# 19 SIMPLE LINEAR EQUATIONS

- Simple Linear Equation
- Root
- Transposition

## Linear Equations

An equation states that the algebraic expression on the left-hand side (LHS) of the equality sign is equal to the algebraic expression on the right-hand side (RHS).

*A simple linear equation is an algebraic statement that involves only one variable, the degree of which is not more than 1.*

Solving an equation involves finding the **root** of the equation or the value of the variable for which the equation is true.

Solving an equation generally involves grouping the variable terms and the constant terms on opposite sides of the equation by **transposition.**

**On transposition of a term to its opposite side,**

- **its sign changes:** $x + 17 = 23 \Rightarrow x = 23 - 17$
- **a multiplier changes into a divisor:**

$$18x = 126 \Rightarrow x = \frac{126}{18}$$

- **a divisor changes into a multiplier:**

$$= 28 \Rightarrow 14x = 28 \times 3$$

***Example 1:*** Solve $5(x-2) + 2(x+4) = 9(x-4)$

$\Rightarrow \quad 5x - 10 + 2x + 8 = 9x - 36$

$\Rightarrow \quad 7x - 2 = 9x - 36$

$\Rightarrow \quad 36 - 2 = 9x - 7x$

(transposing variables and constant terms)

$\Rightarrow \quad 2x = 34$

$\Rightarrow \quad x = \frac{34}{2} = 17$

CHECK: $5(17-2) + 2(17+4) = 9(17-4)$

$\Rightarrow \quad 5 \times 15 + 2 \times 21 = 9 \times 13$

$\Rightarrow \quad 75 + 42 = 117$

$\Rightarrow \quad 117 = 117$

***Example 2:*** Solve $\frac{x+8}{7} + \frac{x-1}{3} = \frac{x}{2}$

$\Rightarrow \quad \frac{3(x+8) + 7(x-1)}{21} = \frac{x}{2}$

$\Rightarrow \quad \frac{3x + 24 + 7x - 7}{21} = \frac{x}{2}$

$\Rightarrow \quad \frac{10x + 17}{21} = \frac{x}{2}$

$\Rightarrow \quad 2(10x + 17) = 21x$

(transposing denominators, or cross-multiplying)

$\Rightarrow \quad 20x + 34 = 21x$

$\Rightarrow \quad 34 = 21x - 20x$

$\Rightarrow \quad x = 34$

***Example 3:*** Solve $\frac{4(2-x) - 3(x+5)}{4-x} = 14$

$\Rightarrow \quad 8 - 4x - 3x - 15 = 14(4 - x)$

$\Rightarrow \quad -7x - 7 = 56 - 14x$

$\Rightarrow \quad 14x - 7x = 56 + 7$

$\Rightarrow \quad 7x = 63$

$\Rightarrow \quad x = \frac{63}{7} = 9$

***Example 4:*** Solve

$$2x-2-\frac{1}{3}\left\{3x-3-\frac{1}{4}(7x+7-\overline{5x-5})\right\}=7$$

$$\Rightarrow \quad 2x-2-\frac{1}{3}\left\{3x-3-\frac{1}{4}(7x+7-5x+5)\right\}=7$$

$$\Rightarrow \quad 2x-2-\frac{1}{3}\left\{3x-3-\frac{1}{4}(2x+12)\right\}=7$$

$$\Rightarrow \quad 2x-2-\frac{1}{3}\left\{3x-3-\frac{x}{2}-3\right\}=7$$

$$\Rightarrow \quad 2x-2-\frac{1}{3}\left\{\frac{6x-x}{2}-6\right\}=7$$

$$\Rightarrow \quad 2x-2-\frac{1}{3}\left\{\frac{5x}{2}-6\right\}=7$$

$$\Rightarrow \quad 2x-2-\frac{5x}{6}+2=7$$

$$\Rightarrow \quad \frac{12x-5x}{6}=7$$

$$\Rightarrow \quad 7x=42$$

$$\Rightarrow \quad x=\frac{42}{7}=6$$

**Try this!**

1. Solve $9a-94=100$
2. Solve $\frac{x+7}{x-5}=17$

## Exercise 19.1

Solve the following equations.

