Chapter 10 FUNDAMENTAL CONCEPTS AND OPERATIONS ON ALGEBRAIC EXPRESSIONS

refresh the

In previous classes, you studied the fundamental concepts of algebra and the operations on algebraic expressions. In this chapter, we shall review these topics and solve a few tougher problems.

FUNDAMENTAL CONCEPTS

In algebra, we use two types of symbols — constants and variables (literals).

Constant. *A symbol which has a fixed value is called a* ***constant****.*

For example, each of $7, -3, \frac{2}{5}, -\frac{7}{2}, \sqrt{2}, 2-\sqrt{3}, \pi$ etc. is a constant.

Variable. *A symbol which can be given various numerical values is called a* ***variable*** *or* ***literal****.*

For example, the formula for circumference of a circle is $C = 2\pi r$, where C is the length of the circumference of the circle and r is its radius. Here 2, π are constants and C, r are variables (or literals).

Algebraic expression

A collection of constants and literals (variables) connected by one or more of the operations of addition, subtraction, multiplication and division is called an ***algebraic expression****.*

The various parts of an algebraic expression separated by '+ or –' sign are called **terms** of the algebraic expression. Various types of algebraic expressions are :

Monomial. *An algebraic expression having only one term is called a* ***monomial****.*

Binomial. *An algebraic expression having two terms is called a* ***binomial****.*

Trinomial. *An algebraic expression having three terms is called a* ***trinomial****.*

Multinomial. *An algebraic expression having two or more terms is called a* ***multinomial****.*

For example :

Algebraic expression	*No. of terms*	*Name*	*Terms*
(*i*) $-7x^2y^3$	1	Monomial	$-7x^2y^3$
(*ii*) $5x^2y - \frac{7x}{y}$	2	Binomial	$5x^2y, -\frac{7x}{y}$
(*iii*) $-3xy^3 + 5xz^2 + \frac{11}{2}$	3	Trinomial	$-3xy^3, 5xz^2, \frac{11}{2}$
(*iv*) $9x^5 - 3x^2 + 4 - \frac{3}{x^3}$	4	Multinomial	$9x^5, -3x^2, 4, -\frac{3}{x^3}$
(*v*) $p^3q^2 + 5pq - \frac{3}{p} + \frac{7p}{q^2}$	4	Multinomial	$p^3q^2, 5pq, -\frac{3}{p}, \frac{7p}{q^2}$

Remark

Multiplication and division do not separate the terms of an algebraic expression. Thus, $-7x^2y^3$ is one term while $-7x^2 + y^3$ has two terms.

Factors. *Each of the quantity (constant or literal) multiplied together to form a product is called a **factor** of the product.*

A constant factor is called a ***numerical factor*** and any factor containing only literals is called a ***literal factor***.

In $-7xy^2$, the numerical factor is -7 and the literal factors are x, y, y^2, xy and xy^2.

Constant term. *The term of an algebraic expression having no literal factors is called its **constant term**.*

In the expression $-3x^2y^3 + \frac{5}{x} - 7$, -7 is the constant term,

while the expression $9x^5 - 3x^2 + \frac{11}{x^3}$ has no constant term.

Coefficient. *Any factor of a (non-constant) term of an algebraic expression is called the **coefficient** of the remaining factor of the term.*

In particular, the constant part is called the ***numerical coefficient*** or simply the ***coefficient*** of the term and the remaining part is called the ***literal coefficient*** of the term.

Consider the expression $7p^3q^2 - 5p^2q - 3p + 2$. In the term $-5p^2q$:

the numerical coefficient $= -5$,
the literal coefficient $= p^2q$,
the coefficient of $p^2 = -5q$,
the coefficient of $5p = -pq$,
the coefficient of $-q = 5p^2$ etc.

Note. When we write x, we mean $1x$. Thus, if no coefficient is written before a literal, then the coefficient is always taken as 1.

Like and unlike terms. *The terms having same literal coefficients are called **like terms**; otherwise, they are called **unlike terms**.*

For example :

(*i*) $5x^2yz$, $-3x^2yz$, $\frac{3}{5}yzx^2$ are like terms

(*ii*) $7ab$, $-3a^2b$, $\frac{2}{3}ab^2$ are unlike terms.

Polynomial

*An algebraic expression is called a **polynomial** if the powers of the variables involved in it in each term are non-negative integers.*

Polynomial in one variable

*An algebraic expression containing only one variable (literal) is called a **polynomial** in that variable if the powers of the variable in each term are non-negative integers.*

*The greatest power of the variable in a polynomial is called its **degree**.*

For example :

(*i*) $7 - 3x$ is a polynomial in x of degree 1.

(*ii*) $5t^2 - \frac{2}{3}t + 8$ is a polynomial in t of degree 2.

(*iii*) $\frac{3}{4}p^3 - 5p^2 - \frac{2}{3}$ is a polynomial in p of degree 3.

(*iv*) $9x^6 - 3x^2 + 4x - \frac{1}{7}$ is a polynomial in x of degree 6.

(*v*) $5x^2 - \frac{3}{x} + 6$ is not a polynomial. Note that this expression is a trinomial.

Polynomial in two or more variables

An algebraic expression containing two or more variables (literals) is called a ***polynomial*** *in those variables if the powers of the variables in each term are non-negative integers. Take the sum of the powers of the variables in each term; the greatest sum is the* ***degree*** *of the polynomial.*

For example :

(*i*) $8x^3y^2 - 5x^2y^3 - \frac{2}{3}xy^3 + 7xy - \frac{1}{5}$ is a polynomial in two variables x and y.

The degrees of its terms are $3 + 2$, $2 + 3$, $1 + 3$, $1 + 1$, 0.

So, the degree of the polynomial is 5.

(*ii*) $5a - 3abc + \frac{7}{2}ab^2 - 9bc^3 + 6$ is a polynomial in three variables a, b, and c.

The degrees of its terms are 1, $1 + 1 + 1$, $1 + 2$, $1 + 3$, 0.

