21 SIMULTANEOUS LINEAR EQUATIONS

- Simultaneous Linear Equation
- Indeterminate
- Truth Table
- Elimination of Variable

Mr Ahuja goes to a 'Fixed Rate' fruit-market to buy bananas and apples. Two fruit-sellers call out to him and since he knows both of them well, he feels obliged to buy from both fruit-sellers. He buys 12 bananas and 8 apples from one and pays him Rs 73. Can you tell how much each fruit costs?

Now, we have 2 variables in the equation $12x + 8y = 73$. For every new value of x, there will be a new value of y. Here, x represents the cost of 1 banana and y represents the cost of 1 apple. Thus, a linear equation with two variables is **indeterminate**. At best we can construct a **truth table** calculating the values of y for different values of x, as shown below:

x	1	1.5	1.75	2
$y = \frac{73-12x}{8}$	$\frac{73-12}{8}$	$\frac{73-18}{8}$	$\frac{73-21}{8}$	$\frac{73-24}{8}$
y	7.625	6.875	6.5	6.125

Now if he buys 6 bananas and 9 apples from the second fruit-seller and pays him Rs 69, we have another indeterminate linear equation with two variables as $6x + 9y = 69$. A truth table for this equation would be:

x	1	1.5	1.75	2
$y = \frac{69-6x}{9}$	$\frac{69-6}{9}$	$\frac{69-9}{9}$	$\frac{69-10.5}{9}$	$\frac{69-12}{9}$
y	7	$6\frac{2}{3}$	6.5	$6\frac{1}{3}$

Now if both equations are considered together, we notice one pair of solutions that match. When $x = 1.75$, in both the equations the value of y is 6.5.

Thus, the bananas were for Rs 1.75 and apples were Rs 6.50 each.

Two or more equations which have only one set of values of the variable involved as a common solution are known as simultaneous equations.

The solution of the simultaneous equations $12x + 8y = 73$; $6x + 9y = 69$ is $x = 1.75$ and $y = 6.5$. No other set of values for x and y will satisfy both the equations simultaneously.

Solving Simultaneous Equations

Case I: *Eliminating one variable by adding or subtracting equations*

Step 1: The numerical coefficient of one variable is made the same in the given simultaneous equations.

Step 2: The equations are added or subtracted to eliminate the above variable.

Step 3: The resultant equation is solved for the other variable.

Step 4: The value of the other variable is substituted in any one of the original simultaneous equations to find the value of the first variable.

Example 1: Solve $x + 4y = 20$; $x - 2y = 2$

Let us select the variable x as its numerical coefficient is the same in both equations.

Now, as the same quantity can be added to or subtracted from both sides of an equation,

$x + 4y - (x - 2y) = 20 - 2$ (subtracting equations)

$\Rightarrow x + 4y - x + 2y = 18$ (eliminating variable x)

$\Rightarrow 6y = 18$

$\Rightarrow y = 3$

$x + (4 \times 3) = 20$ (substituting the value of y in any one equation)

$\Rightarrow x + 12 = 20 \Rightarrow x = 8$

Thus, the solution of $x + 4y = 20$; $x - 2y = 2$ is $x = 8$ and $y = 3$.

Example 2: Let us now solve the equations (for fruit bought) in the example at the beginning of the chapter.

To solve $12x + 8y = 73$; $6x + 9y = 69$, let us make the numerical coefficients of x the same in both the equations.

$(6x \times 2) + (9y \times 2) = 69 \times 2$ (multiplying LHS and RHS by 2)

$\Rightarrow 12x + 18y = 138$

$\Rightarrow 12x + 18y - (12x + 8y) = 138 - 73$ (subtracting equations)

$\Rightarrow 12x + 18y - 12x - 8y = 65$

$\Rightarrow 10y = 65$

$\Rightarrow y = 6.5$

$12x + (8 \times 6.5) = 73$ (substituting the value of y in any one equation)

$\Rightarrow 12x + 52 = 73$

$$\Rightarrow x = \frac{73 - 52}{12} \Rightarrow x = 1.75$$

Thus, the solution of $12x + 8y = 73$; $6x + 9y = 69$ is $x = 1.75$ and $y = 6.5$.

