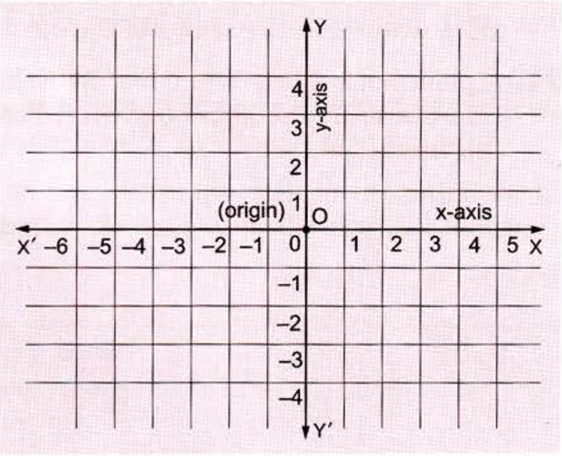
10

Graphs

You are already familiar with the coordinate system. You know how to plot points when their coordinates are given and are also familiar with drawing graphs of linear equations in two variables x and y. Let us revise these things and learn the graphical method of solving simultaneous linear equations.



(i) The horizontal line $X'OX$ is called the x-axis.

(ii) The vertical line $Y'OY$ is called the y-axis.

(iii) The point of intersection of these two lines, that is, the point O, is called the origin.

Coordinate System

To represent the values of a variable, we use a number line. To represent ordered pairs of two variables (e.g., when $x = 0$, $y = 2$, when $x = 1$, $y = 3$), we need a coordinate system or coordinate plane formed by two perpendicular lines intersecting at a point. These lines are called the axes of the system. The figure shows two such lines XOX' and YOY' drawn on graph paper.

Each axis has a scale. Notice the numbers marked along the two axes, showing that the scale is 1 unit length = 1 along both the axes. Numbers to the right of the origin along the x-axis are positive, while numbers to the left of the origin are negative. Similarly, numbers above the origin along the y-axis are positive, while numbers below the origin are negative.

Coordinates of a point

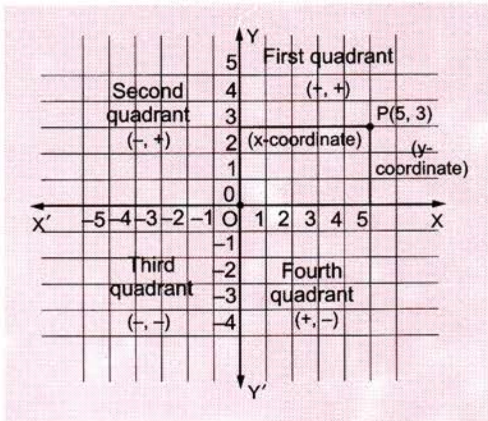
Consider a point P whose distances from the y-axis and the x-axis are 5 units and 3 units respectively.

5 is called the abscissa (denoted by x) or the x-coordinate of the point P. 3 is called the ordinate (denoted by y) or the y-coordinate of the point P.

(5, 3) are the coordinates of the point P, represented as $P(5, 3)$.

Quadrants in the coordinate plane

The x-axis and the y-axis divide the coordinate plane into four regions, called quadrants.



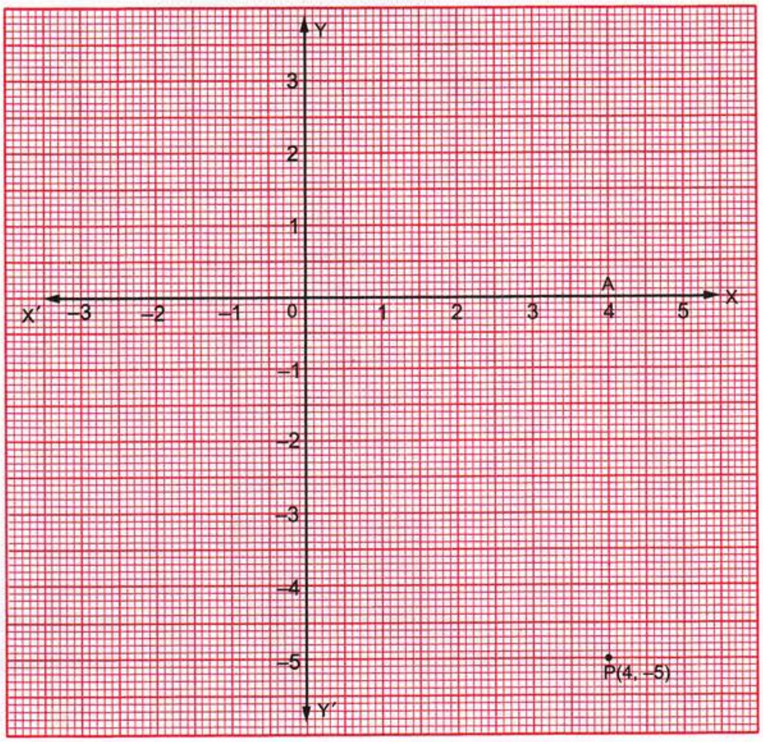
1. In the first quadrant (or XOY), both the x-coordinate and the y-coordinate of a point are positive.
2. In the second quadrant (or YOX'), the x-coordinate of a point is negative and the y-coordinate of the point is positive.
3. In the third quadrant (or $X'OY'$), both the x-coordinate and the y-coordinate of a point are negative.
4. In the fourth quadrant (or $Y'OX$), the x-coordinate of a point is positive, while the y-coordinate of the point is negative.

1st quadrant: $x > 0, y > 0$; 2nd quadrant: $x < 0, y > 0$;
3rd quadrant: $x < 0, y < 0$; 4th quadrant: $x > 0, y < 0$.

