FRACTIONS

(Including Decimals, Rounding off and Significant Figures)

4.1 REVIEW

1. Fraction	(i) A fraction is a part of the whole and is written in the form $\frac{a}{b}$. Such as : $\frac{2}{5}, \frac{3}{8}, \frac{4}{11}$, etc. (ii) The fraction $\frac{2}{5}$ means : 2 parts out of 5 equal parts of the whole (given) quantity. (iii) In any fraction $\frac{a}{b}$, a and b are called its terms. Also, in fraction $\frac{a}{b}$, its upper term ***a*** is called its **numerator** and its lower term ***b*** is called its **denominator** . Thus, $\boxed{\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}}$.

1. The value of a fraction is equal to one (unity), if its numerator is equal to its denominator.
2. The value of a fraction is equal to zero, if its numerator is zero and denominator is not zero.

   Thus, $\frac{0}{7} = 0$, $\frac{0}{-5}$ = ..., $\frac{0}{28}$ = ... and so on. $\boxed{\text{But } \frac{0}{0} \neq 0}$
3. The value of a fraction is not defined, if its denominator is zero.

   Thus, $\frac{5}{0}$ is not defined, $\frac{-8}{0}$ is ...., $\frac{325}{0}$ is ...., $\frac{0}{0}$ is .... and so on.
4. If both the terms of a fraction be multiplied or divided by the same non-zero number, the value of the fraction remains unaltered (unchanged).
5. A fraction should always be expressed in its *lowest terms.*

   To reduce a given fraction to its lowest terms, divide each of its terms by their H.C.F.

   A fraction is said to be in its lowest terms (or, in its simplest form), if its numerator and the denominator have no common factor.

4.2 KINDS OF FRACTIONS

1. Simple fraction	A fraction, whose both the terms are integers, is called a **simple fraction**. *e.g.* $\frac{3}{8}, \frac{-5}{17}, \frac{17}{-53}$, etc.
2. Complex fraction	A fraction, whose one or both the terms are fractional numbers, is called a **complex fraction**. *e.g.* $\frac{\frac{2}{3}}{7}, \frac{5}{\frac{6}{11}}, \frac{2\frac{1}{3}}{7\frac{5}{8}}$, etc.

3. **Decimal fractions**	Fractions, with denominators 10, 100, 1000, etc., are called **decimal fractions.** *e.g.* $\frac{3}{10}, \frac{57}{100}, \frac{9}{1000}, \frac{323}{10^8}$, etc.
4. **Vulgar fractions**	Fractions, whose denominators are not 10, 100, 1000, etc., are called **vulgar fractions.**
5. **Proper fraction**	A fraction, whose numerator is positive and also *less* than its denominator, is called a **proper fraction.** *e.g.* $\frac{5}{12}, \frac{19}{100}, \frac{131}{200}$, etc.
6. **Improper fraction**	A fraction, whose numerator is *greater* than its denominator, is called **improper fraction.** *e.g.* $\frac{7}{5}, \frac{100}{17}, \frac{213}{200}$, etc.
7. **Mixed fraction**	A fraction, which is expressed as a combination of an integer and a proper fraction, is called a **mixed fraction.** *e.g.* $2\frac{3}{5}$, which is the combination of an integer (2) and a proper fraction $\left(\frac{3}{5}\right)$.

1. An improper fraction can always be expressed as a mixed fraction.

   e.g. $\frac{49}{11} = \frac{44+5}{11} = \frac{44}{11} + \frac{5}{11} = 4 + \frac{5}{11} = 4\frac{5}{11}$

2. A mixed fraction can also be converted to an improper fraction by multiplying the integer with the denominator and adding numerator to the product.

   e.g. $5\frac{7}{8} = 5 + \frac{7}{8} = \frac{5 \times 8 + 7}{8} = \frac{47}{8}$

4.3 DECIMAL FRACTION

1. **Decimal**	*It is a fraction whose denominator is 10 or any integral power of 10.* *e.g.* $\frac{7}{10}, \frac{5}{10^3}, \frac{29}{100}, \frac{357}{10^5}$, etc.

2. A dot, called a decimal point is properly placed to remove the denominator of a decimal fraction.

   e.g. (i) $\frac{37}{10} = 3{\cdot}7$; **read as** : three-point-seven.

   (ii) $\frac{37}{100} = 0{\cdot}37$; **read as** : point-three-seven or zero-point-three-seven.

   (iii) $\frac{37}{1000} = 0{\cdot}037$; **read as** : zero-point-zero-three-seven and so on.

3. **Decimal places**	The number of digits which follow the decimal point is called the number of decimal places. *e.g.* (i) 3·47 has 2 decimal places, (ii) 0·0849 has 4 decimal places and so on.

4. The value of a decimal number remains unchanged by annexing cipher (or ciphers) at its extreme right or by removing them.

e.g. (i) 0·3 = 0·30 = 0·300 = 0·3000 and so on.

(ii) 3·7000 = 3·70 = 3·7 and so on.

5. In decimal number 3·47, 3 is its integral part and 0·47 is its decimal part.

6. An integer may be expressed as a decimal number by writing zero (or zeroes) in the decimal part :

e.g. 15 = 15·0 = 15·000 and so on.