Example 3: Solve:

$$\frac{x+3}{3} + \frac{y-2}{4} = 6; \frac{4x+1}{5} - \frac{3y-2}{10} = 1$$

In order to simplify the fractional terms of the equation, we multiply both sides of the equation by the LCM of the denominators.

$$\frac{\overset{4}{\cancel{12}}(x+3)}{\cancel{3}} + \frac{\overset{3}{\cancel{12}}(y-2)}{\cancel{4}} = 6 \times 12$$

$$\text{and } \frac{\overset{2}{\cancel{10}}(4x+1)}{\cancel{5}} - \frac{\overset{1}{\cancel{10}}(3y-2)}{\cancel{10}} = 1 \times 10$$

$\Rightarrow 4x + 12 + 3(y - 2) = 72$

and $8x + 2 - (3y - 2) = 10$

$\Rightarrow 4x + 12 + 3y - 6 = 72$

and $8x + 2 - 3y + 2 = 10$

$\Rightarrow 4x + 3y = 72 - 6$

and $8x - 3y = 10 - 4$

$\Rightarrow 4x + 3y = 66$

and $8x - 3y = 6$

$\Rightarrow 4x + 3y + 8x - 3y = 66 + 6$ (adding equations to eliminate $3y$)

$\Rightarrow 12x = 72 \Rightarrow x = 6$

$(4 \times 6) + 3y = 66$ (substituting the value of x in any one equation)

$\Rightarrow 3y = 66 - 24$

$$\Rightarrow y = \frac{42}{3} \Rightarrow y = 14$$

Thus, the solution of $\frac{x+3}{3} + \frac{y-2}{4} = 6$; $\frac{4x+1}{5} - \frac{3y-2}{10}$ is $x = 6$ and $y = 14$.

Example 4: Solve $\frac{2}{x} + \frac{6}{y} = 2$; $\frac{9}{x} - \frac{9}{y} = 3$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$, then the simultaneous equations become $2a + 6b = 2$; $9a - 9b = 3$

Let us make the numerical coefficient of b the same in both equations.

$(2a \times 3) + (6b \times 3) = 2 \times 3$;

$(9a \times 2) - (9b \times 2) = 3 \times 2$

$\Rightarrow 6a + 18b = 6$;

$18a - 18b = 6$

$6a + 18b + 18a - 18b = 6 + 6$ (adding equations to eliminate $18b$)

$$\Rightarrow 24a = 12 \Rightarrow a = \frac{1}{2}$$

Now $a = \frac{1}{2} = \frac{1}{x} \Rightarrow x = 2$

$\frac{2}{2} + \frac{6}{y} = 2$ (substituting the value of x in any one equation)

$\Rightarrow 1 + \frac{6}{y} = 2 \Rightarrow \frac{6}{y} = 2 - 1$

$\Rightarrow \frac{6}{y} = 1 \Rightarrow 6 = y$

Thus, the solution of $\frac{2}{x} + \frac{6}{y} = 2;\ \frac{9}{x} - \frac{9}{y} = 3$ is $x = 2$ and $y = 6$.

> ***Case II: Eliminating one variable by substitution***
>
> *Step 1:* Make one variable the subject of the formula or bring it to the LHS of the equation.
>
> *Step 2:* Substitute its value (the expression on the RHS) in the other equation.
>
> *Step 3:* Solve the equation obtained to find the value of the other variable.
>
> *Step 4:* Substitute the value of the other variable in any one equation to find the value of the first variable.

Example 5: Solve $x - 7 = y$; $3x - 4y = 8$

Select the first equation as it would be easier to express the value of the variable x.