Plotting a point with given coordinates

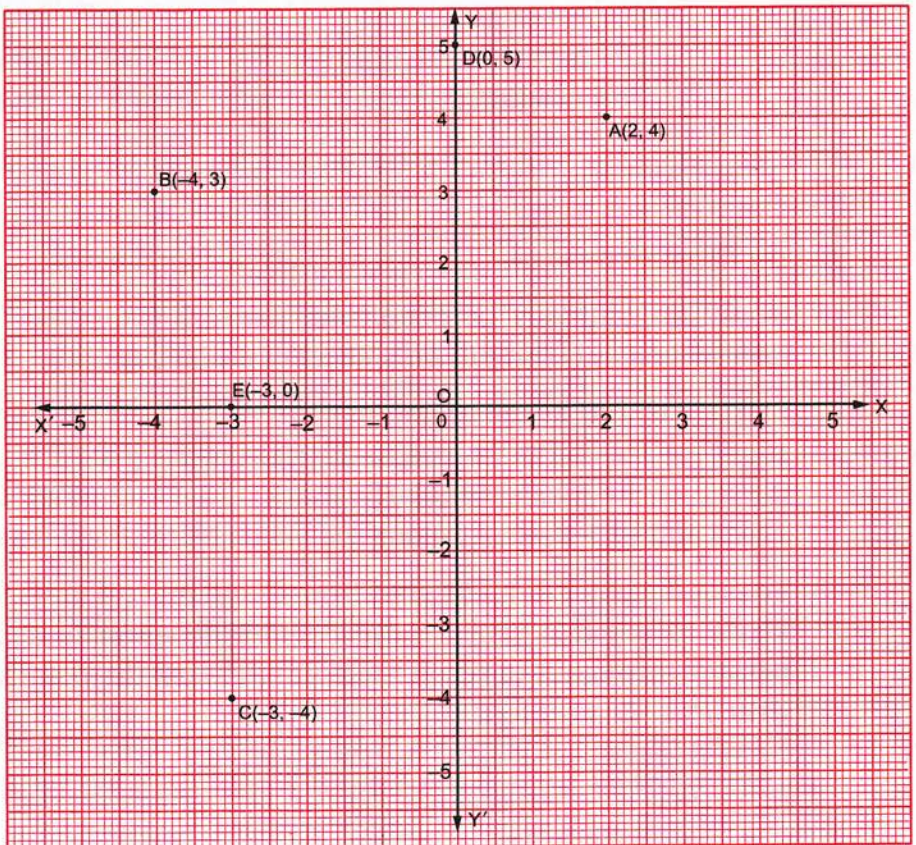
Follow these steps to plot a point with given coordinates.

1. Draw two mutually perpendicular lines, $X'OX$ (x-axis) and $Y'OY$ (y-axis) intersecting at O (origin) on a sheet of graph paper.
2. Starting with 0 at the origin, mark 1, 2, 3, ... at the corners of consecutive small squares on the x-axis to the right of the y-axis. Mark –1, –2, –3, ... on the left of the y-axis. Similarly, mark 1, 2, 3, ... along the y-axis above the x-axis and –1, –2, –3, ... along the y-axis below the x-axis.
3. To plot the point $P(4, -5)$, start from O and move 4 units to the right along the x-axis to reach A. Then move 5 units downwards along the vertical line passing through A to reach the point P. Mark the point with a dark dot and write $P(4, -5)$ next to it.



EXAMPLE **Plot the points (i) (2, 4) (ii) (−4, 3) (iii) (−3, −4) (iv) (0, 5) (v) (−3, 0).**

Solution (i) Both the x- and y-coordinates are positive, so the point lies in the first quadrant. Move 2 units to the right of O(origin) along the x-axis. Then move 4 units up the vertical line passing through the point on the x-axis marked 2. Mark the point you reach with a dark dot and name it, say A. Write (2, 4) next to A.



(ii) The x-coordinate = –4 (negative) and y-coordinate = 3 (positive). So, the point lies in the second quadrant. Mark the point of intersection of the vertical line through the point –4 on the x-axis and the horizontal line through the point 3 on the y-axis. This is the required point. Name it, say B, and write (–4, 3) next to it.

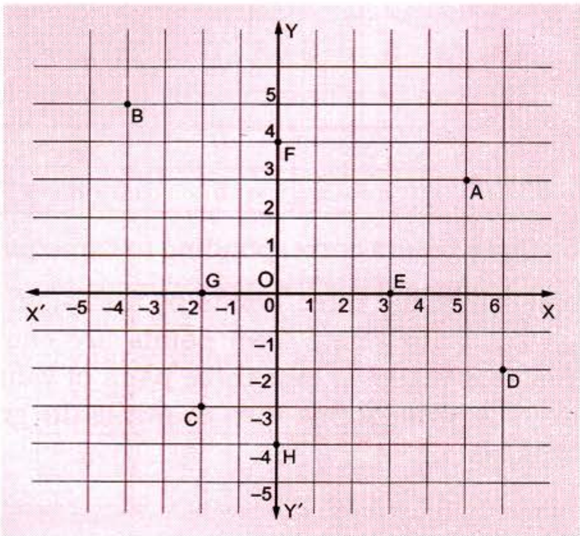
(iii) The x-coordinate = –3 (negative) and the y-coordinate = –4 (negative). So the point lies in the third quadrant. Mark the point of intersection of the vertical line through –3 on the x-axis and the horizontal line through –4 on the y-axis. Name the point, say C, and write (–3, –4) next to it.

(iv) The x-coordinate is 0, so the point lies on the y-axis. The point marked 5 (y-coordinate) on the y-axis is the required point. Name it, say D, and write (0, 5) next to it.

(v) The y-coordinate is 0, so the point lies on the x-axis. The point marked –3 (x-coordinate) on the x-axis is the required point. Name it E, and write (–3, 0) next to it.

When $x = 0$, the point lies on the y-axis, and when $y = 0$, the point lies on the x-axis.

EXAMPLE **Write the coordinates of the plotted points (i) *A*, (ii) *B*, (iii) *C*, (iv) *D*, (v) *E*, (vi) *F*, (vii) *G* and (viii) *H*.**



Solution (i) This is a point in the first quadrant. So, both the coordinates are positive. The x-coordinate = 5 and the y-coordinate = 3. So, the coordinates are (5, 3).

(ii) This is a point in the second quadrant. So. its x-coordinate is negative and its y-coordinate is positive. Its x-coordinate = –4 and its y-coordinate = 5. So, its coordinates are (–4, 5).

(iii) The is a point in the third quadrant. So, both the coordinates are negative. The x-coordinate = –2 and the y-coordinate = –3. So, the coordinates are (–2, –3).

(iv) This is a point in the fourth quadrant. So, its x-coordinate is positive and its y-coordinate is negative. Its x-coordinate = 6 and its y-coordinate = –2. So, its coordinates are (6, –2).

(v) This is a point on the positive side of the x-axis. So, its x-coordinate is positive and its y-coordinate is zero. Its coordinates are (3, 0).

(vi) This is a point on the positive side of the y-axis. So, its x-coordinate is zero and its y-coordinate is positive. Its coordinates are (0, 4).

(vii) This it is a point on the negative side of the x-axis. So, its x-coordinate is negative and its y-coordinate is zero. Its coordinates are (–2, 0).