So, the degree of the polynomial is 4.

(*iii*) $5pq^2 - \frac{3p}{q} + 7q^3 - 8$ is not a polynomial.

Linear polynomial. *A polynomial of degree 1 is called a* ***linear polynomial***.

For example :

(*i*) $3x + 7$, $5 - 9y$, $6p - \frac{2}{3}$, $7a$ are all linear polynomials in one variable.

(*ii*) $3x + 5y$, $2p - 3q + 7$, $5a - \frac{2}{3}b + 11$ are all linear polynomials in two variables.

(*iii*) $2x + 3y - 7z$, $2a - 3b + 5c - 4$ are linear polynomials in three variables.

Quadratic polynomial. *A polynomial of degree 2 is called a* ***quadratic polynomial***.

For example :

(*i*) $3x^2 - 5x + 2$, $7y^2 - 9$, $4p^2 + 3p$ are all quadratic polynomials in one variable.

(*ii*) $3x^2 - 5xy + 7y^2$, $7p^2 - 9pq + 3p - \frac{2}{3}$ are quadratic polynomials in two variables.

(*iii*) $3a^2 + 2b^2 - 3c + 5ab - 4$ is a quadratic polynomial in three variables.

Cubic polynomial. *A polynomial of degree 3 is called a* ***cubic polynomial***.

For example :

(*i*) $5x^3 - 7x^2 + 2$, $1 + 7p - 9p^2 + \frac{2}{3}p^3$ are cubic polynomials in one variable.

(*ii*) $3xy^2 - 7y^3 + 2x^2 - 3xy + \frac{2}{5}y$ is a cubic polynomial in two variables.

Exercise 10.1

1. Identify monomials, binomials and trinomials from the following algebraic expressions :

(i) $5p \times q \times r^2$ (ii) $3x^2 \times y \div 2z$ (iii) $-3 + 7x^2$

(iv) $\frac{5a^2 - 3b^2 + c}{2}$ (v) $7x^5 - \frac{3x}{y}$ (vi) $5p + 3q - 3p^2 \times q^2$

(vii) $m^3 - 2n^2 + 5m - \frac{2}{3}$ (viii) $-9a^3b^3c^3 - 5a^2 + 1$ (ix) $5x^4 + \frac{2x^2 + 3x + 1}{5}$

2. Write the numerical as well as literal coefficient of each of the following monomials :

(i) $-9p^2q^2r$ (ii) $-\frac{7xy^2}{9}$ (iii) $\frac{4}{9}a^2b^2cd$

(iv) $\frac{3}{4x^2y}$ (v) $3x^2 \times y \div 2z$ (vi) $-\frac{2ax}{3by}$

3. In $-\frac{2}{3}p^3q^2r^5$, write down the coefficient of :

(i) p^2 (ii) $-2pq$ (iii) pqr (iv) $\frac{1}{3}p^2qr$

(v) $-2p^2q^2r$ (vi) p^3q^2 (vii) r^5 (viii) $-\frac{2}{3}pqr^2$

4. Group the like terms together :

(i) $3abc, -5ab^2, -\frac{2}{3}cab, 7bac, \frac{2}{7}b^2a$

(ii) $7pq^2, -3p^2q, \sqrt{5}\,qp, \frac{2}{3}q^2p, 4pq, -\pi qp^2$

(iii) $3x^2yz, \sqrt{7}\,xyz^2, -\sqrt{5}\,yzx^2, \frac{2}{5}y^2xz, 9xzy^2, -\frac{4}{3}z^2xy$

5. Identify which of the following expressions are polynomials. If so, write their degrees.

(i) $\frac{2}{5}x^4 - \sqrt{3}\,x^2 + 5x - 1$ (ii) $7x^3 - \frac{3}{x^2} + \sqrt{5}$

(iii) $4a^3b^2 - 3ab^4 + 5ab + \frac{2}{3}$ (iv) $2x^2y - \frac{3}{xy} + 5y^3 + \sqrt{3}$

(v) $2m^2n^4 + \sqrt{3}\,mn^3 - \frac{2}{7}m^7 + 9m^2 + 3\pi mn + 2$

OPERATIONS ON ALGEBRAIC EXPRESSIONS

Addition of like terms

The sum of two or more like terms is a like term whose coefficient is the sum of coefficients of like terms.

For example :

(i) Sum of $3pq, -\frac{2}{5}pq$ and $\frac{7}{10}pq = 3pq - \frac{2}{5}pq + \frac{7}{10}pq$

$$= \left(3 - \frac{2}{5} + \frac{7}{10}\right)pq = \frac{30 - 4 + 7}{10}pq = \frac{33}{10}pq.$$

(*ii*) Sum of $7x^2y$, $-\frac{2}{3}x^2y$, $\frac{3}{4}x^2y$ and $-\frac{5}{6}x^2y = 7x^2y - \frac{2}{3}x^2y + \frac{3}{4}x^2y - \frac{5}{6}x^2y$

$$= \left(7 - \frac{2}{3} + \frac{3}{4} - \frac{5}{6}\right)x^2y = \frac{84 - 8 + 9 - 10}{12}x^2y = \frac{75}{12}x^2y = \frac{25}{4}x^2y.$$

Addition of algebraic expressions

To add two or more algebraic expressions, we collect different groups of like terms and then find the sum of like terms in each group. We may use horizontal method or column method.

Example 1. Add the following :

(*i*) $3a - 2b + 5c + 7$, $5b - 3c - 4$ and $-2a + 3b - 11c$

(*ii*) $3x^2 - 5xy + 4y^2 - 1$, $7y^2 - 9xy + 5$, $7xy - 4x^2 + y^2 - 13$

Solution. (*i*) *Column method :*

Arrange like terms in such a way that they are one below the other

$$\begin{array}{rrrr} 3a & -\,2b & +\,5c & +\,7 \\ & +\,5b & -\,3c & -\,4 \\ -\,2a & +\,3b & -\,11c & \\ \hline a & +\,6b & -\,9c & +\,3, \end{array}$$

which is the required sum.