7. In a decimal number, the first place to the right of decimal is called tenth's place; the second place to the right of decimal is called hundredth's place and so on.

e.g. in number 5·628; 6 is at tenth's place, 2 is at hundredth's place and 8 is at thousandth's place.

TEST YOURSELF

1. **Evaluate :** $\frac{0}{8}$ = ... , $\frac{8}{0}$ = ..., $\frac{0}{-32}$ = ..., $\frac{8}{8}$ = ..., $\frac{-8}{8}$ = ...

2. $\frac{5}{13}$ is a .... fraction, $\frac{13}{5}$ is an .... fraction, $5\frac{3}{13}$ is a .... fraction, $\frac{29}{1000}$ is a .... fraction.

3. 0·562 × 100 = ..., $\frac{0·562}{100}$ = ..., $\frac{97}{1000}$ = ...

4.4 CONVERTING A DECIMAL FRACTION INTO A VULGAR FRACTION

Steps :

1. Remove the decimal point and write the resulting number as numerator. At the same time, in the denominator, write as many zeroes to the right of one (1) as the decimal places in the given number.
2. Reduce the vulgar fraction, obtained in Step 1, to its lowest terms

e.g. $0·9 = \frac{9}{10}$, $3·48 = \frac{348}{100} = \frac{87}{25}$, $0·088 = \frac{88}{1000} = \frac{11}{125}$ and so on.

4.5 FOUR FUNDAMENTAL OPERATIONS

1. Addition and Subtraction :

Example 1 :

Evaluate : $6\frac{2}{5} - 4\frac{4}{15} + 3\frac{5}{9} - 2\frac{3}{10}$

Solution :

$$\frac{32}{5} - \frac{64}{15} + \frac{32}{9} - \frac{23}{10} = \frac{32\times18 - 64\times6 + 32\times10 - 23\times9}{90}$$ [L.C.M. of 5, 15, 9 and 10 is 90]

$$= \frac{576 - 384 + 320 - 207}{90}$$

$$= \frac{896 - 591}{90} = \frac{305}{90} = \frac{61}{18} = 3\frac{7}{18}$$ **(Ans.)**

After simplification, if required :

(i) the fraction should be reduced to its lowest terms.

(ii) an improper fraction should always be expressed as mixed fraction.

Example 2 :

Add : (i) 3·46 , 40·3 and 0·432 (ii) 0·031 , 1·32 and 25

Solution :

1. Write down the numbers under one another with their decimal points in the same vertical line.
2. Add them as it is done in ordinary addition and in the result, place decimal point in the same vertical line *i.e.* just under all decimal points.

(i)

```
   3·46    OR     3·460
  40·3           40·300
   0·432          0·432
 -------        -------
  44·192         44·192  (Ans.)
 -------        -------
```

(ii)

```
   0·031
   1·320
  25·000
 -------
  26·351   (Ans.)
 -------
```

Example 3 :

Subtract : (i) 3·468 from 5·6 (ii) 23·1 from 251·76

Solution :

1. Write down the **subtrahend** (the number to be subtracted) under the **minuend** (the number from which the subtraction is to be done) such that their decimal points are in the same vertical line.
2. Subtract like an ordinary subtraction and in the result, place decimal point just under the other decimals.

(i)

```
  5·6     OR   5·600
  3·468        3·468
 ------       ------
  2·132        2·132  (Ans.)
 ------       ------
```

(ii)

```
 251·76   OR   251·76
  23·1          23·10
 ------        ------
 228·66        228·66  (Ans.)
 ------        ------
```

2. Multiplication :

(a) Multiplication in Fractions :

(i) Multiplication by an integer :

Keeping the denominator unchanged, multiply the numerator by the integer.

e.g. $\frac{2}{7} \times 3 = \frac{2 \times 3}{7} = \frac{6}{7}$, $2\frac{3}{5} \times 4 = \frac{13}{5} \times 4 = \frac{13 \times 4}{5} = \frac{52}{5} = 10\frac{2}{5}$ and so on.

(ii) Multiplication by a fraction :

Multiply the numerators together and denominators separately together, and simplify.

e.g. $4\frac{2}{7} \times 5\frac{1}{4} = \frac{30}{7} \times \frac{21}{4} = \frac{30 \times 21}{7 \times 4} = \frac{45}{2} = 22\frac{1}{2}$

(b) Multiplication in Decimal Fraction :

1. *To multiply a decimal fraction by* 10, 100, 1000, etc. shift the decimal point to the right as many digits as the number of zeroes in the multiplier.
   e.g. (i) 5·62 × 10 = 56·2 (ii) 5·62 × 1000 = 5620 and so on.
2. *To multiply a decimal fraction by a whole number,* first multiply without taking any note of decimal point. In the product, the decimal point should be fixed by count-

ing as many digits from the right as there are decimal places in the multiplicand.

e.g. (i) $5{\cdot}673 \times 8 = 45{\cdot}384$ (ii) $0{\cdot}034 \times 5 = 0{\cdot}170$ and so on.

3. *To multiply a decimal fraction by another decimal fraction,* first multiply without taking any note of the decimal points. In the product, the decimal point is fixed counting from right, the digits equal to the sum of decimal places in the multiplicand and the multiplier.

   e.g. (i) $3{\cdot}6 \times 0{\cdot}25 = 0{\cdot}900 = 0{\cdot}9$ (ii) $0{\cdot}46 \times 1{\cdot}01 = 0{\cdot}4646$ and so on.