(viii) This is a point on the negative side of the y-axis. So, its x-coordinate is zero and its y-coordinate is negative. Its coordinates are (0, –4).

Graphical Representation of Linear Equations

Take the following steps to draw a graph of a linear equation in two variables x and y.

1. Draw the x-axis $X'OX$ and the y-axis $Y'OY$. Mark numbers on the x-axis and the y-axis.
2. Take three values (preferably integers) of x. Substitute these in the equation and find the corresponding values of y. Make a table of values using the pairs of values of x and y.

Example Make a value table for $x + y = 5$, that is, $y = 5 - x$.

When $x = 0$, $y = 5 - 0 = 5$, so, $(x, y) = (0, 5)$.

When $x = 2$, $y = 5 - 2 = 3$, so, $(x, y) = (2, 3)$.

When $x = 5$, $y = 5 - 5 = 0$, so, $(x, y) = (5, 0)$.

This can be represented by the following table.

x	0	2	5
$y = 5 - x$	5	3	0

3. Plot the three points corresponding to three pairs of values of (x, y).
4. Join any two of the three points by a straight line. This line will also pass through the third point. Two points are enough to draw a straight line. However, it is better to take three pairs of values. The third point serves as a check. The straight line thus drawn is the graph of the linear equation in two variables.

> The graph of a linear equation in two variables is a straight line.

EXAMPLE **Draw the graph of $3x + 2y = 5$.**

Solution Here, $3x + 2y = 5$ or $y = \dfrac{5 - 3x}{2}$.

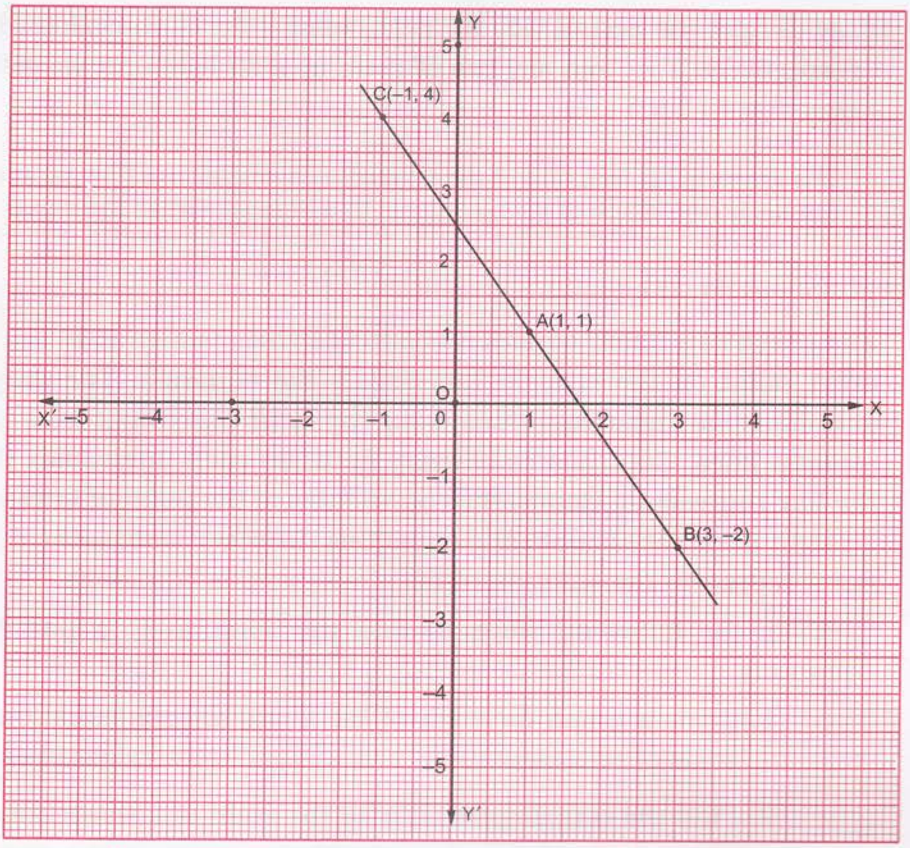
Substituting $x = 1, 3, -1$ we get $y = 1, -2, 4$ respectively.

$\therefore$ the table of values is as follows.

x	1	3	–1
$y = \dfrac{5 - 3x}{2}$	1	–2	4

Let us plot the points (1, 1), (3, –2) and (–1, 4) and name them A, B and C respectively.

A line joining any two of them passes through the third point. This straight line is the graph of the equation $3x + 2y = 5$.

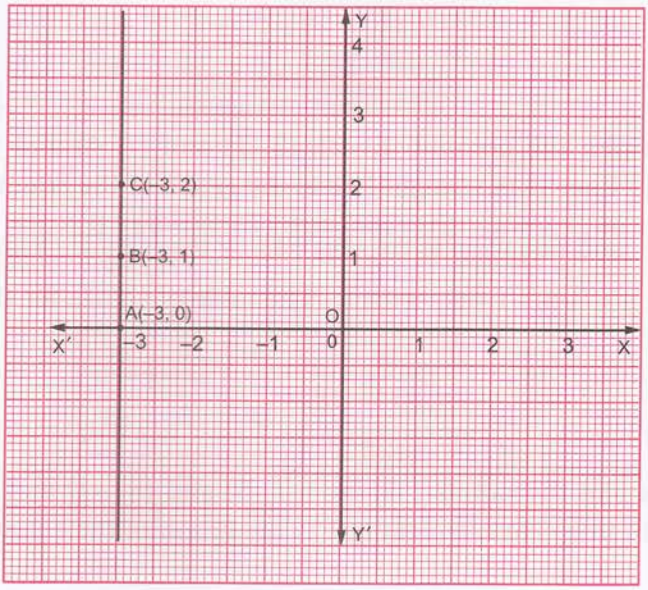


EXAMPLE **Draw the graph of $x + 3 = 0$.**

Solution Here $x + 3 = 0$ or $x = -3$.

The equation does not contain y explicitly. This means $x = -3$ for any value of y. Another way of looking at it is $x + 3 = 0$ which is the same as $x + 0 \cdot y + 3 = 0$. Hence, we can take any three values of y to construct the value table.

x	–3	–3	–3
y	0	1	2



Let us plot the points (–3, 0), (–3, 1) and (–3, 2) and name them *A*, *B* and *C* respectively. If we join any two of them by a straight line, we get the graph of the equation $x + 3 = 0$.

> The graph of an equation of the form x = a constant is a line parallel to the y-axis. Similarly, the graph of y = a constant is a line parallel to the x-axis.