(*ii*)

$$\begin{array}{rrrr} 3x^2 & -\,5xy & +\,4y^2 & -\,1 \\ & -\,9xy & +\,7y^2 & +\,5 \\ -\,4x^2 & +\,7xy & +\,y^2 & -\,13 \\ \hline -\,x^2 & -\,7xy & +\,12y^2 & -\,9, \end{array}$$

which is the required sum.

Example 2. Add $5p + 2q - 7r + 3$, $2r - 4p - 8$, $11q - 8p + 3r - 1$ and $3q - 2r + 5$

Solution.

Arrange like terms in such a way that they are one below the other

$$\begin{array}{rrrr} 5p & +\,2q & -\,7r & +\,3 \\ -4p & & +\,2r & -\,8 \\ -8p & +\,11q & +\,3r & -\,1 \\ & +\,3q & -\,2r & +\,5 \\ \hline -7p & +\,16q & -\,4r & -\,1, \end{array}$$

which is the required sum.

Subtraction of like terms

Change the sign of the term to be subtracted and then add.

Example 3. Subtract :

(*i*) $-7p^2q$ from $3p^2q$ (*ii*) $3p^2q$ from $-7p^2q$

(*iii*) $-3x^2yz$ from $-2x^2yz$ (*iv*) $-\frac{2}{3}p$ from $-\frac{5}{6}p$

Solution. (*i*) $3p^2q - (-7p^2q) = 3p^2q + 7p^2q$

Change the sign of $-7p^2q$ and add

$$= (3 + 7)\,p^2q = 10p^2q.$$

(*ii*) $-7p^2q - 3p^2q = (-7 - 3)\,p^2q = -10p^2q.$

(*iii*) $-2x^2yz - (-3x^2yz) = -2x^2yz + 3x^2yz = (-2 + 3)\,x^2yz = x^2yz.$

(*iv*) $-\frac{5}{6}p - \left(-\frac{2}{3}p\right) = -\frac{5}{6}p + \frac{2}{3}p = \left(-\frac{5}{6} + \frac{2}{3}\right)p = \frac{-5+4}{6}p = -\frac{1}{6}p.$

Subtraction of algebraic expressions

Change the sign of each term of the expression to be subtracted and then add. We may use horizontal method or column method.

Example 4. Subtract :

(*i*) $3a - 5b + 2c - 9$ from $2a - 4b - 7c + 3$

(*ii*) $5x^3 - 3x^2 - 8$ from $2x^3 - 5x^2 - 11x + 2$

Solution. (*i*) *Column method :*

Change the sign of each term to be subtracted and then add

$$\begin{array}{rrrr} 2a & -\ 4b & -\ 7c & +\ 3 \\ 3a & -\ 5b & +\ 2c & -\ 9 \\ - & + & - & + \\ \hline -\ a & +\ b & -\ 9c & +\ 12 \end{array}$$

(*ii*)

$$\begin{array}{rrrr} 2x^3 & -\ 5x^2 & -\ 11x & +\ 2 \\ 5x^3 & -\ 3x^2 & & -\ 8 \\ - & + & & + \\ \hline -\ 3x^3 & -\ 2x^2 & -\ 11x & +\ 10 \end{array}$$

Example 5. Subtract $7p - 2q - 5r$ from the sum of $5p + 2q - 3r + 1$ and $3p - 4r - 3$.

Solution.

Change the sign of each term of the third expression as it is to be subtracted and then add

$$\begin{array}{rrrr} 5p & +\ 2q & -\ 3r & +\ 1 \\ 3p & & -\ 4r & -\ 3 \\ 7p & -\ 2q & -\ 5r & \\ - & + & + & \\ \hline p & +\ 4q & -\ 2r & -\ 2 \end{array}$$

Example 6. How much should $3p^2 - 5pq + 7q^2 + 3$ be increased to get $5q^2 - 2pq + 3p$?

Solution. We have to subtract $3p^2 - 5pq + 7q^2 + 3$ from $5q^2 - 2pq + 3p$.

$$\begin{array}{rrrrr} 5q^2 & -\ 2pq & +\ 3p & & \\ 7q^2 & -\ 5pq & & +\ 3p^2 & +\ 3 \\ - & + & & - & - \\ \hline -\ 2q^2 & +\ 3pq & +\ 3p & -\ 3p^2 & -\ 3 \end{array}$$

Addition/subtraction of unlike terms

You should realise that you cannot add or subtract unlike terms to a single term. All that can be done is simply to connect them by the appropriate sign '+ or–'.

For example :

(*i*) Sum of $3x^2$ and $-5xy$ is $3x^2 - 5xy$.

(*ii*) Subtraction of $-3p^2q$ from $7q^2$ is $7q^2 - (-3p^2q)$ *i.e.* $7q^2 + 3p^2q$.

Exercise 10.2

1. Add :

(i) $7ab, -3ab, \frac{3}{2}ab, -ab$ (ii) $5x, -7y, 9x, 11y, -15x$

(iii) $7pq, -3p^2, -5qp, 2p^2$ (iv) $\frac{2}{3}x, -5x, 7y, -3z, -\frac{4}{5}y.$

2. Subtract :

(i) $-7p^2$ from $5p^2$ (ii) $5xy$ from $-7yx$

(iii) $-\frac{4}{7}p^2q^2r$ from $\frac{2}{3}p^2q^2r$ (iv) $-\frac{3}{2}x^2$ from $5y^2$.

