20 INEQUATIONS

- Inequation
- Replacement Set
- Truth Set
- Operations on Inequations

Introduction

An algebraic statement that shows two algebraic expressions being unequal is known as an algebraic inequation.

When two expressions are compared, one might be less than (<), less than or equal to (≤), greater than (>), or greater than or equal to (≥) the other. Such inequations may have multiple values that satisfy the inequality. For instance, if we are told of a person named Rahul who is not old enough to vote, we can tell that Rahul's age in years can be described by a positive number less than 18.

The values of the variable x that satisfy the inequation $5x - 8 \geq -23$ are infinite in number, being the solution set of real numbers $\left\{-3\ldots -2.78\ldots -1\frac{3}{7}\ldots 0\ldots 5\ldots\right\}$.

This is why inequations are usually accompanied by a **replacement set** which describes the set of values that can be assigned to the variable. If the replacement set to the above inequation is specified as $R = \{x \mid x \in Z_{-ve}\}$, then x can only be a negative integer and the negative integers for which the inequation $5x - 8 \geq -23$ holds 'true' are −3, −2, and −1. Thus, the **truth set** or the set of solutions to the inequation that belong to the replacement set is {−3, −2, −1}.

Remember

- In the set of negative integers, $-1 > -2$, $-2 > -3$, $-3 > -4$ and so on.

Operations on Inequations

The rules for transposition in case of inequations are the same as for equations.

1. Addition of the same quantity to both sides does not change the inequation.

   $11 > 1 \Rightarrow 11 + 4 > 1 + 4 \Rightarrow 15 > 5$

2. Subtraction of the same quantity from both sides does not change the inequation.

   $15 > 5 \Rightarrow 15 - 3 > 5 - 3 \Rightarrow 12 > 2$

3. Multiplication of both sides by the same positive quantity does not change the inequation.

   $12 > 2 \Rightarrow 12 \times 3 > 2 \times 3 \Rightarrow 36 > 6$

4. Division of both sides by the same positive quantity does not change the inequation.

   $36 > 6 \Rightarrow 36 \div 2 > 6 \div 2 \Rightarrow 18 > 3$

Remember

- When both sides of an inequation are multiplied or divided by the same negative quantity, the sign of inequality is reversed.

  $18 > 3 \Rightarrow 18 \times -6 < 3 \times -6 \Rightarrow -108 < -18$ and

  $-108 < -18 \Rightarrow -108 \times -\frac{1}{2} > -18 \times -\frac{1}{2} \Rightarrow 54 > 9$

- If the sides of an inequation are interchanged or if the RHS becomes the LHS and vice-versa, the sign of inequality is reversed.

  $54 > 9 \Rightarrow 9 < 54$ and

  $-108 < -8 \Rightarrow -8 > -108$

On the Number Line

The **solution set** for an inequation can be represented on a number line by marking the true values of solutions with **thick dots just above the number line.**

Example 1: Solve $8x - 5 \geq -45$, $x \in \mathbb{Z}_{-ve}$ and represent the solution set on the number line.

$\Rightarrow \quad 8x \geq -45 + 5$ (adding +5 to LHS and RHS)

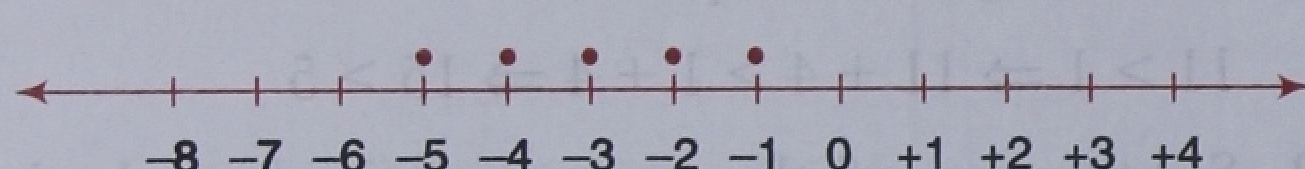
$\Rightarrow \quad x \geq \frac{-40}{8}$ (dividing LHS and RHS by 8)

$\Rightarrow \quad x \geq -5$

As the replacement set is given as the set of negative integers,

$$x = \{-5, -4, -3, -2, -1\}$$

The solution set is represented on the number line as:



Example 2: Solve $7x + 3 < 45$, $x \in W$ and represent the solution set on the number line.