Solving simultaneous linear equations graphically

The graph of a linear equation in two variables is a straight line. Coordinates of every point on the straight line satisfy the equation. The graphs of a pair of simultaneous linear equations using the same coordinate axes are obviously two straight lines. Coordinates of every point on each line satisfy the equation that the line represents. Hence, the coordinates of the point of intersection of the two straight lines satisfy both the equations. In other words, the coordinates of the point represent the solution of the simultaneous equations.

Take the following steps to find the solution to a pair of simultaneous equations.

Steps

1. Draw a graph for each of the simultaneous equations with the same scale of representation. You will get two straight lines.
2. Find the coordinates (x, y) of the point of intersection of the two lines. The values of x and y give the solution to the simultaneous linear equations.

EXAMPLE **Solve the equations $3x - 4y = 5$ and $2x + 5y = 11$ graphically.**

Solution The first equation is $3x - 4y = 5$ or $y = \frac{3x-5}{4}$.

Substituting $x = -1, 3, 7$ in it we get $y = -2, 1, 4$ respectively.

These values are shown in the following table.

x	–1	3	7
$y = \frac{3x-5}{4}$	–2	1	4

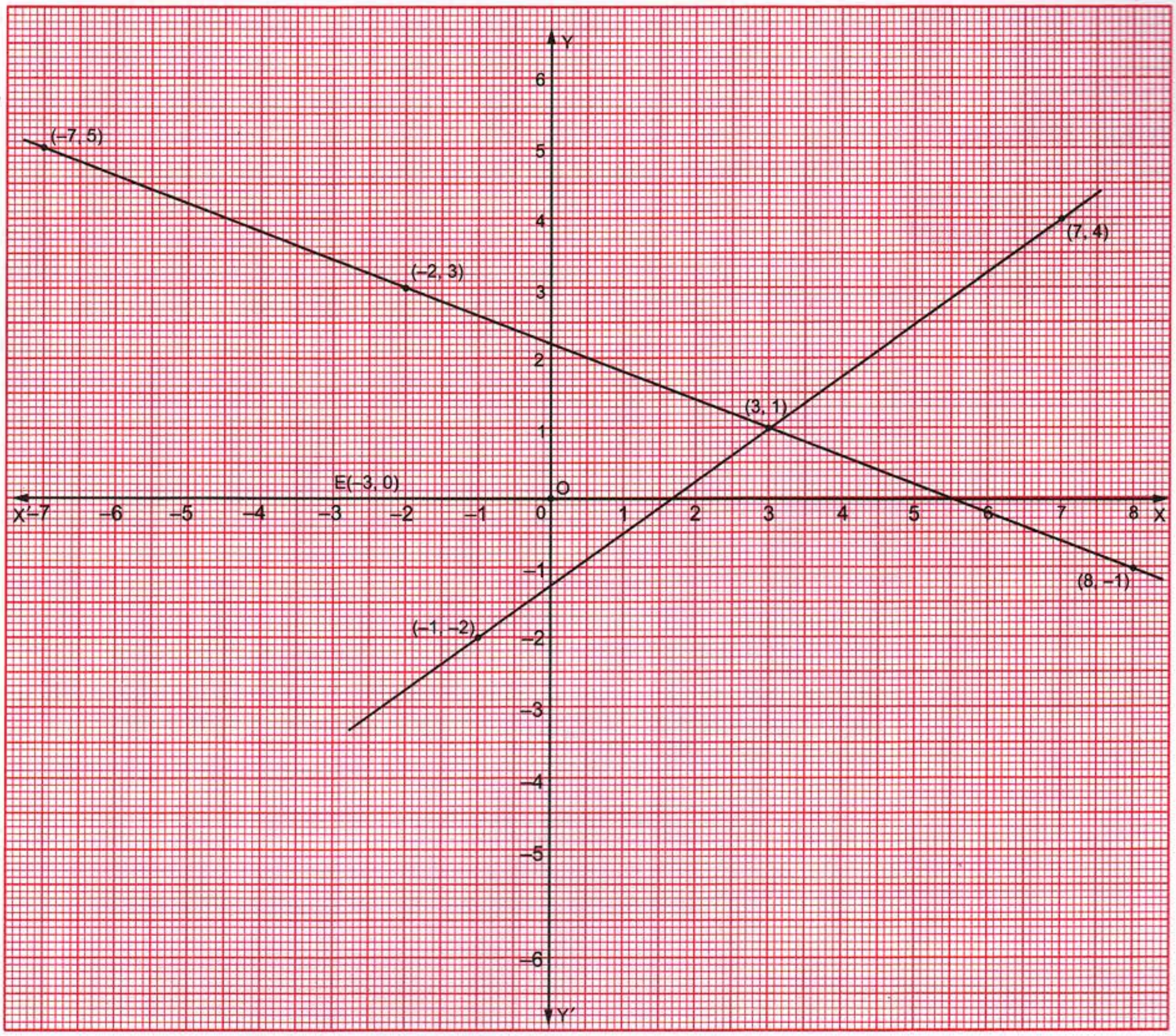
Let us plot the points (–1, –2), (3, 1) and (7, 4) and join any two of them by a straight line. This straight line is the graph of the equation $3x - 4y = 5$.

The second equation is $2x + 5y = 11$ or $y = \frac{11-2x}{5}$. Substituting $x = -2, -7, 8$ in it, we get $y = 3, 5, -1$ respectively. The table of values for this equation is as follows.

x	–2	–7	8
$y = \frac{11-2x}{5}$	3	5	–1

Let us plot the points (–2, 3), (–7, 5) and (8, –1) and join any two of them by a straight line. This straight line is the graph of the equation $2x + 5y = 11$.

The two lines intersect at the point (3, 1). Hence, $x = 3, y = 1$ is the required solution.



