CHAPTER 35

SUBSETS, UNIVERSAL SET
(INCLUDING OPERATIONS ON SETS)

35.1 SUBSET

If each element of set A is also present in set B, then set A is said to be the subset of set B. Conversely, if B is subset of A, then each element of set B is present in set A.

For example :

(i) A = {2, 4, 6} and B = {1, 2, 3, 4, 5, 6, 7} implies that A is subset of B, as each element of set A is in set B also.

(ii) A = {6, 7, 8, 9, 10} and B = {7, 9, 10} ⇒ B is subset of A and so on.

Symbolically, if A is subset of B, we write : **A ⊆ B**.

and, if B is subset of A, we write : **B ⊆ A**.

35.2 SUPER-SET

When set A is the subset of set B, then set B is called the super-set of A.

Symbolically, we write : **B ⊇ A**.

1. A ⊆ B is read as : "A is subset of B" or "A is contained in B".
2. B ⊇ A is read as : "B is super-set of A" or "B contains A".
3. A ⊄ B means 'A is not a subset of B'. And B ⊅ A means "B is not a super-set of A".
4. Every set is a subset of itself, *i.e.*, A ⊆ A, B ⊆ B and so on.
5. Empty set is subset of every set, *i.e.*, φ ⊆ A, φ ⊆ B and so on.

Example 1 :

Find subsets of : (i) { } (ii) {*a*, *b*}

Solution :

Since, every set is subset of itself and the empty set is subset of every set :

∴ (i) **Subset of { } = { }** (Ans.)

(ii) **Subsets of {*a*, *b*} = { }, {*a*}, {*b*} and {*a*, *b*}** (Ans.)

35.3 PROPER SUBSET

All the subsets, other than the set itself, are called proper subsets.

The symbol for proper subset is '⊂', *i.e.*, if A is proper subset of B, we write : A ⊂ B.

1. No set is proper subset of itself.
2. When set A is proper subset of set B :
   (i) each element of set A is in set B,
   (ii) number of elements in set A is less than the number of elements in set B.

Example 2 :

Find, if possible, all proper subsets of : (i) { } (ii) {*a*} (iii) {*a*, *b*}.

Solution :

Since, no set is proper subset of itself, therefore :

(i) **{ } has no proper subset.** (Ans.)

(ii) **Proper subset of {*a*} = { }** (Ans.)

(iii) **Proper subsets of {*a*, *b*} = { }, {*a*} and {*b*}.** (Ans.)

35.4 NUMBER OF SUBSETS AND NUMBER OF PROPER SUBSETS OF A GIVEN SET

If a set has 'n' elements in it,

(i) the number of its subsets = 2^n and

(ii) the number of its proper subsets = $2^n - 1$ [No set is proper subset of itself]

e.g., Number of elements in $\{a, b\}$ = 2 $\Rightarrow n = 2$

$\therefore$ No. of its subsets = $2^n = 2^2 = 4$

and no. of its proper subsets = $2^n - 1 = 2^2 - 1 = 3$

35.5 UNIVERSAL SET

Any set, which contains all the elements of various sets under discussion, is called the universal set.

For example :

If the sets under discussion are :

A = {2, 3, 5}, B = {5, 6, 9, 12} and C = {3, 6, 9, 12}, then form a set which contains every element of sets A, B and C.

Clearly, the set obtained is {2, 3, 5, 6, 9, 12}. So this set is called the *universal set* for the sets A, B and C under discussion.

A universal set is represented by the symbol ξ (read pxi) or U.

Thus, ξ = {2, 3, 5, 6, 9, 12}

Note : *Universal set for the sets under consideration is not unique, i.e., we may have more than one universal set for the same sets under consideration.*

Thus, for the sets A, B and C, given above,

(i) {1, 2, 3, 4, ..., 15} can be taken as universal set, since it contains every element of the sets under discussion.

(ii) Set N, the set of natural numbers, can also be taken as universal set, since each element of the sets under discussion is a natural number and so on.

1. *Every set under discussion is a subset of the universal set.*
2. *$\xi \subseteq \xi$, because every set is a subset of itself.*

35.6 COMPLEMENT OF A SET

The complement of a set A is the set of elements, which are present in the universal set but are not present in set A.

The complement of a set A is written as A′, and is read as *complement of set A*.

Thus, if universal set, ξ = {1, 2, 3, 4, 5, 6, 7, 8},

A = {1, 3, 5, 7} and B = {2, 4, 6};

then : (i) **Complement of set A = A′** = {elements which are in ξ but not in A}

= **{ 2, 4, 6, 8}**

and, (ii) **Complement of set B = B′** = **{1, 3, 5, 7, 8}**

1. *The set and its complement are always disjoint, i.e., sets A and A′ are disjoint, sets B and B′ are disjoint sets and so on.*
2. The complement of the empty set is the universal set, i.e., $\phi' = \xi$.
3. The complement of universal set is the empty set, i.e., $\xi' = \phi$.

