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HCF and LCM

Highest Common Factor

If certain factors are common to two or more expressions then they are called common factors of those expressions. The product of all such common factors is called the highest common factor or HCF of the given expressions.

HCF of monomials

To find the HCF of monomials, take the following steps.

Steps
1. Find the HCF of the numerical coefficients of the monomials.
2. Find the highest power of each of the variables common to the monomials.
3. The product of the number and the powers of the variables obtained in Steps 1 and 2 is the required HCF.

EXAMPLE **Find the HCF of (i) $6ab^3$ and $8ab^2$, and (ii) $2x^2y^2$, $5x^3y$ and $-3y^3$.**

Solution (i) $6ab^3 = 2 \times 3 \times a \times b \times b \times b$, $8ab^2 = 2 \times 2 \times 2 \times a \times b \times b$.

The HCF of the numerical coefficients of the monomials = HCF of 6 and 8 = 2.

The highest power of the variable a common to both = a.

The highest power of the variable b common to both = $b \times b = b^2$.

$\therefore$ the HCF of $6ab^3$ and $8ab^2 = 2 \times a \times b^2 = 2ab^2$.

(ii) $2x^2y^2 = 2 \times x \times x \times y \times y$, $5x^3y = 5 \times x \times x \times x \times y$, $-3y^3 = -3 \times y \times y \times y$

The HCF of 2, 5 and –3 = 1.

The highest power of x common to the monomials = x^0

($\because$ there is no x in $-3y^3$)

The highest power of y common to the monomials = y.

$\therefore$ the HCF of $2x^2y^2, 5x^3y$ and $-3y^3 = 1 \times x^0 \times y = y$.

Note If the monomials do not have any factor in common, the HCF is 1.

HCF of polynomials

Take the following steps to find the HCF of polynomials.

Steps
1. Find the HCF of the common numerical coefficients if any,
2. Factorize each of the given expressions and take the factors common to all of them.
3. The product of the number and factors obtained in Steps 1 and 2 is the required HCF.

EXAMPLE **Find the HCF of $12(a^2 - ab)$ and $16(ab - b^2)$.**

Solution The HCF of the numerical coefficients of the given expressions
= the HCF of 12 and 16 = 4.

Also, $a^2 - ab = a(a-b)$ and $ab - b^2 = b(a-b)$.

We observe that $a-b$ is a factor common to both expressions.

$\therefore$ the HCF of the given expressions $= 4(a-b)$.

Solved Examples

EXAMPLE 1 **Find the HCF of $16a^4b^5c^6$, $20a^5b^4c^3$ and $24a^4b^4$.**

Solution $16a^4b^5c^6 = 2\times2\times2\times2\times a^4\times b^5\times c^6$ $20a^5b^4c^3 = 2\times2\times5\times a^5\times b^4\times c^3$

$24a^4b^4 = 2\times2\times2\times3\times a^4\times b^4$

The HCF of 16, 20 and 24 = $2\times2 = 4$.

The highest power of the variable a common to all $= a^4$.

The highest power of the variable b common to all $= b^4$.

The variable c is absent in the monomial $24a^4b^4$.

$\therefore$ the required HCF $= 4a^4b^4$.

EXAMPLE 2 **Find the HCF of $a^2 - b^2$, $a^2 - b^2 + ac - bc$ and $a^3 - a^2b + ab^2 - b^3$.**

Solution $a^2 - b^2 = (a+b)(a-b)$.

$a^2 - b^2 + ac - bc = (a^2 - b^2) + (ac - bc) = (a+b)(a-b) + c(a-b) = (a-b)(a+b+c)$.

$a^3 - a^2b + ab^2 - b^3 = (a^3 - a^2b) + (ab^2 - b^3) = a^2(a-b) + b^2(a-b) = (a-b)(a^2+b^2)$.

The factor $(a-b)$ is common to all the given expressions.

$\therefore$ the required HCF $= a-b$.

EXAMPLE 3 **Find the HCF of $x^4 + 2x^2 + 1$, $x^6 + x^4 - x^2 - 1$ and $x^4 - 1$.**

Solution $x^4 + 2x^2 + 1 = (x^2)^2 + 2\cdot x^2\cdot 1 + (1)^2 = (x^2+1)^2 = (x^2+1)(x^2+1)$.

$x^6 + x^4 - x^2 - 1 = (x^6 + x^4) - (x^2+1) = x^4(x^2+1) - 1(x^2+1)$

$= (x^2+1)(x^4-1) = (x^2+1)\{(x^2)^2 - 1^2\} = (x^2+1)(x^2+1)(x^2-1)$

$= (x^2+1)(x^2+1)\{(x)^2 - (1)^2\} = (x^2+1)(x^2+1)(x+1)(x-1)$.

$x^4 - 1 = (x^2)^2 - (1)^2 = (x^2+1)(x^2-1) = (x^2+1)\{(x)^2 - (1)^2\} = (x^2+1)(x+1)(x-1)$.

Only (x^2+1) is common to all the given expressions.

$\therefore$ the HCF $= x^2+1$.

EXERCISE 6A

Find the HCF of the following.

