Chapter 14

FACTORISATION

Module 8

You know that the product of $5x^2$ and $2x - 3y = 5x^2(2x - 3y) = 10x^3 - 15x^2y$. We say that $5x^2$ and $2x - 3y$ are factors of $10x^3 - 15x^2y$. We write it as

$$10x^3 - 15x^2y = 5x^2(2x - 3y).$$

Similarly, the product of $3x + 7$ and $3x - 7 = (3x + 7)(3x - 7) = 9x^2 - 49$; we say that $3x + 7$ and $3x - 7$ are factors of $9x^2 - 49$. We write it as

$$9x^2 - 49 = (3x + 7)(3x - 7).$$

Thus, when an algebraic expression can be written as the product of two or more expressions, then each of these expressions is called a ***factor*** *of the given expression.*

To find factors of a given expression means to obtain two or more expressions whose product is the given expression.

The process of finding two or more expressions whose product is the given expression is called ***factorisation****.*

Thus, factorisation is the reverse process of multiplication.

For example :

Product	Factors
(*i*) $7xy(5xy - 3) = 35x^2y^2 - 21xy$	$35x^2y^2 - 21xy = 7xy(5xy - 3)$
(*ii*) $(4a + 5b)(4a - 5b) = 16a^2 - 25b^2$	$16a^2 - 25b^2 = (4a + 5b)(4a - 5b)$
(*iii*) $(p + 3)(p - 7) = p^2 - 4p - 21$	$p^2 - 4p - 21 = (p + 3)(p - 7)$
(*iv*) $(2x + 3)(3x - 5) = 6x^2 - x - 15$	$6x^2 - x - 15 = (2x + 3)(3x - 5)$

In the previous class, you have already learnt the factorisation of polynomials by the following methods :

- Taking out common factors
- Grouping
- Difference of two squares by using $a^2 - b^2 = (a + b)(a - b)$

In this chapter, we shall review the above methods and solve some tougher problems. We shall also find factors of trinomials of the type

(*i*) $x^2 + px + q$, where $p, q \in \mathbf{N}$

(*ii*) $ax^2 + bx + c$, where $a, b, c \in \mathbf{N}$.

Activity

Before taking up factorisation, we would like to introduce the concept of H.C.F.

H.C.F. of two or more polynomials (with integral coefficients) is the largest common factor of the given polynomials.

> ***H.C.F. of two or more monomials = (H.C.F. of their numerical coefficients) × (H.C.F. of their literal coefficients)***
>
> ***H.C.F. of literal coefficients = product of each common literal raised to the lowest power***

Remark

Here it is understood that the numerical coefficients of the monomials (under consideration) are integers and the powers of the literals involved in the monomials are positive integers.

For example :

(*i*) H.C.F. of $6x^2y^2$ and $8xy^3$:

H.C.F. of numerical coefficients = H.C.F. of 6 and 8 = 2

H.C.F. of literal coefficients = H.C.F. of x^2y^2 and xy^3

= product of each common literal raised to the lowest power = xy^2

$\therefore$ H.C.F. of $6x^2y^2$ and $8xy^3 = 2 \times xy^2 = 2xy^2$.

(*ii*) H.C.F. of $15a^3b^2c^3$, $12a^4bc^4$ and $18a^5b^3c^2$:

H.C.F. of numerical coefficients = H.C.F. of 15, 12 and 18 = 3

H.C.F. of literal coefficients = H.C.F. of $a^3b^2c^3$, a^4bc^4 and $a^5b^3c^2$

= product of each common literal raised to the lowest power = a^3bc^2

$\therefore$ H.C.F. of the given monomials = $3 \times a^3bc^2 = 3a^3bc^2$.

FACTORISING BY TAKING OUT COMMON FACTORS

If the different terms/expressions of the given polynomial have common factors, then the given polynomial can be factorised by the following procedure :

(*i*) *Find the H.C.F. of all the terms/expressions of the given polynomial.*

(*ii*) *Divide each term/expression of the given polynomial by H.C.F. Enclose the quotient within the brackets and keep the common factor outside the bracket.*

Example 1. Factorise the following polynomials :

(*i*) $24x^3 - 32x^2$ (*ii*) $15ab^2 - 21a^2b$ (*iii*) $14x^2y^2 - 10x^2y + 8xy^2$.