$x - 7 = y$

$\Rightarrow x = y + 7$

Substitute this value of x in the second equation,

$3(y + 7) - 4y = 8$

$\Rightarrow 3y + 21 - 4y = 8$

$\Rightarrow -y = 8 - 21 = -13$

$\Rightarrow y = 13$ (multiplying both sides by -1)

$x - 7 = 13$ (substituting the value of y in any one equation)

$\Rightarrow x = 13 + 7 \Rightarrow x = 20$

Thus, the solution of $x - 7 = y$; $3x - 4y = 8$ is $x = 20$ and $y = 13$.

Example 6: The example at the beginning of the chapter, solving the equations for fruit bought, can also be solved using this method.

$12x + 8y = 73;\quad 6x + 9y = 69$

$\Rightarrow 12x = 73 - 8y \Rightarrow x = \frac{73 - 8y}{12}$

Substituting the value of x in the second equation,

$$6\left(\frac{73}{12} - \frac{8y}{12}\right) + 9y = 69$$

$\Rightarrow \frac{438}{12} - 4y + 9y = 69$

$\Rightarrow 5y = 69 - \frac{438}{12}$

$\Rightarrow 5y = \frac{65}{2} \Rightarrow y = \frac{65}{10} = 6.5$

$\Rightarrow 12x + (8 \times 6.5) = 73$

$\Rightarrow 12x = 73 - 52 \Rightarrow x = \frac{21}{12} = 1.75$

Thus, the solution of $12x + 8y = 73$; $6x + 9y = 69$ is $x = 1.75$ and $y = 6.5$.

Observe that the formula for x in the above example is not as simple as in the previous example. Thus, the substitution method is used only in equations where one variable can be made the subject in a 'simple' formula.

> **Try this!**
>
> Solve $7x + 8y = 9$; $x - 2y = 0$

Exercise 21.1

1. Solve the following simultaneous equations by eliminating one variable either by adding or by subtracting the equations.

(i) $x + 3y = 8;\quad x - 4y = 1$

(ii) $x + 2y = 22;\ x - 3y = 2$

(iii) $x + 3y = 20;\ x - 6y = -7$

(iv) $x = 33 - 3y;\ x = 5y - 15$
(v) $3x - 4y = 9;\ 3x + 5y = 36$
(vi) $5x - 2y = 5;\ 8x - 2y = 14$
(vii) $2x + 6y = 30;\ 6x - 2y = 50$
(viii) $5x - 6y = 18;\ 2x - y = 17$
(ix) $2y - 4x = 2;\ 8x - 3y = 7$
(x) $6x + 2y = 46;\ 6y + 2x = 42$
(xi) $5x = 67 - 2y;\ 3y = 3x + 6$
(xii) $9x = 62 + 8y;\ 3y = 52 - 2x$
(xiii) $5y - 4x = 32;\ 8y + 2x = 26$
(xiv) $2x - 3y = 29;\ 5x + 2y = 25$
(xv) $3x + 7y = -43;\ 7x - 2y = -27$
(xvi) $4x - 3y = -24;\ 11x - 4y = -15$
(xvii) $\frac{x}{4} + 3y = 18;\ y - \frac{x}{3} = 1$
(xviii) $2x + \frac{2y}{3} = 26;\ 3x - \frac{5y}{6} = 6$
(xix) $\frac{x}{2} + \frac{y}{5} = 5;\ \frac{y}{2} - \frac{x}{3} = 3$
(xx) $\frac{x}{6} + \frac{y}{7} = 6;\ \frac{x}{3} - \frac{y}{2} = 1$

2. Solve the following simultaneous equations by eliminating one variable by substitution.
(i) $x - 2y = 5;\ 2x + 7y = 32$
(ii) $x + 12y = 15;\ 5x - 2y = 13$
(iii) $3x + y = 22;\ 6x - 3y = 9$
(iv) $4x + y = 16;\ 12x - 2y = 8$
(v) $3y - x = 7;\ 4x - 7y = 2$
(vi) $11x - y = 20;\ 3y - 9x = 12$
(vii) $5x + y = -4;\ 7x - 2y = -43$
(viii) $2x - y = 17;\ 8x + 3y = 5$
(ix) $7x - 3y = -14;\ 3x + y = -22$
(x) $4y - 3x = 34;\ x + 6y = 18$