3. Division :

(a) Division in Fractions :

To divide a fraction by a whole number or a fraction; multiply by the ***reciprocal*** of the divisor.

e.g. (i) $\frac{5}{7} \div 3 = \frac{5}{7} \times \frac{1}{3} = \frac{5}{21}$ [reciprocal of 3 is $\frac{1}{3}$]

(ii) $4\frac{1}{5} \div 2\frac{1}{3} = \frac{21}{5} \div \frac{7}{3} = \frac{21}{5} \times \frac{3}{7}$ [reciprocal of $\frac{7}{3}$ is $\frac{3}{7}$]

$= \frac{9}{5} = 1\frac{4}{5}$ and so on.

(b) Division in Decimal Fractions :

1. *To divide a decimal fraction by* 10, 100, 1000, etc. shift the decimal point to the left as many digits as the number of zeroes in the divisor.

   e.g. (i) $43{\cdot}4 \div 100 = 0{\cdot}434$ (ii) $43{\cdot}4 \div 1000 = 0{\cdot}0434$ and so on.

2. *To divide a decimal fraction by an integer,* division is done like an ordinary division and the decimal point is marked in the quotient while just crossing over the decimal point in the dividend.

Example 4 :

Evaluate : (i) $5{\cdot}64 \div 2$ (ii) $0{\cdot}065 \div 5$

Solution :

(i)
```
     2.82
 2 ) 5.64
     4
     --
     16
     16
     ---
     × 4
       4
       -
       ×
```
$\therefore$ **5·64 ÷ 2 = 2·82 (Ans.)**

(ii)
```
      0.013
 5 ) 0.065
        5
       --
       15
       15
       --
        ×
```
$\therefore$ **0·065 ÷ 5 = 0·013** **(Ans.)**

3. *To divide a decimal fraction by another decimal fraction,* remove the decimal point from the denominator (divisor) and divide in the ordinary way.

There are two methods of removing the decimal point from the denominator.

1st method : Shift the decimal points of the numerator (dividend) and the denominator (divisor) by as many equal number of digits, which reduce the denominator (divisor) into a whole number and then divide.

e.g. (i) **$3{\cdot}36 \div 0{\cdot}4$** $= \frac{3{\cdot}36}{0{\cdot}4} = \frac{33{\cdot}6}{4} =$ **$8{\cdot}4$** (ii) **$0{\cdot}845 \div 0{\cdot}05$** $= \frac{0{\cdot}845}{0{\cdot}05} = \frac{84{\cdot}5}{5} =$ **$16{\cdot}9$**

2nd method : Multiply the numerator and the denominator both suitably by 10, 100, 1000 or higher powers of 10, so that the decimal point is removed from the denominator and then divide.

e.g. (i) $3{\cdot}36 \div 0{\cdot}4 = \frac{3{\cdot}36}{0{\cdot}4} = \frac{3{\cdot}36 \times 10}{0{\cdot}4 \times 10} = \frac{33{\cdot}6}{4} = 8{\cdot}4$

(ii) $0{\cdot}845 \div 0{\cdot}05 = \frac{0{\cdot}845}{0{\cdot}05} = \frac{0{\cdot}845 \times 100}{0{\cdot}05 \times 100} = \frac{84{\cdot}5}{5} = 16{\cdot}9$

4.6 USING BODMAS

Example 5 :

Simplify : $10 - 7{\cdot}5 \div 3\frac{1}{3}$ of $\left(2\frac{2}{9} - 1\frac{1}{3}\right) \times 0{\cdot}75 + 3$

Solution :

$= 10 - \frac{75}{10} \div \frac{10}{3}$ of $\left(\frac{20}{9} - \frac{4}{3}\right) \times \frac{75}{100} + 3$

$= 10 - \frac{15}{2} \div \frac{10}{3}$ of $\left(\frac{20-12}{9}\right) \times \frac{3}{4} + 3$ [Removing brackets (B) first]

$= 10 - \frac{15}{2} \div \frac{10}{3}$ of $\frac{8}{9} \times \frac{3}{4} + 3$

$= 10 - \frac{15}{2} \div \frac{80}{27} \times \frac{3}{4} + 3$ [Removing 'of' : $\frac{10}{3}$ of $\frac{8}{9} = \frac{80}{27}$]

$= 10 - \frac{15}{2} \times \frac{27}{80} \times \frac{3}{4} + 3$ [Completing 'division']

$= 10 - \frac{243}{128} + 3$ [Multiplication : $\frac{15}{2} \times \frac{27}{80} \times \frac{3}{4} = \frac{243}{128}$]

$= \frac{1280 - 243 + 384}{128} = \frac{1421}{128} = 11\frac{13}{128}$ **(Ans.)**

Example 6 :

Evaluate : (i) $0{\cdot}4 \times 0{\cdot}05$ of $2{\cdot}4 \div 0{\cdot}02$ (ii) $0{\cdot}7 \times 0{\cdot}6 - 0{\cdot}4 \times 0{\cdot}2 + 0{\cdot}24 \div 0{\cdot}04$

Solution :

(i) $0{\cdot}4 \times 0{\cdot}05$ of $2{\cdot}4 \div 0{\cdot}02$

$= 0{\cdot}4 \times 0{\cdot}12 \div 0{\cdot}02$ [$0{\cdot}05$ of $2{\cdot}4 = 0{\cdot}120 = 0{\cdot}12$]