Solution. (*i*) H.C.F. of $24x^3$ and $32x^2$ is $8x^2$

$24x^3 - 32x^2 = 8x^2(3x - 4)$.

Divide each term by $8x^2$ and keep $8x^2$ outside the bracket

(*ii*) H.C.F. of $15ab^2$ and $21a^2b$ is $3ab$

$\therefore$ $15ab^2 - 21a^2b = 3ab(5b - 7a)$.

(*iii*) H.C.F. of $14x^2y^2$, $10x^2y$ and $8xy^2$ is $2xy$

$\therefore$ $14x^2y^2 - 10x^2y + 8xy^2 = 2xy(7xy - 5x + 4y)$.

Example 2. Factorise the following :

(*i*) $3x(y + 2z) + 5a(y + 2z)$

(*ii*) $10(p - 2q)^3 + 6(p - 2q)^2 - 20(p - 2q)$.

Solution. (*i*) H.C.F. of the expressions

$3x(y + 2z)$ and $5a(y + 2z)$ is $y + 2z$

Divide each expression by $y + 2z$ and keep $y + 2z$ outside the bracket

$\therefore$ $3x(y + 2z) + 5a(y + 2z) = (y + 2z)(3x + 5a)$

(*ii*) H.C.F. of the expressions $10(p - 2q)^3$, $6(p - 2q)^2$ and $20(p - 2q)$ is $2(p - 2q)$

$\therefore$ $10(p - 2q)^3 + 6(p - 2q)^2 - 20(p - 2q)$

$= 2(p - 2q)\,[5(p - 2q)^2 + 3(p - 2q) - 10]$.

Exercise 14.1

Factorise the following (1 to 9) polynomials :

1. (i) $8xy^3 + 12x^2y^2$ (ii) $15ax^3 - 9ax^2$
2. (i) $21py^2 - 56\,py$ (ii) $4x^3 - 6x^2$
3. (i) $2\pi r^2 - 4\pi r$ (ii) $18m + 16n$
4. (i) $25abc^2 - 15a^2b^2c$ (ii) $28p^2q^2r - 42pq^2r^2$
5. (i) $8x^3 - 6x^2 + 10x$ (ii) $14mn + 22m - 62p$
6. (i) $18p^2q^2 - 24pq^2 + 30p^2q$ (ii) $27a^3b^3 - 18a^2b^3 + 75a^3b^2$
7. (i) $15a(2p - 3q) - 10b\,(2p - 3q)$ (ii) $3a(x^2 + y^2) + 6b(x^2 + y^2)$
8. (i) $6(x + 2y)^3 + 8(x + 2y)^2$ (ii) $14(a - 3b)^3 - 21p(a - 3b)$
9. $10a(2p + q)^3 - 15b(2p + q)^2 + 35(2p + q)$

FACTORISING BY GROUPING OF TERMS

When the grouping of terms of the given polynomial gives rise to common factor, then the given polynomial can be factorised by the following procedure:

(i) *Arrange the terms of the given polynomial in groups in such a way that each group has a common factor.*

(ii) *Factorise each group.*

(iii) *Take out the factor which is common to each group.*

Note. Factorisation by grouping is possible only if the given polynomial contains an even number of terms.

Example 1. Factorise the following polynomials :

(i) $ax - ay + bx - by$ (ii) $4x^2 - 10xy - 6xz + 15yz.$

Solution. (i)
$$ax - ay + bx - by = (ax - ay) + (bx - by)$$
$$= a(x - y) + b(x - y)$$
$$= (x - y)\,(a + b).$$

(ii)
$$4x^2 - 10xy - 6xz + 15yz = (4x^2 - 10xy) - (6xz - 15yz)$$
$$= 2x(2x - 5y) - 3z(2x - 5y)$$
$$= (2x - 5y)\,(2x - 3z).$$

Example 2. Factorise the following polynomials :

