CHAPTER 3

H.C.F. AND L.C.M.

3.1 HIGHEST COMMON FACTOR (H.C.F.)

1. Factor	When a number divides another number completely (*i.e.* without leaving any remainder), it is called a factor of the second number. *e.g.* 3 divides 18 exactly, leaving no remainder, so 3 is a factor of 18. Similarly, other factors of 18 are : 1, 2, 6, 9 and 18 as each of these also divides 18 exactly. ∴ Factors of 18 = 1, 2, 3, 6, 9 and 18.
2. Common Factor	A common factor of two or more given numbers is a number which divides each given number completely. Since, factors of 18 = 1, 2, 3, 6, 9 and 18 and, factors of 24 = 1, 2, 3, 4, 6, 8, 12 and 24 ∴ Common factors of 18 and 24 = 1, 2, 3 and 6. (Each of these common factors divides the given numbers 18 and 24 completely)
3. H.C.F.	The **highest common factor** (H.C.F.) of two or more given numbers is the greatest number (factor) which divides each of the given numbers completely. As discussed above, the common factors of 18 and 24 are 1, 2, 3 and 6. Out of these common factors 6 is the greatest. ∴ Highest Common Factor of 18 and 24 = 6 *i.e.* H.C.F. of 18 and 24 = 6

TEST YOURSELF

1. Factors of 16 = ..., ..., ..., ... and .....
2. Factors of 20 = ..., ..., ..., ..., ... and .....
3. Factors of 28 = ..., ..., ..., ..., ... and .....
4. Common factors of 16, 20 and 28 are : ...., ... and .....
   ∴ H.C.F. of 16, 20 and 28 = ...

Example 1 :

Find the H.C.F. of 36 and 84.

Solution :

FIRST METHOD (Factor method) :

Factors of 36 = 1, 2, 3, 4, 6, 9, 12, 18 and 36

Factors of 84 = 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84

Common factors of 36 and 84 = 1, 2, 3, 4, 6, 12

∴ **Highest Common Factors** (*i.e.* H.C.F.) = **12** **(Ans.)**

SECOND METHOD (Prime factor method) :

Since, 36 = $2 \times 2 \times 3 \times 3$

and, 84 = $2 \times 2 \times 3 \times 7$

∴ **H.C.F. of 36 and 84** = Product of common prime factors

= $2 \times 2 \times 3$ = **12** **(Ans.)**

THIRD METHOD (Division method) :

$$\begin{array}{r|l|l} & 84 & 2 \\ 36 & 72 & \\ \hline & 36 & 3 \\ 12 & 36 & \\ \hline & \times & \end{array}$$

∴ **H.C.F. of 36 and 84** = Last divisor = **12** **(Ans.)**

To find the H.C.F. of three or more given numbers by division method :

Steps : (i) First of all, find the H.C.F. of any two of the given numbers.

(ii) Now, find the H.C.F. of the H.C.F. [obtained in step (i)] and the third number. Continue till the last H.C.F. is obtained.

(iii) The last H.C.F. is the H.C.F. of all the given numbers.

Example 2 :

Find (using division method) the H.C.F. of :

(i) 306, 234 and 405 (ii) 525, 420, 1245 and 1080.

Solution :

(i) Step 1 :

Find the H.C.F. of 306 and 234.

$$\begin{array}{r|l|l} & 306 & 1 \\ 234 & 234 & \\ \hline & 72 & 3 \\ 72 & 216 & \\ \hline & 18 & 4 \\ 18 & 72 & \\ \hline & \times & \end{array}$$

∴ H.C.F. of 306 and 234 = 18

Step 2 :

Find the H.C.F. of 18 and 405

$$\begin{array}{r|l|l} & 405 & 22 \\ 18 & 36 & \\ \hline & 45 & 2 \\ 9 & 18 & \\ \hline & \times & \end{array}$$

Hence, **H.C.F. of the given numbers = 9** **(Ans.)**

(ii) Step 1 :

Find the H.C.F. of 525 and 420.

$$\begin{array}{r|l|l} & 525 & 1 \\ 420 & 420 & \\ \hline & 105 & 4 \\ 105 & 420 & \\ \hline & \times & \end{array}$$

H.C.F. of 525 and 420 = 105

Step 2 :

Find the H.C.F. of 105 and 1245.

$$\begin{array}{r|l|l} & 1245 & 11 \\ 105 & 105 & \\ \hline & 195 & 1 \\ 105 & 90 & \\ \hline & 105 & 6 \\ 90 & 90 & \\ \hline & 15 & \\ & \times & \end{array}$$

H.C.F. of 105 and 1245 = 15

Step 3 :

Finally, find the H.C.F. of 15 and 1080.

$$15\overline{)1080}\,72$$
$$105$$
$$30$$
$$30$$
$$\times$$

H.C.F. of 15 and 1080 = 15

∴ **H.C.F. of 525, 420, 1245 and 1080 = 15** **(Ans.)**

Example 3 :

Find the greatest number that will divide 185 and 300 leaving remainders 3 and 6 respectively.