$= 0{\cdot}4 \times 6$ [$0{\cdot}12 \div 0{\cdot}02 = \frac{0{\cdot}12}{0{\cdot}02} = \frac{12}{2} = 6$]

$= 2{\cdot}4$ **(Ans.)**

(ii) $0{\cdot}7 \times 0{\cdot}6 - 0{\cdot}4 \times 0{\cdot}2 + 0{\cdot}24 \div 0{\cdot}04$

$= 0{\cdot}42 - 0{\cdot}08 + 6 = 6{\cdot}42 - 0{\cdot}08 = 6{\cdot}34$ **(Ans.)**

4.7 CONTINUED FRACTIONS

Fractions of the form

$3 + \cfrac{1}{2 - \cfrac{1}{2 + \cfrac{1}{2}}}$, $\cfrac{1}{3 + \cfrac{2}{1 + \cfrac{1}{2}}}$, etc., are called **continued fractions.**

To simplify a continued fraction, begin at the bottom and work upwards.

Thus, $3+\cfrac{1}{2-\cfrac{1}{2+\cfrac{1}{2}}} = 3+\cfrac{1}{2-\cfrac{1}{\cfrac{5}{2}}}$

$$= 3+\cfrac{1}{2-\cfrac{2}{5}} = 3+\cfrac{1}{\cfrac{8}{5}} = 3+\frac{5}{8} = \frac{29}{8} = 3\frac{5}{8}$$ **(Ans.)**

4.8 USING FORMULAE

The following formulae (identities) may be used for evaluating certain types of expressions:

(i) $(a+b)^2 = a^2 + 2ab + b^2$ (ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $a^2 - b^2 = (a+b)(a-b)$ (iv) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(v) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Example 7 :

Evaluate : (i) $\left(1\frac{2}{3}\right)^2 - \left(\frac{3}{4}\right)^2$ (ii) $\frac{(2{\cdot}39)^2 - (1{\cdot}61)^2}{2{\cdot}39 - 1{\cdot}61}$

Solution :

(i) $\left(1\frac{2}{3}\right)^2 - \left(\frac{3}{4}\right)^2 = \left(1\frac{2}{3}+\frac{3}{4}\right)\left(1\frac{2}{3}-\frac{3}{4}\right)$ [$a^2 - b^2 = (a+b)(a-b)$]

$$= \left(\frac{5}{3}+\frac{3}{4}\right)\left(\frac{5}{3}-\frac{3}{4}\right) = \left(\frac{20+9}{12}\right)\left(\frac{20-9}{12}\right)$$

$$= \frac{29}{12} \times \frac{11}{12} = \frac{319}{144} = 2\frac{31}{144}$$ **(Ans.)**

(ii) $\frac{(2{\cdot}39)^2 - (1{\cdot}61)^2}{2{\cdot}39 - 1{\cdot}61} = \frac{a^2 - b^2}{a-b}$ [Let $2{\cdot}39 = a$ and $1{\cdot}61 = b$]

$= \frac{(a+b)(a-b)}{a-b}$ [Since, $a^2 - b^2 = (a+b)(a-b)$]

$= a + b$

$= 2{\cdot}39 + 1{\cdot}61$ [Substituting the values of a and b]

$= 4{\cdot}00$ **(Ans.)**

Example 8 :

Evaluate : $\frac{6{\cdot}3 \times 6{\cdot}3 \times 6{\cdot}3 + 3{\cdot}7 \times 3{\cdot}7 \times 3{\cdot}7}{6{\cdot}3 \times 6{\cdot}3 - 6{\cdot}3 \times 3{\cdot}7 + 3{\cdot}7 \times 3{\cdot}7}$

Solution :

$\frac{6{\cdot}3 \times 6{\cdot}3 \times 6{\cdot}3 + 3{\cdot}7 \times 3{\cdot}7 \times 3{\cdot}7}{6{\cdot}3 \times 6{\cdot}3 - 6{\cdot}3 \times 3{\cdot}7 + 3{\cdot}7 \times 3{\cdot}7} = \frac{a \times a \times a + b \times b \times b}{a \times a - a \times b + b \times b}$ [Let $6{\cdot}3 = a$ and $3{\cdot}7 = b$]

$$= \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$= \frac{(a+b)\ (a^2 - ab + b^2)}{a^2 - ab + b^2}$$

$= a + b = 6{\cdot}3 + 3{\cdot}7 \quad = 10$ **(Ans.)**

4.9 L.C.M. AND H.C.F.

(a) L.C.M. and H.C.F. of Fractions

For the fractions in the lowest terms, their :

1. $\text{L.C.M.} = \dfrac{\textit{L.C.M. of numerators}}{\textit{H.C.F. of denominators}}$ 2. $\text{H.C.F.} = \dfrac{\textit{H.C.F. of numerators}}{\textit{L.C.M. of denominators}}$

Example 9 :

Find the L.C.M. of $\frac{3}{5}$, $\frac{21}{25}$ and $\frac{14}{15}$.

Solution :

Since, L.C.M. of numerators 3, 21 and 14 = 42

and, H.C.F. of denominators 5, 25 and 15 = 5

$\therefore$ **L.C.M. of given fractions** $= \frac{42}{5} = 8\frac{2}{5}$ **(Ans.)**

Example 10 :

Find the H.C.F. of $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{9}{10}$.

