8

Linear Equations and Inequations

Equations

An algebraic equation represents the equality of two mathematical expressions involving at least one variable (literal).

Examples (i) $7-3x=4$ (ii) $2x+5=9$ (iii) $x^2+6x=7$ (iv) $y=0$

Linear equations

A linear equation in one variable, say x, is an equation in which the exponent of x is 1. The general form of a linear equation is $ax+b=0$.

Examples (i) $2x+3=6$ (ii) $3z-7=\frac{z}{2}+1$

Solution of an equation

The solution or root of an equation is a number, which when substituted for the variable in the equation makes the left-hand side (LHS) of the equation equal to the right-hand side (RHS).

Examples (i) The solution of the equation $x-2=6$ is 8 because $8-2=6$.

(ii) 5 is not the solution of the equation $2x+3=3x-1$ because $2\times5+3\neq3\times5-1$.

Laws of equality

1. If the same number or quantity is added to or subtracted from both sides of an equation, the two sides remain equal. We can express this symbolically as follows.

If $x=y$ then $x+k=y+k$ and $x-c=y-c$.

2. If both sides of an equation are multiplied or divided by the same nonzero quantity, the sides remain equal.

If $x=y$ then $ax=ay$ where $a\neq0$

and $\frac{x}{c}=\frac{y}{c}$ where $c\neq0$.

Transposition

Any term on one side of an equation can be shifted to the other side by changing the sign of the term. This process is called transposition.

Example If $a+b=c-d$ then $a+b-c=-d$ or $a+b-c+d=0$.

Note An equation remains unchanged if all the terms on the LHS are shifted to the RHS and all the terms on RHS are shifted to LHS.

Solving linear equations

To solve a linear equation, proceed step by step.

Steps

1. Simplify both sides of the equation. Use the distributive law (if necessary) to separate terms containing the variable and the constant terms.
2. If the equation involves fractions, multiply both sides by the LCM of the denominators to clear the fractions.
3. If decimals are present, multiply both sides by a suitable power of 10 to eliminate the decimals.
4. Collect all the terms containing the variable on one side of the equation (generally, LHS) and all constant terms on the other side.
5. Divide both sides of the equation by the resulting coefficient of the variable.

Verification of the solution

Substitute the value of the variable on both sides of the equation. If the values of both sides are equal, the solution of the equation is correct.

EXAMPLE **Solve $2-3(2x+3)=9-2x$ and verify the solution.**

Solution The equation is $2-(6x+9)=9-2x$.

Simplifying, $2-6x-9=9-2x$ or $-6x-7=9-2x$.

Collecting terms containing x on the LHS and constants on the RHS,

$-6x+2x=9+7$ or $-4x=16$.

Dividing both sides by the coefficient of x,

$\frac{-4x}{-4}=\frac{16}{-4}$ or $x=-4$.

Verification

Substituting $x=-4$ on the LHS, we get

$2-3(2x+3)=2-3(-8+3)=2+15=17$.

Substituting $x=-4$ on the RHS, we get

$9-2x=9-2\times(-4)=9+8=17$.

$\therefore$ LHS = RHS. So, the solution $x=-4$ is correct.

Solved Examples

EXAMPLE 1 **Solve $3(z+1)-2(4z-3)=4z$.**

Solution The equation is $3z+3-(8z-6)=4z$.

$\therefore$ $3z+3-8z+6=4z$ or $-5z+9=4z$ or $9=4z+5z$ or $4z+5z=9$ or $9z=9$.

$\therefore$ $z=\frac{9}{9}=1$.

EXAMPLE 2 **Solve $3x - \frac{2(x+3)}{3} = 16 - \frac{x+2}{2}$.**

Solution Multiplying both sides by the LCM of 3 and 2, that is, 6,

$$6 \times \left[3x - \frac{2(x+3)}{3}\right] = 6 \times \left[16 - \frac{x+2}{2}\right].$$

$\therefore \quad 6 \times 3x - 6 \times \frac{2}{3}(x+3) = 6 \times 16 - 6 \times \frac{x+2}{2}$

or $18x - 4(x+3) = 96 - 3(x+2)$ or $18x - 4x - 12 = 96 - 3x - 6$

or $14x - 12 = 90 - 3x$ or $14x + 3x = 90 + 12$ or $17x = 102$.

$\therefore \quad x = \frac{102}{17} = 6.$

EXAMPLE 3 **Solve $\frac{1}{4} + \frac{9}{x} = 1 \left(\text{or } 0.25 + \frac{9}{x} = 1\right)$.**

Solution The given equation is $\frac{1}{4} + \frac{9}{x} = 1.$

$\therefore \quad \frac{9}{x} = 1 - \frac{1}{4}$ or $\frac{9}{x} = \frac{3}{4}$.

By cross multiplication, $9 \times 4 = 3 \times x$ or $3 \times x = 9 \times 4$.

$\therefore \quad x = \frac{9 \times 4}{3} = 12.$

EXAMPLE 4 **Solve $\frac{4}{x-2} = \frac{9}{x+8}$.**

Solution The given equation is $\frac{4}{x-2} = \frac{9}{x+8}$.

By cross multiplication,

$4 \times (x+8) = 9 \times (x-2)$ or $4x + 32 = 9x - 18$

or $32 + 18 = 9x - 4x$ or $5x = 50$.