Solution :

Since on dividing 185 by the required number, the remainder is 3.

∴ 185 – 3 = 182 will be exactly divisible by the required number.

Similarly, 300 – 6 = 294 will also be exactly divisible by the required number.

∴ The required number is H.C.F. of 182 and 294.

∴ $182 = 2 \times 7 \times 13$ and $294 = 2 \times 3 \times 7 \times 7$ [Resolving into prime factors]

∴ **Required greatest no.** $= 2 \times 7 = 14$ **(Ans.)**

If two numbers are co-prime, *i.e.*, they do not have any common divisor except one; their H.C.F. = 1

e.g. (i) H.C.F. of 16 and 25 is 1

(ii) H.C.F. of 9 and 80 is 1 and so on.

3.2 LOWEST COMMON MULTIPLE (L.C.M.)

1. Multiple	When a number divides another number completely, the second number is called a multiple of the first number. *e.g.* 3 divides 18 completely, so 18 is a multiple of 3. Multiples of 3 = 3, 6, 9, 12, 15, 18, 21, ... Multiples of 5 = 5, 10, 15, 20, 25, ... and so on.
2. Common Multiple	A number, which can be divided completely by the given two or more numbers, is called their **common multiple**. Since, multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ... multiples of 6 = 6, 12, 18, 24, 30, 36, ... and, multiples of 12 = 12, 24, 36, 48, ... ∴ Common multiples of 4, 6 and 12 = 12, 24, 36, ... (Each of these common multiples is completely divisible by the given numbers 4, 6 and 12).
3. L.C.M.	It is the lowest number which is completely divisible by each of the given numbers. Since, the common multiples of 4, 6 and 12 = 12, 24, 36, ... ∴ The lowest common multiple of 4, 6 and 12 is **12**. And, 12 is the smallest number which is completely divisible by each of the given numbers 4, 6 and 12.

TEST YOURSELF

5. Multiples of 3 = ..., ..., ..., ..., ..., ..., ..., ..., ..., ...

multiples of 4 = ..., ..., ..., ..., ..., ..., ..., ...

and, multiples of 6 = ..., ..., ..., ..., ..., ..., ...

∴ **Common multiples** of 3, 4 and 6 = ..., ..., ...

and, **L.C.M.** of 3, 4 and 6 = ...

Example 4 :

Find L.C.M. of 24, 36 and 40.

Solution :

First Method (Index method) :

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$

∴ **L.C.M.** = Product of each prime factor with highest index used.

$= 2^3 \times 3^2 \times 5$

$= 8 \times 9 \times 5 =$ **360** **(Ans.)**

Second Method (Division method) :

2	24, 36, 40
2	12, 18, 20
2	6, 9, 10
3	3, 9, 5
	1, 3, 5

L.C.M. = Product of each divisor and quotients

$= 2 \times 2 \times 2 \times 3 \times 1 \times 3 \times 5$

= **360** **(Ans.)**

Example 5 :

Find the smallest number which when divided by 48 and 60 respectively, leaves a remainder 7 in each case.

Solution :

Find the L.C.M. of the given numbers 48 and 60.

Since, $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

and, $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

∴ L.C.M. $= 2^4 \times 3 \times 5 = 240$

L.C.M. of 48 and 60 is 240 means, 240 is the smallest number which is exactly divisible by 48 and 60.

So, **the required smallest number**, which on dividing by 48 and 60, will leave remainder 7 in each case

$= 240 + 7 =$ **247** **(Ans.)**

Example 6 :

What is the smallest number which when increased by 3 is divisible by 27, 35, 25 and 21.

Solution :

The smallest number which is divisible by 27, 35, 25 and 21 is the L.C.M. of 27, 35, 25 and 21.

∴ The required number = L.C.M. of the given numbers *minus* 3.

