CHAPTER 14

SPECIAL PRODUCTS AND EXPANSIONS

14.1 REVIEW

1. Special Products

The multiplications of certain types of expressions can be obtained by direct or short cut method. Such multiplications are known as ***special products***.

For example (The product of two binomials) :

1. $(x + a)(x + b) = x(x + b) + a(x + b)$
$= x^2 + bx + ax + ab$
$= x^2 + ax + bx + ab = \mathbf{x^2 + (a + b)x + ab}$

2. $(x + a)(x - b) = x(x - b) + a(x - b)$
$= x^2 - bx + ax - ab$
$= x^2 + ax - bx - ab = \mathbf{x^2 + (a - b)x - ab}$

3. $(x - a)(x + b) = x(x + b) - a(x + b)$
$= x^2 + bx - ax - ab$
$= x^2 - ax + bx - ab = \mathbf{x^2 - (a - b)x - ab}$

4. $(x - a)(x - b) = x(x - b) - a(x - b)$
$= x^2 - bx - ax + ab$
$= x^2 - ax - bx + ab = \mathbf{x^2 - (a + b)x + ab}$

Examples (Using direct method) :

1. $(x + 5)(x + 3) = x^2 + (5 + 3)x + 5 \times 3 = \mathbf{x^2 + 8x + 15}$
2. $(x + 5)(x - 3) = x^2 + (5 - 3)x - 5 \times 3 = \mathbf{x^2 + 2x - 15}$
3. $(x - 5)(x + 3) = x^2 - (5 - 3)x - 5 \times 3 = \mathbf{x^2 - 2x - 15}$
4. $(x - 5)(x - 3) = x^2 - (5 + 3)x + 5 \times 3 = \mathbf{x^2 - 8x + 15}$

TEST YOURSELF

1. $(x + 15)(x + 4)$ = ... = ...
2. $(x + 15)(x - 4)$ = ... = ...
3. $(x - 15)(x + 4)$ = ... = ...
4. $(x - 15)(x - 4)$ = ... = ...

14.2 IMPORTANT

While using direct method, the product of two binomials gives three terms :

(i) The first term = Product of the first terms of the two binomials

(ii) The middle term = (First term of first binomial × second term of second binomial) + (second term of first binomial × first term of second binomial)
= Product of outer terms + Product of inner terms

(iii) The third term = Product of the second terms of the two binomials.

Example 1 :

Evaluate :

(i) $(2x + 3y)(3x + 4y)$ (ii) $(2a + 3)(5a - 7)$

(iii) $(4a - 3b)(2a + 5b)$ (iv) $(7x - 3)(2x - 9)$.

Solution :

(i) $(2x + 3y)(3x + 4y) = (2x \times 3x) + (2x \times 4y + 3y \times 3x) + (3y \times 4y)$

$= 6x^2 + (8xy + 9xy) + (12y^2)$

$= \mathbf{6x^2 + 17xy + 12y^2}$ **(Ans.)**

(ii) $(2a + 3)(5a - 7) = (2a \times 5a) + (2a \times -7 + 3 \times 5a) + (3 \times -7)$

$= 10a^2 + (-14a + 15a) + (-21)$

$= \mathbf{10a^2 + a - 21}$ **(Ans.)**

(iii) $(4a - 3b)(2a + 5b) = (4a \times 2a) + (4a \times 5b + -3b \times 2a) + (-3b \times 5b)$

$= 8a^2 + (20ab - 6ab) + (-15b^2)$

$= \mathbf{8a^2 + 14ab - 15b^2}$ **(Ans.)**

(iv) $(7x - 3)(2x - 9) = (7x \times 2x) + (7x \times -9 + -3 \times 2x) + (-3 \times -9)$

$= 14x^2 + (-63x - 6x) + (27)$

$= \mathbf{14x^2 - 69x + 27}$ **(Ans.)**

14.3 PRODUCT OF SUM AND DIFFERENCE OF TWO TERMS

Consider the two terms 5x and 4y.

the sum of these two terms = 5x + 4y and

the difference of these terms = 5x – 4y.

And, **the product of their sum and their difference**

$= (5x + 4y)(5x - 4y)$

$= 5x(5x - 4y) + 4y(5x - 4y)$

$= 25x^2 - 20xy + 20xy - 16y^2$

$= 25x^2 - 16y^2$

$= (5x)^2 - (4y)^2 =$ **(First Term)^2 – (Second Term)^2**

TEST YOURSELF

5. $(x + 3)(x - 3)$ = ... = ...
6. $(3x + 4y)(3x - 4y)$ = ... = ...
7. $(1{\cdot}6x^2 - 5)(1{\cdot}6x^2 + 5)$ = ... = ...
8. $(5a^2 + 8b)(5a^2 - 8b)$ = ... = ...