3. Simplify the following :

(i) $3x^2 - 4y^2 + 5xy + 6x^2 - 9xy + 3y^2 - 8x^2$

(ii) $\frac{2}{3}x^3 - 5x^2 + 3x + \frac{4}{5} - 2x^3 - \frac{7}{2}x^2 + 5x - 1 + 2x^4$

(iii) $\frac{1}{2}p - 3q + \frac{2}{3}r + \frac{4}{5}p + \frac{3}{2}q - 2r + \frac{2}{3}p - 7r.$

4. Add the following expressions :

(i) $2a - 7b + 5c - 2, -3a + 2b + 9c, 4a - 6b + 5c - 4$

(ii) $3x - 4y + z, 2x + 5y - z, 6x - 7z, 9y + 2z - 3x + 5$

(iii) $4x^3 - 7x^2 + 9, 3x^2 - 5x + 4, 7x^3 - 11x + 1, 6x^2 - 13x$

(iv) $5ax - 3by + 7cz, 7by - 11ax - 3cz, 12cz - ax - 3by.$

5. The two adjacent sides of a rectangle are $3x^2 - 2y^2$ and $x^2 + 3xy$. Find its perimeter.

6. Subtract :

(i) $5a - 3b + 11c - 2$ from $3a + 5b - 9c + 3$

(ii) $3x^2 - 9xy + y^2$ from $7x^2 - 10xy - y^2 + 5$

(iii) $4p^2 + 5q^2 - 7r^2 + 9$ from $3p^2 - 4q^2 - 8r^2$

(iv) $5m^4 - 3m^3 + 2m^2 - 3$ from $9m^4 + 5m^2 - 7m + 6$.

7. Take away $3p - 2q + 3r + 5$ from $5p + 7q - 2r + 4$.

8. Subtract $3x + 5y - 4z + 3$ from the sum of $5x - 3y + 7z - 2$ and $7y - 6x + 5z + 11$.

9. Subtract the sum of $3x^2 + 5xy + 7y^2 + 3$ and $2x^2 - 4xy - 3y^2 + 7$ from $9x^2 - 8xy + 11y^2$.

10. What must be added to $5x^2 - 3x + 1$ to get $3x^3 - 7x^2 + 8$?

11. What must be subtracted from $3a^2 - 5ab - 2b^2 - 3$ to get $5a^2 - 7ab - 3b^2 + 3a$?

12. Find the excess of $3y^3 - 5y + 7$ over $5y^3 - 3y^2 + 9$.

13. If $x = 2a - 3b + c$ and $y = 5a + 2b - 3c$, calculate the following in terms of a, b and c :

(i) $x + 2y$ (ii) $3x + y$ (iii) $3x - 2y$ (iv) $2x - 3y.$

14. The perimeter of a triangle is $7p^2 - 5p + 11$ and two of its sides are $p^2 + 2p - 1$ and $3p^2 - 6p + 3$. Find the third side of the triangle.

MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

If x is a literal and m, n are positive integers, then $x^m \times x^n = x^{m+n}$.

Multiplication of two monomials

Product of two monomials = (product of their numerical coefficients) × (product of their literal coefficients)

Example 1. Find the product of :

(i) $\frac{2}{3}x^2y^3$ and $-\frac{6}{7}xy^2$ (ii) $7ab$, $-\frac{4}{5}a^2b^2c$ and $-\frac{5}{7}bc^3$.

Solution. (i) $\left(\frac{2}{3}x^2y^3\right) \times \left(-\frac{6}{7}xy^2\right) = \frac{2}{3} \times \left(-\frac{6}{7}\right) \times x^2y^3 \times xy^2 = -\frac{4}{7}x^3y^5$.

(ii) $(7ab) \times \left(-\frac{4}{5}a^2b^2c\right) \times \left(-\frac{5}{7}bc^3\right)$

$= 7 \times \left(-\frac{4}{5}\right) \times \left(-\frac{5}{7}\right) \times ab \times a^2b^2c \times bc^3 = 4a^3b^4c^4$.

Multiplication of a polynomial by a monomial

Multiply each term of the polynomial by the monomial.

Example 2. (i) Multiply $5p^2q - 3pq^2 + 2pq - 5$ by $7p^2q$.

(ii) Simplify $-3xy^2\,(7x^3 - 5xy^2 - 2y^3 + 3xy + 2x - 5)$.

Solution. (i) $7p^2q\,(5p^2q - 3pq^2 + 2pq - 5)$

$= 7p^2q \times 5p^2q - 7p^2q \times 3pq^2 + 7p^2q \times 2pq - 7p^2q \times 5$

$= 35p^4q^2 - 21p^3q^3 + 14p^3q^2 - 35p^2q$

Column method :

$$\begin{array}{r} 5p^2q - 3pq^2 + 2pq - 5 \\ 7p^2q \\ \hline 35p^4q^2 - 21p^3q^3 + 14p^3q^2 - 35p^2q \end{array}$$

(Multiply each term of the polynomial by the monomial $7p^2q$)

(ii)
$$\begin{array}{r} 7x^3 - 5xy^2 - 2y^3 + 3xy + 2x - 5 \\ -3xy^2 \\ \hline -21x^4y^2 + 15x^2y^4 + 6xy^5 - 9x^2y^3 - 6x^2y^2 + 15xy^2 \end{array}$$

Multiplication of two polynomials

Multiply each term of one polynomial with each term of the other polynomial and combine the like terms in the product.

Example 3. Simplify : $(5x - 8y)(9x + 4y)$.

Solution. $(5x - 8y)(9x + 4y) = 5x(9x + 4y) - 8y(9x + 4y)$

$= 45x^2 + 20xy - 72xy - 32y^2 = 45x^2 - 52xy - 32y^2$.

Column method :

$$\begin{array}{rrrl}
5x & -\ 8y & & \\
9x & +\ 4y & & \\
\hline
45x^2 & -\ 72xy & & \text{(Multiply first polynomial by } 9x\text{)} \\
 & 20xy & -\ 32y^2 & \text{(Multiply first polynomial by } 4y\text{)} \\
\hline
45x^2 & -\ 52xy & -\ 32y^2 & \text{(Add like terms)}
\end{array}$$

Example 4. Multiply :

(*i*) $5x + 3x^2 - 7$ by $2 + 3x$

(*ii*) $2x^2 - 3x + 5$ by $5x^2 - 2x - 7$.