Solution :

Since, H.C.F. of numerators 4, 5 and 9 = 1

and, L.C.M. of denominators 5, 6 and 10 = 30

$\therefore$ **H.C.F. of given fractions** $= \frac{1}{30}$ **(Ans.)**

(b) L.C.M. and H.C.F. of Decimals

Steps :
1. Make the number of decimal places in all the given fractions equal (add zeroes, if required).
2. Find L.C.M. or H.C.F., ignoring the decimal points.
3. In the result, mark the decimal point after as many decimal places as there were in step (1).

Example 11 :

Find L.C.M. and H.C.F. of 1·2, 0·60 and 0·144.

Solution :

Step 1 : Making the number of decimal places in all the given fractions equal, we get : 1·200, 0·600 and 0·144.

Step 2 : Ignoring decimal points, we get : 1200, 600 and 144, whose L.C.M. = 3600 and H.C.F. = 24.

Step 3 : $\therefore$ **L.C.M.** of 1·2, 0·60 and 0·144 = 3·600 = **3·6** and their **H.C.F. = 0·024** **(Ans.)**

TEST YOURSELF

4. $5\frac{3}{4}$ = ..., $\frac{32}{7}$ = ..., $\frac{93}{25} = \frac{93 \times \text{.......}}{25 \times 4} = \frac{\text{.......}}{100}$ = ...

5. $\begin{array}{r} 5{\cdot}03 \\ +\,0{\cdot}356 \\ \hline \\ \hline \end{array}$ $\quad\begin{array}{r} 5{\cdot}03 \\ -\,0{\cdot}0356 \\ \hline \\ \hline \end{array}$ $\quad\begin{array}{r} 4{\cdot}7 \\ +\,2{\cdot}093 \\ \hline \\ \hline \end{array}$ $\quad\begin{array}{r} 4{\cdot}7 \\ -\,2{\cdot}093 \\ \hline \\ \hline \end{array}$

6. 6·32 × 2 = ..., 6·32 × 20 = ..., 6·32 × 200 = ..., 6·32 × 2000 = ...

7. $\frac{6{\cdot}32}{2}$ = ..., $\frac{6{\cdot}32}{20}$ = ..., $\frac{6{\cdot}32}{200}$ = ..., $\frac{6{\cdot}32}{2000}$ = ...

8. 0·8 of 2·4 ÷ 0·16 × 3 = ... and

   0·8 ÷ 2·4 of 0·16 × 3 = ...

9. $\frac{5^2 - 3^2}{2}$ = ..., $(0{\cdot}6)^2 - (0{\cdot}4)^2$ = ..., $\frac{12^2 - 7^2}{8^2 - 3^2}$ = ...

10. For $\frac{2}{5}$ and $\frac{6}{25}$, L.C.M. = ..., and H.C.F. = ...

11. For 0·8 and 0·24, L.C.M. = ..., and H.C.F. = ...

EXERCISE 4 (A)

1. Evaluate :

(i) $2{\cdot}2 + 1\frac{4}{7} - 4\cdot5 + 2\frac{8}{35}$

(ii) $0.8 \times \frac{7}{12} \div \frac{5}{24}$

(iii) $0.8 \div \frac{7}{12} \times \frac{5}{24}$

(iv) $4{\cdot}375 \div 2\frac{1}{11}$ of $2\frac{9}{23}$

(v) $11 \div 1\frac{9}{22} \times 5\frac{7}{11}$

(vi) (5 ÷ 2·25) ÷ (9 ÷ 2·25)

(vii) $3{\cdot}75 \div \left\{\frac{5}{6} \text{ of } \frac{2}{3}\left(\frac{1}{3} - \overline{\frac{1}{4} - \frac{1}{6}}\right)\right\}$

(viii) $37\cdot5 - \left[3{\cdot}75 \div \left\{\frac{5}{6} \text{ of } \frac{2}{3}\left(\frac{1}{3} - \overline{\frac{1}{4} - \frac{1}{6}}\right)\right\}\right]$

(ix) 0·5 + 0·5 ÷ (0·5 + 0·5 ÷ 1)

(x) 1 ÷ {1 + 1 ÷ (1 + 1 ÷ 2)}

(xi) $0{\cdot}4 \times \frac{7}{3} + \frac{15}{8}$ of $\left(\frac{7}{5} - \frac{4}{3}\right)$

(xii) $3\frac{1}{3} \div 2{\cdot}5 \times 0{\cdot}75 \div \frac{1}{3}$ of $20 \times \frac{7}{6}$

(xiii) $1 + \frac{1}{1+\frac{1}{2}}$ (xiv) $1 - \frac{1}{1-\frac{1}{1-\frac{1}{3}}}$

2. Evaluate :

(i) 0·8 × 0·7 – 0·5 × 0·3 + 0·16 ÷ 0·04

(ii) 4·4 – [2 – 0·64 {3 – 1·2 ÷ (0·63 – 0·03)}]

(iii) $\frac{9{\cdot}75 \text{ of } 3{\cdot}6}{0{\cdot}039} + \frac{3{\cdot}75}{0{\cdot}015}$