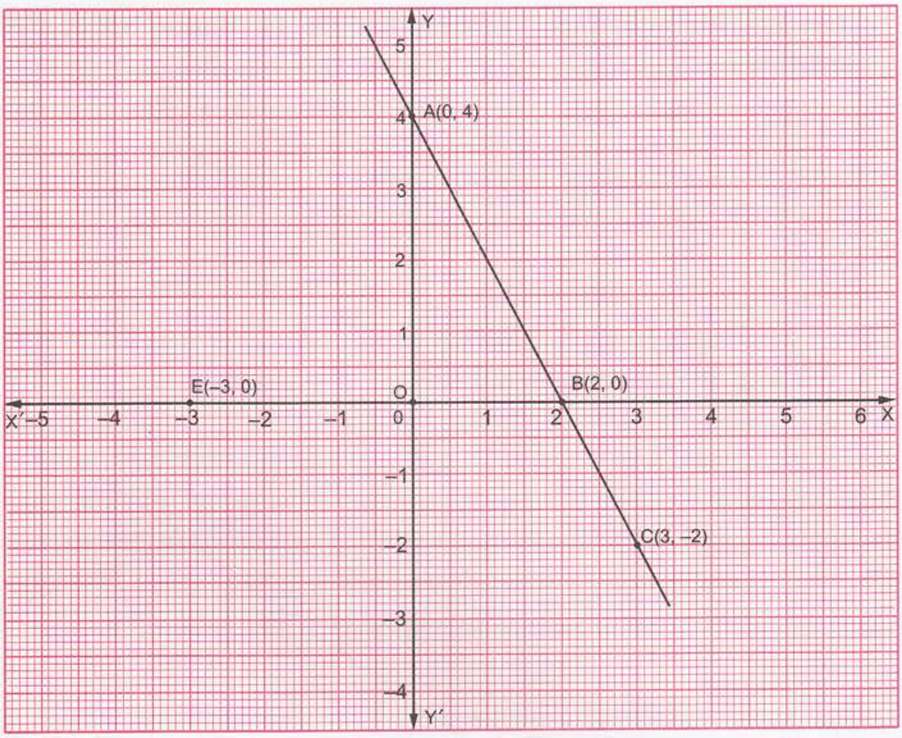
Solved Examples

EXAMPLE 1 **Draw the graph of $y = 4 - 2x$.**

Solution Substituting $x = 0, 2, 3$ in the given equation, we get $y = 4, 0, -2$ respectively. So the table of values is as follows.

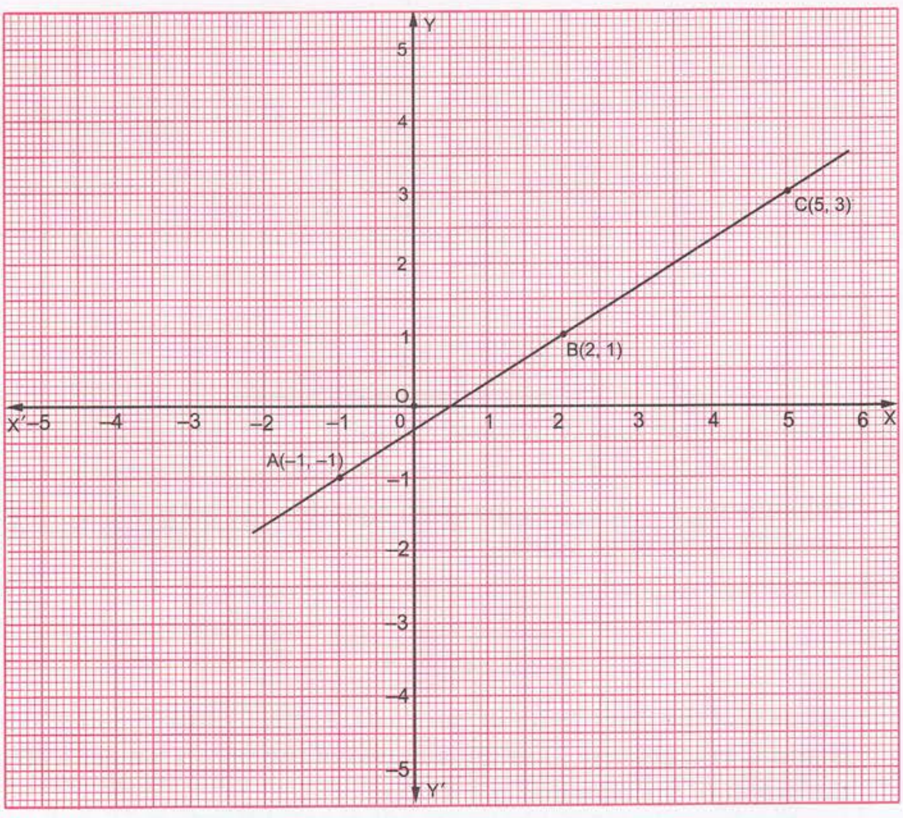
x	0	2	3
$y = 4 - 2x$	4	0	–2

Let us plot the three pairs of values (0, 4), (2, 0) and (3, –2) and name the points A, B and C respectively. Joining any two of the points we get a straight line which is the graph of the equation $y = 4 - 2x$.



EXAMPLE 2 **Draw the graph of $2x - 3y = 1$.**

Solution Given, $2x - 3y = 1$ or $3y = 2x - 1$ or $y = \dfrac{2x-1}{3}$.



Substituting $x = -1, 2, 5$ in the equation we get $y = -1, 1, 3$ respectively. So the table of values is as follows.

x	-1	2	5
$y = \frac{2x-1}{3}$	-1	1	3

Let us plot the points $(-1, -1)$, $(2, 1)$ and $(5, 3)$ and name them A, B and C respectively. Joining any two of the points by a straight line we get the graph of the equation $2x - 3y = 1$.

EXAMPLE 3 **Solve the equations $5y - 3x = 1$ and $y = 2x + 3$ graphically.**

Solution Given, $5y - 3x = 1$ or $y = \frac{3x+1}{5}$.

Substituting $x = 3, -2, -7$ in the equation we get $y = 2, -1, -4$ respectively. These values are shown in the following table.