Solution. (*i*) Arranging the terms of the given polynomials in descending powers of x and then multiplying, we get

$$\begin{array}{rrrrl}
3x^2 & +\ 5x & -\ 7 & & \\
 & 3x & +\ 2 & & \\
\hline
9x^3 & +\ 15x^2 & -\ 21x & & \text{(Multiply by } 3x\text{)} \\
 & 6x^2 & +\ 10x & -\ 14 & \text{(Multiply by 2)} \\
\hline
9x^3 & +\ 21x^2 & -\ 11x & -\ 14 & \text{(Add like terms)}
\end{array}$$

(*ii*)

$$\begin{array}{rrrrr}
2x^2 & -\ 3x & +\ 5 & & \\
5x^2 & -\ 2x & -\ 7 & & \\
\hline
10x^4 & -\ 15x^3 & +\ 25x^2 & & \\
 & -\ 4x^3 & +\ 6x^2 & -\ 10x & \\
 & & -\ 14x^2 & +\ 21x & -\ 35 \\
\hline
10x^4 & -\ 19x^3 & +\ 17x^2 & +\ 11x & -\ 35
\end{array}$$

Example 5. Multiply :

(*i*) $5x^4 - 2x^3 + 3x - 4$ with $2 - 3x - 7x^2$

(*ii*) $3x^2 - 2xy - 4y^2$ with $2x - 3y - 5$

Solution. (*i*) Arranging the terms of the given polynomials in descending powers of x and then multiplying, we get

$$\begin{array}{rrrrrrr}
 & 5x^4 & -\ 2x^3 & & +\ 3x & -\ 4 & \\
 & & -\ 7x^2 & -\ 3x & +\ 2 & & \\
\hline
-\ 35x^6 & +\ 14x^5 & & -\ 21x^3 & +\ 28x^2 & & \\
 & -\ 15x^5 & +\ 6x^4 & & -\ 9x^2 & +\ 12x & \\
 & & 10x^4 & -\ 4x^3 & & +\ 6x & -\ 8 \\
\hline
-\ 35x^6 & -\ x^5 & +\ 16x^4 & -\ 25x^3 & +\ 19x^2 & +\ 18x & -\ 8
\end{array}$$

(*ii*)

$$\begin{array}{rrrrrrr}
3x^2 & -\ 2xy & -\ 4y^2 & & & & \\
2x & -\ 3y & -\ 5 & & & & \\
\hline
6x^3 & -\ 4x^2y & -\ 8xy^2 & & & & \\
 & -\ 9x^2y & +\ 6xy^2 & +\ 12y^3 & & & \\
 & & & & -\ 15x^2 & +\ 10xy & +\ 20y^2 \\
\hline
6x^3 & -\ 13x^2y & -\ 2xy^2 & +\ 12y^3 & -\ 15x^2 & +\ 10xy & +\ 20y^2
\end{array}$$

Exercise 10.3

1. Find the product of :

   (i) $-5x^2$ and $\frac{7}{10}x^3y^2$ (ii) $-\frac{2}{3}p^2q$, $\frac{3}{4}pq^2$ and $5pqr$

   (iii) $-7ab$, $-3a^3$ and $-\frac{2}{7}ab^2$ (iv) $-\frac{1}{2}x^2$, $-\frac{3}{5}xy$, $\frac{2}{3}yz$ and $\frac{5}{7}xyz$.

2. Multiply :

   (i) $3x - 5y + 7z$ by $-3xyz$ (ii) $2p^2 - 3pq + 5q^2 + 5$ by $-2pq$

   (iii) $\frac{2}{3}a^2b - \frac{4}{5}ab^2 + \frac{2}{7}ab + 3$ by $35ab$ (iv) $7x^5 - 4x^3 + 5x^2 - \frac{2}{3}x + 6$ by $-6x^3$.

3. Simplify the following :

   (i) $(5x - 2)(3x + 4)$ (ii) $(ax + b)(cx + d)$

   (iii) $(4p - 7)(2 - 3p)$ (iv) $(2x^2 + 3)(3x - 5)$

   (v) $2\left(a - \frac{1}{2}\right)\left(a - \frac{1}{3}\right)$ (vi) $(3x - 5y)(5x - 3y)$.

4. Multiply :

   (i) $2x^2 + 3x - 1$ by $3x + 1$ (ii) $x - 2y + 3$ by $x + 2y$

   (iii) $2 - 3x + 5x^2$ by $2x - 3$ (iv) $3x^3 - 2x^2 + 5$ by $5 - 3x$.

5. Multiply :

   (i) $3x^2 - 2x - 1$ by $2x^2 + x - 5$ (ii) $2 - 3y - 5y^2$ by $2y - 1 + 3y^2$

   (iii) $2p - 3q + 5$ by $5p + 2q - 3$.

6. Simplify :

   (i) $(x + 2)(x + 3)(x + 4)$ (ii) $(x + 3)(x - 3)(x + 4)(x - 4)$.

7. If two adjacent sides of a rectangle are $5x^2 + 25xy + 4y^2$ and $2x^2 - 2xy + 3y^2$, find its area.

DIVISION OF ALGEBRAIC EXPRESSIONS

If x is a literal and m, n are positive integers, then

$$x^m \div x^n = x^{m-n}, \text{ when } m > n.$$

Division of a monomial by a monomial

Quotient of two monomials = (quotient of their numerical coefficients) × (quotient of their literal coefficients)

Example 1. Divide :

(i) $-24x^2y^3z^5$ by $6xy^5z^4$ (ii) $-52pqr^3$ by $-8p^3q^4r$.

Solution. (i) $-24x^2y^3z^5 \div 6xy^5z^4 = \frac{-24x^2y^3z^5}{6xy^5z^4} = \left(-\frac{24}{6}\right) \times \frac{x^2y^3z^5}{xy^5z^4}$

$$= -4 \times \frac{x^{2-1} \times z^{5-4}}{y^{5-3}} = -4\frac{xz}{y^2}.$$

$$(ii)\ -52pqr^3 \div -8p^3q^4r = \frac{-52pqr^3}{-8p^3q^4r} = \left(\frac{-52}{-8}\right) \times \frac{pqr^3}{p^3q^4r}$$

$$= \frac{13}{2} \times \frac{r^{3-1}}{p^{3-1}q^{4-1}} = \frac{13r^2}{2p^2q^3}.$$

Division of a polynomial by a monomial

Divide each term of the polynomial by the monomial.