3. Solve the following simultaneous equations.
(i) $6x + 9y = 6;\ 12y - 6x = 1$
(ii) $15x + 6y = 7;\ 9y - 5x = 5$
(iii) $30x - 35y = 13;\ 10x + 7y = 7$
(iv) $21y + 5x = -1;\ 20x + 28y = -12$
(v) $\frac{x+4}{3} + \frac{y-1}{5} = 5;\ \frac{x-2}{4} - \frac{y-3}{6} = 1$
(vi) $\frac{x+2}{7} + \frac{y-1}{2} = 6;\ \frac{x-3}{3} - \frac{y+1}{5} = 1$
(vii) $\frac{12}{x} - \frac{6}{y} = 1;\ \frac{6}{x} + \frac{10}{y} = 7$
(viii) $\frac{12}{x} - \frac{6}{y} = 6;\ \frac{16}{x} + \frac{4}{y} = 2$
(ix) $\frac{2}{x} + \frac{3}{y} = 18;\ \frac{6}{x} - \frac{2}{y} = 10$
(x) $\frac{6}{x} + \frac{4}{y} = 16;\ \frac{3}{x} - \frac{6}{y} = -4$

Solving Word Problems Using Simultaneous Equations

Word problems involving two unknown quantities can be easily solved by formulating a pair of simultaneous equations.

Example 7: Mr. Karkare bought $2\frac{1}{2}$ kg of mangoes and $3\frac{1}{4}$ kg of apples from a fruit-seller for Rs 226.25 while Rekha bought $1\frac{3}{4}$ kg of mangoes and $1\frac{1}{2}$ kg of apples for Rs 123.50. At what price was the fruit-seller selling each kg of mangoes and apples?

Let the price of 1 kg of mangoes be Rs x and the price of 1 kg of apples be Rs y.

Given $\frac{5x}{2} + \frac{13y}{4} = 226.25$; $\frac{7x}{4} + \frac{3y}{2} = 123.50$

$\Rightarrow \quad \frac{5x \times 4}{2} + \frac{13y \times 4}{4} = 226.25 \times 4$;

$\frac{7x \times 4}{4} + \frac{3y \times 4}{2} = 123.50 \times 4$

$\Rightarrow \quad 10x + 13y = 905; \quad 7x + 6y = 494$

$\Rightarrow \quad (10x \times 7) + (13y \times 7) = 905 \times 7$;

$(7x \times 10) + (6y \times 10) = 494 \times 10$

$\Rightarrow \quad 70x + 91y = 6335; \quad 70x + 60y = 4940$

$\Rightarrow \quad 70x + 91y - (70x + 60y) = 6335 - 4940$

$\Rightarrow \quad 70x + 91y - 70x - 60y = 1395$

$\Rightarrow \quad 31y = 1395 \Rightarrow y = \frac{1395}{31} = 45$

$10x + (13 \times 45) = 905$ (substituting value of y in any one equation)

$\Rightarrow \quad 10x = 905 - 585 \Rightarrow x = \frac{320}{10} = 32$

Thus, mangoes were being sold for Rs 32 per kg while apples cost Rs 45 per kg.

Example 8: Renu's age is double the age of her cousin Roy. 8 years hence, her age will be 1.5 times Roy's age. Find the present ages of the cousins.