Since, $27 = 3 \times 3 \times 3 = 3^3$; $35 = 5 \times 7$;

$25 = 5 \times 5 = 5^2$ and $21 = 3 \times 7$

∴ L.C.M. $= 3^3 \times 5^2 \times 7 = 4725$

And, **the required no.** $= 4725 - 3 =$ **4722** **(Ans.)**

1. For any two co-prime numbers, their L.C.M. is equal to their product.
   e.g. (i) L.C.M. of 8 and 15 = 8 × 15 = 120
   (ii) L.C.M. of 12 and 35 = 12 × 35 = 420 and so on.
2. For any two numbers :
   The product of their L.C.M. and H.C.F. = The product of the numbers.

Example 7 :

The L.C.M. of two numbers is 2079 and their H.C.F. is 27. If one of the numbers is 189, find the other.

Solution :

For any two numbers;

One no. × other no. = Their L.C.M. × their H.C.F.

$\Rightarrow$ 189 × other no. = 2079 × 27

$\Rightarrow$ **The other no.** = $\frac{2079 \times 27}{189}$ = **297** **(Ans.)**

Example 8 :

Two persons take steps of 80 cm and 90 cm respectively. If they start in step, how far will they walk before they are in step again ?

Solution :

The required distance = The L.C.M. of 80 cm and 90 cm
= $2^4 \times 3^2 \times 5$ cm
= **720 cm** **(Ans.)**

$80 = 2^4 \times 5$
$90 = 2 \times 3^2 \times 5$

Example 9 :

Find the smallest number which when divided by 35, 45 and 55 leaves the remainders 17, 27 and 37 respectively.

Solution :

For divisor 35, remainder = 17 and 35 – 17 = 18.

For divisor 45, remainder = 27 and 45 – 27 = 18

For divisor 55, remainder = 37 and 55 – 37 = 18

$\therefore$ Every pair of divisor and remainder has same difference *i.e.* 18

Hence, the required number = L.C.M. of 35, 45 and 55 **minus** 18.

Since, 35 = 5 × 7; 45 = 3 × 3 × 5 and 55 = 5 × 11

L.C.M. of 35, 45 and 55 = 3 × 3 × 5 × 7 × 11
= 3465

$\therefore$ **The required number** = 3465 – 18 = **3447** **(Ans.)**

EXERCISE 3 (A)

1. Find the H.C.F. of :
   (i) 540, 720 and 450
   (ii) 805, 1127 and 1449
   (iii) 1836, 810, 1296 and 702.
2. What is the greatest number which when divides 398 and 436, leaves 7 and 11 respectively as remainders.
3. Find the greatest number which divides 1750 and 2000 leaving 48 and 2 respectively as remainders.
4. Find L.C.M. of :
   (i) 72, 84 and 126
   (ii) 294, 420 and 504
   (iii) 24, 36, 108 and 192

5. Find the least number which when divided by 36, 48 and 112; leaves no remainder.
6. Find the smallest number which when divided by 12, 20, 30 and 60; leaves a remainder 5 each time.
7. What is the least number which when increased by 3 is exactly divisible by 27, 35 and 21?
8. (i) Find the least number that can be divided exactly by all the even numbers between 10 and 20.
   (ii) Find the least number that can be divided exactly by all odd numbers between 20 and 30.
9. What is the least number which when decreased by 5 is divisible by 36, 48, 21 and 28 ?
10. The H.C.F. of two numbers is 119 and their L.C.M. is 11781. If one of the numbers is 1071, find the other.
11. The product of two numbers is 20736 and their H.C.F. is 24. Find their L.C.M.
12. Two persons take steps of 64 cm and 84 cm respectively. If they start in step, how far will they walk before they are in step again ?
13. Four bells are ringing at intervals of 12, 16, 24 and 36 minutes. They start ringing simultaneously at 12 O'clock. Find when will they again ring together ?
14. Show that 280 and 297 are prime to each other.

Show that the H.C.F. of the given numbers is 1.

15. A farmer has 2494 sheep and 2193 lambs. He farms them into flocks, keeping sheep and lambs separate and having the same number of animals in each flock. If these flocks are as large as possible, find :
   (i) the maximum number of animals in each flock and
   (ii) total number of flocks required for the purpose.
16. Find the smallest number which when divided by 42 and 54 leaves the remainders 34 and 46 respectively
17. Find the smallest number which when divided by 24, 36 and 48 leaves the remainders 21, 33 and 45 respectively.

Example 10 :

Find the greatest number which when divides 590, 908 and 1014 leaves the same remainder each time.

Solution :

Since, $908 - 590 = 318$ and $1014 - 908 = 106$

$\therefore$ **Required greatest number** = H.C.F. of 318 and 106 = **106** **(Ans.)**

Explanation :

Let the remainder in each case be x.