Example 2. Divide :

(i) $4x^5 - 3x^4 - 6x^2 + 5x + 8$ by $2x^2$

(ii) $30x^3y^2 - 20xy^3 + 12xy - 13y$ by $-5xy$

Solution. (i) $$\frac{4x^5 - 3x^4 - 6x^2 + 5x + 8}{2x^2} = \frac{4x^5}{2x^2} - \frac{3x^4}{2x^2} - \frac{6x^2}{2x^2} - \frac{5x}{2x^2} + \frac{8}{2x^2}$$

$$= 2x^3 - \frac{3}{2}x^2 - 3 + \frac{5}{2x} + \frac{4}{x^2}$$

(ii) $$\frac{30x^3y^2 - 20xy^3 + 12xy - 13y}{-5xy} = \frac{30x^3y^2}{-5xy} - \frac{20xy^3}{-5xy} + \frac{12xy}{-5xy} - \frac{13y}{-5xy}$$

$$= -6x^2y + 4y^2 - \frac{12}{5} + \frac{13}{5x}.$$

Division of a polynomial by a polynomial

For dividing one polynomial by another, proceed as under :

(i) *Arrange the terms of the dividend and the divisor in descending order of their degrees.*

(ii) *Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient.*

(iii) *Multiply all terms of the divisor by the first term of the quotient; write these terms below the corresponding terms of the dividend; carry out subtraction and find the remainder.*

(iv) *Take down the remaining terms of the dividend next to the remainder so obtained. Treat it as new dividend and repeat steps (ii) and (iii) till we obtain a remainder which is either 0 or a polynomial of degree less than that of divisor.*

Remember the formula :

$$\boxed{\text{Dividend} = \text{divisor} \times \text{quotient} + \text{remainder}}$$

Example 3. Divide $6x^3 - 11x^2 + 7x + 5$ by $2x - 3$ and verify your answer.

Solution.

$$\begin{array}{r|rrrr}
 & 3x^2 & -\ x & +\ 2 & \\
\hline
2x - 3 & 6x^3 & -\ 11x^2 & +\ 7x & +\ 5 \\
 & 6x^3 & -\ 9x^2 & & \\
 & - & + & & \\
\hline
 & & -\ 2x^2 & +\ 7x & +\ 5 \\
 & & -\ 2x^2 & +\ 3x & \\
 & & + & - & \\
\hline
 & & & 4x & +\ 5 \\
 & & & 4x & -\ 6 \\
 & & & - & + \\
\hline
 & & & & 11
\end{array}$$

$\therefore$ Quotient $= 3x^2 - x + 2$ and remainder $= 11$

Verification :

$$\begin{aligned}\text{Divisor} \times \text{quotient} + \text{remainder} &= (2x-3)(3x^2-x+2)+11\\ &= 6x^3-2x^2+4x-9x^2+3x-6+11\\ &= 6x^3-11x^2+7x+5 = \text{dividend.}\end{aligned}$$

Example 4. Divide $3 - 4x - 32x^2 - 19x^3 + 10x^4$ by $3x - 1 + 5x^2$

Solution. Arranging the terms of the dividend and the divisor in descending order of powers of x and then on dividing, we get

$$\begin{array}{r|l}
 & 2x^2 - 5x - 3 \\ \hline
5x^2+3x-1 & 10x^4 - 19x^3 - 32x^2 - 4x + 3 \\
 & 10x^4 + 6x^3 - 2x^2 \\
 & - \quad - \quad + \\ \hline
 & \quad -25x^3 - 30x^2 - 4x + 3 \\
 & \quad -25x^3 - 15x^2 + 5x \\
 & \quad + \quad + \quad - \\ \hline
 & \qquad -15x^2 - 9x + 3 \\
 & \qquad -15x^2 - 9x + 3 \\
 & \qquad + \quad + \quad - \\ \hline
 & \qquad\qquad 0
\end{array}$$

$\therefore$ Quotient $= 2x^2 - 5x - 3$ and remainder $= 0$.

Example 5. Divide $3x^5 + 7x^4 - 11x^3 + 8x^2 - 32x + 5$ by $2 + 3x + x^2$.

Solution. Arranging the terms of the divisor in descending order of powers of x and then on dividing, we get

$$\begin{array}{r|l}
 & 3x^3 - 2x^2 - 11x + 45 \\ \hline
x^2+3x+2 & 3x^5 + 7x^4 - 11x^3 + 8x^2 - 32x + 5 \\
 & 3x^5 + 9x^4 + 6x^3 \\
 & - \quad - \quad - \\ \hline
 & \quad -2x^4 - 17x^3 + 8x^2 - 32x + 5 \\
 & \quad -2x^4 - 6x^3 - 4x^2 \\
 & \quad + \quad + \quad + \\ \hline
 & \qquad -11x^3 + 12x^2 - 32x + 5 \\
 & \qquad -11x^3 - 33x^2 - 22x \\
 & \qquad + \quad + \quad + \\ \hline
 & \qquad\qquad 45x^2 - 10x + 5 \\
 & \qquad\qquad 45x^2 + 135x + 90 \\
 & \qquad\qquad - \quad - \quad - \\ \hline
 & \qquad\qquad\quad -145x - 85
\end{array}$$

$\therefore$ Quotient $= 3x^3 - 2x^2 - 11x + 45$ and remainder $= -145x - 85$.

Example 6. Divide $3y^3 + 10xy^2 - 17x^2y + 6x^3$ by $2x - 3y$.