Let Roy present age = x
and Renu's age = y
8 years hence, Roy's age = $x + 8$ and
Renu's age = $y + 8$

$y = 2x; \quad y + 8 = 1.5(x + 8)$

Substituting the value of y in the second equation,

$2x + 8 = 1.5x + 12$

$\Rightarrow \quad 2x - 1.5x = 12 - 8$

$\Rightarrow \quad 0.5x = 4 \quad \Rightarrow \quad x = 8$

Substituting the value of x in $y = 2x$,

$y = 2 \times 8 = 16$

Thus, Renu is 16 years old while her cousin Roy is 8 years old.

Example 9: If Sheetal gives 15 berries to Suresh, they will have the same number of berries, but if Suresh gives 15 berries to Sheetal, she will have 4 times as many berries as him. How many berries do Sheetal and Suresh have?

Let Suresh have x berries and Sheetal have y berries.
When Sheetal gives 15 berries to Suresh,

$x + 15 = y - 15$

When Suresh gives 15 berries to Sheetal,

$4(x - 15) = y + 15$

Thus $\quad x + 15 = y - 15; \; 4x - 60 = y + 15$

$\Rightarrow \quad x - y = -15 - 15; \; 4x - y = +15 + 60$

$\Rightarrow \quad x = y - 30; \; 4x - y = 75$

Substituting the value of x in the second equation

$4(y - 30) - y = 75 \quad \Rightarrow \quad 4y - 120 - y = 75$

$\Rightarrow \quad 3y = 75 + 120 \quad \Rightarrow \quad y = \frac{195}{3} = 65$

Substituting the value of y in $x = y - 30$, we get

$x = 65 - 30 = 35$

Thus, Suresh has 35 berries while Sheetal has 65 berries.

Example 10: If 1 is added to the numerator of a fraction and subtracted from its denominator, the fraction becomes 1, but if 2 is subtracted from the numerator and 1 is added to the denominator the fraction becomes $\frac{1}{2}$. Find the fraction.

Let the numerator and the denominator of the fraction be x and y. When 1 is added to the numerator and subtracted from the denominator respectively,

$\frac{x+1}{y-1} = 1$

$\Rightarrow \quad x + 1 = y - 1 \Rightarrow x = y - 2$

When 2 is subtracted from the numerator and 1 is added to the denominator,

$\frac{x-2}{y+1} = \frac{1}{2} \Rightarrow 2(x - 2) = y + 1$

$\Rightarrow \quad 2x - 4 = y + 1 \Rightarrow 2x - y = 5$

Substituting $x = y - 2$ in the above equation, we get

$2(y - 2) - y = 5$

$\Rightarrow \quad 2y - 4 - y = 5 \quad \Rightarrow \quad y = 5 + 4 = 9$

Substituting the value of y in $x = y - 2$, we get

$$x = 9 - 2 = 7$$

Thus, the fraction is $\frac{7}{9}$.

Example 11: A 2-digit number is 7 times the sum of its digits. If 27 is subtracted from the number, its digits are reversed. Find the number.

Let the digit in the ones place be x and the digit in the tens place be y.
Then the 2-digit number is $10y + x$.
Given that the number is 7 times the sum of its digits,

Or $\quad 10y + x = 7(y + x)$

$\Rightarrow \quad 10y + x = 7y + 7x$

$\Rightarrow \quad 10y - 7y = 7x - x$

$\Rightarrow \quad 3y = 6x \Rightarrow y = 2x$

When 27 is subtracted from $10y + x$, its digits are reversed or the new number has x in the tens place and y in the ones place.

Or $\quad 10y + x - 27 = 10x + y$

$\Rightarrow \quad 10y - y = 10x - x + 27$

$\Rightarrow \quad 9y = 9x + 27$

Substituting $y = 2x$ in the above equation, we get

$$9 \times 2x = 9x + 27$$

$\Rightarrow \quad 18x - 9x = 27$

$\Rightarrow \quad 9x = 27 \Rightarrow x = \frac{27}{9} = 3$

Substituting the value of x in $y = 2x$, we get

$$y = 2 \times 3 = 6$$

Thus, the number is 63.