(iv) $\frac{0{\cdot}12 \text{ of } (0{\cdot}0104 - 0{\cdot}002) + 0{\cdot}36 \times 0{\cdot}002}{0{\cdot}12 \times 0{\cdot}12}$

3. Evaluate (using formulae) :

(i) $\frac{1{\cdot}25 \times 1{\cdot}25 - 0{\cdot}87 \times 0{\cdot}87}{1 - 0{\cdot}62}$

(ii) $\frac{0{\cdot}87 \times 0{\cdot}87 + 2 \times 0{\cdot}87 \times 0{\cdot}13 + 0{\cdot}13 \times 0{\cdot}13}{0{\cdot}87 + 0{\cdot}13}$

(iii) $\frac{2{\cdot}43 \times 2{\cdot}43 - 2 \times 2{\cdot}43 \times 1{\cdot}67 + 1{\cdot}67 \times 1{\cdot}67}{2{\cdot}43 - 1{\cdot}67}$

(iv) $\frac{(0{\cdot}4)^3 - (0{\cdot}24)^3}{(0{\cdot}4)^2 + 0{\cdot}4 \times 0{\cdot}24 + (0{\cdot}24)^2}$

(v) $\frac{0{\cdot}3 \times 0{\cdot}3 \times 0{\cdot}3 + 0{\cdot}1 \times 0{\cdot}1 \times 0{\cdot}1}{0{\cdot}3 \times 0{\cdot}3 - 0{\cdot}3 \times 0{\cdot}1 + 0{\cdot}1 \times 0{\cdot}1}$

(vi) $\frac{5{\cdot}7 \times 5{\cdot}7 - 2{\cdot}3 \times 2{\cdot}3}{5{\cdot}7 \times 5{\cdot}7 + 2 \times 5{\cdot}7 \times 2{\cdot}3 + 2{\cdot}3 \times 2{\cdot}3}$

(vii) $\frac{0{\cdot}46 \times 0{\cdot}46 + 2 \times 0{\cdot}46 \times 0{\cdot}14 + 0{\cdot}14 \times 0{\cdot}14}{0{\cdot}46 \times 0{\cdot}46 - 0{\cdot}14 \times 0{\cdot}14}$

4. Find the L.C.M. of :
(i) $\frac{5}{8}, \frac{15}{32}$ and $\frac{25}{64}$ (ii) $\frac{8}{9}, \frac{20}{27}$ and $\frac{25}{54}$

5. Find the H.C.F. of
(i) $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{4}$ (ii) $\frac{4}{9}, \frac{16}{45}$ and $\frac{64}{81}$

6. Find the L.C.M. and the H.C.F. of :
(i) 0·8, 1·6 and 2·4
(ii) 2·5, 0·15 and 0·05
(iii) 0·48, 6·4 and 0·256

7. Evaluate :
(i) 0·4 of ₹ 55 + $\frac{1}{7}$ of ₹ 42
(ii) $\frac{5}{9}$ of 810 gm – 0·05 times of 2 kg
(iii) $\frac{2}{3}$ of 72 cm + 0·75 of 2 m – 0·4 of 3 m

4.10 PROBLEMS BASED ON FRACTIONS

Example 12 :

Ram bought 240 toffees. He gave 0·4 times of these toffees to his sister and $\frac{5}{12}$ of the remaining to his younger brother. Find, how many toffees are still left with him ?

Solution :

Toffees given to his sister = 0·4 times of 240 = $\frac{4}{10} \times 240 = 96$

Remaining toffees = 240 – 96 = 144

Toffees given to younger brother = $\frac{5}{12}$ of 144 = $\frac{5 \times 144}{12} = 60$

∴ **No. of toffees still left with him** = 144 – 60 = **84** **(Ans.)**

Example 13 :

A man spends $\frac{3}{8}$ of his monthly income on food and 0·4 times of the remaining on clothes. If money left is ₹ 1,200; find his monthly income.

Solution :

Spent on food = $\frac{3}{8}$ of his income

Remaining = $1 - \frac{3}{8} = \frac{8-3}{8} = \frac{5}{8}$ of his income

Spent on clothes = 0·4 of $\frac{5}{8} = \frac{4}{10} \times \frac{5}{8} = \frac{1}{4}$ of his income

∴ **Fraction of income left with him** = $\frac{5}{8} - \frac{1}{4} = \frac{5-2}{8} = \mathbf{\frac{3}{8}}$

Since, $\frac{3}{8}$ of his monthly income = ₹ 1,200

∴ **His monthly income** = ₹ 1,200 × $\frac{8}{3}$ = **₹ 3,200** **(Ans.)**

EXERCISE 4 (B)

1. On a particular day, $\frac{2}{15}$ th of the total number of students in a school were absent. If 1950 were present on that day, find the total strength of the school.

2. Geeta read $\frac{3}{8}$ of a book on one day and $\frac{4}{5}$ of the remainder on another day. Find :
(i) portion of the book left unread after one day.
(ii) portion of the book read on another (second) day.
(iii) portion of the book left after two days.
(iv) total number of pages in the book, if 60 pages are left unread after second day.

3. Peter spent $\frac{2}{5}$ of his money on food and $\frac{1}{3}$ on books.
   (i) What fraction did he spend altogether ?
   (ii) What fraction was he left with ?
   (iii) If he had ₹ 9,000 initially, how much was left with him ?