Solution. Arranging the terms of the dividend in descending order of powers of x and then dividing, we get

$$
\begin{array}{r|rrrr}
 & 3x^2 & -\ 4xy & -\ y^2 & \\
\cline{2-5}
2x - 3y & 6x^3 & -\ 17x^2y & +\ 10xy^2 & +\ 3y^3 \\
 & 6x^3 & -\ 9x^2y & & \\
 & - & + & & \\
\cline{2-5}
 & & -\ 8x^2y & +\ 10xy^2 & +\ 3y^3 \\
 & & -\ 8x^2y & +\ 12xy^2 & \\
 & & + & - & \\
\cline{3-5}
 & & & -\ 2xy^2 & +\ 3y^3 \\
 & & & -\ 2xy^2 & +\ 3y^3 \\
 & & & + & - \\
\cline{4-5}
 & & & & 0
\end{array}
$$

$\therefore$ Quotient $= 3x^2 - 4xy - y^2$ and remainder $= 0$.

Exercise 10.4

1. Divide :
   (i) $15a^3$ by $-3a^2$
   (ii) $-21x^2y^3z$ by $14xyz^3$
   (iii) $-39pq^2r^5$ by $-24p^3q^3r$
   (iv) $-\frac{3}{4}a^2b^3$ by $\frac{6}{7}a^3b^2$.

2. Divide :
   (i) $12x - 9y + 21z$ by -3
   (ii) $9x^4 - 8x^3 - 12x + 3$ by $3x$
   (iii) $24a^3b^2 - 36a^4b^3 + 12ab - 2$ by $6a^2b^2$
   (iv) $14p^2q^3 - 32p^3q^2 + 15pq^2 - 22p + 18q$ by $-2p^2q$.

3. Divide :
   (i) $6x^2 + 13x + 5$ by $2x + 1$
   (ii) $1 + y^3$ by $1 + y$
   (iii) $12x^2 + 5x + 7$ by $3x - 1$
   (iv) $5 + x - 2x^2$ by $x + 1$
   (v) $6p^2 + p - 15$ by $3p + 5$
   (vi) $x^3 - 6x^2 + 12x - 8$ by $x - 2$.

4. Divide :
   (i) $6x^3 + x^2 - 26x - 25$ by $3x - 7$
   (ii) $3y^3 - 4y^2 - 3y + 25$ by $5 + 3y$
   (iii) $m^3 - 6m^2 + 7$ by $m - 1$.

5. Divide :
   (i) $a^3 + 2a^2 + 2a + 1$ by $a^2 + a + 1$
   (ii) $12x^3 - 17x^2 + 26x - 18$ by $3x^2 - 2x + 5$
   (iii) $3x^4 - 14x^3 + 12x^2 + 6x + 5$ by $x^2 - 4x - 1$.

6. If the area of a rectangle is $8x^2 - 45y^2 + 18xy$ and one of its sides is $4x + 15y$, find the length of adjacent side.

REMOVAL OF BRACKETS AND USE OF RULE OF BODMAS

Removal of brackets

Brackets are used as grouping symbols. Brackets are removed in order of :

(*i*) line bracket (*ii*) common brackets

(*iii*) curly brackets and lastly (*iv*) rectangular brackets

Example 1. Simplify : $5ab - 3\{2a^2 - 5a(b - 3a)\} - \{5b^2 - b(2a - 3b)\}$

Solution.
$$5ab - 3\{2a^2 - 5a(b - 3a)\} - \{5b^2 - b(2a - 3b)\}$$
$$= 5ab - 3\{2a^2 - 5ab + 15a^2\} - \{5b^2 - 2ab + 3b^2\}$$
$$= 5ab - 3\{17a^2 - 5ab\} - \{8b^2 - 2ab\}$$
$$= 5ab - 51a^2 + 15ab - 8b^2 + 2ab = 22ab - 51a^2 - 8b^2.$$

Example 2. Simplify : $7z - 2[3(x - 2y) - 4\{2z - (3x - \overline{2y - 3x - 5z})\}]$

Solution.
$$7z - 2[3(x - 2y) - 4\{2z - (3x - \overline{2y - 3x - 5z})\}]$$
$$= 7z - 2[3(x - 2y) - 4\{2z - (3x - 2y + 3x + 5z)\}]$$
$$= 7z - 2[3x - 6y - 4\{2z - (6x - 2y + 5z)\}]$$
$$= 7z - 2[3x - 6y - 4\{2z - 6x + 2y - 5z\}]$$
$$= 7z - 2[3x - 6y - 4\{-3z - 6x + 2y\}]$$
$$= 7z - 2[3x - 6y + 12z + 24x - 8y\}]$$
$$= 7z - 2[27x - 14y + 12z]$$
$$= 7z - 54x + 28y - 24z = -17z - 54x + 28y.$$

Use of rule of BODMAS

In chapter 3 (on Number Systems), you have learnt the rule of BODMAS. The same rule is applicable in algebra.

Thus, $a \div b + b = \frac{a}{b} + b$, while $a \div (b + b) = a \div 2b = \frac{a}{2b}$.

Example 3. Simplify the following expressions :

(*i*) $a^2 \div (b \times a) - a + a^2 \div a$

(*ii*) $(a^2 \div b) \times (a - a) + a^2 \div a$

(*iii*) $13x - 12ax \div 3a + \frac{1}{2}$ of $10y - 27y^2 \div 9y$.