$\Rightarrow \quad 7x < 45 - 3$

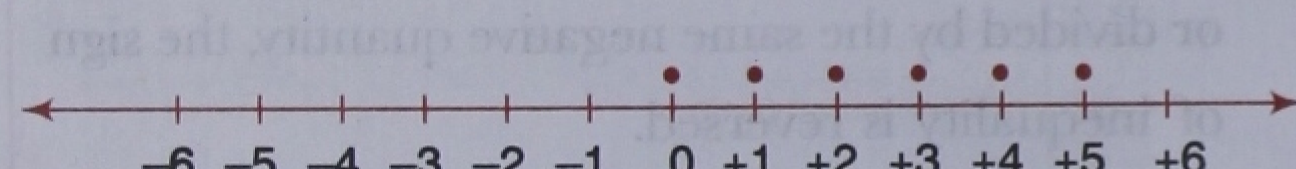
$\Rightarrow \quad x < \frac{42}{7}$

$\Rightarrow \quad x < 6$

As the replacement set is given as the set of whole numbers,

$$x = \{0, 1, 2, 3, 4, 5\}$$

The solution set is represented on the number line as:

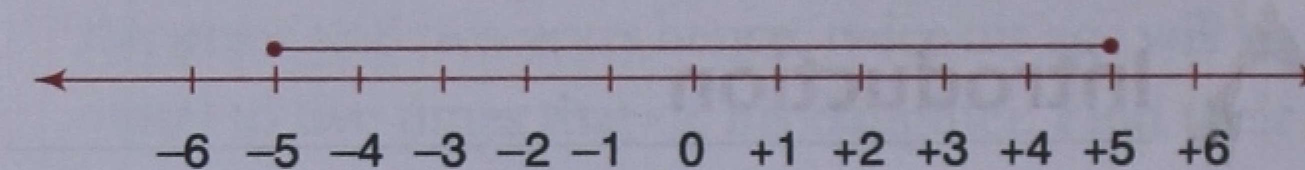


Example 3: Solve $-22 \leq 3x - 7 \leq 8$, $x \in R$ and represent the solution set on the number line.

Notice that when the given inequation is a combination of two inequations, they need to be solved separately.

$-22 \leq 3x - 7$	$3x - 7 \leq 8$
$\Rightarrow \quad -22 + 7 \leq 3x$	$\Rightarrow \quad 3x \leq 8 + 7$
$\Rightarrow \quad \frac{-15}{3} \leq x$	$\Rightarrow \quad x \leq \frac{15}{3}$
$\Rightarrow \quad x \geq -5$ (sides reversed)	$\Rightarrow \quad x \leq 5$

As the replacement set is given as the set of real numbers, $x = \{-5, \ldots, 0, \ldots +5\}$, the solution set is the infinite set of real numbers between –5 and +5. It is represented on the number line as a line segment with its end points being –5 and +5.



Example 4: Solve $4(2x - 1) \geq 2(2x + 1)$, $x \in N$ and represent the solution set on the number line.

$4(2x - 1) \geq 2(2x + 1)$

$\Rightarrow \quad 8x - 4 \geq 4x + 2$

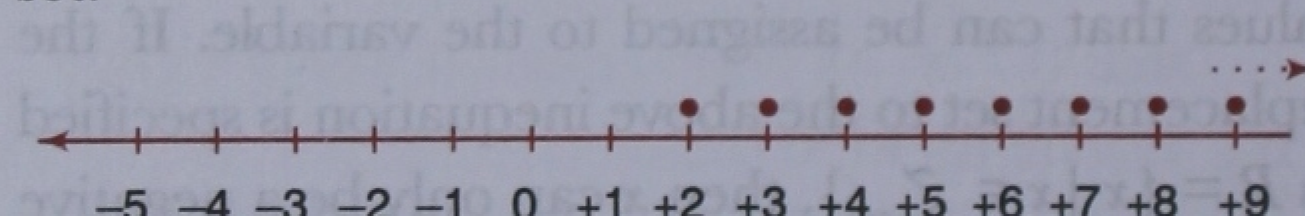
$\Rightarrow \quad 8x - 4x \geq 2 + 4$

$\Rightarrow \quad 4x \geq +6$

$\Rightarrow \quad x \geq \frac{6}{4}$

$\Rightarrow \quad x \geq 1.5$

As the replacement set is given as the set of natural numbers, $x = \{2, 3, 4, 5\ldots\}$, the solution set is the infinite set of natural numbers greater than or equal to 2. It is represented on the number line with thick dots above numbers greater than 2 and a dotted arrow on the extreme right to signify an infinite truth set.



Try this!

Solve $8x + 10 < 50$, $x \in W$ and represent the solution set on the number line.