Example 12: A boat first travels 36 km upstream and 30 km downstream in $8\frac{1}{2}$ hours. Then it travels for $5\frac{1}{2}$ hours going 24 km downstream and 21 km upstream. Find the speed of the boat in still water and the speed of the water current.

Let the speed of the boat in still water be x and speed of the water current be y.
Then speed upstream is $x - y$ and speed downstream $= x + y$.

As speed $= \frac{\text{distance}}{\text{time}}$, $\quad$ time $= \frac{\text{distance}}{\text{speed}}$

Thus, in first journey $\frac{36}{x-y} + \frac{30}{x+y} = 8.5$ and in second journey $\frac{24}{x+y} + \frac{21}{x-y} = 5.5$

I. Let $\frac{1}{x+y} = a$ and $\frac{1}{x-y} = b$, then the simultaneous equations become

$$36b + 30a = 8.5$$

and $\quad 24a + 21b = 5.5$

The LCM of 30 and 24 which are the numerical coefficients of a, is 120.

$\Rightarrow \quad (36b \times 4) + (30a \times 4) = 8.5 \times 4$

and $\quad (24a \times 5) + (21b \times 5) = 5.5 \times 5$

$\Rightarrow \quad 144b + 120a = 34$

and $\quad 120a + 105b = 27.5$

$\Rightarrow \quad 144b + 120a - 120a - 105b = 34 - 27.5$ (subtracting equations)

$\Rightarrow \quad 39b = 6.5$

$\Rightarrow \quad \frac{39}{x-y} = 6.5 \quad \left(\text{substituting } b = \frac{1}{x-y}\right)$

$\Rightarrow \quad x - y = \frac{39}{6.5} = 6$ [Eq. (i)]

II. $36b + 30a = 8.5$ and $24a + 21b = 5.5$

The LCM of 36 and 21, which are the numerical coefficients of b, is 252.

$\Rightarrow \quad (36b \times 7) + (30a \times 7) = 8.5 \times 7$

and $\quad (24a \times 12) + (21b \times 12) = 5.5 \times 12$

$\Rightarrow \quad 252b + 210a = 59.5$

and $\quad 288a + 252b = 66$

$\Rightarrow \quad 288a + 252b - 252b - 210a = 66 - 59.5$ (subtracting equations)

$\Rightarrow \quad 78a = 6.5$

$\Rightarrow \quad \frac{78}{x+y} = 6.5 \quad \left(\text{substituting } a = \frac{1}{x+y}\right)$

$\Rightarrow \quad x + y = \frac{78}{6.5} = 12$ [Eq. (ii)]

Downloaded from https:// www.studiestoday.com

Thus, the simultaneous equations are simplified to:

$$x + y = 12 \quad \text{[from Eq. (ii)]}$$

and $$x - y = 6 \quad \text{[from Eq. (i)]}$$

$$x + y - x + y = 12 - 6$$

(subtracting equations)

$\Rightarrow \quad 2y = 6 \Rightarrow y = 3$

Substituting the value of y in $x + y = 12$

$$x + 3 = 12$$

$\Rightarrow \quad x = 12 - 3 = 9$

Thus, the speed of the boat in still water is 9 km/h and that of the water current is 3 km/h.

Try this!

Solve : $2x + 4y = 34$

$4x + 3y = 33$

Exercise 21.2

1. Half the sum of two numbers is 5, while three times their difference is 12. Find the numbers.
2. One-third the sum of two numbers is 6 while half their difference is also 6. Find the numbers.
3. Half the difference of two numbers is 3 while one-sixth their sum is 5. Find the numbers.
4. 6 pencils and 4 erasers cost Rs 42 while 2 pencils and 6 erasers cost Rs 21. Find the price of a pencil and an eraser.
5. Ramesh bought 5 books and 7 notebooks for Rs 492, while Neetu bought 2 books and 3 notebooks for Rs 204. Find the price of a book and a notebook.