1. $7x+35=84$
2. $14x-32=80$
3. $11x-14=8x+22$
4. $13x+23=16x-16$
5. $6(x-6)=4x$
6. $5(x+7)=7.5x$
7. $8(x+9)=20(x-3)$
8. $11(x-5)=6(x+5)$
9. $4(x-3)+2(x+5)=8(x-4)$
10. $5(x+3)-2(x-2)=5(x-5)$
11. $5(x+7)-2(x-3)=7(x-1)$
12. $3(x-2)+2(x-5)=\frac{9x}{2}$
13. $\frac{x+8}{x-4}=2$
14. $\frac{x+18}{x-18}=5$
15. $\frac{x-2}{7}+\frac{x+2}{3}=\frac{x}{2}$
16. $\frac{x-3}{5}+\frac{x+2}{6}=\frac{5x}{14}$
17. $\frac{x}{4}-\frac{x}{7}=\frac{x-2}{9}$
18. $\frac{x}{2}-\frac{x}{3}=\frac{x-4}{5}$
19. $\frac{5x+3}{3}-\frac{3x+4}{4}-\frac{2x+9}{3}=0$
20. $0.75x-0.5x+3=0.3x$
21. $0.4x+1.7x=2.15x-1.1$
22. $\frac{x}{3}+\frac{x}{4}=\frac{x}{5}+13\frac{4}{5}$
23. $\frac{x}{6}-\frac{x}{7}=\frac{x}{8}-4\frac{1}{4}$
24. $\frac{3x-5(1-x)}{x-4}=5$
25. $\frac{3(3x-5)-3(x-3)}{x+3}=4$
26. $\frac{x-(4-6x)}{9x-(x-4)}=\frac{2}{3}$
27. $\frac{4(3-x)-2(x+10)}{2-x}=10$

**Challenge**

1. $\frac{2x+6}{11}+\frac{3x-6}{6}-\frac{4x-7}{5}=0$
2. $x+12+[3x+1+\{9x-5-(5x+3-\overline{2x-5})\}]=30$
3. $\frac{1}{5}\left\{2x+2-\frac{1}{7}(6x+6-\overline{3x+2})\right\}=5$

## Solving Word Problems with Equations

***Example 5:*** Rajat is 4 years elder to his wife. If one-half of his age exceeds one-third of his wife's age by 9 years, find how old Rajat and his wife are.

Let Rajat's wife's age = $x$

Then Rajat's age = $x + 4$

$$\text{Given } \frac{1}{2}(x+4) - \frac{x}{3} = 9$$

$$\Rightarrow \quad \frac{x+4}{2} - \frac{x}{3} = 9$$

$$\Rightarrow \quad \frac{3x+12-2x}{6} = 9$$

$$\Rightarrow \quad x + 12 = 54$$

$$\Rightarrow \quad x = 54 - 12 = 42$$

Thus, Rajat's wife is 42 years old and Rajat is $42 + 4 = 46$ years old.

***Example 6:*** Divide Rs 7100 between A and B such that one-fifth of A's share exceeds one-sixth of B's share by Rs 155.

If A's share = $x$, then B's share = $7100 - x$

$$\text{Now } \frac{x}{5} - \frac{1}{6}(7100 - x) = 155$$

$$\Rightarrow \quad \frac{x}{5} - \frac{7100-x}{6} = 155$$

$$\Rightarrow \quad \frac{6x - 35500 + 5x}{30} = 155$$

$$\Rightarrow \quad 11x - 35500 = 155 \times 30$$

$$\Rightarrow \quad 11x = 4650 + 35500$$

$$\Rightarrow \quad 11x = 40150$$

$$\Rightarrow \quad x = \frac{40150}{11} = 3650$$

Thus, A's share = Rs 3650 and B's share
= Rs 7100 – 3650 = Rs 3450

***Example 7:*** If the length of a square is decreased by 7 cm, its area decreases by 259 cm$^2$. What was the original length of the square?

Let the original length of the square be $x$.