(i) $x^3 + 2x^2 + x + 2$ (ii) $1 + p + pq + p^2q.$

Solution. (i)
$$x^3 + 2x^2 + x + 2 = (x^3 + 2x^2) + (x + 2)$$
$$= x^2(x + 2) + 1(x + 2)$$
$$= (x + 2)\,(x^2 + 1).$$

(ii)
$$1 + p + pq + p^2q = (1 + p) + (pq + p^2q)$$
$$= 1(1 + p) + pq(1 + p)$$
$$= (1 + p)\,(1 + pq).$$

Example 3. Factorise the following expressions :

(i) $xy - pq + qy - px$ (ii) $a^2 + bc + ab + ca$

(iii) $ab(x^2 + y^2) + xy(a^2 + b^2)$.

Solution. (i) Since xy and pq have nothing in common, we do not group the terms in pairs in the order in which the given expression is written. Here we interchange $-pq$ and $-px$.

$\therefore$ $xy - pq + qy - px = (xy - px) + (qy - pq) = x(y - p) + q(y - p)$

$= (y - p)(x + q)$.

(ii) $a^2 + bc + ab + ca = a^2 + ab + bc + ca$

Interchange the positions of bc and ab

$= a(a + b) + c(b + a)$

$= a(a + b) + c(a + b)$

$= (a + b)(a + c)$.

(iii) $ab(x^2 + y^2) + xy(a^2 + b^2) = abx^2 + aby^2 + a^2xy + b^2xy$

$= (abx^2 + a^2xy) + (aby^2 + b^2xy)$

$= ax(bx + ay) + by(ay + bx)$

$= ax(bx + ay) + by(bx + ay)$

$= (bx + ay)(ax + by)$.

Example 4. Factorise the following expressions :

(i) $ax + by + bx + az + ay + bz$ (ii) $p^3 - 3p^2 + 2p - 6 - pq + 3q$

(iii) $x^3 - x^2 + ax + x - a - 1$ (iv) $p(x - y)^2 - qy + qx + 3x - 3y$.

Solution. (i) $ax + by + bx + az + ay + bz = (ax + ay + az) + (bx + by + bz)$

$= a(x + y + z) + b(x + y + z)$

$= (x + y + z)(a + b)$.

(ii) $p^3 - 3p^2 + 2p - 6 - pq + 3q = (p^3 - 3p^2) + (2p - 6) + (-pq + 3q)$

$= p^2(p - 3) + 2(p - 3) - q(p - 3)$

$= (p - 3)(p^2 + 2 - q)$.

(iii) $x^3 - x^2 + ax + x - a - 1 = (x^3 - x^2) + (ax - a) + (x - 1)$

$= x^2(x - 1) + a(x - 1) + 1(x - 1)$

$= (x - 1)(x^2 + a + 1)$.

(iv) $p(x - y)^2 - qy + qx + 3x - 3y = p(x - y)^2 + (qx - qy) + (3x - 3y)$

$= p(x - y)^2 + q(x - y) + 3(x - y)$

$= (x - y)[p(x - y) + q + 3]$.

Exercise 14.2

Factorise the following (1 to 11) polynomials :

1. (i) $x^2 + xy - x - y$ (ii) $y^2 - yz - 5y + 5z$
2. (i) $5xy + 7y - 5y^2 - 7x$ (ii) $5p^2 - 8pq - 10p + 16q$
3. (i) $a^2b - ab^2 + 3a - 3b$ (ii) $x^3 - 3x^2 + x - 3$
4. (i) $6xy^2 - 3xy - 10y + 5$ (ii) $3ax - 6ay - 8by + 4bx$
5. (i) $x^2 + xy(1 + y) + y^3$ (ii) $y^2 - xy(1 - x) - x^3$
6. (i) $ab^2 + (a - 1)b - 1$ (ii) $2a - 4b - xa + 2bx$

