

SQUARES AND SQUARE ROOTS

5.1 REVIEW

1. Square	<p>If a number is multiplied by itself, the product obtained is called the square of that number.</p> <p><i>e.g.</i> (i) Since, $5 \times 5 = 25$; \therefore 25 is square of 5 and we write $(5)^2 = 25$. (ii) 0.04 is square of 0.2 as $0.2 \times 0.2 = 0.04$ and so on.</p>
2. Square root	<p>The square root of a given number x is the number whose square is x. <i>e.g.</i> square root of 36 is 6 as square of 6 is 36 <i>i.e.</i> $6^2 = 36$.</p> <p>The symbol of square root is radical sign $\sqrt{\quad}$.</p> <p>Thus, square root of 64 = $\sqrt{64} = 8$; square root of 1.44 = $\sqrt{1.44} = 1.2$ and so on.</p> <p>The sign $\sqrt{\quad}$ is of the form of letter r, the first letter of the Latin word radix meaning a root.</p>

1. $4^2 = 16$ is also read as; 4 **raised to the power 2** is 16.
2. Squares of even numbers are always even.
e.g. $2^2 = 2 \times 2 = 4$; $6^2 = 6 \times 6 = 36$; $14^2 = 14 \times 14 = 196$ and so on.
3. Squares of odd numbers are always odd.
e.g. $3^2 = 3 \times 3 = 9$; $7^2 = 49$; $15^2 = 225$ and so on.
4. Whether the number is negative or positive, its square is always positive.
e.g. $(3)^2 = 3 \times 3 = 9$, **which is a positive number.**
 $(-3)^2 = -3 \times -3 = 9$, **which is also a positive number.**
Similarly, $(-5)^2 = 25$ and $(5)^2 = 25$, $(-8)^2 = 64$ and $8^2 = 64$.
5. Since, the square of every number is positive, the square root of a positive number can be obtained, but the square root of a negative number is not possible.

TEST YOURSELF

1. The square of 0.5 = and the square root of 0.49 =
2. The square root of 81 = and the square of 25 =
3. Square of an odd number is always an number.
4. Square of an even number is always an number.
5. Square of every integer is
6. Square of every negative number is

5.2 PERFECT SQUARE

A number, whose exact square root can be obtained, is called a **perfect square**.

e.g. 16, 49, 1.21, $\frac{9}{16}$, etc. are perfect squares as $\sqrt{16} = 4$, $\sqrt{49} = 7$, $\sqrt{1.21} = 1.1$ and so on.

To find out whether a given number is a perfect square or not, express the number as a product of its prime factors. If the number is a perfect square, you would be able to group all the factors in pairs in such a way that both the factors in each pair are equal.

Example 1 :

Is 196 a perfect square ?

Solution :

$$196 = 2 \times 2 \times 7 \times 7 = \overline{2 \times 2} \times \overline{7 \times 7}$$

\therefore The prime factors of 196 can be grouped in pairs; **196 is a perfect square. (Ans.)**

Example 2 :

Is 180 a perfect square ?

Solution :

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = \overline{2 \times 2} \times \overline{3 \times 3} \times 5$$

Since, all the prime factors of 180 cannot be grouped in pairs. [One factor (*i.e.* 5) is left]

\therefore **180 is not a perfect square. (Ans.)**

5.3 TO FIND THE SQUARE ROOT OF A PERFECT SQUARE NUMBER (Using Prime Factor Method)

Example 3 :

Find the square root of 484.

Solution :

$$\text{Square root of 484} = \sqrt{484}$$

- Steps :**
1. Resolve the number into prime factors : $= \sqrt{2 \times 2 \times 11 \times 11}$
 2. Make pairs such that both the factors in each pair are equal : $= \sqrt{(2 \times 2) \times (11 \times 11)}$
 3. Take one factor from each pair : $= 2 \times 11$
 4. **The product is the square root of the given number = 22 (Ans.)**

Example 4 :

Find the smallest number by which 980 be multiplied so that the product is a perfect square.

Solution :

$$980 = \overline{2 \times 2} \times 5 \times \overline{7 \times 7}$$

Since, the prime factor 5 is not in pair.

\therefore **The given number should be multiplied by 5. (Ans.)**

$$980 \times 5 = \overline{2 \times 2} \times \overline{5 \times 5} \times \overline{7 \times 7}, \therefore \sqrt{980 \times 5} = 2 \times 5 \times 7 = 70$$

Example 5 :

Find the smallest number by which 3150 be divided, so that the quotient is a perfect square.