Then the original area of the square = $x^2$

New length of the square = $x - 7$

New area of the square = $(x-7)^2 = x^2 - 14x + 49$

Given that original area – new area = 259 cm$^2$

$$\Rightarrow \quad x^2 - (x^2 - 14x + 49) = 259$$

$$\Rightarrow \quad x^2 - x^2 + 14x - 49 = 259$$

$$\Rightarrow \quad 14x = 259 + 49$$

$$\Rightarrow \quad x = \frac{308}{14} = 22$$

Thus, the original length of the square was 22 cm.

*If the face values of the two digits in a two-digit number are $x$ and $y$, the number can be either $10x + y$ or $10y + x$.*

For instance, $3 \times 10 + 9 = 39$ or $9 \times 10 + 3 = 93$

***Example 8:*** The sum of the two digits in a 2-digit number is 13. If 45 is added to the number, the digits are reversed. Find the original 2-digit number.

Now let the digit in the ones place of the 2-digit number in this example be $x$.

Then the digit in the tens place = $13 - x$ and the number = $10(13 - x) + x$

When the number is reversed,
the digit in the ones place = $13 - x$,
the digit in the tens place = $x$ and
the new number = $10x + (13 - x)$

$$\text{Given } \{10x + (13 - x)\} - \{10(13 - x) + x\} = 45$$

$$\Rightarrow \quad (10x + 13 - x) - (130 - 10x + x) = 45$$

$$\Rightarrow \quad (9x + 13) - (130 - 9x) = 45$$

$$\Rightarrow \quad 9x + 13 - 130 + 9x = 45$$

$$\Rightarrow \quad 18x - 117 = 45$$

$$\Rightarrow \quad x = \frac{45 + 117}{18} = \frac{162}{18} = 9$$

Thus, the digit in the ones place of the original number is 9 and the digit in the tens place = $13 - 9 = 4$ or the original number = 49

CHECK: $49 + 45 = 94$ (digits reversed)

***Example 9:*** Mihir starts from a jetty and swims against the current of the Hooghly for $17\frac{1}{2}$ minutes to reach the temple ghat. On the return trip, he

swims the same distance in $7\frac{1}{2}$ minutes. If the water current was 2.16 km/h on that day, find how fast can Mihir swim in still water.

Water current = $2.16 \times \frac{5}{18} = 0.6$ m/s

Let Mihir's speed in still water = $x$
Then Mihir's speed upstream = $x - 0.6$
and Mihir's speed downstream = $x + 0.6$

At a speed of $x - 0.6$, Mihir swam for $17\frac{1}{2}$ min or 1050 seconds.

Thus, distance from jetty to temple ghat
$= 1050(x - 0.6)$ (as $d = st$)

At a speed of $x + 0.6$, Mihir swam for $7\frac{1}{2}$ min or 450 seconds.

Thus, distance from temple ghat to jetty
$= 450(x + 0.6)$

As the distances are the same,

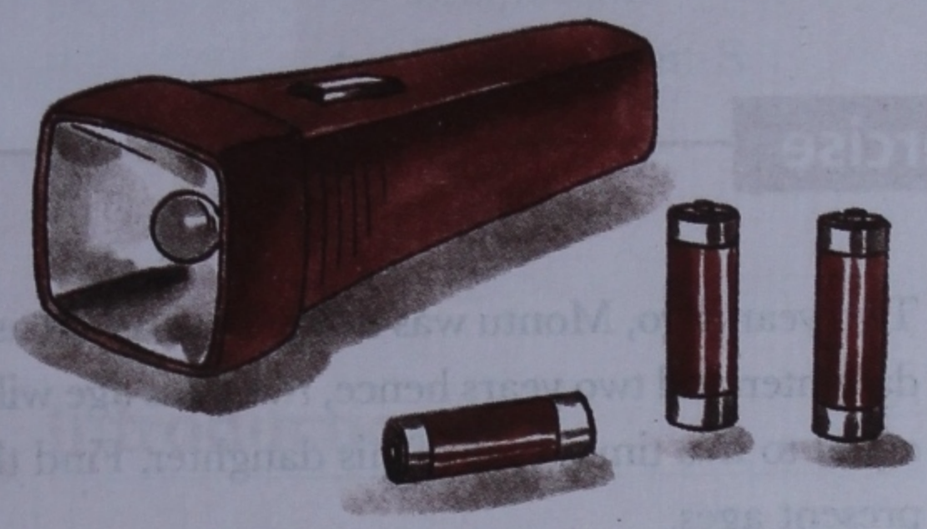
$$1050(x - 0.6) = 450(x + 0.6)$$
$$\Rightarrow \quad 1050x - 630 = 450x + 270$$
$$\Rightarrow \quad 1050x - 450x = 270 + 630$$
$$\Rightarrow \quad 600x = 900$$
$$\Rightarrow \quad x = \frac{900}{600} = 1\frac{1}{2}$$