7. (i) $5ph - 10qk + 2rph - 4qrk$ (ii) $x^2 - x(a + 2b) + 2ab$
8. (i) $ab(x^2 + y^2) - xy(a^2 + b^2)$ (ii) $(ax + by)^2 + (bx - ay)^2$
9. (i) $a^3 + ab(1 - 2a) - 2b^2$ (ii) $3x^2y - 3xy + 12x - 12$.
[**Hint.** (ii) 3 is a common factor, first take 3 outside.]
10. (i) $a^2b + ab^2 - abc - b^2c + axy + bxy$ (ii) $ax^2 - bx^2 + ay^2 - by^2 + az^2 - bz^2$
11. (i) $x - 1 - (x - 1)^2 + ax - a$ (ii) $ax + a^2x + aby + by - (ax + by)^2$

DIFFERENCE OF TWO SQUARES

Module 3 Applying identities

When the given polynomial is expressible as the difference of two squares, then it can be factorised by using the formula

$$a^2 - b^2 = (a + b)(a - b)$$

Example 1. Factorise the following polynomials :

(i) $25a^2 - 64b^2$ (ii) $\frac{4}{25}x^2 - \frac{9}{49}y^2$.

Solution. (i) $25a^2 - 64b^2 = (5a)^2 - (8b)^2 = (5a + 8b)(5a - 8b)$.

(ii) $\frac{4}{25}x^2 - \frac{9}{49}y^2 = \left(\frac{2}{5}x\right)^2 - \left(\frac{3}{7}y\right)^2 = \left(\frac{2}{5}x + \frac{3}{7}y\right)\left(\frac{2}{5}x - \frac{3}{7}y\right)$.

Example 2. Factorise the following polynomials:

(i) $9(a - 2b)^2 - 16(2a - 3b)^2$ (ii) $16y^3 - 4y$

Solution. (i) $9(a - 2b)^2 - 16(2a - 3b)^2 = [3(a - 2b)]^2 - [4(2a - 3b)]^2$
$= (3a - 6b)^2 - (8a - 12b)^2$
$= [(3a - 6b) + (8a - 12b)][(3a - 6b) - (8a - 12b)]$
$= (11a - 18b)(6b - 5a)$.

(ii) $16y^3 - 4y = 4y(4y^2 - 1) = 4y[(2y)^2 - 1^2]$
$= 4y(2y + 1)(2y - 1)$.

Example 3. Factorise the following polynomials:

(i) $1 - 25(2x - 3y)^2$ (ii) $7(3x - 4y)^2 - 28(2x - y)^2$
(iii) $4x^2 - y^2 + 6y - 9$.

Solution. (i) $1 - 25(2x - 3y)^2 = 1^2 - (5(2x - 3y))^2$
$= [1 + 5(2x - 3y)][1 - 5(2x - 3y)]$
$= (1 + 10x - 15y)(1 - 10x + 15y)$.

(ii) $7(3x - 4y)^2 - 28(2x - y)^2$
$= 7[(3x - 4y)^2 - 4(2x - y)^2]$
$= 7[(3x - 4y)^2 - (2(2x - y))^2]$
$= 7[(3x - 4y) + 2(2x - y)][(3x - 4y) - 2(2x - y)]$
$= 7(3x - 4y + 4x - 2y)(3x - 4y - 4x + 2y)$
$= 7(7x - 6y)(-x - 2y) = -7(7x - 6y)(x + 2y)$.

7 is common factor, first take 7 out

(iii) $4x^2 - y^2 + 6y - 9 = (2x)^2 - (y^2 - 6y + 9) = (2x)^2 - (y - 3)^2$
$= [2x + (y - 3)][2x - (y - 3)]$
$= (2x + y - 3)(2x - y + 3)$.

Example 4. Factorise the following :

(i) $16a^4 - \frac{1}{81}$ (ii) $3x^5 - 48x$ (iii) $a^2b^2 - a^2 - b^2 + 1$.

Solution. (i) $16a^4 - \frac{1}{81} = (4a^2)^2 - \left(\frac{1}{9}\right)^2 = \left(4a^2 + \frac{1}{9}\right)\left(4a^2 - \frac{1}{9}\right)$

$= \left(4a^2 + \frac{1}{9}\right)\left[(2a)^2 - \left(\frac{1}{3}\right)^2\right] = \left(4a^2 + \frac{1}{9}\right)\left(2a + \frac{1}{3}\right)\left(2a - \frac{1}{3}\right)$.