6. 9 kg of rice and 5 kg of flour cost Rs 191.75, while 3 kg of rice and 4 kg of flour cost Rs 92.50. Find the price of a kg of rice and a kg of flour.
7. One-fifth a litre of mustard oil and half a litre of milk cost Rs 18, while half a litre of mustard oil and one-third a litre of milk cost Rs 34. Find the price of mustard oil and milk per litre.
8. The price of 5 brushes is Rs 27 more than the price of 3 pencils while the price of 3 brushes and 5 pencils is Rs 40. Find the prices of one brush and one pencil.
9. The sum of an uncle's and his nephew's ages is 46. Three years hence the uncle will be 3 times the nephew's age. Find their present ages.
10. Today a father's age is 6 times his son's age. 4 years hence the father's age will be 4 times his son's age. Find their present ages.
11. Five years ago a mother was 5 times her son's age. 6 years hence the sum of their ages will be 52. Find their present ages.
12. A brother asks his sister for Rs 100 so that he can be 3 times richer than her, but the sister tells her brother to give her Rs 400, so that she can be 7 times richer than him. How much money do the brother and sister have now?
13. Vinay has 16 marbles more than Vijay. If Vijay gives Vinay 8 marbles, Vinay will have 3 times as many marbles as Vijay. How many marbles do each have?
14. When 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{5}$ but when 1 is subtracted from its numerator, it becomes $\frac{1}{7}$. Find the fraction.

15. When 1 is added to the numerator of a fraction, the fraction becomes $\frac{1}{2}$, but when 3 is added to its denominator it becomes $\frac{1}{3}$. Find the fraction.
16. If 4 is added to the numerator of a fraction and 5 is subtracted from its denominator, the fraction becomes 3. If 1 is subtracted from the numerator and 2 is added to the denominator, the fraction becomes $\frac{2}{5}$. Find the fraction.
17. If 2 is added to the numerator of a fraction and 1 is subtracted from its denominator, the fraction becomes $\frac{4}{5}$. If 1 is added to the denominator of the fraction, the fraction becomes $\frac{1}{2}$. Find the fraction.
18. The sum of the digits in a 2-digit number is 7. If 9 is subtracted from the number, its digits are reversed. Find the number.
19. The digit in the tens place of a 2-digit number is half the digit in the ones place. If the sum of the digits is 12, find the number.
20. The sum of the digits in a 2-digit number is 7. If 27 is added to the number, its digits are reversed. Find the number.
21. A 2-digit number is 5 times the sum of its digits. If 9 is added to the number, its digits are reversed. Find the number.
22. A 2-digit number is 3 times the sum of its digits. If 45 is added to the number, its digits are reversed. Find the number.
23. A boat travels 72 km upstream in 6 hours and then returns 54 km downstream in 3 hours. Find the speed of the boat in still water and the speed of the water current.
24. A boat first travels 48 km downstream and 24 km upstream in 7 hours. Then it travels for 9 hours, going 48 km upstream and 36 km downstream. Find the speed of the boat in still water and the speed of the water current.

25. Bula first swims 600 m against the river current and 400 m with the current in $13\frac{1}{3}$ minutes. Then she swims for 5 minutes, swimming 350 m with the current and 125 m against the current. Find the speed of the river current and Bula's speed in still water.

Revision Exercise

1. Solve $x + 8y = 40$; $x - 4y = 4$.
2. Solve $x - 14 = y$; $2x - 4y = 16$.
3. Solve $\frac{x+1}{2} + \frac{y-2}{4} = 8$; $\frac{x-4}{4} - \frac{y-1}{6} = 1$.
4. If 2 is added to the numerator of a fraction and subtracted from its denominator, it becomes 2, but if 4 is subtracted from the numerator and added to the denominator it becomes $\frac{1}{2}$. Find the fraction.
5. 4 chocolates and 2 pastries cost Rs 56 while 2 chocolates and 6 pastries cost Rs 68. Find the price of a chocolate and a pastry.