Thus, Mihir can swim through still water at $1\frac{1}{2}$ m/s or 5.4 km/h.

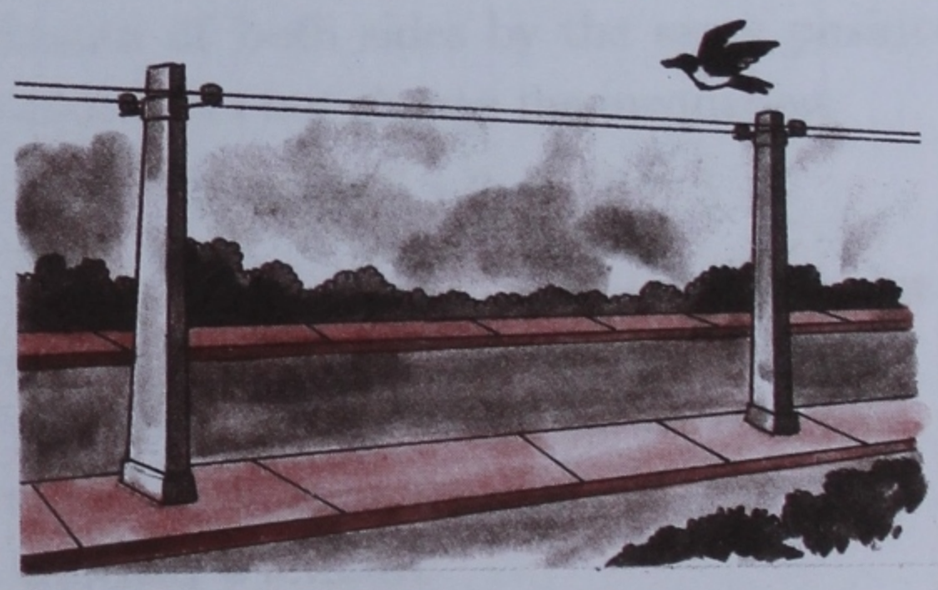
## Exercise 19.2

1. If 21 is subtracted from a certain number and the result is multiplied by 6, the product is 120. Find the number.
2. If a number is increased by 23 and the sum is multiplied by 7, the product is 364. Find the number.
3. Sheela thought of a number, halved it and then added 3 to it. The product of this sum and 6 came to 150. What was the number Sheela thought of?
4. Abhishek thought of a number and then divided it by 7. To the quotient, he added 4 and multiplied the sum obtained by 8 to get 104. What was the number Abhishek thought of?
5. If 11 is subtracted from one-fourth of a certain number, the difference is equal to 1 more than one-sixth of that number. Find the number.
6. Ritu thought of a number. One-seventh of the number exceeds one-eighth of the number by 12. What is the number Ritu thought of?
7. Find three consecutive even numbers such that the sum of twice the least number and thrice the greatest number is 622.
8. Find three consecutive odd numbers such that 5 times the middle number is greater than the sum of the first and last numbers by 495.
9. If Bansi adds his present age to 4 times his age 5 years ago, it exceeds 3 times his present age by 16 years. What is Bansi's present age?
10. Mr A is 3 years elder to Mrs A. One-fifth of Mr A's age is 3 years more that one-eighth of Mrs A's age. How old are Mr and Mrs A?
11. Purnima's grandfather is 7 years elder to her grandmother. The sum of one-eighth of her grandfather's age and one-ninth of her grandmother's age equals 20 years. How old are Purnima's grandparents?
12. A teacher is 18 years elder to her student. 3 years ago one-tenth of the teacher's age was equal to one-fourth of the student's age. Find their present ages.
13. The denominator of a common fraction exceeds the numerator by 7. If 3 is deducted from the numerator as well as the denominator, the fraction $\frac{2}{3}$ is obtained. Find the original common fraction.
14. The denominator of a common fraction exceeds the numerator by 16. If 5 is added to the numerator as well as the denominator, the fraction $\frac{3}{5}$ is obtained. Find the original common fraction.

