▲18▲ FORMULA

- Framing a Formula
- Subject of a Formula
- Substitution

▲ Introduction

A formula is a short algebraic statement that describes the relation between variable measures. The variable on the left-hand side of the statement, whose value is expressed by the relation between the other variables on the right-hand side, is known as the **subject** of the formula.

▲ Framing a Formula

If an article costing Rs C is marked up by x% and then sold at a discount of y%, where $x > y$, find the profit made.

As Rs C is marked up by x%, the marked price will be Rs $C + \dfrac{xC}{100}$

Now the article is sold at Rs $C + \dfrac{xC}{100}$ less discount of y% on the above price.

Thus, S.P. $= C + \dfrac{xC}{100} - \dfrac{y}{100}$ of $\left(C + \dfrac{xC}{100}\right)$

$= C + \dfrac{xC}{100} - \dfrac{yC}{100} - \dfrac{Cxy}{10000}$

Thus, S.P. $- C = \dfrac{Cx}{100} - \dfrac{Cy}{100} - \dfrac{Cxy}{10000}$

Thus, Profit $= \dfrac{100Cx - 100Cy - Cxy}{10000}$

CHECK:

Let us find the profit made when a ceiling fan costing Rs 1580 is first marked up by 40% and then sold at a 25% discount.

Applying the formula for profit,

Profit $= \dfrac{(100 \times 1580 \times 40) - (100 \times 1580 \times 25) - (1580 \times 40 \times 25)}{10000}$

$= \dfrac{790000}{10000} = $ Rs 79

Applying percentages,

$1580 + 40\% = 1580 + 632 = 2212$
$2212 - 25\% = 2212 - 553 = 1659$
S.P. $-$ C.P. $=$ Profit $= 1659 - 1580 = 79$

Thus, the formula has been correctly framed.

Find the number of hours in x weeks and y days.

In 1 day there are 24 hours
In y days there will be $24y$ hours
In 1 week there are 7 days $\times$ 24 = 168 hours
In x weeks there will be $168x$ hours

Thus, in x weeks and y days there will be $168x + 24y$ hours.

Example 1: The speed of a car for $\dfrac{1}{3}$ of a journey of 'x' km was 'a' km/h and for the rest of the journey it was 'b' km/h. Find the average speed of the car.

Time taken to cover $\dfrac{1}{3} \times x$ km $= \dfrac{\text{distance}}{\text{speed}}$

$$= \dfrac{x}{3} \div a = \dfrac{x}{3a} \text{ h}$$

Remaining distance $= \left(1 - \dfrac{1}{3}\right) x = \dfrac{2}{3} x$

Time taken to cover $\dfrac{2}{3} x = \dfrac{2x}{3} \div b = \dfrac{2x}{3b} \text{ h}$

Total time taken to cover 'x' km

$$= \dfrac{x}{3a} + \dfrac{2x}{3b} = \dfrac{bx + 2ax}{3ab} \text{ h}$$

Average speed $= \dfrac{\text{total distance}}{\text{total time taken}} = x \div \dfrac{bx + 2ax}{3ab}$

$$= x \times \dfrac{3ab}{bx + 2ax} = \dfrac{3abx}{bx + 2ax}$$

Try this!

A scooterist covers a certain distance at 30 kmph. How many metres does he cover in 2 minutes?
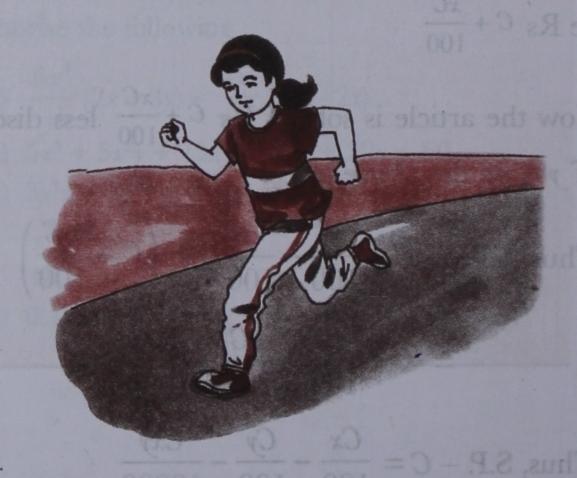

Exercise 18.1

1. Find the cost price C of an article selling at S after a 30% loss.

2. Find the profit percentage P when an article costing Rs x is sold at Rs y.

3. Find the wages w earned by a worker in a month(of 30 days), given that he earns Rs m per day and he was absent from work for y days.

4. Find the number of seconds s in x hours, y minutes, and z seconds.

5. Find the profit P made when p bananas costing Rs x each are sold at Rs y each.

6. Find the profit percentage p when 12 apples costing Rs x each are sold at Rs y each.

7. The maximum temperature recorded on Saturday was x °C while the minimum temperature was y °C. What was the average temperature A on Saturday?

8. A piggy bank contains e number of Rs 5 coins, f number of Rs 2 coins, g Re 1 coins, h 50 p coins, i 25 p coins, and j 10 p coins. Find the total amount (A) of money in the piggy bank.

9. The average weight of n number of boys in a team is p kg. If one boy weighing x kg leaves and another boy weighing y kg joins the team, find the new average weight A of the team.

10. Frame a formula to find the radius r of a cylinder, given that the square root of the quotient, when its volume v is divided by the product of π and its height h, equals its radius.

11. Reena ran the first half of an x metre long race at a m/s and the second half at b m/s. Find her average speed S for the race.