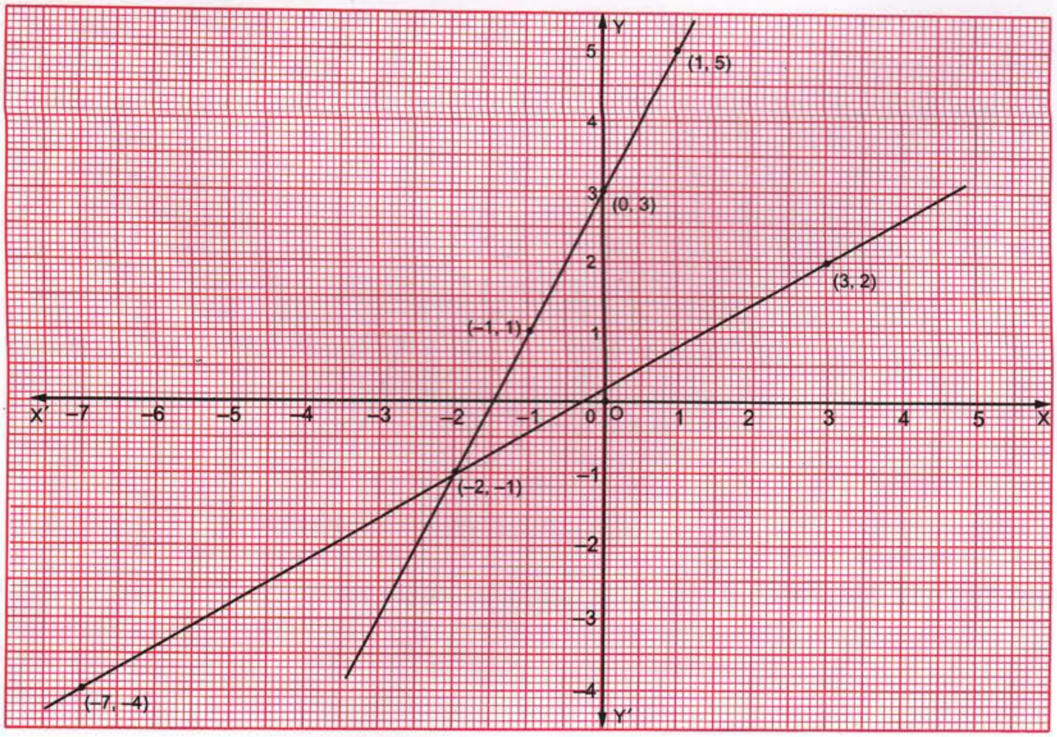
x	3	-2	-7
$y = \frac{3x+1}{5}$	2	-1	-4

Let us plot the points $(3, 2)$, $(-2, -1)$ and $(-7, -4)$ and join any two of them by a straight line. This line is the graph of the equation $5y - 3x = 1$.

Again, $y = 2x + 3$. Substituting $x = 0, 1, -1$ in the equation we get $y = 3, 5, 1$ respectively. These values are shown in the following table.

x	0	1	-1
$y = 2x + 3$	3	5	1

Let us plot the points $(0, 3)$, $(1, 5)$ and $(-1, 1)$ and join any two of them by a straight line. This straight line is the graph of the equation $y = 2x + 3$.



We find that the two lines intersect at the point (–2, –1). Hence, $x = -2$, $y = -1$ is the required solution.

EXAMPLE 4 **Solve the equations $y - x = 3$ and $2x + y = 0$ graphically.**

Solution Given, $y - x = 3$ or $y = x + 3$.

Substituting $x = 0, 1, 2$ in the equation we get $y = 3, 4, 5$ respectively. These values are shown in the following table.

x	0	1	2
$y = x + 3$	3	4	5

Let us plot the points (0, 3), (1, 4) and (2, 5) and join any two of them by a straight line. This line is the graph of the equation $y - x = 3$.

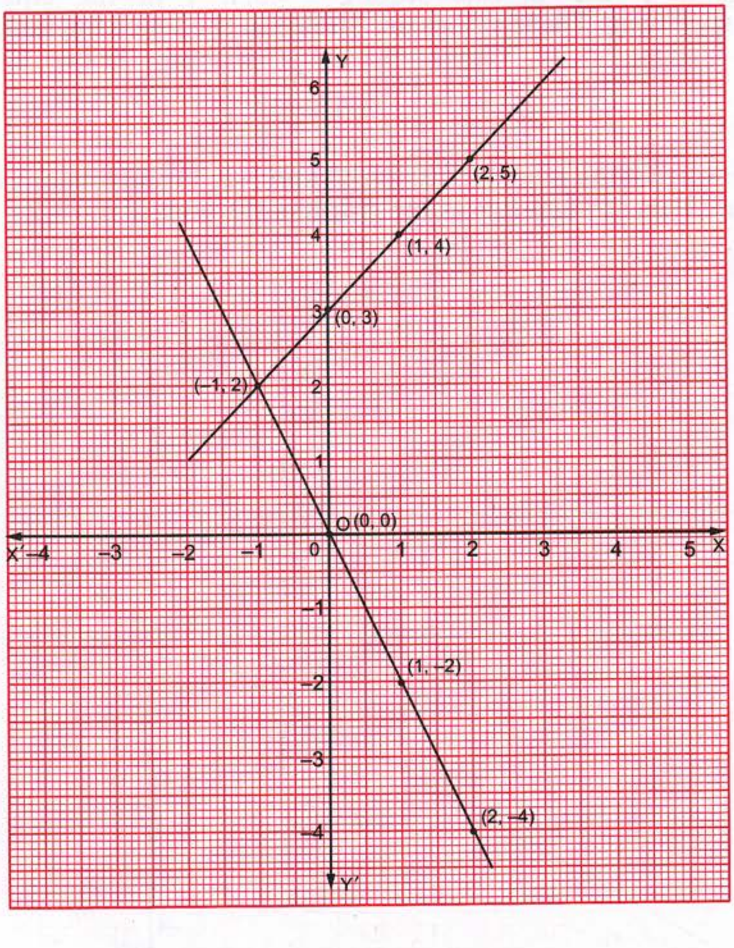
Again, $2x + y = 0$ or $y = -2x$.

Substituting $x = 0, 1, 2$ in the equation we get $y = 0, -2, -4$. These values are shown in the following table.

x	0	1	2
$y = -2x$	0	–2	–4

Let us plot the points (0, 0), (1, –2) and (2, –4) and join any two of them by a straight line. This line is the graph of the equation $2x + y = 0$.

We find that the two lines intersect at the point (–1, 2). Hence, $x = -1$, $y = 2$ is the required solution.