$\therefore \quad x = \frac{50}{5} = 10.$

EXAMPLE 5 **Solve $\frac{3x+7}{5x+16} = \frac{3x-2}{5x-2}$.**

Solution The given equation is $\frac{3x+7}{5x+16} = \frac{3x-2}{5x-2}$.

By cross multiplication, $(3x+7)(5x-2) = (3x-2)(5x+16)$

or $15x^2 + 35x - 6x - 14 = 15x^2 - 10x + 48x - 32$

or $15x^2 + 29x - 14 = 15x^2 + 38x - 32$

or $15x^2 + 38x - 15x^2 - 29x = 32 - 14$.

$\therefore \quad 9x = 18$ or $x = \frac{18}{9} = 2.$

EXAMPLE 6 **Solve $\frac{1}{x+3} - \frac{x}{x^2-9} = \frac{2}{3-x}$.**

Solution The given equation is

$\frac{1}{x+3} - \frac{x}{(x+3)(x-3)} = \frac{-2}{x-3}$ $\quad [\because \; x^2 - 9 = (x+3)(x-3) \text{ and } 3 - x = -(x-3)]$

Multiplying both sides by the LCM of the denominators or $(x+3)(x-3)$,

$$(x+3)(x-3)\left[\frac{1}{x+3}-\frac{x}{(x+3)(x-3)}\right]=\frac{-2(x+3)(x-3)}{x-3}$$

or $x-3-x=-2(x+3)$ or $-3=-2(x+3)$ or $3=2x+6$ or $2x=3-6=-3$.

Thus, $x=-\frac{3}{2}$.

EXAMPLE 7 **Solve** $\frac{x+6}{x+5}-\frac{2x-1}{x-4}+\frac{x+4}{x-2}=0.$

Solution Multiplying both sides by the LCM of the denominators, i.e., $(x+5)(x-4)(x-2)$,

$$(x+6)(x-4)(x-2)-(2x-1)(x+5)(x-2)+(x+4)(x+5)(x-4)=0$$

or $(x+6)\{x^2-(4+2)x+4\times 2\}-(2x-1)\{x^2+(5-2)x-5\times 2\}+(x+5)(x^2-16)=0$

or $(x+6)(x^2-6x+8)-(2x-1)(x^2+3x-10)+(x+5)(x^2-16)=0$

or $x(x^2-6x+8)+6(x^2-6x+8)-2x(x^2+3x-10)+x^2+3x-10+x(x^2-16)$
$+5(x^2-16)=0$

or $x^3-6x^2+8x+6x^2-36x+48-2x^3-6x^2+20x+x^2+3x-10+x^3-16x$
$+5x^2-80=0$

or $8x-36x+48+20x+3x-10-16x-80=0$

or $-21x-42=0$ or $21x=-42$.

$\therefore$ $x=\frac{-42}{21}=-2.$

Remember These

1. A linear equation in one variable x is of the form $ax+b=c$.
2. Remember the following while solving equations.
   (i) If $a=b$ then (a) $a+k=b+k$, (b) $a-k=b-k$, (c) $ak=bk\ (k\neq 0)$, (d) $\frac{a}{k}=\frac{b}{k}\ (k\neq 0)$.
   (ii) If $x+y=z$ then $x=z-y$.
   (iii) If $x-y=z$ then $x=z+y$.

EXERCISE 8A

1. Solve: (i) $3x-5=x+3$ (ii) $7y-1.5=8.5-3y$
2. Solve the following equations.
   (i) $2(x-3)=14-3x$ (ii) $5x+7(x-2)=3-4(x+6)$ (iii) $7(2x+5)-6(x+8)=7$
3. Solve: (i) $\frac{x-3}{2}+4=\frac{3x+7}{4}$ (ii) $\frac{1}{2}(x+3)-\frac{2}{5}(x-3)=x+9$ (iii) $\frac{x+7}{4}=\frac{x-3}{5}+2$
4. Solve: (i) $\frac{3}{1-x}=\frac{4}{x}$ (ii) $5=\frac{7}{2x+1}$ (iii) $\frac{6}{x+7}=\frac{2}{x-3}$ (iv) $\frac{2}{x+2}=\frac{3}{4x-2}$

5. Solve: (i) $\frac{x+4}{2x+1}=\frac{x+2}{2x-1}$ (ii) $\frac{4x+7}{2x+6}=\frac{6x+5}{3x+5}$

6. Solve: (i) $\frac{3}{x-1}+\frac{1}{x-3}=\frac{4}{x-2}$ (ii) $\frac{3}{x+2}-\frac{1}{x+1}=\frac{2}{x+3}$ (iii) $\frac{1}{x+3}+\frac{1}{x+2}=\frac{2}{x+4}$

7. Solve: (i) $\frac{3x-7}{x-4}+\frac{2x+5}{x+2}=5$ (ii) $\frac{4x+3}{2x-1}+\frac{3x-8}{x-2}=5$

8. Solve: (i) $\frac{3}{x+1}-\frac{2}{x-1}=\frac{5}{x^2-1}$ (ii) $\frac{2}{x-3}+\frac{x}{x^2-9}=\frac{4}{x+3}$

ANSWERS

1. (i) 4 (ii) 1
2. (i) 4 (ii) $-\frac{7}{16}$ (iii) $\frac{5}{2}$
3. (i) 3 (ii) −7 (iii) −7
4. (i) $\frac{4}{7}$ (ii) $\frac{1}{5}$ (iii) 8 (iv) 2
5. (i) 3 (ii) 1
6. (i) 4 (ii) 1 (iii) $-\frac{8}{3}$
7. (i) −1 (ii) 8
8. (i) 10 (ii) 18

Problems leading to linear equations

Many word problems can be solved easily with the help of linear equations. To solve a word problem this way, take the following steps.