12. A fruit-seller buys x number of bananas at Rs r each. If he sells all the bananas at Rs t each, find his profit percentage P.

152

Changing the Subject of a Formula

Given the interest earned (I), the rate of interest (R), and the principal borrowed (P), if we have to calculate the time period (T) over which the interest was earned, in the formula

$$I = \frac{PRT}{100}$$

the variable T is taken to the left-hand side and made the subject of the formula, bringing all the other variables to the right hand side.

$$T = \frac{I \times 100}{PR}$$

Thus, here the subject has been changed from I to T.

Example 2: The famous Archimedes' principle describes the relative density of a body as a relation between its weight in air and in water as:

$$D_r = \frac{w_1}{w_1 - w_2}$$

Change the subject of the formula to w_2.

$$D_r(w_1 - w_2) = w_1$$
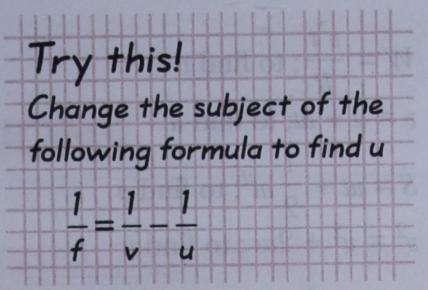$$\Rightarrow \quad D_r w_1 - D_r w_2 = w_1$$
$$\Rightarrow \quad -D_r w_2 = w_1 - D_r w_1$$
$$\Rightarrow \quad D_r w_2 = D_r w_1 - w_1$$

(multiplying LHS and RHS by -1)

$$\Rightarrow \quad w_2 = \frac{D_r w_1 - w_1}{D_r}$$

$$\Rightarrow \quad w_2 = w_1 \left(\frac{D_r - 1}{D_r} \right)$$

> **Try this!**
> Change the subject of the following formula to find u
>
> $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Substitution

The process of substitution involves finding the value of the subject by substituting the other variables in the formula with the given values.

Example 3: Given $m = 50$, $K = 900$ in the formula $K = \frac{1}{2} mv^2$, find the value of v.

The subject of the given formula needs to be changed to v.

$$K = \frac{1}{2} mv^2 \Rightarrow mv^2 = 2K$$

$$\Rightarrow \quad v^2 = \frac{2K}{m} \Rightarrow v = \sqrt{\frac{2K}{m}}$$

Substituting the values of m and K, we have

$$\Rightarrow \quad v = \sqrt{\frac{2 \times 900}{50}} = \sqrt{\frac{1800}{50}} = \sqrt{36} \Rightarrow v = 6$$

> **Try this!**
> Find the value of x in the formula
> where $a = 3$, $b = 2$ $c = 5$ and $d = 7$.
>
> $$\frac{a}{b} = \frac{x - c}{x - d}$$

Exercise 18.2

1. Change the subject of the given formulae as indicated.

   (i) $F = ma$; to find m

   (ii) $V = IR$; to find I

   (iii) $e = mc^2$; to find c

   (iv) $h = \frac{u^2}{2g}$; to find g

   (v) $K = \frac{1}{2} mv^2$; to find m

   (vi) $f = \frac{x + y}{2}$; to find x

   (vii) $S = \frac{v^2}{2a}$; to find v

(viii) $W = \dfrac{v^2 t}{R}$; to find v

(ix) $v = u + at$; to find a

(x) $S = ut + \dfrac{1}{2} at^2$; to find a

(xi) $v^2 = u^2 + 2as$; to find u

(xii) $\dfrac{F_2}{F_1} = \dfrac{A_2}{A_1}$; to find A_1

2. Given $s = \dfrac{d}{t}$, find s when $d = 1984$ and $t = 248$.

3. Given $I = \dfrac{PRT}{100}$, find R when $P = 15500$, $I = 4533.75$, and $T = 3$.

4. Given $A = 2(\ell b + \ell h + bh)$, find ℓ when $A = 73$, $b = 3$, and $h = 4$.

5. Given $S = \dfrac{n}{2}[2a + (n-1)d]$, find d when $S = 800$, $n = 16$, and $a = 5$.

6. Given $A = \pi r^2 h$, find r when $\pi = \dfrac{22}{7}$, $h = 5$, and $A = 192.5$.

7. Given $w = \dfrac{1}{2} mv^2 - \dfrac{1}{2} mu^2$, find w when $m = 6$, $v = 8$, and $u = 5$

8. Given $R = \dfrac{1}{2} mv^2$, find m when $R = 625$ and $v = 500$.

9. Given $y = 3x^2 - 6x + 5$, find y when the values of x are $-2, -1, +1,$ and $+2$.

10. Given $y = x^4 - 2x^3 + 4x^2 - 3x + 5$, find y when the values of x are -2 and $+2$.

Challenge

1. $F = \dfrac{1}{2\ell} \sqrt{\dfrac{T}{M}}$; to find T

2. $S = \dfrac{n}{2}[2a + (n-1)d]$; to find a

Revision Exercise

1. Find the cost price C of an article selling at S after a 20% profit.

2. A fruit seller buys y number of oranges at Rs b each. If he sells all the oranges at Rs a each, find his profit percentage P.

3. Given $P = 2\pi rh$, find h when $\pi = \dfrac{22}{7}$, $r = 14$, $P = 1056$.

4. Given $A = 2(l + b)h$, find l, when $b = 7$, $A = 180$, $h = 6$.

5. Given $y = 4x^2 - 3x + 7$, find y when the values of x are $-3, -1, 1$ and 3.