15. Kirti bought some pencil batteries costing Rs 7.50 each and some torch bulbs costing Rs 8 each, spending Rs 137 in all. If the number of bulbs she bought was two-sevenths of the number of batteries, how many of each did Kirti buy?



16. A bus conductor's purse has Rs 156.75 in all, made up of Rs 5, Re 1, 50 p, and 25 p coins. The number of 50 p coins is $1\frac{4}{5}$ times the number of 25 p coins while the number of Re 1 coins is $\frac{2}{3}$ times the number of 25 p coins. If the number of Rs 5 coins is half the number of Re 1 coins, find how many Rs 5, Re 1, 50 p, and 25 p coins are there in the bus conductor's purse.
17. Divide Rs 5880 between A and B in such a way that $\frac{1}{7}$ of A's share equals $\frac{1}{5}$ of B's share.
18. Divide Rs 7056 between A and B in such a way that $\frac{1}{4}$ of A's share exceeds $\frac{1}{5}$ of B's share by Rs 211.50.
19. Divide Rs 3984 between A and B in such a way that 0.7 of A's share equals 0.5 of B's share.
20. Divide Rs 4450 between A and B in such a way that 0.6 of A's share exceeds 0.4 of B's share by Rs 520.
21. If the length of a square is increased by 3 cm, its area increases by 99 sq. cm. What was the length of the square before being increased?
22. If the length of a square is decreased by 6 cm, its area decreases by 192 sq. cm. What was the length of the square before being decreased?
23. In a 2-digit number, the face value of the digit in the tens place is double the face value of the digit in the ones place. If the sum of the two face values is 12, find the 2-digit number.
24. The sum of the two digits of a 2-digit number is 13. If 9 is subtracted from the number, the digits are reversed. What is the number?
25. The sum of the two digits of a 2-digit number is 10. If 36 is added to the number, the digits are reversed. What is the number?
26. The sum of the two digits of a 2-digit number is 7. When the digits are reversed, the number increases by 27. Find the original number.
27. The sum of the two digits of a 2-digit number is 9. When the digits are reversed, the number decreases by 9. Find the original number.
28. A man was walking home at a steady speed for 5 minutes, when it suddenly started raining forcing him to run the rest of the distance at double the speed in only 2 minutes, covering 1350 m in all. At what speed did the man run?
29. A crow takes 150 seconds to fly from pole A to pole B against a strong breeze blowing at 1.5 m/s. Flying back from pole B to pole A with the breeze, it takes just 114 seconds. How fast can the crow fly through still air?



30. A rowboat is travelling downstream a river in which the water current is 1 m/s. 2.8 km away, a steamboat is chugging upstream 6 m/s faster than the rowboat. If the two boats cross each other after 3 minutes 20 seconds, what was the speed of the rowboat?

## Mental maths

Find the value of x, y and z as the case may be.

1. $8x + 10 = 50$
2. $3z - 20 = 70$
3. $150\,y = 1 - 151$

## Revision Exercise

1. Solve: $\frac{12x+14}{6x+4} = \frac{8x+10}{4x+6}$
2. Solve: $\frac{0.6+1.4x}{2x} = 1.70$
3. Two numbers are in the ratio 7 : 9. If the sum of the numbers is 272, find the numbers.
4. Two years ago, Montu was three times as old as his daughter and two years hence, twice his age will be equal to five times that of his daughter. Find their present ages.
5. Find three consecutive even numbers whose sum is 246.