Steps
1. Read the problem carefully to analyse the facts given.
2. Denote the unknown quantity by x or by any other variable.
3. Express all other quantities mentioned in the problem in terms of the variable.
4. Frame an equation by using the conditions of the problems.
5. Solve the equation to find the value of the variable or the unknown. If the conditions of the given problem are satisfied by the value of the unknown, the solution is correct.

Solved Examples

EXAMPLE 1 **Four less than three times a number is 8 more than twice the number. Find the number.**

Solution Let the number $= x$.

Then four less than three times the number $= 3x-4$

and eight more than twice the number $= 2x+8$.

From the question, $3x-4=2x+8$ or $3x-2x=8+4$ or $x=12$.

$\therefore$ the required number is 12.

EXAMPLE 2 **The difference between the squares of two consecutive numbers is 101. Find the numbers.**

Solution Let the two consecutive numbers be x and $x+1$.

Then, $(x+1)^2 - x^2 = 101$ or $x^2 + 2x + 1 - x^2 = 101$

or $2x + 1 = 101$ or $2x = 101 - 1 = 100$

or $x = \frac{100}{2} = 50$. $\therefore$ $x + 1 = 50 + 1 = 51$.

Hence, the numbers are 50 and 51.

EXAMPLE 3 **Two numbers add up to 70. One-third of the larger number is 10 more than one-seventh of the smaller number. Find the numbers.**

Solution Let the larger number be x. Then the smaller number $= 70 - x$.

From the question, $\frac{1}{3}x = \frac{1}{7}(70 - x) + 10$ or $\frac{1}{3}x = 10 - \frac{1}{7}x + 10$.

or $\frac{1}{3}x + \frac{1}{7}x = 20$ or $\frac{7x + 3x}{21} = 20$ or $\frac{10x}{21} = 20$.

$\therefore$ $x = 20 \div \frac{10}{21} = 20 \times \frac{21}{10} = 2 \times 21 = 42$.

$\therefore$ the larger number $= 42$.

So, the smaller number $= 70 - x = 70 - 42 = 28$.

EXAMPLE 4 **If the same number be added to the numbers 5, 11, 15 and 31 then the resultant numbers are in proportion. Find the number.**

Solution Let the number be x.

Then $5 + x, 11 + x, 15 + x$ and $31 + x$ are in proportion.

That is $\frac{5 + x}{11 + x} = \frac{15 + x}{31 + x}$ or $(5 + x)(31 + x) = (11 + x)(15 + x)$

or $155 + 36x + x^2 = 165 + 26x + x^2$ or $36x - 26x = 165 - 155$

or $10x = 10$ or $x = \frac{10}{10} = 1$.

Hence, the required number $= 1$.

EXAMPLE 5 **If a number is subtracted from the numerator of the fraction $\frac{3}{5}$ and thrice that number is added to the denominator, the fraction becomes $\frac{1}{4}$. Find the number.**

Solution Let the number be x.

From the question, $\frac{3 - x}{5 + 3x} = \frac{1}{4}$ or $4 \times (3 - x) = 5 + 3x$

or $12 - 4x = 5 + 3x$ or $4x + 3x = 12 - 5$.

$\therefore$ $7x = 7$ or $x = \frac{7}{7} = 1$.

$\therefore$ the required number $= 1$.

EXAMPLE 6 **The sum of the digits of a two-digit number is 7. If 2 is subtracted from the number formed by interchanging the digits, the result is double the original number. Find the original number.**

Solution Let the digit in the units place be x.

Then the digit in the tens place $= 7 - x$ $\{\because$ sum of digits $= 7\}$.

$\therefore$ the number $= 10(7 - x) + x$.

Then the number formed by interchanging the digits $= 10x + (7 - x)$.

From the question, $10x + (7 - x) - 2 = 2[10(7 - x) + x]$

or $9x + 5 = 140 - 18x$ or $18x + 9x = 140 - 5$ or $27x = 135$ or $x = \frac{135}{27} = 5$.

So, the digit in the units place $= 5$.

$\therefore$ the digit in the tens place $= 7 - x = 2$.

Hence, the original number $= 25$.

EXAMPLE 7 **In a two-digit number, the digit in the tens place exceeds the digit in the units place by 5. If 3 more than six times the sum of the digits is subtracted from the number, the digits are reversed. Find the original number.**

Solution Let the digit in the units place be x.

Then the digit in the tens place $= x + 5$.

So, the number $= 10(x + 5) + x$ and the sum of the digits $= x + 5 + x = 2x + 5$.

After reversing the digits, the new number $= 10x + (x + 5)$.