Solution :

$$3150 = 2 \times \overline{5 \times 5} \times \overline{3 \times 3} \times 7$$

Since, the prime factors 2 and 7 cannot be paired.

\therefore **The given number should be divided by $2 \times 7 = 14$ (Ans.)**

$$\frac{3150}{14} = \frac{\overline{2 \times 5 \times 5} \times \overline{3 \times 3} \times 7}{2 \times 7} = \overline{5 \times 5} \times \overline{3 \times 3}$$

Example 6 :

Find the square root of : (i) $2\frac{7}{9}$ (ii) 4.41

Solution :

$$(i) \text{ Square root of } 2\frac{7}{9} = \sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3} = 1\frac{2}{3} \quad (\text{Ans.})$$

$$\text{Square root of a fraction} = \frac{\text{Square root of its numerator}}{\text{Square root of its denominator}}$$

$$(ii) \sqrt{4.41} = \sqrt{\frac{441}{100}} = \sqrt{\frac{3 \times 3 \times 7 \times 7}{2 \times 2 \times 5 \times 5}} = \frac{3 \times 7}{2 \times 5} = \frac{21}{10} = 2.1 \quad (\text{Ans.})$$

1. Instead of writing the prime factors of the given number in pairs, we can write them in index form and then in order to find the required square root, take half of each index value.

$$\text{e.g. } \sqrt{784} = \sqrt{2 \times 2 \times 2 \times 2 \times 7 \times 7} = \sqrt{2^4 \times 7^2} = 2^2 \times 7^1 = 28$$

2. $\sqrt{9} = 3$, but $\sqrt{0.9} \neq 0.3$

$$\text{Reason : } (0.3)^2 = 0.3 \times 0.3 = 0.09 \quad \therefore \sqrt{0.09} = 0.3$$

In the same way, $\sqrt{144} = 12$, but $\sqrt{14.4} \neq 1.2$

$$\text{Reason : } (1.2)^2 = 1.2 \times 1.2 = 1.44 \quad \therefore \sqrt{1.44} = 1.2$$

3. Square root of a perfect square even number is always an even number and square root of a perfect square odd number is always an odd number.

$$\text{e.g. (i) } \sqrt{4} = 2, \sqrt{16} = 4, \sqrt{36} = 6, \sqrt{64} = 8, \sqrt{100} = 10 \text{ and so on.}$$

$$(ii) \sqrt{9} = 3, \sqrt{25} = 5, \sqrt{49} = 7, \sqrt{81} = 9, \sqrt{121} = 11 \text{ and so on.}$$

Example 7 :

A man plants his orchard with 5625 trees and arranges them so that there are as many rows as there are trees in each row. How many rows are there ?

Solution :

Let the number of rows be x .

\therefore Number of trees in each row = x

and, total number of trees planted = $x \times x = x^2$

$$\begin{aligned} \text{Given : } x^2 = 5625 &\Rightarrow x = \sqrt{5625} = \sqrt{5 \times 5 \times 5 \times 5 \times 3 \times 3} \\ &= 5 \times 5 \times 3 = 75 \end{aligned}$$

\therefore The number of rows = 75 (Ans.)

Example 8 :

In a basket there are 50 flowers. A man goes to worship and puts as many flowers in each temple as there are temples in the city. Thus, he needs 8 baskets of flowers. Find the number of temples in the city.

Solution :

Let the number of temples in the city = x

\therefore The number of flowers put in each temple = x

According to the given statement :

$$x^2 = 50 \times 8 \Rightarrow x = \sqrt{50 \times 8} = \sqrt{5 \times 5 \times 2 \times 2 \times 2 \times 2} = 5 \times 2 \times 2 = 20$$

\therefore The number of temples in the city = 20

(Ans.)

EXERCISE 5 (A)

1. Find the square of :

- (i) 59 (ii) 6.3 (iii) $15\frac{2}{3}$

2. By splitting into prime factors, find the square root of :

- (i) 11025 (ii) 396900 (iii) 194481

3. (i) Find the smallest number by which 2592 be multiplied so that the product is a perfect square.

(ii) Find the smallest number by which 12748 be multiplied so that the product is a perfect square.

4. Find the smallest number by which 10368 be divided, so that the result is a perfect square.

Also, find the square root of the resulting number.