$\therefore$ $590 - x$, $908 - x$ and $1014 - x$ are completely divisible by the required greatest number.

It must be noted here that if any three numbers are divisible by the same number, their difference is also divisible by that number.

Since, difference between $590 - x$ and $908 - x$ $= (908 - x) - (590 - x)$

$= 908 - x - 590 + x = 318$

and, difference between $908 - x$ and $1014 - x$ $= (1014 - x) - (908 - x)$

$= 1014 - x - 908 + x = 106$

$\Rightarrow$ Differences 318 and 106 are completely divisible by the required greatest number.

$\Rightarrow$ Required greatest number = H.C.F. of 318 and 106.

Important :

H.C.F. of two or more numbers is the largest number that divides each of the numbers and their L.C.M. completely.

e.g., the H.C.F. and the L.C.M. of numbers 12, 18, and 24 are 6 and 72 respectively. Clearly, the H.C.F. 6 is the largest number that divides each of the numbers 12, 18 and 24 and also their L.C.M. (72).

Example 11 :

Two numbers are in the ratio 5 : 8. If their H.C.F. is 6, find the numbers.

Solution :

Let numbers be $5x$ and $8x$.

The H.C.F. of $5x$ and $8x = x$

and, their L.C.M. $= 5 \times 8 \times x = 40x$

Given, H.C.F. $= 6 \Rightarrow x = 6$

$\therefore$ **Nos.** $= 5x$ and $8x$

$= 5 \times 6$ and $8 \times 6 =$ **30 and 48** **Ans.**

EXERCISE 3 (B)

1. Find the greatest number which when divides 253, 568 and 813 leaves the same remainder each time.
2. If the L.C.M. of two numbers is divisible by each of 1, 2, 4, 5, 10 and 20, what is the H.C.F. of these numbers ?

Required H.C.F. of two numbers
= Largest number that divides their L.C.M. completely
= **20**

3. Two numbers are in the ratio 3 : 4. If their H.C.F. is 36, find :
   (i) the numbers (ii) their L.C.M.
4. The L.C.M. of two numbers is six-times their H.C.F. If the difference between L.C.M. and H.C.F. is 60 and one of the two numbers is 36; find the other number.
5. The L.C.M. of two numbers is 15 times that of their H.C.F. If the sum of the L.C.M. and the H.C.F. is 288, find :
   (i) the H.C.F. and the L.C.M.
   (ii) the other number, if one number is 54
6. (i) Find the largest number of four digits which is completely divisible by 24, 36 and 48.
   (ii) Find the smallest number of five digits which is completely divisible by 24, 36 and 48.

Since, the L.C.M. of 24, 36 and 48 is 144 and largest number of four-digits is 9999.

Divide 9999 by the L.C.M. 144 to get quotient = 69 and remainder = 63. Then :

(i) **Required largest number of four-digits** = 9999 – remainder

(ii) **Required smallest number of five-digits** = 9999 – remainder + L.C.M.
or = (Quotient + 1) × L.C.M.

7. (i) Find the largest number of three digits which is completely divisible by 8 and 18.
   (ii) Find the smallest number of four digits which is completely divisible by 8 and 18.

ANSWERS

TEST YOURSELF

1. 1, 2, 4, 8 and 16 **2.** 1, 2, 4, 5, 10 and 20 **3.** 1, 2, 4, 7, 14 and 28 **4.** 1, 2 and 4; 4 **5.** 3, 6, 9, 12, 15, 18, 21, 24, 27, ... ; 4, 8, 12, 16, 20, 24, 28, ... and 6, 12, 18, 24, 30, 36, ... ; 12, 24, 36, ... ; 12

EXERCISE 3 (A)

1. (i) 90 (ii) 161 (iii) 54 **2.** 17 **3.** 74 **4.** (i) 504 (ii) 17640 (iii) 1728 **5.** 1008 **6.** 65 **7.** 942
8. (i) 1008 (ii) 3151575 **9.** 1013 **10.** 1309 **11.** 864 **12.** 1344 cm = 13·44 m **13.** 2·24 P.M.
15. (i) 43 (ii) 109 **16.** 370 **17.** 141

EXERCISE 3 (B)

1. 35 **2.** 20 **3.** (i) 108 and 144 (ii) 432 **4.** 24 **5.** (i) H.C.F. = 18 and L.C.M. = 270 (ii) 90
6. (i) 9936 (ii) 10080 **7.** (i) 936 (ii) 1008