From the question, $[10(x + 5) + x] - [3 + 6(2x + 5)] = 10x + (x + 5)$

or $11x + 50 - (12x + 33) = 11x + 5$ or $12x = 50 - 33 - 5 = 12$ or $x = \frac{12}{12} = 1$.

So, the digit at the units place $= 1$ and the digit at the tens place $= x + 5 = 1 + 5 = 6$.

$\therefore$ the original number $= 61$.

EXAMPLE 8 **In 7 years from now, Ram will be twice as old as he was 8 years ago. How old is he now?**

Solution Let Ram's present age be x years. Then in 7 years he will be $(x + 7)$ years old and 8 years ago he was $(x - 8)$ years old.

From the question, $x + 7 = 2(x - 8)$ or $x + 7 = 2x - 16$

or $2x - x = 16 + 7$ or $x = 23$.

$\therefore$ Ram's present age $= 23$ years.

EXAMPLE 9 **The present age of a man is twice that of his son. Eight years hence their ages will be in the ratio 7 : 4. Find the son's present age.**

Solution Let the present age of the son be x years.

Then, the present age of the father $= 2x$ years.

After 8 years, age of the son $= (x + 8)$ years and age of the father $= (2x + 8)$ years.

From the question, $\frac{2x + 8}{x + 8} = \frac{7}{4}$

or $4(2x + 8) = 7(x + 8)$ or $8x + 32 = 7x + 56$

or $8x - 7x = 56 - 32$ or $x = 24$.

Hence, the present age of the son $= 24$ years.

EXAMPLE 10 **Raj has some coins. In these the number of one-rupee coins is four times the number of five-rupee coins and twice the number of two-rupee coins. If the value of all the coins is Rs 910, find the number of five-rupee coins.**

Solution Let the number of 5-rupee coins be x.

Then the number of 1-rupee coins = $4x$ and the number of 2-rupee coins = $\frac{4x}{2} = 2x$.

The total value of the coins = Rs $(5 \times x + 1 \times 4x + 2 \times 2x)$ = Rs $13x$.

From the question, Rs $13x$ = Rs 910 or $13x = 910$ or $x = \frac{910}{13} = 70$.

$\therefore$ the number of 5-rupee coins = 70.

EXAMPLE 11 **A car covers the distance between two cities in 6 hours. A van covers the same distance in 5.5 hours by travelling 4 km/h faster than the car. What is the distance between the two cities? Find the speeds of the car and the van.**

Solution Let the distance between the cities be x km.

Then the speed of the car = $\frac{\text{distance}}{\text{time}} = \frac{x}{6}$ km/h.

Similarly, the speed of the van = $\frac{x}{5.5}$ km/h = $\frac{10x}{55}$ km/h = $\frac{2x}{11}$ km/h.

From the question, $\frac{2x}{11} = \frac{x}{6} + 4$ or $\frac{2x}{11} - \frac{x}{6} = 4$ or $\frac{12x - 11x}{66} = 4$

or $\frac{x}{66} = 4$ or $x = 264$. $\therefore$ the distance between two cities = 264 km.

$\therefore$ $\frac{x}{6} = \frac{264}{6} = 44$, $\frac{2x}{11} = \frac{2}{11} \times 264 = 2 \times 24 = 48$.

$\therefore$ the speed of the car = $\frac{x}{6}$ km/h = 44 km/h and that of the van = $\frac{2x}{11}$ km/h = 48 km/h.

EXAMPLE 12 **The length of a rectangular field is 11 m more than its width. If the length is decreased by 12 m and the width is increased by 10 m, the area decreases by 24 m^2. Find the length and the width of the field.**

Solution Let the width of the field be x m.

Then the length of the field = $(x + 11)$ m

and the area of the field = $x(x + 11)$ m^2.

The length of the new field = $\{(x + 11) - 12\}$ m = $(x - 1)$ m

and the width of the new field = $(x + 10)$ m.

$\therefore$ the area of the new field = $(x - 1)(x + 10)$ m^2.

From question, $x(x + 11) - (x - 1)(x + 10) = 24$ or $x^2 + 11x - (x^2 + 9x - 10) = 24$

or $2x + 10 = 24$ or $2x = 24 - 10 = 14$ or $x = \frac{14}{2} = 7$. $\therefore$ $x + 11 = 7 + 11 = 18$.

$\therefore$ the width of the field = 7 m and its length = 18 m.

EXERCISE 8B

1. Six more than four times a number is four less than five times the number. Find the number.

2. (i) The difference between the squares of two consecutive numbers is 141. Find the numbers.

   (ii) The difference between the squares of two consecutive even numbers is 36. Find the numbers.

(iii) The difference between the squares of two consecutive odd numbers is 56. Find the numbers.

3. The sum of two numbers is 90. One third of the larger number is 9 more than twice the smaller one. What are the numbers?

4. (i) If the same number be added to the numbers 2, 4, 14 and 22 then the resultant numbers are in proportion. Find the number.

(ii) Find a number such that if 25, 7 and 1 are added to it, the product of the first and third results is equal to the square of the second.

5. (i) What must be added to the numerator and the denominator of the fraction $\frac{5}{12}$ to make it equal to $\frac{2}{3}$?

(ii) The denominator of a fraction is greater than the numerator by 3. If 7 is added to the numerator, the fraction increases by unity. Find the fraction.