EXAMPLE 5 **Solve the equation $2x + 3y = 4$ and $3x + 2y = 11$ graphically.**

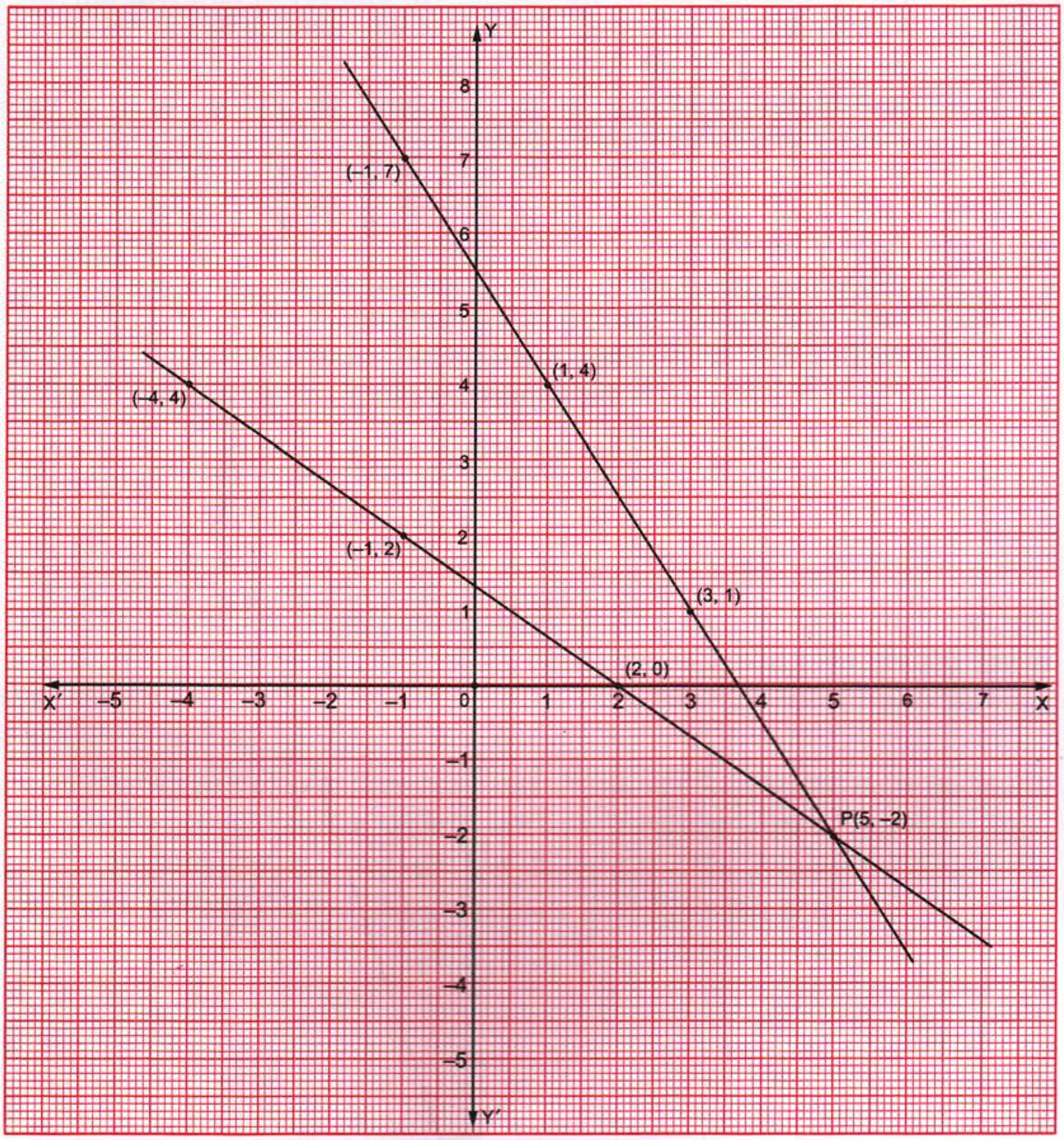
Solution Given, $2x + 3y = 4$ or $y = \frac{4-2x}{3}$.

Substituting $x = -1, -4, 2$ in the equation we get $y = 2, 4, 0$ respectively. These values are shown in the following table.

x	-1	-4	2
$y = \frac{4-2x}{3}$	2	4	0

Let us plot the points (–1, 2), (–4, 4) and (2, 0) and join any two of them by a straight line. This line is the graph of the equation $2x + 3y = 4$.

Again, $3x + 2y = 11$ or $y = \frac{11-3x}{2}$.



Substituting $x = -1, 1, 3$ in the equation we get $y = 7, 4, 1$ respectively. The following table shows these values.

x	-1	1	3
$y = \frac{11-3x}{2}$	7	4	1

Let us plot the points (–1, 7), (1, 4) and (3, 1) and join any two of them by a straight line. This line is the graph of the equation $3x + 2y = 11$.

We find that the two lines intersect at the point $P(5, -2)$.

Hence, $x = 5, y = -2$ is the required solution.

EXAMPLE 6 **Solve the equations $3x - y = 6$ and $2x + y - 4 = 0$ graphically.**

Solution Given, $3x - y = 6$ or $y = 3x - 6$.

Substituting $x = 0, 1, 3$ in the equation we get $y = -6, -3, 3$ respectively. The following table shows these values.

x	0	1	3
$y = 3x - 6$	–6	–3	3

Let us plot the points (0, –6), (1, –3) and (3, 3) and join any two of them by a straight line. This line is the graph of the equation $3x - y = 6$.

Again, $2x + y - 4 = 0$ or $y = 4 - 2x$.

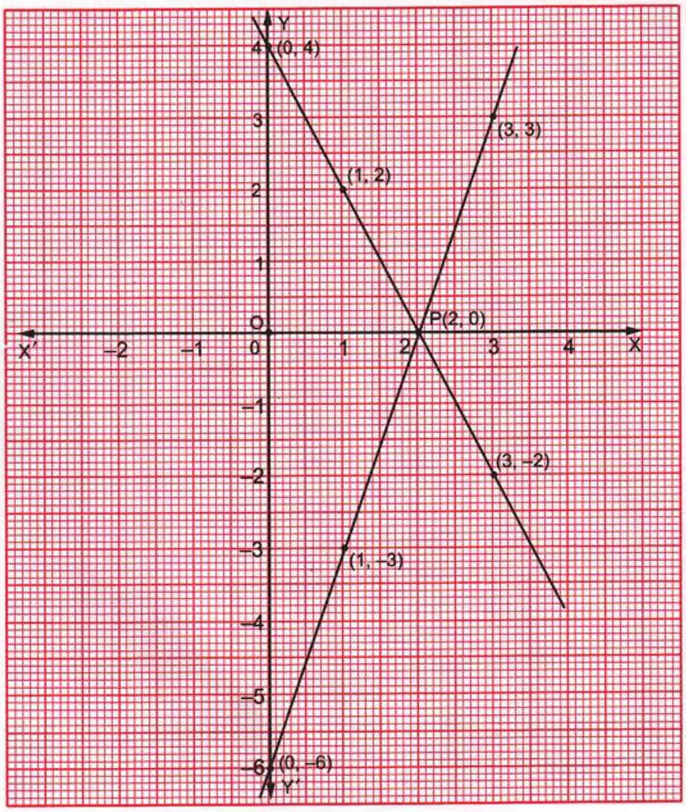
Substituting $x = 0, 1, 3$ in the equation we get $y = 4, 2, -2$ respectively. The following table shows these values.

x	0	1	3
$y = 4 - 2x$	4	2	–2

Let us plot the points (0, 4), (1, 2) and (3, –2) and join any two of them by a straight line. This line is the graph of the equation $2x + y - 4 = 0$.