Example 2 :

Evaluate :

(i) $(x - 2)(x + 2)(x^2 + 4)$ (ii) $(2a - 5b)(2a + 5b)(4a^2 + 25b^2)$

Solution :

(i) $(x - 2)(x + 2)(x^2 + 4) = [(x - 2)(x + 2)](x^2 + 4)$

$= (x^2 - 2^2)(x^2 + 4)$

$= (x^2 - 4)(x^2 + 4)$

$= (x^2)^2 - (4)^2 = \mathbf{x^4 - 16}$ **(Ans.)**

(ii) $(2a - 5b)(2a + 5b)(4a^2 + 25b^2) = [(2a - 5b)(2a + 5b)](4a^2 + 25b^2)$

$= [(2a)^2 - (5b)^2](4a^2 + 25b^2)$

$= (4a^2 - 25b^2)(4a^2 + 25b^2)$

$= (4a^2)^2 - (25b^2)^2 = \mathbf{16a^4 - 625b^4}$ **(Ans.)**

Example 3 :

Use of the formula $(a + b)(a - b) = a^2 - b^2$ to find the value of :

(i) 107×93 (ii) 30.8×29.2

Solution :

(i) $107 \times 93 = (100 + 7)(100 - 7)$

$= (100)^2 - (7)^2 = 10000 - 49 = \mathbf{9951}$ **(Ans.)**

(ii) $30.8 \times 29.2 = (30 + 0.8)(30 - 0.8)$

$= (30)^2 - (0.8)^2 = 900 - 0.64 = \mathbf{899.36}$ **(Ans.)**

EXERCISE 14 (A)

1. Use direct method to evaluate the following products :

(i) $(x + 8)(x + 3)$ (ii) $(y + 5)(y - 3)$

(iii) $(a - 8)(a + 2)$ (iv) $(b - 3)(b - 5)$

(v) $(3x - 2y)(2x + y)$ (vi) $(5a + 16)(3a - 7)$

(vii) $(8 - b)(3 + b)$

2. Use direct method to evaluate :

(i) $(x + 1)(x - 1)$ (ii) $(2 + a)(2 - a)$

(iii) $(3b - 1)(3b + 1)$ (iv) $(4 + 5x)(4 - 5x)$

(v) $(2a + 3)(2a - 3)$ (vi) $(xy + 4)(xy - 4)$

(vii) $(ab + x^2)(ab - x^2)$

(viii) $(3x^2 + 5y^2)(3x^2 - 5y^2)$

(ix) $\left(z - \frac{2}{3}\right)\left(z + \frac{2}{3}\right)$

(x) $\left(\frac{3}{5}a + \frac{1}{2}\right)\left(\frac{3}{5}a - \frac{1}{2}\right)$

(xi) $(0{\cdot}5 - 2a)(0{\cdot}5 + 2a)$

(xii) $\left(\frac{a}{2} - \frac{b}{3}\right)\left(\frac{a}{2} + \frac{b}{3}\right)$

3. Evaluate :

(i) $(a + 1)(a - 1)(a^2 + 1)$

(ii) $(a + b)(a - b)(a^2 + b^2)$

(iii) $(2a - b)(2a + b)(4a^2 + b^2)$

(iv) $(3 - 2x)(3 + 2x)(9 + 4x^2)$

(v) $(3x - 4y)(3x + 4y)(9x^2 + 16y^2)$

4. Use the product $(a + b)(a - b) = a^2 - b^2$ to evaluate :

(i) 21×19 (ii) 33×27

(iii) 103×97 (iv) $9{\cdot}8 \times 10{\cdot}2$

(v) $7{\cdot}7 \times 8{\cdot}3$ (vi) $4{\cdot}6 \times 5{\cdot}4$

5. Evaluate :

(i) $(6 - xy)(6 + xy)$

(ii) $\left(7x + \frac{2}{3}y\right)\left(7x - \frac{2}{3}y\right)$

(iii) $\left(\frac{a}{2b} + \frac{2b}{a}\right)\left(\frac{a}{2b} - \frac{2b}{a}\right)$

(iv) $\left(3x - \frac{1}{2y}\right)\left(3x + \frac{1}{2y}\right)$

(v) $(2a + 3)(2a - 3)(4a^2 + 9)$

(vi) $(a + bc)(a - bc)(a^2 + b^2c^2)$

(vii) $(5x + 8y)(3x + 5y)$

(viii) $(7x + 15y)(5x - 4y)$

(ix) $(2a - 3b)(3a + 4b)$

(x) $(9a - 7b)(3a - b)$

14.4 EXPANSIONS

In expansion, we study the multiplication of an expression by itself to obtain its second, third or higher power.