(iii) If the same integer is added to the numerator of the fraction $\frac{2}{7}$ and subtracted from the denominator, the fraction becomes 2. What is the integer?

6. In a two-digit number, the digit in the tens place is four times the digit in the units place. If the sum of the digits is 10, find the number.

7. In a two-digit number, the digit in the tens place is three times the digit in the units place. If the digits are reversed, the number decreases by 36. Find the number.

8. The sum of the digits of a two-digit number is 6. If 18 is subtracted from the number, the digits of the number are interchanged. Find the number.

9. (i) In five years from now, Shyam will be thrice as old as he was 9 years ago. Find his present age.

(ii) 4 years ago, Lal was twice as old as he was 9 years ago. Find his present age.

10. A man's age is seven times that of his son. In 5 years, he will be four times as old as his son. What are the present ages of the man and his son?

11. (i) A man's age is 42 years and his son's age is 12 years. After how many years will the man be thrice as old as his son?

(ii) A man's age is 32 years and his son's age is 5 years. In how many years will the man's age be four times that of his son?

12. (i) A purse contains some coins. Of these, the number of one-rupee coins is three times the number of five-rupee coins and twice the number of two-rupee coins. If the total number of coins is 66, what is the value of the five-rupee coins?

(ii) Anuj has an equal number of two-rupee and five-rupee coins. He has also some one-rupee coins. If the total number of coins is 40 and their value is Rs 120, how many five-rupee and two-rupee coins does he have?

13. A plane takes 2 hours and 45 minutes to cover the distance between two airports. If it travels 20 km/h slower then it takes 3 hours. Find the speed of the plane. Also, find distance between the two airports.

14. Two cyclists make the same trip. The first one takes 3.5 hours and the second, with a speed that is 1 km/h greater than that of the first, takes 3 hours. Find the speeds of the two cyclists. Also, find the distance covered in the trip.

15. The length of a rectangle is 1 m greater than its width. If the length is increased by 2 m and the width is decreased by 6 m, the area decreases by 50 m^2. What are the dimensions of the rectangle?

16. The length of a rectangle is 5 m greater than its width. If the perimeter of the rectangle is 50 m, find its length.

17. In a right-angled triangle, one of the acute angles is one fifth the other. Find the measure of the smallest angle.

18. The base of an isosceles triangle is 3 cm longer than each of the other sides. If the perimeter is 54 cm, find the lengths of the sides.

ANSWERS

1. 10 **2.** (i) 70, 71 (ii) 8, 10 (iii) 13, 15 **3.** 81, 9
4. (i) 2 (ii) 2 **5.** (i) 9 (ii) $\frac{4}{7}$ (iii) 4 **6.** 82 **7.** 62
8. 42 **9.** (i) 16 years (ii) 14 years
10. Man's age = 35 years, son's age = 5 years **11.** (i) 3 (ii) 4 **12.** (i) Rs 60 (ii) 16 each
13. 240 km/h, 660 km **14.** 6 km/h and 7 km/h, 21 km **15.** length = 9 m, width = 8 m
16. 15 m **17.** 15° **18.** 17 cm, 17 cm, 20 cm

Inequations

An inequality is a mathematical statement showing that two expressions are not equal, for example, $9 > 4$ and $6 < 7$. The symbols of inequality are represented in the following table.

Table 8.1 Symbols of inequality

Symbol	Meaning
$>$	is greater than
$<$	is less than
$\geq$	is greater than or equal to
$\leq$	is less than or equal to

An inequality involving at least one variable which can take on some values to make the statement true, is called an inequation.

Linear inequations

A linear inequation in one variable, say x, is of the form $ax + b > c$, where the symbol $>$ can be replaced by any of the other symbols of inequality shown in Table 8.1.

Examples (i) $5x < 7$ (ii) $12x + 6 \leq 2 - 3x$ (iii) $3 - 6x > 9$ (iv) $3m - 2 \geq 4$

Replacement set

The set from which the values of the variable are to be selected to make an inequation true, is called the replacement set or domain of the variable.

Examples (i) If $x < 2$ and $x \in W$ (the set of whole numbers) then the values of x are to be selected from the set W, which is the replacement set for the inequation.

(ii) If $x \geq 4$ and $x \in \{2, 3, 4, 5, 6, 7\}$ then the set $\{2, 3, 4, 5, 6, 7\}$ is the replacement set from which the values of x are to be selected.

Solution set

The solution set or truth set of an inequation is a set of numbers each element of which, when substituted for the variable, makes the inequality true.

Examples (i) Consider the inequation $x \geq 2$, $x \in N$.

The replacement set $= N = \{1, 2, 3, \ldots\}$.

Out of the set N, the values 2, 3, 4, ... satisfy the inequation, that is, make the inequality true.

$\therefore$ the solution set $= \{2, 3, 4, \ldots\}$.

(ii) Consider the inequation $x \leq 2$, $x \in \{-3, -2, 2, 4, 5\}$.

The replacement set $= \{-3, -2, 2, 4, 5\}$.

Only the values –3, –2 and 2 from the replacement set satisfy the inequation.

$\therefore$ the solution set $= \{-3, -2, 2\}$.

Laws of inequality

The following laws hold for inequations.