(ii) $3x^5 - 48x = 3x(x^4 - 16) = 3x[(x^2)^2 - (4)^2]$

$= 3x(x^2 + 4)(x^2 - 4) = 3x(x^2 + 4)(x + 2)(x - 2)$.

(iii) $a^2b^2 - a^2 - b^2 + 1 = (a^2b^2 - a^2) - b^2 + 1$

$= a^2(b^2 - 1) - 1(b^2 - 1) = (b^2 - 1)(a^2 - 1)$

$= (b + 1)(b - 1)(a + 1)(a - 1)$.

Example 5. Evaluate :

(i) $(501)^2 - (499)^2$ (ii) $(99{\cdot}9)^2 - (0{\cdot}1)^2$.

Solution. (i) $(501)^2 - (499)^2 = (501 + 499)(501 - 499)$

$= 1000 \times 2 = 2000$.

(ii) $(99{\cdot}9)^2 - (0{\cdot}1)^2 = (99{\cdot}9 + 0{\cdot}1)(99{\cdot}9 - 0{\cdot}1)$

$= 100 \times 99{\cdot}8 = 9980$.

Exercise 14.3

Factorise the following (1 to 11) expressions :

1. (i) $4p^2 - 9$ (ii) $4x^2 - 169y^2$
2. (i) $9x^2y^2 - 25$ (ii) $16x^2 - \frac{1}{144}$
3. (i) $20x^2 - 45y^2$ (ii) $\frac{9}{16} - 25a^2b^2$
4. (i) $(2a + 3b)^2 - 16c^2$ (ii) $1 - (b - c)^2$
5. (i) $9(x + y)^2 - x^2$ (ii) $(2m + 3n)^2 - (3m + 2n)^2$
6. (i) $25(a + b)^2 - 16(a - b)^2$ (ii) $9(3x + 2)^2 - 4(2x - 1)^2$
7. (i) $x^3 - 25x$ (ii) $63p^2q^2 - 7$
8. (i) $32a^2b - 72b^3$ (ii) $9(a + b)^3 - 25(a + b)$
9. (i) $x^2 - y^2 - 2y - 1$ (ii) $a^2 - 2ab + b^2 - c^2$
10. (i) $9x^2 - y^2 + 4y - 4$ (ii) $4a^2 - 4b^2 + 4a + 1$
11. (i) $625 - p^4$ (ii) $5y^5 - 405y$.
12. Evaluate the following :

    (i) $(992)^2 - 8^2$ (ii) $(678)^2 - (322)^2$

    (iii) $(8{\cdot}6)^2 - (1{\cdot}4)^2$ (iv) $\left(5\frac{2}{3}\right)^2 - \left(3\frac{1}{3}\right)^2$.
13. Factorise $ab^2 - ac^2$. Hence show that $10(7{\cdot}5)^2 - 10(2{\cdot}5)^2 = 500$.

FACTORISING OF TRINOMIALS

Case I. When the trinomial is of the form $x^2 + px + q$, where p and $q \in \mathbf{N}$:

Let $x^2 + px + q = (x + a)(x + b) = x^2 + (a + b)x + ab$.

Thus, if we want to factorise the trinomial of the form $x^2 + px + q$, we need to find two integers a and b such that $a + b = p$ and $ab = q$.

Therefore, split p (the coefficients of x) into two parts such that the algebraic sum of these two parts is p and their product is q

Case II. When the trinomial is of the form $ax^2 + bx + c$, where a, b and $c \in \mathbf{N}$:

We want to find two integers A and B such that

$$A + B = b \text{ and } AB = ac.$$

Therefore, split b (the coefficient of x) into two parts such that the algebraic sum of these two parts is b and their product is ac.

Example 1. Factorise the following trinomials :

(*i*) $x^2 + 5x + 6$ (*ii*) $x^2 - 9x + 20$

(*iii*) $x^2 + 2x - 63$ (*iv*) $y^2 - 16y - 105$.