1. $(a + b)^2 = (a + b)(a + b)$

$= a^2 + ab + ab + b^2 = \mathbf{a^2 + 2ab + b^2}$

$$(\text{Sum of two terms})^2 = (\text{1st term})^2 + 2 \times \text{1st term} \times \text{2nd term} + (\text{2nd term})^2$$

2. $(a - b)^2 = (a - b)(a - b)$

$= a^2 - ab - ab + b^2 = \mathbf{a^2 - 2ab + b^2}$

$$(\text{Difference of two terms})^2 = (\text{1st term})^2 - 2 \times \text{1st term} \times \text{2nd term} + (\text{2nd term})^2$$

Examples :

1. $(3x + 4y)^2 = (\text{Ist term})^2 + 2 \times \text{Ist term} \times \text{2nd term} + (\text{2nd term})^2$
$= (3x)^2 + 2 \times 3x \times 4y + (4y)^2$
$= 9x^2 + 24xy + 16y^2$ **(Ans.)**

2. $\left(\frac{3x}{2y} - \frac{2y}{3x}\right)^2 = (\text{Ist term})^2 - 2 \times \text{Ist term} \times \text{2nd term} + (\text{2nd term})^2$
$= \left(\frac{3x}{2y}\right)^2 - 2 \times \frac{3x}{2y} \times \frac{2y}{3x} + \left(\frac{2y}{3x}\right)^2 = \frac{9x^2}{4y^2} - 2 + \frac{4y^2}{9x^2}$ **(Ans.)**

3. $(208)^2 = (200 + 8)^2$
$= (200)^2 + 2 \times 200 \times 8 + (8)^2$
$= 40000 + 3200 + 64 = 43264$ **(Ans.)**

4. $(9{\cdot}7)^2 = (10 - 0{\cdot}3)^2$
$= (10)^2 - 2 \times 10 \times 0{\cdot}3 + (0.3)^2 = 100 - 6 + 0{\cdot}09 = 94{\cdot}09$ **(Ans.)**

TEST YOURSELF

Using expansions, evaluate :

9. $\left(2a - \frac{3}{2}\right)^2$ = ... = ...

10. $\left(x + \frac{1}{2x}\right)^2$ = ... = ...

11. $(2x^2 - 3y)^2$ = ... = ...

12. $(107)^2$ = ... = ... = ...

13. $(97)^2$ = ... = ... = ...

14. $(10{\cdot}6)^2$ = ... = ... = ...

15. $(19{\cdot}8)^2$ = ... = ... = ...

14.5 IMPORTANT FORMULAE TO BE MEMORISED

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$
4. $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2$
5. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
6. $(a + b - c)^2 = a^2 + b^2 + (-c)^2 + 2(a \times b) + 2(b \times -c) + 2(-c \times a)$
$= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

Example 4 :

Expand : (i) $\left(2x+\frac{1}{2x}\right)^2$ (ii) $\left(3a-\frac{1}{a}\right)^2$

(iii) $(a + 2b - 5c)^2$ (iv) $(a - 2b - 5c)^2$

Solution :

(i) $\left(2x+\frac{1}{2x}\right)^2 = (2x)^2 + \left(\frac{1}{2x}\right)^2 + 2 \times 2x \times \frac{1}{2x}$

$= 4x^2 + \frac{1}{4x^2} + 2$ **(Ans.)**

(ii) $\left(3a-\frac{1}{a}\right)^2 = (3a)^2 + \left(\frac{1}{a}\right)^2 - 2 \times 3a \times \frac{1}{a}$

$= 9a^2 + \frac{1}{a^2} - 6$ **(Ans.)**

(iii) $(a + 2b - 5c)^2 = (a)^2 + (2b)^2 + (-5c)^2 + 2(a \times 2b) + 2(2b \times -5c) + 2(-5c \times a)$

$= a^2 + 4b^2 + 25c^2 + 4ab - 20bc - 10ca$ **(Ans.)**

(iv) $(a - 2b - 5c)^2 = (a)^2 + (-2b)^2 + (-5c)^2 + 2(a \times -2b) + 2(-2b \times -5c) + 2(-5c \times a)$

$= a^2 + 4b^2 + 25c^2 - 4ab + 20bc - 10ca$ **(Ans.)**

14.6 CUBES OF BINOMIALS

1. $(a + b)^3 = (a + b)(a + b)^2$

$= (a + b)(a^2 + 2ab + b^2)$

$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$

$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3 = a^3 + 3a^2b + 3ab^2 + b^3$