4. Peter spent $\frac{2}{5}$ of his money on food and $\frac{1}{3}$ of the remainder on books.
   (i) What fraction did he spend altogether ?
   (ii) What fraction was left with him ?
   (iii) If he had ₹ 9,000 initially, how much was left with him ?

5. A man leaves ₹ 1,15,500; $\frac{1}{3}$ of it he leaves to his wife, $\frac{2}{7}$ of the rest to each of his two daughters and the remaining to his son. How much does his son get ?

4.11 ROUNDING OFF DECIMAL NUMBERS

Many times we require decimal numbers correct to a certain number of decimal places. For this the following steps are adopted :

1. Retain as many digits after decimal point as are required and omit the remainings.
2. Out of the omitted digits, if the first digit is 5 or more than 5; increase the last retained digit by one, otherwise no change is made.

Example 14 :

Round off the following decimal numbers correct to given decimal places :

(i) 3·9642 correct to two decimal places, (ii) 43·685 correct to one decimal place.

Solution :

(i) Retaining two digits after the decimal point and omitting the remainings, we get 3·96. Since, the first digit in the omitted digits is 4, which is less than 5; therefore no change is made in the last retained digit (*i.e.* 6).

∴ **3·9642 correct to two decimal places = 3·96** **(Ans.)**

(ii) Retaining one digit after the decimal and omitting the remainings, we get 43·6 Since, the first digit in the omitted digits is 8, therefore increase 6 by 1.

∴ **43·685 correct to one decimal place = 43·7** **(Ans.)**

4.12 SIGNIFICANT FIGURES

Significant figures are the number of digits used to express the number (quantity) with precision.

In order to find the number of significant digits in the given number, note that :

(i) all the numerals from 1 to 9 including any number of zeroes between them or after them are counted.

(ii) the position of the decimal point is disregarded and the zeroes preceding the first numeral are not counted.

For example :

Numbers	4036	40·36	0·4036	0·0004036	4·03600	$4{\cdot}00 \times 10^{8}$	$7{\cdot}9 \times 10^{-10}$
Number of significant digits	4	4	4	4	6	3	2

Example 15 :

Express : (i) $3\frac{2}{7}$ correct to one decimal place. (ii) $\frac{3}{11}$ correct to 3 significant figures.

Solution :

(i) Since, $3\frac{2}{7} = 3{\cdot}28......$

$\therefore 3\frac{2}{7}$ **correct to one decimal place figures = 3·3 (Ans.)**

(ii) Since, $\frac{3}{11} = 0{\cdot}2727.....$

$\therefore \frac{3}{11}$ **correct to three significant figures = 0·273 (Ans.)**

4.13 TERMINATING AND NON-TERMINATING DECIMALS

1. Terminating decimals :

Consider the following divisions :

(i) $\frac{1}{4} = 0{\cdot}25$ (ii) $3{\cdot}66 \div 6 = 0{\cdot}61$ (iii) $14{\cdot}4 \div 1{\cdot}2 = 12$

In each example, given above, the division is exact *i.e.* no remainder is left. The quotients of such divisions are called *terminating decimals.*

2. Non-terminating decimals :

Now, consider the following divisions :

(i) $\frac{4}{7} = 0{\cdot}5714285$ .... (ii) $\frac{14}{19} = 0{\cdot}736842$ ....

Here, the division never ends, no matter how long it continues. The quotients of such divisions are called *non-terminating decimals.*

4.14 RECURRING DECIMALS

Consider the fraction $\frac{1}{3}$. If we convert it into a decimal, we get $\frac{1}{3} = 0{\cdot}3333$ ....

It is obvious that the digit 3 will occur again and again, and the process will never end (terminate).

A decimal, which contains digits that repeat at regular intervals is called a recurring or repeating decimal. The digit or the group of digits that repeats itself is called the period of the recurring decimal.

The period of recurring decimals is indicated as follows :

(i) If only one digit is repeated, a dot or a bar is put above it.

Thus, 0·3333 .... = $0{\cdot}\dot{3}$ or $0{\cdot}\overline{3}$.

(ii) If two digits are repeated, a dot or a bar is put above both the repeating digits.

Thus, 0·818181 .... = $0{\cdot}\dot{8}\dot{1}$ or $0{\cdot}\overline{81}$.

(iii) If three or more digits are repeated, a dot is put above the first digit and another dot is put above the last digit or a bar is put above all the repeating digits.

Thus, 0·243243 ..... = $0{\cdot}\dot{2}4\dot{3}$ or $0{\cdot}\overline{243}$.

1. When all the digits in a recurring decimal are repeated, it is called a *pure recurring decimal*
   as :0·333...... = $0{\cdot}\overline{3}$, 0·818181....... = $0{\cdot}\overline{81}$, 5·131313.......... = $5{\cdot}\overline{13}$ and so on.
2. When some digit or digits that are immediately followed by the decimal point do not repeat, but other digits are repeated, it is called a *mixed recurring decimal* as :
   0·5333...... = $0{\cdot}5\overline{3}$, 3·9474747....... = $3{\cdot}9\overline{47}$, 10·846666.......... = $10{\cdot}84\overline{6}$ and so on.