Solution. (*i*) We want to find two integers whose sum is 5 and product is 6. By trial, we see that $2 + 3 = 5$ and $2 \times 3 = 6$

Use grouping method

$$\therefore \quad x^2 + 5x + 6 = x^2 + 2x + 3x + 6 = x(x + 2) + 3(x + 2) = (x + 2)(x + 3).$$

(*ii*) We want to find two integers whose sum is -9 and product is 20. By trial, we see that $(-4) + (-5) = -9$ and $(-4)(-5) = 20$

$$\therefore \quad x^2 - 9x + 20 = x^2 - 4x - 5x + 20 = x(x - 4) - 5(x - 4) = (x - 4)(x - 5).$$

(*iii*) We want to find two integers whose sum is 2 and product is -63. By trial, we see that $9 + (-7) = 2$ and $9 \times (-7) = -63$

$$\therefore \quad x^2 + 2x - 63 = x^2 + 9x - 7x - 63 = x(x + 9) - 7(x + 9) = (x + 9)(x - 7).$$

(*iv*) We want to find two integers whose sum is -16 and product is -105. By trial, we see that $(-21) + 5 = -16$ and $(-21) \times 5 = -105$

$$\therefore \quad y^2 - 16y - 105 = y^2 - 21y + 5y - 105 = y(y - 21) + 5(y - 21) = (y - 21)(y + 5).$$

Example 2. Factorise the following trinomials :

(*i*) $3x^2 - 10x + 8$ (*ii*) $2p^2 - 17p - 30$.

Solution. (*i*) We want to find two integers whose sum is -10 and product is 3×8 *i.e.* 24.

By trial, we see that $(-6) + (-4) = -10$ and $(-6)(-4) = 24$

$$\therefore \quad 3x^2 - 10x + 8 = 3x^2 - 6x - 4x + 8 = 3x(x - 2) - 4(x - 2) = (x - 2)(3x - 4).$$

(*ii*) We want to find two integers whose sum is -17 and product is $2 \times (-30)$ *i.e.* -60.

By trial, we see that $(-20) + 3 = -17$ and $(-20) \times 3 = -60$

$\therefore\ 2p^2 - 17p - 30 = 2p^2 - 20p + 3p - 30$
$= 2p(p - 10) + 3(p - 10)$
$= (p - 10)(2p + 3).$

Example 3. Factorise the following expressions:

(*i*) $2x^2 + 13xy - 24y^2$ (*ii*) $15(x - 2y)^2 - 8(x - 2y) - 16$

Solution. (*i*) We want to find two integers whose sum is 13 and product is $2 \times (-24)$ *i.e.* -48.

By trial, we see that $16 + (-3) = 13$ and $16 \times (-3) = -48$

$\therefore\ 2x^2 + 13xy - 24y^2 = 2x^2 + 16xy - 3xy - 24y^2$
$= 2x(x + 8y) - 3y(x + 8y)$
$= (x + 8y)(2x - 3y).$

(*ii*) Let $x - 2y = p$, then

$15(x - 2y)^2 - 8(x - 2y) - 16 = 15p^2 - 8p - 16.$

We want to find two integers whose sum is -8 and product is $15 \times (-16)$ *i.e.* -240.

By trial, we see that $12 + (-20) = -8$ and $12 \times (-20) = -240$

$\therefore\ 15p^2 - 8p - 16 = 15p^2 + 12p - 20p - 16$
$= 3p(5p + 4) - 4(5p + 4) = (5p + 4)(3p - 4).$

Substituting $p = x - 2y$, we get

$15(x - 2y)^2 - 8(x - 2y) - 16 = [5(x - 2y) + 4][3(x - 2y) - 4]$
$= (5x - 10y + 4)(3x - 6y - 4).$

Exercise 14.4

Factorise the following (1 to 14) polynomials :