2. $(a - b)^3 = (a - b)(a - b)^2$

$= (a - b)(a^2 - 2ab + b^2)$

$= a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2)$

$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3 = a^3 - 3a^2b + 3ab^2 - b^3$

1. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + b^3 + 3ab(a + b)$

2. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - b^3 - 3ab(a - b)$

Example 5 :

Expand : (i) $(3x + 2y)^3$ (ii) $(5y - 3x)^3$

Solution :

(i) Since, $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\therefore$ $(3x + 2y)^3 = (3x)^3 + 3 \times (3x)^2 \times 2y + 3 \times 3x \times (2y)^2 + (2y)^3$

$= 27x^3 + 54x^2y + 36xy^2 + 8y^3$ **(Ans.)**

(ii) Since, $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$\therefore$ $(5y - 3x)^3 = (5y)^3 - 3 \times (5y)^2 \times 3x + 3 \times 5y \times (3x)^2 - (3x)^3$

$= 125y^3 - 225y^2x + 135yx^2 - 27x^3$ **(Ans.)**

EXERCISE 14 (B)

1. Expand :

(i) $(2a + b)^2$ (ii) $(a - 2b)^2$

(iii) $\left(a+\frac{1}{2a}\right)^2$ (iv) $\left(2a-\frac{1}{a}\right)^2$

(v) $(a + b - c)^2$ (vi) $(a - b + c)^2$

(vii) $\left(3x+\frac{1}{3x}\right)^2$ (viii) $\left(2x-\frac{1}{2x}\right)^2$

2. Find the square of :

(i) $x + 3y$ (ii) $2x - 5y$

(iii) $a + \frac{1}{5a}$ (iv) $2a - \frac{1}{a}$

(v) $x - 2y + 1$ (vi) $3a - 2b - 5c$

(vii) $2x + \frac{1}{x} + 1$ (viii) $5 - x + \frac{2}{x}$

(ix) $2x - 3y + z$ (x) $x + \frac{1}{x} - 1$

3. Evaluate using expansion of $(a + b)^2$ or $(a - b)^2$:

(i) $(208)^2$ (ii) $(92)^2$

(iii) $(415)^2$ (iv) $(188)^2$

(v) $(9.4)^2$ (vi) $(20.7)^2$

4. Expand :

(i) $(2a + b)^3$ (ii) $(a - 2b)^3$

(iii) $(3x - 2y)^3$ (iv) $(x + 5y)^3$

(v) $\left(a+\frac{1}{a}\right)^3$ (vi) $\left(2a-\frac{1}{2a}\right)^3$

5. Find the cube of :

(i) $a + 2$ (ii) $2a - 1$

(iii) $2a + 3b$ (iv) $3b - 2a$

(v) $2x + \frac{1}{x}$ (vi) $x - \frac{1}{2}$

14.7 APPLICATION OF FORMULAE

Example 6 :

(i) If $a + b = 8$ and $ab = 15$, find $a^2 + b^2$.

(ii) If $a - b = 3$ and $a^2 + b^2 = 29$, find ab.

Solution :

(i) $(a + b)^2 = a^2 + b^2 + 2ab = a^2 + b^2 + 2ab$

$\Rightarrow (8)^2 = a^2 + b^2 + 2 \times 15$

$\Rightarrow 64 - 30 = a^2 + b^2 \quad \therefore a^2 + b^2 = 34$ **(Ans.)**

(ii) $(a - b)^2 = a^2 + b^2 - 2ab = a^2 + b^2 - 2ab$

$\Rightarrow (3)^2 = 29 - 2ab$

$\Rightarrow 2ab = 29 - 9 = 20 \quad \therefore ab = \frac{20}{2} = 10$ **(Ans.)**

Example 7 :

If $a^2 + b^2 = 73$ and $ab = 24$; find : (i) $a + b$ (ii) $a - b$

Solution :

(i) $(a + b)^2 = a^2 + b^2 + 2ab \Rightarrow (a + b)^2 = 73 + 2 \times 24 = 73 + 48 = 121$

$\therefore a + b = \pm\sqrt{121} = \pm 11$ **(Ans.)**

(ii) $(a - b)^2 = a^2 + b^2 - 2ab \Rightarrow (a - b)^2 = 73 - 2 \times 24 = 25$

$\therefore a - b = \pm\sqrt{25} = \pm 5$ **(Ans.)**

Example 8 :

If $a^2 + \frac{1}{a^2} = 2$; find : (i) $a + \frac{1}{a}$ (ii) $a - \frac{1}{a}$

Solution :

(i) $\left(a+\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}+2 \Rightarrow \left(a+\frac{1}{a}\right)^2 = 2+2=4$

$\therefore \quad a+\frac{1}{a} = \pm\sqrt{4} = \pm 2$ **(Ans.)**

(ii) $\left(a-\frac{1}{a}\right)^2 = a^2+\frac{1}{a^2}-2 \Rightarrow \left(a-\frac{1}{a}\right)^2 = 2-2=0$

$\therefore \quad a-\frac{1}{a} = \sqrt{0} = 0$ **(Ans.)**

Example 9 :

If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 29$, find $ab + bc + ca$.