Exercise 20.1

1. Given that $x \geq y$, write the signs of inequalities after the following operations are performed on the given inequation.
   (i) $x + 6 \square y + 6$
   (ii) $x - 8 \square y - 8$
   (iii) $x + 1\frac{1}{2} - 5\frac{1}{3} \square y + 1\frac{1}{2} - 5\frac{1}{3}$
   (iv) $9x \square 9y$
   (v) $-17x \square -17y$
   (vi) $\frac{x}{11} \square \frac{y}{11}$ (vii) $y \square x$
   (viii) $\frac{x}{y} \square 1$ (ix) $1 \square \frac{y}{x}$
   (x) $-\frac{3x}{4} \square -\frac{3y}{4}$ (xi) $x^2 \square yx$
   (xii) $-xy \square -y^2$ (xiii) $0 \square y - x$
   (xiv) $x - y \square 0$
2. Solve the following inequations, given their respective replacement sets.
   (i) $x - 6 < 1, x \in N$
   (ii) $x - 9 > -15, x \in Z_{-ve}$
   (iii) $2x + 3 \leq 13, x \in W$
   (iv) $8x - 11 \leq 21, x \in N$
   (v) $6x - 7 \geq 35, x \in Z$
   (vi) $11x - 5 \leq -49, x \in Z$
   (vii) $-29 < 9x + 2 < 29, x \in Z$
   (viii) $42 > 12x - 7 > -42, x \in W$
   (ix) $\frac{6x + 3}{13} > -3, x \in Z_{-ve}$
   (x) $\frac{7x - 5}{6} < 5, x \in Z_{-ve}$
   (xi) $8(x - 1) \geq 10(x + 1), x \in Z$
   (xii) $6(x + 6) \geq 5(x - 3), x \in W$
   (xiii) $\frac{2(x + 1)}{5} \geq \frac{9(x - 1)}{15}, x \in N$
   (xiv) $\frac{5(2x - 3)}{9} \leq \frac{5(3x - 2)}{11}, x \in Z_{-ve}$
3. Represent the solution sets of the following inequations on the number line.
   (i) $3x + 4 \geq -14, x \in Z_{-ve}$
   (ii) $7x - 3 < 47, x \in W$
   (iii) $17 < x < 21, x \in N$
   (iv) $-21\frac{1}{3} < x < -16\frac{2}{11}, x \in Z$
   (v) $7 < 3x - 2, x \in N$
   (vi) $21 > 5x - 4, x \in Z$
   (vii) $25 \leq 5(2x + 7) \leq 75, x \in Z$
   (viii) $-147 < 3(6x - 7) < 15, x \in Z$
   (ix) $-5 \leq \frac{6x - 1}{7} \leq 5, x \in Z$
   (x) $-3 \leq \frac{5x - 3}{9} \leq 3, x \in Z$
   (xi) $-\frac{2}{3} \leq \frac{2x + 3}{3} \leq 3\frac{1}{3}, x \in R$
   (xii) $5\frac{2}{3} \leq \frac{2x + 3}{3} \leq 8\frac{1}{3}, x \in R$
4. Solve $-666 \leq 6(12x - 3) \leq -378$, if the replacement set is given as: $\{5, -4, -6, 9, -3, -9, 3, -5\}$
5. Solve $\frac{8x - 17}{19} > 1$, if the replacement set is given as $\{2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5\}$.

Challenge

1. An artist can spend anywhere from Rs 80 to Rs 200 on brushes. If brushes cost Rs 4.50 each but only come in sealed packets of 6 each, how many packets of brushes can the artist buy?
2. A tram has 6 speed settings. The lowest setting sets the speed at 3 m/s and each higher setting increases the speed by 3 m/s. If the speed limit on a road is 40 km/h, how many speed settings can the tram driver choose to drive at?

Revision Exercise

1. Solve $6x-9 \geq -33, x \in Z_{-ve}$ and represent the solution set on the number line.
2. Solve $5x+3<27, x \in W$ and represent the solution set on the number line.
3. Solve $-26 \leq 2x-8 \leq 10$, $x \in$ R and represent the solution set on the number line.
4. Solve $5(9x-2) \geq 10(3x+2), x \in$ N and represent the solution set on the number line.
5. Given that $x \leq y$, write the signs of inequalities after the following operations are performed on the given inequation.
   (i) $x+9$ ____ $y+9$
   (ii) $x-3$ ____ $15y$
   (iii) $-15x$ ____ $-15y$
   (iv) $\frac{x}{13}$ ____ $\frac{x}{13}$