EXERCISE 35(A)

1. Fill in the blanks :
   (i) If each element of set P is also an element of set Q, then P is said to be .... of Q and Q is said to be .... of P.
   (ii) Every set is a .... of itself.
   (iii) The empty set is a .... of every set.
   (iv) If A is proper subset of B, then $n(A)$ .... $n(B)$.
2. If A = {5, 7, 8, 9}, then which of the following are subsets of A ?
   (i) B = {5, 8} (ii) C = {0} (iii) D = {7, 9, 10}
   (iv) E = { } (v) F = {8, 7, 9, 5}
3. If P = {2, 3, 4, 5}, then which of the following are proper subsets of P ?
   (i) A = {3, 4} (ii) B = { } (iii) C = {23, 45}
   (iv) D = {6, 5, 4} (v) E = {0}
4. If A = {even numbers less than 12},
   B = {2, 4}, C = {1, 2, 3}, D = {2, 6} and E = { 4 }
   State, which of the following statements are **true** :
   (i) $B \subset A$ (ii) $C \subseteq A$ (iii) $D \subset C$
   (iv) $D \not\subset A$ (v) $E \supseteq B$ (vi) $A \supseteq B \supseteq E$
5. Give A = {*a*, *c*}, B = {*p*, *q*, *r*} and C = Set of digits used to form number 1351. Write all the subsets of sets A, B and C.
6. (i) If A = {*p*, *q*, *r*}, then number of subsets of A = ...
   (ii) If B = {5, 4, 6, 8}, then number of proper subsets of B = ...
   (iii) If C = {0}, then number of subsets of C = ...
   (iv) If M = {$x : x \in N$ and $x < 3$}, then M has .... proper subsets.
7. For the universal set {4, 5, 6, 7, 8, 9, 10, 11, 12, 13}, find its subsets A, B, C and D such that :
   (i) A = {even numbers} (ii) B = {odd numbers greater than 8}
   (iii) C = {prime numbers} (iv) D = {even numbers less than 10}.
   Also, find compliments of each set, *i.e.*, find A′, B′, C′ and D′.
8. If ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {even numbers less than 10},
   B = {odd numbers less than 10} and C = {1, 2, 3, 7, 8, 9}
   Find : (i) A′ (ii) B′ (iii) C′
9. Given, universal set ξ = {Natural numbers upto 30}, P = {multiples of 3 upto 30},
   Q = {multiples of 5 upto 30} and R = {multiples of 10 upto 30}
   Find : (i) P′ (ii) Q′ (iii) R′
   Also, find : $n(P)$, $n(Q)$, $n(R)$, $n(P')$, $n(Q')$ and $n(R')$.
10. If P represents the set of people with heights more than 5 m, what is the number of elements in P ? Also, write the number of subsets and number of proper subsets of P.

35.7 OPERATIONS OF SETS

1. Union of two sets :

Union of two sets A and B is a set whose elements are either in set A or in set B or in both the sets A and B.

In other words, *union of sets A and B contains all the elements of set A, all the elements of set B and no other elements.*

The union of two sets is represented by the symbol '$\cup$'

Thus, the union of sets A and B = A $\cup$ B (read : A *union* B).

(i) If A = {1, 2, 3, 4, 5} and B = {2, 4, 6, 8};
then, union of sets A and B = A $\cup$ B = {1, 2, 3, 4, 5, 6, 8}

And, (ii) If P = {*a, b, c, d*} and Q = {*x* : *x* is a letter in the word ABBAS};
then : P $\cup$ Q = {*a, b, c, d, s*} [As, Q = {*a, b, s*}]

> 1. *Union of sets is commutative, i.e., A $\cup$ B = B $\cup$ A, P $\cup$ Q = Q $\cup$ P and so on.*
> 2. *A and B both are subsets of A $\cup$ B, i.e., A $\subseteq$ (A $\cup$ B) and B $\subseteq$ (A $\cup$ B).*
> *Similarly, P $\subseteq$ (P $\cup$ Q), Q $\subseteq$ (P $\cup$ Q) and so on.*

2. Intersection of two sets :

The intersection of two sets A and B is the set of all elements which are common to sets A and B both.

The intersection of two sets is represented by the symbol '$\cap$'

Thus, intersection of sets A and B = A $\cap$ B (read : A *intersection* B).

(i) If A = {5, 6, 7, 8} and B = {4, 6, 8, 10, 12};
intersection of sets A and B = A $\cap$ B = {6, 8}

And, (ii) If M = {*a, b, c, d, e*} and N = {*a, e, i, u*},
then : M $\cap$ N = {*a, e*}

> 1. *Intersection of sets is commutative, i.e., A $\cap$ B = B $\cap$ A; M $\cap$ N = N $\cap$ M and so on.*
> 2. *A $\cap$ B is subset of both the sets A and B, i.e., A $\cap$ B $\subseteq$ A and A $\cap$ B $\subseteq$ B.*
> *Similarly, P $\cap$ Q $\subseteq$ P, P $\cap$ Q $\subseteq$ Q and so on.*

3. Difference of two sets :

For any two sets A and B, their difference is : A – B or B – A.

where, A – B = Set of elements of A which are not in B

and, B – A = Set of elements of B which are not in A.