Solution :

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow \quad 9^2 = 29 + 2(ab + bc + ca)$ [By substituting the given values]

$\Rightarrow \quad 81 - 29 = 2(ab + bc + ca)$

$\Rightarrow \quad 52 = 2(ab + bc + ca)$

$\Rightarrow \quad ab + bc + ca = \frac{52}{2} = 26$ **(Ans.)**

Example 10 :

If $a + b = 5$ and $ab = 6$, find $a^3 + b^3$.

Solution :

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$\Rightarrow \quad (5)^3 = a^3 + b^3 + 3 \times 6 \times 5$

$\Rightarrow \quad 125 - 90 = a^3 + b^3 \quad \therefore \quad a^3 + b^3 = 35$ **(Ans.)**

Example 11 :

If $a - \frac{1}{a} = 3$; find $a^3 - \frac{1}{a^3}$

Solution :

Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$\Rightarrow \quad \left(a-\frac{1}{a}\right)^3 = a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a}\left(a-\frac{1}{a}\right)$

$\Rightarrow \quad (3)^3 = a^3 - \frac{1}{a^3} - 3 \times 3$ [Given $a - \frac{1}{a} = 3$]

$\Rightarrow \quad 27 + 9 = a^3 - \frac{1}{a^3} \quad \Rightarrow \quad a^3 - \frac{1}{a^3} = 36$ **(Ans.)**

Example 12 :

The sum of two numbers is 4 and their product is 3. Find :

(i) the sum of their squares. (ii) the sum of their cubes.

Solution :

Let the numbers be x and y.

$\therefore \quad x + y = 4$ and $xy = 3$

To find (i) $x^2 + y^2$ (ii) $x^3 + y^3$.

(i) $(x + y)^2 = x^2 + y^2 + 2xy$

$\Rightarrow 4^2 = x^2 + y^2 + 2 \times 3 \Rightarrow x^2 + y^2 = 10$ **(Ans.)**

(ii) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow 4^3 = x^3 + y^3 + 3 \times 3 \times 4 \Rightarrow x^3 + y^3 = 64 - 36 = 28$ **(Ans.)**

TEST YOURSELF

16. If $x^2 + y^2 = 25$ and $xy = 12$; $(x + y)^2$ = ... = ... and $x + y$ = ...

17. If $x^2 + y^2 = 74$ and $xy = 35$; $(x - y)^2$ = ... = ... and $x - y$ = ...

18. If $x + \frac{1}{x} = \frac{5}{2}$; $x^2 + \frac{1}{x^2}$ = ... = ...;

$(x - \frac{1}{x})^2$ = ... = ... and $x - \frac{1}{x}$ = ...

19. If $a^2 + b^2 + c^2 = p$ and $ab + bc + ca = q$; $(a + b + c)^2$ = ...
= ..... and $a + b + c$ = ...

EXERCISE 14 (C)

1. If $a + b = 5$ and $ab = 6$, find $a^2 + b^2$
2. If $a - b = 6$ and $ab = 16$, find $a^2 + b^2$
3. If $a^2 + b^2 = 29$ and $ab = 10$, find :
   (i) $a + b$ (ii) $a - b$
4. If $a^2 + b^2 = 10$ and $ab = 3$; find :
   (i) $a - b$ (ii) $a + b$
5. If $a + \frac{1}{a} = 3$, find : $a^2 + \frac{1}{a^2}$
6. If $a - \frac{1}{a} = 4$, find : $a^2 + \frac{1}{a^2}$
7. If $a^2 + \frac{1}{a^2} = 23$, find : $a + \frac{1}{a}$
8. If $a^2 + \frac{1}{a^2} = 11$, find : $a - \frac{1}{a}$
9. If $a + b + c = 10$ and $a^2 + b^2 + c^2 = 38$, find :
   $ab + bc + ca$
10. Find : $a^2 + b^2 + c^2$, if $a + b + c = 9$ and $ab + bc + ca = 24$.
11. Find : $a + b + c$, if $a^2 + b^2 + c^2 = 83$ and $ab + bc + ca = 71$.
12. If $a + b = 6$ and $ab = 8$, find : $a^3 + b^3$.
13. If $a - b = 3$ and $ab = 10$, find : $a^3 - b^3$.
14. Find : $a^3 + \frac{1}{a^3}$, if $a + \frac{1}{a} = 5$.
15. Find : $a^3 - \frac{1}{a^3}$, if $a - \frac{1}{a} = 4$.
16. If $2x - \frac{1}{2x} = 4$, find :
   (i) $4x^2 + \frac{1}{4x^2}$ (ii) $8x^3 - \frac{1}{8x^3}$
17. If $3x + \frac{1}{3x} = 3$, find :
   (i) $9x^2 + \frac{1}{9x^2}$ (ii) $27x^3 + \frac{1}{27x^3}$
18. The sum of the squares of two numbers is 13 and their product is 6. Find :
   (i) the sum of the two numbers.
   (ii) the difference between them.