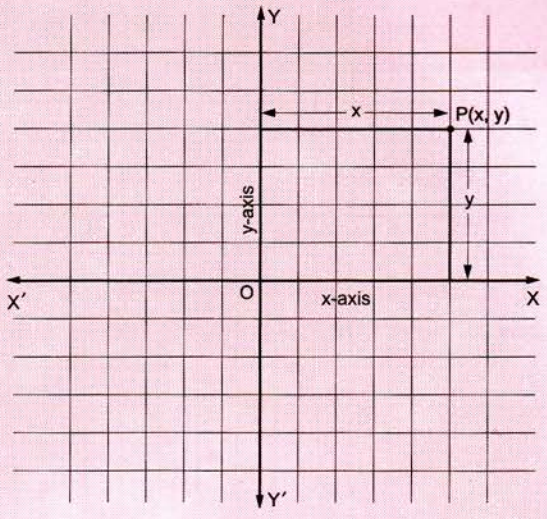
We find that the two lines intersect at the point $P(2, 0)$.

Hence, $x = 2, y = 0$ is the required solution.

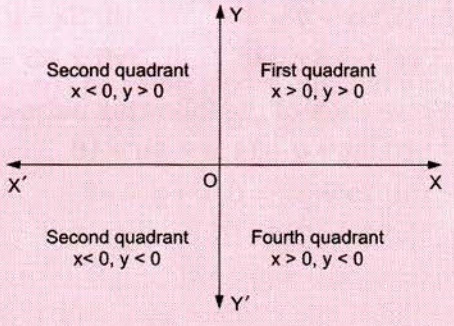


Remember These

1. A point P has the coordinates (x, y) if its x-coordinate (or abscissa) $= x$ and its y-coordinate (or ordinate) $= y$. The distane of P from the y-axis $= x$ units, the distance of P from the x-axis $= y$ units.



2. A point $P(x, y)$ lies in the
   (i) first quadrant if $x > 0, y > 0$,
   (ii) second quadrant if $x < 0, y > 0$,
   (iii) third quadrant if $x < 0, y < 0$,
   (iv) fourth quadrant if $x > 0, y < 0$.



3. A point lies on the y-axis if its x-coordinate is 0, and on the x-axis if its y-coordinate is 0.
4. The graph of a linear equation in two variables is a straight line. To draw such a graph, plot three points with coordinates (x, y), where the values of x and y satisfy the linear equation and then join any two of them by a straight line. The line should pass through the third point.
5. To find the solution of a pair of simultaneous linear equations graphically, draw the graph of each of the linear equations in the same figure with the same scale of representation. The point of intersection of the two straight lines gives the required solution. Its x-coordinate gives the value of x and its y-coordinate gives the value of y.

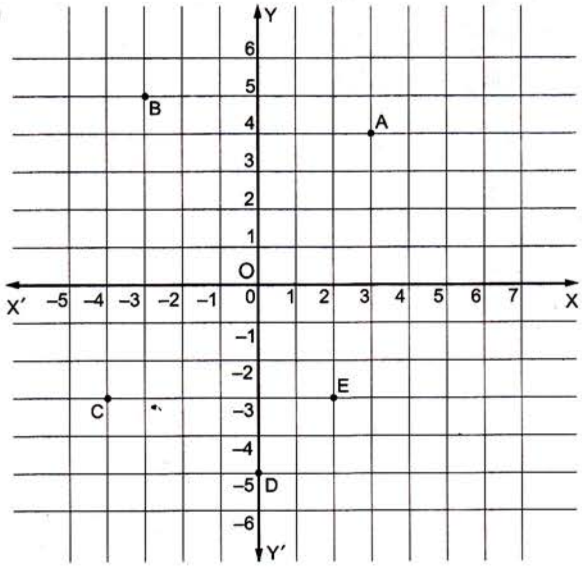
EXERCISE 10

1. Plot the following points on a graph paper.

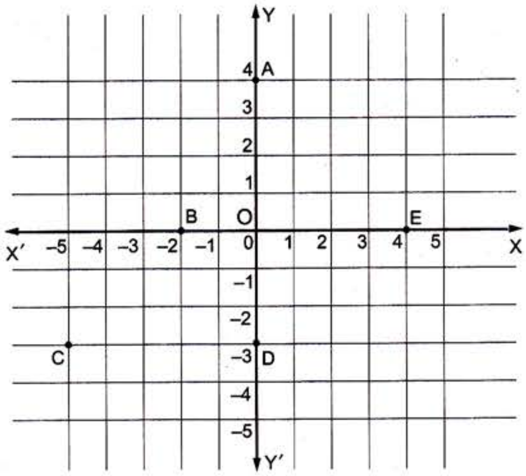
(i) (6, 3) (ii) (3, –5) (iii) (–4, 2) (iv) (–1, –3)
(v) (0, 3) (vi) (5, 0) (vii) (0, –4) (viii) (–2, 0)

2. Write the coordinates of the points *A*, *B*, *C*, *D* and *E*.

(i)



(ii)



3. Draw the graphs of the following equations.

(i) $3x + y = 4$ (ii) $3x - y + 5 = 0$ (iii) $x = 2y + 6$ (iv) $y = 2x$

(v) $y + 4x = 0$ (vi) $x + 5 = 0$ (vii) $y = 2$

4. Solve each of the following pairs of equations graphically.

(i) $2x + y = 11, x + 4y = 16$ (ii) $5x + 6y = 1, 3x + 2y + 1 = 0$

(iii) $2x - 3y = 6, x + y = -7$ (iv) $y = 7 - 2x, 2x - y = 13$

(v) $3x + 2y = 10, 5x + y = 5$ (vi) $3x - y = 3, y = 7(x - 1)$

ANSWERS

2. (i) $A(3, 4)$, $B(-3, 5)$, $C(-4, -3)$, $D(0, -5)$, $E(2, -3)$ (ii) $A(0, 4)$, $B(-2, 0)$, $C(-5, -3)$, $D(0, -3)$, $E(4, 0)$

4. (i) $x = 4, y = 3$ (ii) $x = -1, y = 1$ (iii) $x = -3, y = -4$ (iv) $x = 5, y = -3$ (v) $x = 0, y = 5$ (vi) $x = 1, y = 0$

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