EXERCISE 4 (C)

1. Round off each of the following as required:
   (i) 5·5493 correct to two decimal places.
   (ii) 5·5493 correct to three decimal places.
   (iii) 0·0829 correct to three decimal places.
   (iv) 0·0829 correct to one decimal place.
2. Multiply 4·28 and 0·67. Round off the product obtained correct to three decimal places.
3. Write the number of significant digits in :
   (i) 0·07 (ii) 3·005
   (iii) 23·4 (iv) 805·060
   (v) $5{\cdot}16 \times 10^8$ (vi) 16·000
   (vii) 0·0016 (viii) $0{\cdot}97 \times 10^{-2}$
4. Multiply 4·28 and 0·67. Express the product obtained in three significant digits.
5. Divide 7 by 11 and express the result in two significant digits.
6. Divide :
   (i) 3 by 8 correct to 2 places of decimal.
   (ii) 1·38305 by 11 correct to 3 places of decimal.
   (iii) 9·4628 by 17 correct to 3 places of decimal.
   (iv) 0·3297 by 0·07 correct to 2 significant digits.
   (v) 8·283 by 1·6 correct to 3 significant digits.
7. Find, whether each of the following is a terminating or a non-terminating decimal :
   (i) 5 ÷ 8 (ii) 7 ÷ 11
   (iii) 0·3 ÷ 0·09 (iv) 1·2 ÷ 0·16
   (v) 2·4 ÷ 0·072 (vi) 3·2 ÷ 2·24
8. Express as recurring decimal :
   (i) $\frac{4}{9}$ (ii) $\frac{3}{11}$ (iii) $2\frac{1}{6}$ (iv) $\frac{2}{7}$
9. After converting each of the following simple fractions into recurring decimals; state which are pure recurring decimals :
   (i) $\frac{1}{6}$ (ii) $\frac{2}{15}$ (iii) $\frac{4}{9}$ (iv) $\frac{5}{6}$

ANSWERS

TEST YOURSELF

1. 0, not-defined, 0, 1, –1 **2.** proper, improper, mixed, decimal **3.** 56·2, 0·00562, 0·097 **4.** $\frac{23}{4}$, $4\frac{4}{7}$, 93 × 4, 372, 3·72 **5.** 5·386, 4·9944, 6·793, 2·607 **6.** 12·64, 126·4, 1264, 12640, **7.** 3·16, 0·316, 0·0316, 0·00316 **8.** $\frac{1{\cdot}92}{0{\cdot}16} \times 3 = 36$, $0{\cdot}8 \div 0{\cdot}384 \times 3 = \frac{0{\cdot}8}{0{\cdot}384} \times 3 = 6{\cdot}25$ **9.** 8, 0·2, $1\frac{8}{11}$ **10.** $\frac{6}{5}$, $\frac{2}{25}$ **11.** 2·40, 0·08

EXERCISE 4(A)

1. (i) 1·5 (ii) $2\frac{6}{25}$ (iii) $\frac{2}{7}$ (iv) $\frac{7}{8}$ (v) 44 (vi) $\frac{5}{9}$ (vii) 27 (viii) 10·5 (ix) 1 (x) $\frac{3}{5}$ (xi) $7\frac{7}{15}$ (xii) $\frac{7}{40} = 0{\cdot}175$ (xiii) $1\frac{2}{3}$ (xiv) 3 **2.** (i) 4·41 (ii) 3·04 (iii) 1150 (iv) 0·12 **3.** (i) 2·12 (ii) 1 (iii) 0·76 (iv) 0·16 (v) 0·4 (vi) 0·425 (vii) 1·875 **4.** (i) $9\frac{3}{8}$ (ii) $22\frac{2}{9}$ **5.** (i) $\frac{1}{12}$ (ii) $\frac{4}{405}$ **6.** (i) L.C.M. = 4·8 and H.C.F. = 0·8 (ii) L.C.M. = 7·5 and H.C.F. = 0·05 (iii) L.C.M. = 19·2 and H.C.F. = 0·032 **7.** (i) ₹ 28 (ii) 350 gm (iii) 78 cm

EXERCISE 4(B)

1. 2250 **2.** (i) $\frac{5}{8}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{8}$ (iv) 480 **3.** (i) $\frac{11}{15}$ (ii) $\frac{4}{15}$ (iii) ₹ 2,400 **4.** (i) $\frac{3}{5}$ (ii) $\frac{2}{5}$ (iii) ₹ 3,600 **5.** ₹ 33,000

EXERCISE 4(C)

1. (i) 5·55 (ii) 5·549 (iii) 0·083 (iv) 0·1 **2.** 2·8676 = 2·868 **3.** (i) 1 (ii) 4 (iii) 3 (iv) 6 (v) 3 (vi) 5 (vii) 2 (viii) 2 **4.** 2·8676 = 2·87 **5.** 0·64 **6.** (i) 0·38 (ii) 0·126 (iii) 0·557 (iv) 4·7 (v) 5·18 **7.** (i) terminating (ii) non-terminating (iii) non-terminating (iv) terminating (v) non-terminating (vi) non-terminating **8.** (i) $0{\cdot}\overline{4}$ (ii) $0{\cdot}\overline{27}$ (iii) $2{\cdot}1\dot{6}$ (iv) $0{\cdot}\overline{285714}$ **9.** (i) $0{\cdot}1\dot{6}$ (ii) $0{\cdot}1\overline{3}$ (iii) $0{\cdot}\overline{4}$ = pure recurring (iv) $0{\cdot}8\overline{3}$