1. If the same quantity is added to or subtracted from both sides of an inequation, the sign of inequality between the two sides does not change. Symbolically, we can express this as follows.

   If $a > b$ then $a + c > b + c$, $a - c > b - c$

Note This holds for other inequalities as well.

Examples (i) If $x - 3 > 5$ then $x - 3 + 3 > 5 + 3$ or $x > 8$.

(ii) If $m + 1 < 3$ then $m + 1 - 1 < 3 - 1$ or $m < 2$.

Note In other words, the rule of transposition holds for inequations as well. Thus, we can shift a term from one side of an inequation to the other by changing its sign.

2. If both sides of an inequation are multiplied or divided by the same positive quantity, the symbol of inequality does not change. Symbolically, this can be expressed as follows. It holds for all inequalities.

   If $a < b$ then $ac < bc$, $\frac{a}{c} < \frac{b}{c}$, where c is a positive quantity.

Examples (i) If $\frac{x}{4} \geq 3$ then $\frac{x}{4} \times 4 \geq 3 \times 4$ or $x \geq 12$.

(ii) If $3y < 9$ then $\frac{3y}{3} < \frac{9}{3}$ or $y < 3$.

3. If both sides of an inequality are multiplied or divided by the same negative quantity, the symbol of inequality is reversed. Symbolically, this can be represented as follows. This holds for other inequalities as well.

If $a > b$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$ where c is a negative quantity.

Examples (i) If $\frac{-x}{3} > 5$ then $-3 \times \frac{-x}{3} < 5 \times (-3)$ or $x < -15$.

(ii) If $-2x < -4$ then $\frac{-2x}{-2} > \frac{-4}{-2}$ or $x > 2$.

Solving inequations

The steps for solving a linear inequation are similar to those for solving a linear equation.

1. If there are fractions involved, multiply both sides of the inequality by the LCM of the denominators.
2. Simplify both sides and separate the terms with the variable and constant terms.
3. Collect the terms containing the variable on the LHS and the constants on the RHS.
4. Divide both sides of the inequation by the coefficient of the variable and remember the replacement set while writing the solutions set.

EXAMPLE **Solve $\frac{3x-2}{2} < \frac{2x+1}{5}$, $x \in W$.**

Solution Multiplying both sides of the given inequation by 10 (the LCM of the denominators 2 and 5),

$5(3x-2) < 2(2x+1)$ or $15x - 10 < 4x + 2$ or $15x - 4x < 2 + 10$

or $11x < 12$ or $x < \frac{12}{11}$.

But $x \in W$ (the set of whole numbers). So, x = whole numbers less than $\frac{12}{11}$.

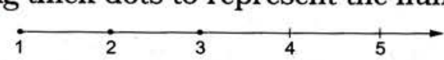
$\therefore$ the solution set = {0, 1}.

Graphical representation

The solution set of an inequation can be graphically represented on the number line.

EXAMPLE **Represent the solution set of the inequation $x \leq 3$, $x \in N$ on the number line.**

Solution The solution set for the inequation = {1, 2, 3}, which can be represented on the number line by making thick dots to represent the numbers 1, 2 and 3.



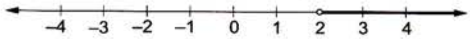
EXAMPLE **Represent the solution set of $x > 4$, $x \in W$ on the number line.**

Solution The solution set for the inequation $x > 4$, $x \in W$ is {5, 6, 7, ...}. This can be represented on the number line as follows.

The three dots (or arrow) above the number line indicate that all other subsequent integers are included in the solution set.

EXAMPLE **Represent the solution set of $x > 2, x \in R$ on the number line.**

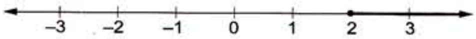
Solution The graph of the solution set $x > 2, x \in R$ can be represented by drawing a dark line over the part of the number line that represents the soluton set.



The open circle at 2 indicates that 2 is not included in the solution set.

EXAMPLE **Represent the solution set of $x \geq 2, x \in R$ on the number line.**

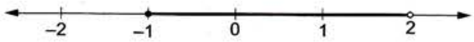
Solution The solution set of the given inequation may be represented as follows.



The closed circle at 2 indicates that 2 is included in the solution set.

EXAMPLE **Represent the solution set of $-1 \leq x < 2, x \in R$ graphically.**

Solution The solution set of the inequation $-1 \leq x < 2, x \in R$ can be graphically represented by a dark line between –1 and 2 on the number line.



The closed circle at –1 indicates that –1 is included in the solution set. The open circle at 2 indicates that 2 is not included in the solution set.

Solved Examples

EXAMPLE 1 **If the replacement set = {0, 1, 2, 3, 4, 5}, find the solution set of each of the following inequalities.**

(i) $x + 5 < 10$ (ii) $2x - 1 \geq 6$ (iii) $2x + 3 \leq 27 - 4x$

Solution (i) Given, $x + 5 < 10$ or $x + 5 - 5 < 10 - 5$ or $x < 5$.

The replacement set = {0, 1, 2, 3, 4, 5}. So, the possible values of x from the replacement set are 0, 1, 2, 3 and 4.

$\therefore$ the solution set = {0, 1, 2, 3, 4}.

(ii) Given, $2x - 1 \geq 6$ or $2x \geq 6 + 1$ or $2x \geq 7$ or $\frac{2x}{2} \geq \frac{7}{2}$ or $x \geq 3.5$.