1. (i) $2x^4y, 3xy^4$ (ii) $2a^2b^3, 6a^4b^2$
2. (i) $a^4b^2c^5, ab^3c^2, a^2b^4c^4$ (ii) abc, bcd, cde
   (iii) $12m^2np, 16mn^2p, 20mnp^2$ (iv) $24a^3x^3y^4, 36a^2x^5y^7, 48a^4x^4y^5$
   (v) $10m^4n^2p^6, 15m^3n^3p^5, 25m^5n^5p^7$

3. (i) $x^2 - y^2, (x+y)^2$ (ii) $a^2 + ab, b^2 + ab$
(iii) $x^2 - xy, x^3 - x^2y$ (iv) $3(a^5b^3 - a^3b^5), 6(a^7b - b^7a)$

4. $a^2 + ab, ab + b^2, a^2 - b^2$

5. $(a+b)(a^2 - ab + b^2), a^2b + ab^2, a^2 - b^2$

6. $(x-y)(x^2 + xy + y^2), x^2 - y^2, x^2 + 5xy - 6y^2$

7. $x^2 - x - 2, x^2 + x - 6$

8. $x^2 - x - 12, x^2 + 2x - 3, 5x^2 + 11x - 12$

9. $x^2 + 3x + 2, 2x^2 + x - 1, x^2 - 2x - 3$

10. $x^3 - 5x^2 + 6x, x^3 + 4x^2 - 12x, x^3 - 9x^2 + 14x$

11. $5x^2(x-2), 10x^2(x^2-4), 15x^4(x-2)(x^2+2x+4)$

12. $x^2 - xy + yz - zx, y^2 - xy - yz + xz$

ANSWERS

1. (i) xy (ii) $2a^2b^2$ 2. (i) ab^2c^2 (ii) c (iii) $4mnp$ (iv) $12a^2x^3y^4$ (v) $5m^3n^2p^5$
3. (i) $x + y$ (ii) $a + b$ (iii) $x(x - y)$ (iv) $3ab(a + b)(a - b)$ 4. $a + b$
5. $a + b$ 6. $x - y$ 7. $x - 2$ 8. $x + 3$
9. $x + 1$ 10. $x(x - 2)$ 11. $5x^2(x - 2)$ 12. $x - y$

Lowest Common Multiple (LCM)

The LCM of two or more algebraic expressions is the lowest expression which is exactly divisible by all the expressions.

LCM of monomials

To find the LCM of monomials, take the following steps.

Steps
1. Find the LCM of the numerical coefficients of the given monomials.
2. Take the highest powers of each of the variables in the monomials.
3. The product of the number and the powers of the variables obtained in Steps 1 and 2 is the required LCM.

EXAMPLE **Find the LCM of $2x^2$ and $3y$.**

Solution The LCM of the numerical coefficients 2 and 3 = 6.
The highest power of the variable $x = x^2$.
The highest power of the variable $y = y$.
$\therefore$ LCM $= 6 \times x^2 \times y = 6x^2y$.

Note The LCM of monomials that have no factor in common is the product of the monomials.

EXAMPLE **Find the LCM of $2a^2b$, $3b^2c$ and $6abc^2$.**

Solution The LCM of the numerical coefficients 2, 3 and 6 = 6.

The highest power of the variable $a = a^2$.

The highest power of the variable $b = b^2$.

The highest power of the variable $c = c^2$.

$\therefore$ LCM $= 6 \times a^2 \times b^2 \times c^2 = 6a^2b^2c^2$.

LCM of polynomials

To find the LCM of polynomials, take the following steps.

Steps
1. Find the LCM of the numerical coefficients (if any) of the polynomials.
2. Factorize the given polynomials.
3. Take the highest power of each of the factors (including the ones in common).
4. The product of the number and the powers of the factors obtained in Steps 1 and 3 is the LCM of the given polynomials.

EXAMPLE **Find the LCM of $a^2 - b^2$ and $a^2 + 2ab + b^2$.**

Solution $a^2 - b^2 = (a+b)(a-b)$.

$a^2 + 2ab + b^2 = (a)^2 + 2 \cdot a \cdot b + (b)^2 = (a+b)^2$.

$\therefore$ the required LCM $= (a+b)^2(a-b)$.

Solved Examples

EXAMPLE 1 **Find the LCM of $8a^4b^5c^6$, $12a^3b^4c^5$ and $16a^2b^3c^4$.**

Solution The LCM of the numerical coefficients, 8, 12 and 16 = 48.

The highest power of the variable $a = a^4$.

The highest power of the variable $b = b^5$.

The highest power of the variable $c = c^6$.

$\therefore$ the LCM of the given expressions $= 48a^4b^5c^6$.