1. (*i*) $x^2 + 3x + 2$ (*ii*) $z^2 + 10z + 24$
2. (*i*) $x^2 + 15x + 56$ (*ii*) $p^2 + 22p + 85$
3. (*i*) $x^2 - 10x + 24$ (*ii*) $m^2 - 23m + 42$
4. (*i*) $x^2 - 3x - 54$ (*ii*) $a^2 - 7a - 30$
5. (*i*) $y^2 - 5y - 24$ (*ii*) $t^2 + 23t - 108$
6. (*i*) $3x^2 + 14x + 8$ (*ii*) $3y^2 + 10y + 8$
7. (*i*) $14x^2 - 23x + 8$ (*ii*) $12x^2 - x - 35$
8. (*i*) $5x^2 + 7x - 6$ (*ii*) $3x^2 - 7x - 6$
9. (*i*) $6x^2 + 11x - 10$ (*ii*) $5 - 4x - 12x^2$
10. (*i*) $1 - 18y - 63y^2$ (*ii*) $3x^2 - 5xy - 12y^2$
11. (*i*) $x^2 - 3xy - 40y^2$ (*ii*) $10p^2q^2 - 21pq + 9$
12. (*i*) $2a^2b^2 + ab - 45$ (*ii*) $x(12x + 7) - 10$
    [Hint. (*ii*) $x(12x + 7) - 10 = 12x^2 + 7x - 10$.**]**
13. (*i*) $(a + b)^2 - 11(a + b) - 42$ (*ii*) $8 + 6(p + q) - 5(p + q)^2$
14. (*i*) $(x - 2y)^2 - 6(x - 2y) + 5$ (*ii*) $7 + 10(2x - 3y) - 8(2x - 3y)^2$

Summary

- When an algebraic expression can be written as the product of two or more expressions, then each of these expressions is called a **factor** of the given expression.
- To find factors of a given expression means to find two or more expressions whose product is the given expression.
- The process of finding factors of the given expression is called **factorisation**.
- H.C.F. of two or more polynomials is the largest common factor of the given polynomials.
- You have learnt the factorisation of polynomials by the following methods :
  - ❑ Taking out common factors
  - ❑ Grouping
  - ❑ Difference of two squares by using $a^2 - b^2 = (a + b)(a - b)$
- To find factors of the trinomial $x^2 + px + q$, where p and $q \in \mathbf{N}$:
  Split b(the coefficient of x) into two parts such that the algebraic sum of these parts is p and their product is q. Then use grouping method.
- To find factors of the trinomial $ax^2 + bx + c$, where a, b and $c \in \mathbf{N}$:
  Split b(the coefficient of x) into two parts such that the algebraic sum of these parts is b and their product is ac. Then use grouping method.

Check Your Progress

Factorise the following (1 to 11) polynomials :

1. (*i*) $21x^2y^3 - 12x^3y$ (*ii*) $24pq^2 - 18p^2q - 60pq$
2. (*i*) $15(2x - 3)^3 - 10(2x - 3)$ (*ii*) $a(b - c)(b + c) - d(c - b)$
3. (*i*) $2a^2x - bx + 2a^2 - b$ (*ii*) $p^2 - (a + 2b)p + 2ab$
4. (*i*) $(x^2 - y^2)z + (y^2 - z^2)x$ (*ii*) $5a^4 - 5a^3 + 30a^2 - 30a$
   [Hint. (*ii*) $5a$ is a common factor, first take $5a$ outside.**]**
5. (*i*) $b(c - d)^2 + a(d - c) + 3c - 3d$ (*ii*) $p^2 - 16q^2$
6. (*i*) $12p^3 - 3p$ (*ii*) $9x^2 - 4(x + y)^2$
7. (*i*) $ax^2 + b^2y - ab^2 - x^2y$ (*ii*) $9x^2 - (y^2 - 8y + 16)$
8. (*i*) $x^2 + 2x - 48$ (*ii*) $p^2 - 7p - 120$
9. (*i*) $3x^2 - 4x - 4$ (*ii*) $15a^2b^2 - 26ab + 8$
10. (*i*) $x^2 + 2xy - 99y^2$ (*ii*) $\pi a^5 - \pi^3ab^2$
11. (*i*) $3(a - b)^2 - (a - b) - 44$ (*ii*) $a^4 - 10a^2 + 9$
12. Find the value of :
    (*i*) $(100002)^2 - (99998)^2$ (*ii*) $\left(8\frac{3}{4}\right)^2 - \left(7\frac{1}{4}\right)^2$.