But $x \in \{0, 1, 2, 3, 4, 5\}$. So, $x = 4, 5$.

$\therefore$ the solution set = {4, 5}.

(iii) Given, $2x + 3 \leq 27 - 4x$ or $2x + 4x \leq 27 - 3$

or $6x \leq 24$ or $\frac{6x}{6} \leq \frac{24}{6}$

or $x \leq 4$. But $x \in \{0, 1, 2, 3, 4, 5\}$.

$\therefore$ $x = 0, 1, 2, 3, 4$. So, the solution set = {0, 1, 2, 3, 4}.

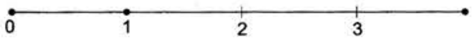
EXAMPLE 2 **Solve $5(x - 1) < 2x + 1, x \in W$ and represent the solution set on the number line.**

Solution Given, $5(x - 1) < 2x + 1$ or $5x - 5 < 2x + 1$ or $5x - 2x < 1 + 5$

or $3x < 6$ or $x < \frac{6}{3}$ or $x < 2$.

Since $x \in W$, the solution set = {0, 1}.

The solution set of the given inequation may be represented on the number line as follows.



EXAMPLE 3 **If the replacement set is the set of positive integers, find the truth set of the inequation $1 - 2(x - 8) > 5 + x$.**

Solution Given, $1 - 2(x - 8) > 5 + x$ or $1 - 2x + 16 > 5 + x$ or $17 - 2x > 5 + x$

or $17 - 5 > x + 2x$ or $12 > 3x$ or $3x < 12$ or $x < \frac{12}{3} = 4$.

Since the replacement set is the set of positive integers, the solution set = {1, 2, 3}.

EXAMPLE 4 **Find the solution set of the inequation $2(x - 3) \le 5x$, given that x is a negative integer.**

Solution Given, $2(x - 3) \le 5x$ or $2x - 6 \le 5x$ or $2x - 5x \le 6$ or $-3x \le 6$

or $\frac{-3x}{-3} \ge \frac{6}{-3}$ [$\because$ on dividing by –*ve* quantity, symbol is reversed]

or $x \ge -2$.

Since the replacement set is the set of negative integers, the solution set = {–2, –1}.

EXAMPLE 5 **Solve $\frac{2}{3}(9x - 15) + 4 \le 6 + \frac{3}{4}(4 - 12x)$, $x \in N$.**

Solution Here, $\frac{2}{3} \times 9x - \frac{2}{3} \times 15 + 4 \le 6 + \frac{3}{4} \times 4 - \frac{3}{4} \times 12x$

or $6x - 10 + 4 \le 6 + 3 - 9x$ or $6x - 6 \le 9 - 9x$ or $6x + 9x \le 9 + 6$

or $15x \le 15$ or $\frac{15x}{15} \le \frac{15}{15}$ or $x \le 1$.

$\because$ $x \in N$, the solution set = {1}.

EXAMPLE 6 **If the replacement set = {3, 4, 5, 6, 7} then find the solution set of $7 - \frac{x}{2} > \frac{5x}{3} - 6$.**

Solution Here, $7 - \frac{x}{2} > \frac{5x}{3} - 6$ or $6\left(7 - \frac{x}{2}\right) > 6\left(\frac{5x}{3} - 6\right)$

or $42 - 3x > 10x - 36$ or $42 + 36 > 10x + 3x$ or $78 > 13x$

or $13x < 78$ or $\frac{13x}{13} < \frac{78}{13}$ or $x < 6$.

The replacement set = {3, 4, 5, 6, 7}, so the solution set = {3, 4, 5}.

EXAMPLE 7 **Solve the inequality $-3 < 5 - 2x \le 1$ and represent the solution graphically.**

Solution Given, $-3 < 5 - 2x \le 1$ or $-3 - 5 < 5 - 2x - 5 \le 1 - 5$

or $-8 < -2x \le -4$ or $\frac{-8}{-2} > \frac{-2x}{-2} \ge \frac{-4}{-2}$ or $4 > x \ge 2$.

$\therefore$ the solution set is the set of real numbers less than 4 and greater than or equal to 2, which may be represented as shown.

The closed circle at $x = 2$ indicates that 2 is included in the solution set.
The open circle at 4 indicates that 4 is not included in the solution set.

Remember These

1. A linear inequation in x is of the form $ax > b$, $ax < b$, $ax \geq b$, $ax \leq b$.
2. The rules for solving linear inequations are similar to those for solving linear equations. But there are important exceptions. When multiplying or dividing by a negative number, the direction of the inequality changes.
3. The solution set of an inequation consists of those values of the variable from the solution of the inequation which belong to the replacement set.
4. The solution set of an inequation may be represented by dark dots or a dark line on the number line, as the case may be. An open circle at a particular number indicates that the number is not included in the solution set.