EXAMPLE 2 **Find the LCM of $2x^2 - x - 6$, $3x^2 - 7x + 2$ and $6x^2 + 7x - 3$.**

Solution

$$2x^2 - x - 6 = 2x^2 - 4x + 3x - 6$$
$$= (2x^2 - 4x) + (3x - 6)$$
$$= 2x(x-2) + 3(x-2) = (x-2)(2x+3).$$

$$3x^2 - 7x + 2 = 3x^2 - 6x - x + 2$$
$$= (3x^2 - 6x) - (x - 2)$$
$$= 3x(x-2) - 1(x-2) = (x-2)(3x-1).$$

$$6x^2 + 7x - 3 = 6x^2 + 9x - 2x - 3$$
$$= (6x^2 + 9x) - (2x + 3)$$
$$= 3x(2x+3) - 1(2x+3) = (2x+3)(3x-1).$$

$\therefore$ the LCM of the given expressions $= (x-2)(2x+3)(3x-1)$.

EXERCISE 6B

1. Find the LCM of the following monomials.

(i) $2abc$ and $3xy$
(ii) $6x^2y^2$ and $3x^3y^3$
(iii) $24a^4b^3c^2$ and $30a^2b^2c^4$
(iv) xy, yz and zx
(v) $2abc$, $3bcd$ and $4cde$
(vi) $18a^2m^3n^4$, $24a^3m^2n^3$ and $36a^2m^4n^5$
(vii) $14m^2n^2p^2$, $21m^3np^3$ and $28mn^3p^2$

2. Find the LCM of the following polynomials.

(i) $x^3y - xy^3$ and $x^3y^2 + x^2y^3$
(ii) $6xy^2(x+y)^2$ and $4x^2y(x^2 - y^2)$
(iii) $x(y-z)$, $y(z-x)$ and $z(x-y)$
(iv) $(x-y)(y-z)$, $(y-z)(z-x)$ and $(z-x)(x-y)$
(v) $2x^2 - x - 1$, $2x^2 + 3x + 1$ and $x^2 - 1$
(vi) $3x^2 - 10x - 8$, $4x^2 - 20x + 9$ and $6x^2 + x - 2$
(vii) $x^2 - 3xy - 10y^2$, $x^2 + 2xy - 35y^2$ and $x^2 - 8xy + 15y^2$
(viii) $x^2 - 7x + 12$, $3x^2 - 6x - 9$ and $2x^3 - 6x^2 - 8x$

ANSWERS

1. (i) $6abcxy$ (ii) $6x^3y^3$ (iii) $120a^4b^3c^4$ (iv) xyz (v) $12abcde$ (vi) $72a^3m^4n^5$ (vii) $84m^3n^3p^3$

2. (i) $x^2y^2(x+y)(x-y)$ (ii) $12x^2y^2(x+y)^2(x-y)$ (iii) $xyz(x-y)(y-z)(z-x)$ (iv) $(x-y)(y-z)(z-x)$ (v) $(2x+1)(x+1)(x-1)$ (vi) $(3x+2)(x-4)(2x-1)(2x-9)$ (vii) $(x-5y)(x+2y)(x+7y)(x-3y)$ (viii) $6x(x+1)(x-3)(x-4)$

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Revision Exercise 4

1. Factorize the following.

(i) $30x^4y+12x^2y^3$ (ii) $9ab^2c^5-15a^2b^3c^4-21a^3b^4c^3$ (iii) $4-5a-4b+5ab$

(iv) $6a-96ab^4$ (v) $8(x+y)^2-18(2x-y)^2$

(vi) $9m^2-p^2-10pq-25q^2$ (vii) $x^2-x(2b+c)+2bc$

(viii) $a^3+a^2+a+a^2b+ab+b$ (ix) $3x^2+7x-6$ (x) $12+x-6x^2$

(xi) $(y+5)^2+7(y+5)(y-2)+12(y-2)^2$ (xii) $3(x+2)^2-2(x+2)(x-1)-(x-1)^2$

2. Find the HCF and LCM.

(i) p^2-q^2, p^2-pq (ii) $16a^2-4b^2, 6b^2-6ab-36a^2$

(iii) $m^2-3m-10, m^2-6m+5$ (iv) $1-x^2, 4-4x, 6x-6x^2$

(v) $a^4-x^4, a^2-x^2, ax-x^2$

(vi) $x^2-9, x^2-6x+9, x^2+2x-15, x^2-8x+15$

ANSWERS

1. (i) $6x^2y(5x^2+2y^2)$ (ii) $3ab^2c^3(3c^2-5abc-7a^2b^2)$ (iii) $(4-5a)(1-b)$ (iv) $6a(1+4b^2)(1+2b)(1-2b)$ (v) $2(8x-y)(5y-4x)$ (vi) $(3m+p+5q)(3m-p-5q)$ (vii) $(x-2b)(x-c)$ (viii) $(a+b)(a^2+a+1)$ (ix) $(3x-2)(x+3)$ (x) $(3-2x)(4+3x)$ (xi) $(4y-1)(5y-3)$ (xii) $3(4x+5)$

2. (i) $p-q, p(p+q)(p-q)$ (ii) $2(2a+b), 12(2a+b)(2a-b)(b-3a)$ (iii) $m-5, (m-5)(m-1)(m+2)$ (iv) $1-x, 12x(1+x)(1-x)$ (v) $a-x, x(a^2+x^2)(a+x)(a-x)$ (vi) $x-3, (x+3)(x-3)^2(x+5)(x-5)$

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