Thus, if A = {2, 4, 6, 8, 10} and B = {2, 3, 7, 9, 5}, then :

(i) A – B = {elements of A which are not in B} = {4, 6, 8, 10}

(ii) B – A = {elements of B which are not in A} = {3, 5, 7, 9}

Similarly, if M = {*b, c, d, e, f*} and N = {*a, c, f, h*}
then, M – *N* = {*b, d, e*} and N – M = {*a, h*}.

EXERCISE 35(B)

1. Given A = {*a, b, c, d*} and B = {*b, c, d, f*}. Find :
   (i) A $\cup$ B (ii) A $\cap$ B (iii) Is A $\cap$ B a subset of A $\cup$ B ?
2. If A = {4, 6, 8, 10, 12} and B = {3, 6, 9, 12, 15}. Find :
   (i) A $\cap$ B (ii) A $\cup$ B (iii) A $\cup$ ϕ (iv) B $\cap$ ϕ .
3. Given : A = {*x* : *x* is multiple of 2 less than 28}
   B = {*x* : *x* is a square of the natural number and is less than 28}

C = {x : x is a multiple of 3 and is less than 28}
D = {x : x is a prime number less than 28}

Express A, B, C and D in roster form, and find :

(i) $B \cup C$ (ii) $C \cup A$ (iii) $D \cup B$ (vi) $A \cup D$
(v) $A \cap B$ (vi) $B \cap C$ (vii) $C \cap A$ (viii) $B \cap D$

4. Given : A = {0, 1, 2, 3, 4, 5, 6, 7, 8}
B = {3, 5, 7, 9, 11} and C = {0, 10, 20, 30}

(a) List the elements of the sets $A \cup B$, $B \cup C$, $A \cup C$, $A \cap B$, $B \cap C$ and $C \cap A$. Also, find :

(i) $n(A \cup B)$ (ii) $n(A \cap B)$ (iii) $n(B \cup C)$
(iv) $n(B \cap C)$ (v) $n(A \cap C)$ (vi) $n(C \cup A)$

(b) For the same sets A, B and C (given above), state whether the following statements are **true** or **false** :

(i) $8 \in A$ (ii) $3 \in B \cap A$ (iii) $7 \notin A \cup C$
(iv) $0 \in B \cap C$ (v) $5 \in A \cup B$ (vi) $9 \in B \cup A$
(vii) $10, 20 \in A \cup C$

5. Given ξ = {a, b, c, d, e, f, g, h}, A = {b, d, f, h}, B = {a, b, c, d} and C = {d, f, g, h}. Find :

(i) $A - B$ (ii) $B - A$ (iii) $B - C$
(iv) $C - A$ (v) $A' - B$ (vi) $A' - B'$
(vii) $(A \cup B) - (A \cap B)$

6. If universal set, ξ = {3, 4, 5, 6, 7, 8, 9, 10, 11, 12},
A = {even numbers}, B = {odd numbers} and C = {3, 7, 8, 9}. Find :

(i) $(A \cup B)'$ (ii) $(A \cap C)'$ (iii) $A \cap B'$
(iv) $C' \cap A$ (v) $B' \cup C$ (vi) $B' - A$

7. Given, ξ = {Natural numbers between 10 and 40},
P = {multiples of 3}, Q = {multiples of 5} and R = {multiples of 6}

Find : (i) $n(P) + n(Q)$ (ii) $n(P \cup Q) + n(P \cap Q)$
(iii) Is $n(P) + n(Q) = n(P \cup Q) + n(P \cap Q)$?

For any two sets P and Q,
$n(P) + n(Q)$ is always equal to $n(P \cup Q) + n(P \cap Q)$
Similarly, for sets A and B,
$n(A) + n(B) = n(A \cup B) + n(A \cap B)$ and so on.

8. If A = {4, 8, 10, 11}, B = {4, 6, 11} and C = {6, 8, 9, 10}, then find :

(i) $A \cup B$ (ii) $(A \cup B) \cap C$ (iii) $A \cap C$ and $B \cap C$
(iv) $(A \cap C) \cup (B \cap C)$ (v) Is $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$?

9. If A = {a, b, c, d, e}, B = {b, d, f, g} and C = {a, c, d, f}, then find :

(i) $A \cap B$ (ii) $(A \cap B) \cup C$ (iii) $(A \cup C)$ and $(B \cup C)$
(iv) $(A \cup C) \cap (B \cup C)$ (v) Is $(A \cup C) \cap (B \cup C) = (A \cap B) \cup C$?