EXERCISE 8C

1. If the replacement set is {0, 1, 2, 3, 4, 5, 6, 7}, find the solution set for each inequation.

(i) $x < 2$ (ii) $x > 0$ (iii) $x \leq 4$ (iv) $x + 4 \geq 8$

(v) $4 - x \leq -3$ (vi) $12 - x > 16$ (vii) $3x < 9$ (viii) $\frac{x}{2} \geq 3$

(ix) $-2x \leq -8$ (x) $-2x + \frac{7}{2} < \frac{1}{2}$ (xi) $11 - 3x \geq 9$ (xii) $\frac{15}{2} < 2x + \frac{5}{2}$

(xiii) $4(x - 2) < 3x + 1$ (xiv) $n + 2 > \frac{3n - 1}{2}$

2. Represent the solution set graphically.

(i) $x - 2 < 3, x \in W$ (ii) $-3 + x \geq \frac{1}{2}, x \in N$ (iii) $2x < 6, x \in W$

(iv) $4p > -12, p \in I$ (v) $-2m \leq -8, m \in N$ (vi) $\frac{5}{2} - 2x \geq \frac{1}{2}, x \in W$

3. Solve the following inequations.

(i) $\frac{1}{2}x + 2 \geq x - 1, x \in N$ (ii) $4(x - 3) < x + 3, x \in W$

(iii) $3(2x - 1) \geq 2(2x + 3), x \in I$ (iv) $2(4 - 3x) \leq 4(x - 5), x \in W$

(v) $2(4x - 1) < 4(3x + 1), x \in$ {set of negative integers}

4. Represent the solution set graphically.

(i) $2x - 3(x - 4) \geq 4 - 2(x - 7), x \in W$ (ii) $5(x - 2) > 9x - 3(2x - 4), x \in N$

(iii) $3[4(n - 2) - (1 - n)] \leq n + 1, n \in W$

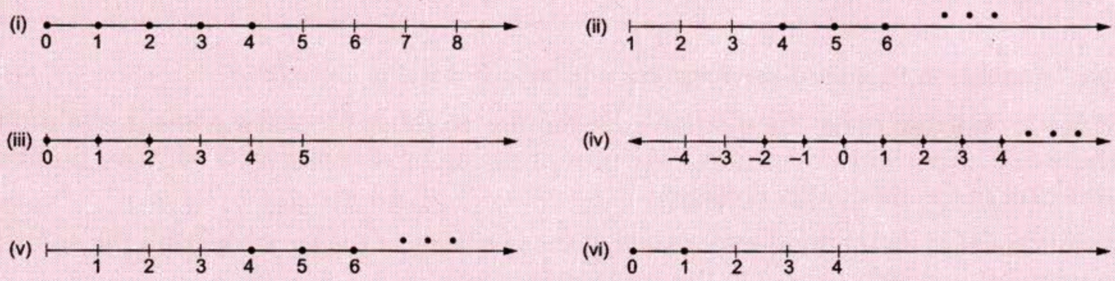
5. Find the solution set and represent graphically.

(i) $4 < -(x + 2) < 9, x \in I$ (ii) $15 > 2x - 7 > 7, x \in R$

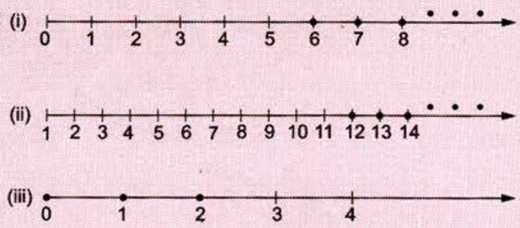
(iii) $0 \leq \frac{4 - x}{2} < 2, x \in R$ (iv) $-4 \leq -2(x + 8) < 8, x \in R$

ANSWERS

1. (i) {0, 1} (ii) {1, 2, 3, 4, 5, 6, 7} (iii) {0, 1, 2, 3, 4} (iv) {4, 5, 6, 7} (v) {7} (vi) No solution (vii) {0, 1, 2} (viii) {6, 7} (ix) {4, 5, 6, 7} (x) {2, 3, 4, 5, 6, 7} (xi) {0} (xii) {3, 4, 5, 6, 7} (xiii) {0, 1, 2, 3, 4, 5, 6, 7} (xiv) {0, 1, 2, 3, 4}

2.


3. (i) {1, 2, 3, 4, 5, 6} (ii) {0, 1, 2, 3, 4} (iii) {5, 6, 7, ...} (iv) {3, 4, 5, ...} (v) {–1}

4.


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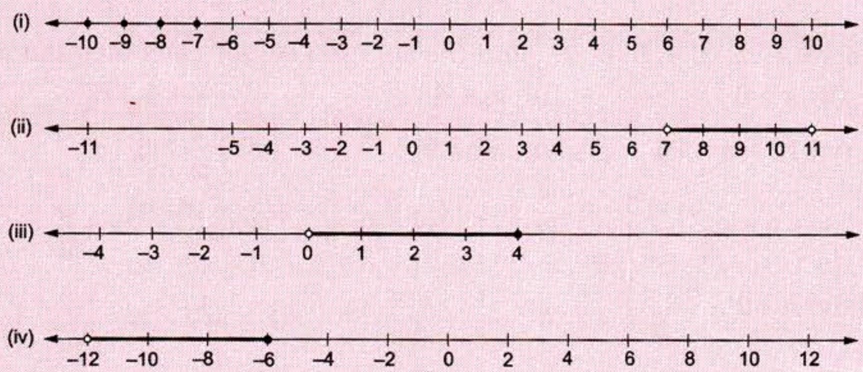

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