5. Find the square root of :

- (i) 0.1764 (ii) $96\frac{1}{25}$ (iii) 0.0169

6. Evaluate :

- (i) $\sqrt{\frac{14.4}{22.5}}$ (ii) $\sqrt{\frac{0.225}{28.9}}$

(iii) $\sqrt{\frac{25}{32} \times 2\frac{13}{18} \times 0.25}$

(iv) $\sqrt{1\frac{4}{5} \times 14\frac{21}{44} \times 2\frac{7}{55}}$

7. Evaluate :

(i) $\sqrt{3^2 \times 6^3 \times 24}$ (ii) $\sqrt{(0.5)^3 \times 6 \times 3^5}$

(iii) $\sqrt{\left(5 + 2\frac{21}{25}\right) \times \frac{0.169}{1.6}}$

(iv) $\sqrt{5\left(2\frac{3}{4} - \frac{3}{10}\right)}$

(v) $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$

8. A man, after a tour, finds that he had spent every day as many rupees as the number of days he had been on tour. How long did his tour last, if he had spent in all ₹ 1,296?

9. Out of 745 students, maximum are to be arranged in the school field for a P.T. display, such that the number of rows is equal to the number of columns. Find the number of rows if 16 students were left out after the arrangement.

10. 13 and 31 is a strange pair of numbers such that their squares 169 and 961 are also mirror images of each other. Find two more such pairs.

11. Find the smallest perfect square divisible by 3, 4, 5 and 6.

L.C.M. of 3, 4, 5 and 6 = 60.

Also, $60 = 2 \times 2 \times 5 \times 3$ in which 5 and 3 are not in pairs.

So, 60 should be multiplied by 5×3 to get a perfect square number.

\therefore The required least square number

$= 60 \times 5 \times 3 = 900$ **Ans.**

12. If $\sqrt{784} = 28$, find the value of :

(i) $\sqrt{7.84} + \sqrt{78400}$

(ii) $\sqrt{0.0784} + \sqrt{0.000784}$

5.4 TO FIND THE SQUARE ROOT OF A PERFECT SQUARE NUMBER

(Using Division Method)

Example 9 :

Find the square root of 276676

Solution :

Steps :

- Group the digits in pairs starting from right to left, thus $276676 = \overline{27} \overline{66} \overline{76}$. Take the first pair (i.e. 27) and find the largest whole number which when multiplied by itself gives 27 or just less than 27. Such a number is 5. Write 5 in the quotient and also in the divisor.

Examine the following results :

$$\therefore (0.7)^2 = 0.49$$

$$\therefore \sqrt{0.49} = 0.7$$

$$\therefore (0.03)^2 = 0.0009$$

$$\therefore \sqrt{0.0009} = 0.03$$

$$\therefore (3.2)^2 = 10.24$$

$$\therefore \sqrt{10.24} = 3.2$$

$$\therefore (0.015)^2 = 0.000225$$

$$\therefore \sqrt{0.000225} = 0.015$$

It is clear from these results that the square of any decimal number contains even number of decimal places and that the number of decimal places in the square is double the number of decimal places in the square root. Hence, a decimal number (or a mixed decimal number) can be a perfect square only when it has an even number of digits in its decimal part.

5.5 TO FIND THE SQUARE ROOT OF A NUMBER WHICH IS NOT A PERFECT SQUARE (Using Division Method)

Example 12 :

Find the square root of 24.729 correct to two places of decimal.

Solution :

When the square root is required correct to two places of decimal, we shall find the square root up to three places of decimal and then round it off upto two places of decimal.

Similarly, if the square root is required correct to three places of decimal, find the square root up to four places and then round it off upto three places and so on.

In order to find square root upto three places of decimal, we must have three pairs of digits after decimal.

$$\text{For this purpose, } 24.729 = \overline{24.729000}$$

(Addition of any number of zeroes on the right of a decimal fraction does not change its value)

$$\begin{array}{r}
 4 \ .9 \ 7 \ 2 \\
 4 \ \overline{) 24.729000} \\
 \underline{16} \\
 8 \ 72 \\
 \underline{8 \ 01} \\
 71 \ 90 \\
 \underline{69 \ 09} \\
 281 \ 00 \\
 \underline{198 \ 84} \\
 82 \ 16
 \end{array}$$

$$\therefore \sqrt{24.729} = 4.972 \text{ upto three places of decimal}$$

$$= 4.97 \text{ correct to two places of decimal}$$

(Ans.)**Example 13 :**

Find the square root of :

- (i) 3 correct to three places of decimal. (ii) 0.07688 correct to two places of decimal.