EXERCISE 14 (D)

1. Evaluate :
   (i) $\left(3x + \frac{1}{2}\right)\left(2x + \frac{1}{3}\right)$
   (ii) $(2a + 0.5)(7a - 0.3)$
   (iii) $(9 - y)(7 + y)$
   (iv) $(2 - z)(15 - z)$

(v) $(a^2 + 5)(a^2 - 3)$

(vi) $(4 - ab)(8 + ab)$

(vii) $(5xy - 7)(7xy + 9)$

(viii) $(3a^2 - 4b^2)(8a^2 - 3b^2)$

2. Evaluate :

(i) $\left(2x - \frac{3}{5}\right)\left(2x + \frac{3}{5}\right)$

(ii) $\left(\frac{4}{7}a + \frac{3}{4}b\right)\left(\frac{4}{7}a - \frac{3}{4}b\right)$

(iii) $(6 - 5xy)(6 + 5xy)$

(iv) $\left(2a + \frac{1}{2a}\right)\left(2a - \frac{1}{2a}\right)$

(v) $(4x^2 - 5y^2)(4x^2 + 5y^2)$

(vi) $(1.6x + 0.7y)(1.6x - 0{\cdot}7y)$

(vii) $(m + 3)(m - 3)(m^2 + 9)$

(viii) $(3x + 4y)(3x - 4y)(9x^2 + 16y^2)$

(ix) $(a + bc)(a - bc)(a^2 + b^2c^2)$

(x) 203×197

(xi) 20.8×19.2

3. Find the square of :

(i) $3x + \frac{2}{y}$ (ii) $\frac{5a}{6b} - \frac{6b}{5a}$

(iii) $2m^2 - \frac{2}{3}n^2$ (iv) $5x + \frac{1}{5x}$

(v) $8x + \frac{3}{2}y$ (vi) 607

(vii) 391 (viii) 9.7

4. If $a + \frac{1}{a} = 2$, find :

(i) $a^2 + \frac{1}{a^2}$ (ii) $a^4 + \frac{1}{a^4}$

5. If $m - \frac{1}{m} = 5$, find :

(i) $m^2 + \frac{1}{m^2}$ (ii) $m^4 + \frac{1}{m^4}$

(iii) $m^2 - \frac{1}{m^2}$

6. If $a^2 + b^2 = 41$ and $ab = 4$, find :

(i) $a - b$ (ii) $a + b$

7. If $2a + \frac{1}{2a} = 8$, find :

(i) $4a^2 + \frac{1}{4a^2}$ (ii) $16a^4 + \frac{1}{16a^4}$

8. If $3x - \frac{1}{3x} = 5$, find :

(i) $9x^2 + \frac{1}{9x^2}$ (ii) $81x^4 + \frac{1}{81x^4}$

9. Expand :

(i) $(3x - 4y + 5z)^2$ (ii) $(2a - 5b - 4c)^2$

(iii) $(5x + 3y)^3$ (iv) $(6a - 7b)^3$

10. If $a + b + c = 9$ and $ab + bc + ca = 15$, find : $a^2 + b^2 + c^2$.

11. If $a + b + c = 11$ and $a^2 + b^2 + c^2 = 81$, find : $ab + bc + ca$.

12. If $3x - 4y = 5$ and $xy = 3$, find : $27x^3 - 64y^3$.

13. If $a + b = 8$ and $ab = 15$, find : $a^3 + b^3$.

14. If $3x + 2y = 9$ and $xy = 3$, find : $27x^3 + 8y^3$.

15. If $5x - 4y = 7$ and $xy = 8$, find : $125x^3 - 64y^3$.

16. The difference between two numbers is 5 and their product is 14. Find the difference between their cubes.

ANSWERS

TEST YOURSELF

1. $x^2 + (15 + 4)x + 15 \times 4$; $x^2 + 19x + 60$ **2.** $x^2 + (15 - 4)x + 15 \times (-4)$; $x^2 + 11x - 60$