Use rule of BODMAS

Solution. (*i*) $a^2 \div (b \times a) - a + a^2 \div a = a^2 \div ba - a + a^2 \div a$
$$= \frac{a^2}{ba} - a + \frac{a^2}{a} = \frac{a}{b} - a + a = \frac{a}{b}.$$

(*ii*) $(a^2 \div b) \times (a - a) + a^2 \div a = \frac{a^2}{b} \times 0 + \frac{a^2}{a} = 0 + a = a.$

(*iii*) $13x - 12ax \div 3a + \frac{1}{2}$ of $10y - 27y^2 \div 9y$
$$= 13x - \frac{12ax}{3a} + \frac{1}{2} \times 10y - \frac{27y^2}{9y}$$
$$= 13x - 4x + 5y - 3y = 9x + 2y.$$

Exercise 10.5

Simply the following (1 to 3) algebraic expressions :

1. (*i*) $3p - 7\{2(p - 3q) - 3(q - 2p)\}$ (*ii*) $7(2x - 3) - 5x\{3 - 2x(1 - 5x)\}$
2. (*i*) $16a - [7b - 2\{a - (3b - 5a) - 4b\} - 6a]$
   (*ii*) $9x - [5x - y - \{4y - 2(3x - \overline{x - y})\}]$
3. (*i*) $16p - 6p \div 2 + 2\,(3p - 1)$ (*ii*) $a^3 \div a + a^2 - \frac{1}{3}$ of $6a^2$
   (*iii*) $a^3 \div (a + a) - \frac{2}{3}$ of $(5a + 8a^2 \div 2a)$

Summary

- A symbol which has a fixed value is called a *constant* and a symbol which can take various numerical values is called a *variable* (or *literal*).
- A collection of constants and literals connected by one or more of the operations of '+, –, × or ÷' is called an *algebraic expression*.
- The various parts of an algebraic expression separated by '+ or –' sign are called *terms* of the algebraic expression.
- *Monomials*, *binomials* and *trinomials* have one, two and three terms respectively.
- *Multinomial* has two or more terms.
- A term having no literals is called a *constant term*.
- Any factor of a (non-constant) term is called the *coefficient* of the remaining factor of the term.
- Terms having same literal coefficients are called *like terms*, otherwise the terms are called *unlike terms*.
- An algebraic expression in which the powers of the variables (literals) in each term are non-negative integers is called a *polynomial*.
- The *degree* of a polynomial in one variable is the greatest power of the variable present in the polynomial.
- In case of a polynomial in two or more variables, take the sum of the powers of all variables in each term. The greatest sum is the *degree* of the polynomial.
- Polynomials of degree 1, 2 and 3 are called *linear*, *quadratic* and *cubic* respectively.
- The *sum* of two or more like terms is a like term whose coefficient is the sum of coefficients of like terms.
- To *add* two or more algebraic expressions, we collect different groups of like terms and then find the sum of like terms in each group.
- To *subtract* one like term from another — change the sign of the term to be subtracted and then add.
- To *subtract* one algebraic expression from another — change the sign of each term of the expression to be subtracted and then add.
- *Product* of two monomials = (product of their numerical coefficients) × (product of their literal coefficients).

- To *multiply* a polynomial by a monomial — multiply each term of the polynomial by the monomial.
- To *multiply* two polynomials — multiply each term of one polynomial with each term of the other polynomial and combine the like terms in the product.
- Quotient of two monomials = (quotient of their numerical coefficients) × (quotient of their literal coefficients).
- To divide a polynomial by a monomial — divide each term of the polynomial by the monomial.
- The division of a polynomial by another polynomial can be checked by using the formula :

  dividend = divisor × quotient + remainder
- Brackets are grouping symbols. When removing the brackets, we start from the inner most grouping symbol and work our way outwards.
- Rule of BODMAS is applicable in algebraic expressions also.

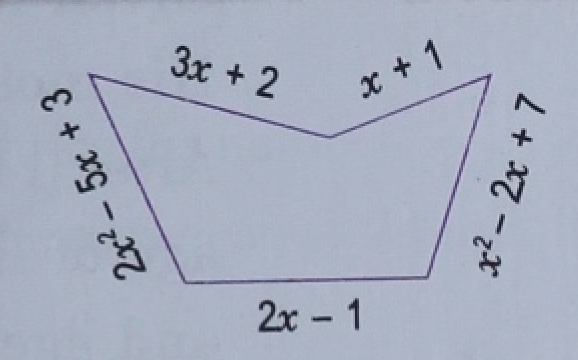
Check Your Progress

1. Identify which of the following algebraic expressions are polynomials. If so, write their degrees.

   (*i*) $3 - 7t + 8t^3 - \frac{3}{2}t^4$ (*ii*) $5p^4 - \frac{4}{5}p^2 + \frac{3}{2p} - \frac{7}{p^3}$

   (*iii*) $3x^3yz - 8x^2y + \frac{3}{5}y^2z^3 + \sqrt{2}\,z^4 + 3$ (*iv*) $\frac{3}{7}abc^2 - \frac{2}{3}b^2c^3 + 5abc - \frac{2}{b^2} + 9$

2. Find the perimeter of the adjoining polygon.



3. By how much does $5x^4 - 3x^3 - 7x + 3$ exceed $7x^4 - 5x^3 + 11x^2 + 9$?
4. By how much is $7x^3 - 3x^2y - 5xy^2 + 3y^3$ less than $5x^3 + 4x^2y - 8xy^2 + 13y^3$?
5. Multiply : $3 - 5x + 7x^2 - 2x^3$ with $1 - 3x + 2x^2$.
6. Divide $x^2 - \frac{1}{4}$ by $x + \frac{1}{2}$.
7. Divide $10x^4 - 19x^3 + 17x^2 + 15x - 42$ by $2x^2 - 3x + 5$.
8. The length and breadth of a rectangle are $(3a - b)$ units and $(a + 3b)$ units respectively. If the perimeter of a square is equal to the perimeter of the rectangle, by how much does the area of the square exceed the area of rectangle?

   [**Hint.** Perimeter of the rectangle = $2(3a - b + a + 3b) = 8a + 4b$

   ∴ Perimeter of square = $8a + 4b$

   ∴ The length of each side of square = $(8a + 4b) \div 4 = 2a + b$

   Area of square = $(2a + b)^2$ and area of rectangle = $(3a - b)(a + 3b)$.]
9. Simplify the following expressions :

   (*i*) $23x - [15y - \{4y - 2(3x - 2y) - 3(5x - \overline{2x - y})\}]$

   (*ii*) $7x^2 - 3x \times 2x + 6x^3 \div 3x + \frac{2}{5}$ of $(10x^2 + 3)$.