11. (x – a) (x + 3) **12.** (x – b) (x + 2) **13.** (b – c) (a + d) **14.** (b – 1) (ab + c) **15.** $(a - c)(b^2 + ac)$
16. $(a - 1)(a^2 - b + 1)$ **17.** (ac – bd) (bc – ad) **18.** (2b – y) (ab + cy) **19.** (a + 2b + 3c) (x – 3)
20. $(b^2c - 1)(2a + 3b - 4c)$

EXERCISE 15(C)

1. (4 + 3x) (4 – 3x) **2.** (1 + 10a) (1 – 10a) **3.** (2x + 9y) (2x – 9y) **4.** $\left(\frac{2}{5} + 5b\right)\left(\frac{2}{5} - 5b\right)$ **5.** 4b(a + b)
6. (5a + b) (5a – 7b) **7.** $3b^2(2a^2 - 3b^2)$ **8.** (7a – 3b) (3a – b) **9.** (1 + 5a + 5b) (1 – 5a – 5b)
10. 3(5a + b) (a + b) **11.** 3(14x + y) (2x + 3y) **12.** (13x – 4y) (x – 10y) **13.** 39 **14.** $48\frac{22}{25}$
15. 0·4 **16.** 18 **17.** 3(9x + y) (x + 9y) **18.** (a + 2 + b) (a + 2 – b) **19.** (a + b + 1) (a – b – 1)
20. (x + 3 + 2y) (x + 3 – 2y)

EXERCISE 15(D)

1. (x + 4) (x + 2) **2.** (x + 3) (x + 1) **3.** (a + 3) (a + 2) **4.** (a – 3) (a – 2) **5.** (a + 6) (a – 1)
6. (x + 4y) (x + y) **7.** (a – 8) (a + 5) **8.** (x – 9) (x + 8) **9.** (x – 6y) (x – 4y) **10.** (2a + 3) (a + 2)
11. (a – 1) (3a – 2) **12.** (b – 1) (7b – 1) **13.** (2a – 13b) (a – 2b) **14.** (x + 2y) (2x – 3y)
15. (c + 2) (4c – 5) **16.** (2x + 1) (7x – 3) **17.** (2 + 3b) (3 – b) **18.** (1 + 2x) (5 – 3x) **19.** (1 + 2y) (4 – 7y)
20. (1 + 2a) (5 – 7a) **21.** (2a + b + 3) (2a + b + 2) **22.** (1 – 6x – 9y) (1 + 4x + 6y)
23. (x – 2y – 8) (x – 2y – 4) **24.** (4 + 5a + 5b) (2 – a – b) **25.** (x + 2y – 2) (2x + 4y – 1)

EXERCISE 15(E)

1. (i) Yes (ii) Yes (iii) Yes (iv) No (v) No (vi) No **2.** 2(1 + 2x) (1 – 2x) **3.** 2y(2x + 3y) (2x – 3y)
4. a(x + y) (x – y) **5.** x(5x + 1) (5x – 1) **6.** $(a^2 + b^2)(a + b)(a - b)$ **7.** $(4x^2 + 9y^2)(2x + 3y)(2x - 3y)$
8. $(25 + x^2)(5 + x)(5 - x)$ **9.** (x + y) (x – y – 3) **10.** (x – y) (x + y – 2) **11.** 3(x + 8) (x – 3)
12. 2(a – 8) (a + 4) **13.** 5(b + 6) (b + 3) **14.** y(x + 3) (3x + 2) **15.** a(p + 2) (5p + 1) **16.** (a + b + c) (a + b – c)
17. (x + 3y) (x + 3y + 1) **18.** (2a – 3b) (2a – 3b + 2) **19.** $2b^2(a + 7b)(a - 7b)$ **20.** (a + 4b) (a – 4b – 2)

EXERCISE 15(F)

1. (i) $2x^2(3x - 4)$ (ii) $7ab^2c(5a^2 + 6c)$ (iii) $6x^2y^2(6 - 5xy + 8x)$ (iv) $4(2a + 3b)^2 (4a + 6b - 3)$ (v) $3a(x - 2y)^3 (3x - 6y - 4)$ **2.** (i) $(a - b) (a - 3)$ (ii) $(x - y) (xy + 5)$ (iii) $(a - b) (a + b^2)$ (iv) $(y + 1) (xy - 1)$ (v) $(a^2 + b^2) (x^2 + y^2)$ (vi) $(bx - ay) (ax - by)$ (vii) $(m - 1) (2 - m + a)$

3. (i) $(a + b - c) (a - b + c)$ (ii) $3(14x - 13y) (2x + y)$ (iii) $11(29x + 10) (x + 2)$ (iv) $(3x + \frac{1}{4}) (3x - \frac{1}{4})$ (v) $(5x - 10y + 2) (5x - 10y - 2)$ **4.** (i) $(a - 21) (a - 2)$ (ii) $(a - 27) (a + 4)$ (iii) $(1 - 21x) (1 + 3x)$ (iv) $(x - 2y) (5x + 6y)$ (v) $(x + 4) (3x + 2)$ (vi) $(1 - 2x) (5 + 6x)$ (vii) $(xy - 8) (xy + 5)$ (viii) $(3x - 2y - 8) (3x - 2y + 3)$ (ix) $(4a + 4b - 7) (3a + 3b + 5)$

5. (i) $5(5x - 4) (5x - 14)$ (ii) $(3a^2 - b) (x + a)$ (iii) $(c - d) (bc - bd - a + 3)$ (iv) $(a - y) (x + b) (x - b)$ (v) $(1 - 4x - 4y) (1 + x + y)$ **6.** (i) $2a(a + 5) (a - 5)$ (ii) $6(3ab + 1) (3ab - 1)$ (iii) $16b(2a + 3b) (2a - 3b)$ (iv) $(2x - y) (2x - y + 1) (2x - y - 1)$ (v) $(x - y + z) (x - y - z)$ (vi) $(x + y + z) (x - y - z)$ (vii) $7a(a^2 + 9) (a + 3) (a - 3)$ (viii) $5x^2\left(1+\frac{2x}{3}\right)\left(1-\frac{2x}{3}\right)$ **7.** $x(y + z) (y - z)$ (i) 540 (ii) 400

8. (i) $(a + 3b) (a - 9b)$ (ii) $(7a - 31b) (3a - 19b)$ (iii) $(a + 0{\cdot}6\,b) (a - 0{\cdot}6\,b)$ (iv) $(a^2 + 25) (a + 5) (a - 5)$ (v) $(x^2 + 4) (x + 3) (x - 3)$ (vi) $(6x - 3y - 5) (10x - 5y + 3)$ **9.** $b(a + b) (a - b)$ and 20100 **10.** 180300.