3. $x^2 + (-15 + 4)x + (-15) \times 4$; $x^2 - 11x - 60$ **4.** $x^2 + (-15 - 4)x + (-15) \times (-4)$; $x^2 - 19x + 60$

5. $(x)^2 - (3)^2$; $x^2 - 9$ **6.** $(3x)^2 - (4y)^2$; $9x^2 - 16y^2$ **7.** $(1{\cdot}6x^2)^2 - (5)^2$; $2{\cdot}56x^4 - 25$

8. $(5a^2)^2 - (8b)^2$; $25a^4 - 64b^2$ **9.** $(2a)^2 - 2 \times 2a \times \frac{3}{2} + (\frac{3}{2})^2$; $4a^2 - 6a + \frac{9}{4}$

10. $(x)^2 + 2x \times \frac{1}{2x} + (\frac{1}{2x})^2$; $x^2 + 1 + \frac{1}{4x^2}$ **11.** $(2x^2)^2 - 2 \times 2x^2 \times 3y + (3y)^2 = 4x^4 - 12x^2y + 9y^2$

12. $(100 + 7)^2$; $(100)^2 + 2 \times 100 \times 7 + (7)^2$; $10000 + 1400 + 49 = 11449$

13. $(100-3)^2$; $(100)^2 - 2\times100\times3 + (3)^2$; $10000 - 600 + 9 = 9409$

14. $(10+0{\cdot}6)^2$; $(10)^2 + 2\times10\times0{\cdot}6 + (0{\cdot}6)^2$; $100 + 12 + 0{\cdot}36 = 112{\cdot}36$

15. $(20-0{\cdot}2)^2$; $(20)^2 - 2\times20\times0{\cdot}2 + (0{\cdot}2)^2$; $400 - 8 + 0{\cdot}04 = 392{\cdot}04$

16. $x^2 + y^2 + 2xy$; 49; ±7 **17.** $x^2 + y^2 - 2xy$; 4 ; ±2 **18.** $(x + \frac{1}{x})^2 - 2$; $\frac{17}{4}$; $x^2 + \frac{1}{x^2} - 2$; $\frac{9}{4}$; $\pm\frac{3}{2}$

19. $a^2 + b^2 + c^2 + 2(ab + bc + ca)$; $p + 2q$; $\pm\sqrt{p+2q}$

EXERCISE 14(A)

1. (i) $x^2 + 11x + 24$ (ii) $y^2 + 2y - 15$ (iii) $a^2 - 6a - 16$ (iv) $b^2 - 8b + 15$ (v) $6x^2 - xy - 2y^2$
(vi) $15a^2 + 13a - 112$ (vii) $24 + 5b - b^2$ **2.** (i) $x^2 - 1$ (ii) $4 - a^2$ (iii) $9b^2 - 1$ (iv) $16 - 25x^2$ (v) $4a^2 - 9$
(vi) $x^2y^2 - 16$ (vii) $a^2b^2 - x^4$ (viii) $9x^4 - 25y^4$ (ix) $z^2 - \frac{4}{9}$ (x) $\frac{9}{25}a^2 - \frac{1}{4}$ (xi) $0{\cdot}25 - 4a^2$
(xii) $\frac{a^2}{4} - \frac{b^2}{9}$ **3.** (i) $a^4 - 1$ (ii) $a^4 - b^4$ (iii) $16a^4 - b^4$ (iv) $81 - 16x^4$ (v) $81x^4 - 256y^4$ **4.** (i) 399 (ii) 891
(iii) 9991 (iv) 99·96 (v) 63·91 (vi) 24·84 **5.** (i) $36 - x^2y^2$ (ii) $49x^2 - \frac{4}{9}y^2$ (iii) $\frac{a^2}{4b^2} - \frac{4b^2}{a^2}$
(iv) $9x^2 - \frac{1}{4y^2}$ (v) $16a^4 - 81$ (vi) $a^4 - b^4c^4$ (vii) $15x^2 + 49xy + 40y^2$ (viii) $35x^2 + 47xy - 60y^2$
(ix) $6a^2 - ab - 12b^2$ (x) $27a^2 - 30ab + 7b^2$

EXERCISE 14(B)