Solution :

(i) $3 = 3.\overline{00\ 00\ 00\ 00}$

	1 7 3 2 0
1	3.00 00 00 00
	1
27	2 00
	1 89
343	11 00
	10 29
3462	71 00
	69 24
34640	17600

$\therefore \sqrt{3} = 1.7320$
 $= 1.732$ (Ans.)

(ii) $0.07688 = 0.\overline{07\ 68\ 80}$

	0 2 7 7
2	0.07 68 80
	4
47	3 68
	3 29
547	39 80
	38 29
	151

$\therefore \sqrt{0.07688} = 0.277$
 $= 0.28$ (Ans.)

Example 14 :

Find the least number that must be subtracted from 2433 so that the remainder is a perfect square.

Solution :

	49
4	24 33
	16
89	8 33
	8 01
	32

Clearly, if 32 subtracted from 2433 the remainder will be a perfect square. (Ans.)

Since, $2433 - 32 = 2401$
 and, $\sqrt{2401} = 49$

Example 15 :

Find the least number which must be added to 18,265 to obtain a perfect square.

Solution :

	135
1	1 82 65
	1
23	82
	69
265	13 65
	13 25
	40

Clearly, 18265 is greater than 135^2 .
 \therefore On adding the required number to 18265, we shall be getting 136^2 i.e. 18496

Hence, the required number = $18496 - 18265$
 $= 231$ (Ans.)

TEST YOURSELF

7. Since, $\sqrt{85} = 9.22$; $\sqrt{0.85} = \dots\dots\dots$, $\sqrt{8500} = \dots\dots\dots$, $\sqrt{0.0085} = \dots\dots\dots$
8. Since, $\sqrt{6.4} = 2.53$; $\sqrt{640} = \dots\dots\dots$, $\sqrt{0.064} = \dots\dots\dots$, $\sqrt{64000} = \dots\dots\dots$
9. $(3.2)^2 = 10.24$; $(32)^2 = \dots\dots\dots$, $(0.32)^2 = \dots\dots\dots$

EXERCISE 5 (B)

- Find the square root of :
 (i) 4761 (ii) 7744 (iii) 15129
 (iv) 0.2916 (v) 0.001225
 (vi) 0.023104 (vii) 27.3529
- Find the square root of :
 (i) 4.2025 (ii) 531.7636 (iii) 0.007225
- Find the square root of :
 (i) 245 correct to two places of decimal.
 (ii) 496 correct to three places of decimal.
 (iii) 82.6 correct to two places of decimal.
 (iv) 0.065 correct to three places of decimal.
 (v) 5.2005 correct to two places of decimal.
 (vi) 0.602 correct to two places of decimal.
- Find the square root of each of the following correct to two decimal places :
 (i) $3\frac{4}{5}$ (ii) $6\frac{7}{8}$

(i) $3\frac{4}{5} = 3.8$ (ii) $6\frac{7}{8} = 6.875$
- For each of the following, find the least number that must be subtracted so that the resulting number is a perfect square.
 (i) 796 (ii) 1886 (iii) 23497
- For each of the following, find the least number that must be added so that the resulting number is a perfect square.
 (i) 511 (ii) 7172 (iii) 55078
- Find the square root of 7 correct to two decimal places; then use it to find the value of $\sqrt{\frac{4+\sqrt{7}}{4-\sqrt{7}}}$ correct to three significant digits.

$$\sqrt{\frac{4+\sqrt{7}}{4-\sqrt{7}}} = \sqrt{\frac{(4+\sqrt{7})(4+\sqrt{7})}{(4-\sqrt{7})(4+\sqrt{7})}}$$

$$= \sqrt{\frac{(4+\sqrt{7})^2}{16-7}} = \frac{4+\sqrt{7}}{3}$$
- Find the value of $\sqrt{5}$ correct to 2 decimal places; then use it to find the square root of $\frac{3-\sqrt{5}}{3+\sqrt{5}}$ correct to 2 significant digits.

5.6 PROPERTIES OF SQUARE NUMBERS**1st Property :**

The ending digit (*i.e.* the digit at unit's place) of the square of a number is 0, 1, 4, 5, 6 or 9. In other words, the square of any number does not have 2, 3, 7 or 8 at its unit's place.

For example :

- | | | | |
|--------------------|-------------------|---------------------|----------------------|
| (i) $11^2 = 121$ | (ii) $22^2 = 484$ | (iii) $53^2 = 2809$ | (iv) $30^2 = 900$ |
| (v) $4^2 = 16$ | (vi) $25^2 = 625$ | (vii) $46^2 = 2116$ | (viii) $37^2 = 1369$ |
| (ix) $68^2 = 4624$ | (x) $19^2 = 361$ | | |

2nd Property :

If a number has 1 or 9 at its unit's place, then square of this number always has 1 (one) at its unit place :

For example :

- | | | |
|---------------------|-------------------|-------------------------------|
| (i) Square of 1 = 1 | (ii) $11^2 = 121$ | (iii) $31^2 = 961$ |
| (iv) $9^2 = 81$ | (v) $29^2 = 841$ | (vi) $49^2 = 2401$ and so on. |

3rd Property :

If the digit at the unit's place of a number is 4 or 6, then its square will always have 6 at its unit's place.

For example :

- | | | |
|--------------------|-------------------|-------------------------------|
| (i) $4^2 = 16$ | (ii) $6^2 = 36$ | (iii) $24^2 = 576$ |
| (iv) $36^2 = 1296$ | (v) $84^2 = 7056$ | (vi) $96^2 = 9216$ and so on. |

4th Property :

If a number ends with n zeroes; its square ends with $2n$ zeroes.

For example :

- | | |
|------------------------|---------------------------------------|
| (i) Square of 30 = 900 | (ii) Square of 300 = 90000 and so on. |
|------------------------|---------------------------------------|

EXERCISE 5 (C)

- | | |
|--|---|
| <p>1. Seeing the value of the digit at unit's place, state which of the following can be square of a number :</p> <p>(i) 3051 (ii) 2332 (iii) 5684
(iv) 6908 (v) 50699</p> <p>2. Squares of which of the following numbers will have 1(one) at their unit's place :</p> <p>(i) 57 (ii) 81 (iii) 139
(iv) 73 (v) 64</p> <p>3. Which of the following numbers will not have 1(one) at their unit's place :</p> <p>(i) 32^2 (ii) 57^2 (iii) 69^2
(iv) 321^2 (v) 265^2</p> | <p>4. Squares of which of the following numbers will not have 6 at their unit's place :</p> <p>(i) 35 (ii) 23 (iii) 64
(iv) 76 (v) 98</p> <p>5. Which of the following numbers will have 6 at their unit's place :</p> <p>(i) 26^2 (ii) 49^2 (iii) 34^2
(iv) 43^2 (v) 244^2</p> <p>6. If a number ends with 3 zeroes, how many zeroes will its square have ?</p> <p>7. If the square of a number ends with 10 zeroes, how many zeroes will the number have ?</p> <p>8. Is it possible for the square of a number to end with 5 zeroes ? Give reason.</p> |
|--|---|

ANSWERS**TEST YOURSELF**

1. 0.25, 0.7 2. 9, 625 3. odd 4. even 5. a positive number 6. positive 7. 0.922, 92.2, 0.0922 8. 25.3, 0.253, 253 9. 1024, 0.1024

EXERCISE 5(A)

1. (i) 3481 (ii) 39.69 (iii) $245\frac{4}{9}$ 2. (i) 105 (ii) 630 (iii) 441 3. (i) 2 (ii) 3187 4. 2 ; 72 5. (i) 0.42
(ii) $9\frac{4}{5}$ (iii) 0.13 6. (i) 0.8 (ii) $\frac{3}{34}$ (iii) $\frac{35}{48}$ (iv) $7\frac{49}{110}$ 7. (i) 216 (ii) 13.5 (iii) 0.91 (iv) $3\frac{1}{2}$ (v) 16
8. 36 days 9. 27 10. 12 and 21; 102 and 201 12. (i) 282.8 (ii) 0.308

EXERCISE 5(B)

1. (i) 69 (ii) 88 (iii) 123 (iv) 0.54 (v) 0.035 (vi) 0.152 (vii) 5.23 2. (i) 2.05 (ii) 23.06 (iii) 0.085
3. (i) 15.65 (ii) 22.271 (iii) 9.09 (iv) 0.255 (v) 2.28 (vi) 0.78 4. (i) 1.95 (ii) 2.62 5. (i) 12 (ii) 37 (iii) 88
6. (i) 18 (ii) 53 (iii) 147 7. 2.65; 2.22 8. 2.24, 0.38

EXERCISE 5(C)

1. (i), (iii) and (v) 2. (ii) and (iii) 3. (i), (ii) and (v) 4. (i), (ii) and (v) 5. (i), (iii) and (v) 6. six zeroes
7. five zeroes 8. no; it will always have an even number of zeroes.