1. (i) $4a^2+4ab+b^2$ (ii) $a^2-4ab+4b^2$ (iii) $a^2+1+\frac{1}{4a^2}$ (iv) $4a^2-4+\frac{1}{a^2}$ (v) $a^2+b^2+c^2+2ab-2bc-2ca$
(vi) $a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$ (vii) $9x^2 + 2 + \frac{1}{9x^2}$ (viii) $4x^2 - 2 + \frac{1}{4x^2}$
2. (i) $x^2 + 6xy + 9y^2$ (ii) $4x^2 - 20xy + 25y^2$ (iii) $a^2 + \frac{2}{5} + \frac{1}{25a^2}$ (iv) $4a^2 - 4 + \frac{1}{a^2}$
(v) $x^2+4y^2+1-4xy-4y+2x$ (vi) $9a^2+4b^2+25c^2-12ab+20bc-30ca$ (vii) $4x^2+\frac{1}{x^2}+5+\frac{2}{x}+4x$
(viii) $21 + x^2 + \frac{4}{x^2} - 10x + \frac{20}{x}$ (ix) $4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4zx$ (x) $x^2 + \frac{1}{x^2} + 3 - \frac{2}{x} - 2x$
3. (i) 43264 (ii) 8464 (iii) 172225 (iv) 35344 (v) 88·36 (vi) 428·49 **4.** (i) $8a^3 + 12a^2b + 6ab^2 + b^3$
(ii) $a^3 - 6a^2b + 12ab^2 - 8b^3$ (iii) $27x^3 - 54x^2y + 36xy^2 - 8y^3$ (iv) $x^3 + 15x^2y + 75xy^2 + 125y^3$
(v) $a^3 + 3a + \frac{3}{a} + \frac{1}{a^3}$ (vi) $8a^3 - 6a + \frac{3}{2a} - \frac{1}{8a^3}$ **5.** (i) $a^3 + 6a^2 + 12a + 8$ (ii) $8a^3 - 12a^2 + 6a - 1$
(iii) $8a^3 + 36a^2b + 54ab^2 + 27b^3$(iv) $27b^3 - 54b^2a + 36ba^2 - 8a^3$ (v) $8x^3 + 12x + \frac{6}{x} + \frac{1}{x^3}$
(vi) $x^3 - \frac{3x^2}{2} + \frac{3x}{4} - \frac{1}{8}$

EXERCISE 14(C)

1. 13 **2.** 68 **3.** (i) ±7 (ii) ±7 **4.** (i) ±2 (ii) ±4 **5.** 7 **6.** 18 **7.** ±5 **8.** ±3 **9.** 31 **10.** 33 **11.** ±15
12. 72 **13.** 117 **14.** 110 **15.** 76 **16.** (i) 18 (ii) 76 **17.** (i) 7 (ii) 18 **18.** (i) ±5 (ii) ±1

EXERCISE 14(D)

1. (i) $6x^2 + 2x + \frac{1}{6}$ (ii) $14a^2 + 2{\cdot}9a - 0{\cdot}15$ (iii) $63 + 2y - y^2$ (iv) $30 - 17z + z^2$ (v) $a^4 + 2a^2 - 15$ (vi) $32 - 4ab - a^2b^2$ (vii) $35x^2y^2 - 4xy - 63$ (viii) $24a^4 - 41a^2b^2 + 12b^4$ **2.** (i) $4x^2 - \frac{9}{25}$ (ii) $\frac{16}{49}a^2 - \frac{9}{16}b^2$ (iii) $36 - 25x^2y^2$ (iv) $4a^2 - \frac{1}{4a^2}$ (v) $16x^4 - 25y^4$ (vi) $2{\cdot}56x^2 - 0{\cdot}49y^2$ (vii) $m^4 - 81$ (viii) $81x^4 - 256y^4$ (ix) $a^4 - b^4c^4$ (x) 39991 (xi) 399·36 **3.** (i) $9x^2 + \frac{12x}{y} + \frac{4}{y^2}$ (ii) $\frac{25a^2}{36b^2} - 2 + \frac{36b^2}{25a^2}$ (iii) $4m^4 - \frac{8}{3}m^2n^2 + \frac{4}{9}n^4$ (iv) $25x^2 + 2 + \frac{1}{25x^2}$ (v) $64x^2 + 24xy + \frac{9}{4}y^2$ (vi) 368449 (vii) 152881 (viii) 94·09 **4.** (i) 2 (ii) 2 **5.** (i) 27 (ii) 727 (iii) $5\sqrt{29}$ **6.** (i) $\sqrt{33}$ (ii) 7 **7.** (i) 62 (ii) 3842 **8.** (i) 27 (ii) 727 **9.** (i) $9x^2 + 16y^2 + 25z^2 - 24xy - 40yz + 30zx$ (ii) $4a^2 + 25b^2 + 16c^2 - 20ab + 40bc - 16ca$ (iii) $125x^3 + 225x^2y + 135xy^2 + 27y^3$ (iv) $216a^3 - 756a^2b + 882ab^2 - 343b^3$ **10.** 51 **11.** 20 **12.** 665 **13.** 152 **14.** 243 **15.** 3703 **16.** 335