| UNIT – 7 SET THEORY | CHAPTER 34 |

SET CONCEPTS

34.1 BASIC CONCEPT

In our day to day life, different collective nouns are used to describe collection of objects, such as : *a group of students playing cricket, a pack of cards, a bunch of flowers,* etc.

In mathematics, *such collections of objects are named as* **sets**.

> *A set is a collection of well-defined objects, things or symbols, etc.*

The phrase 'well-defined' means, it must be possible to know, without any doubt, whether a given object, thing or symbol belongs to the set under consideration or not.

For example :

"The set of tall boys of Class 10" is not *well-defined*, since it is not possible to know that which boys are to be included and exactly what is the limit.

But when we say, "The set of boys of Class 10, which are taller than Peter", now we can compare the heights of different boys with the height of Peter and can know exactly that which boys are to be included in the required set. Thus, *the objects are well-defined.*

34.2 ELEMENTS OF A SET

The objects (things, symbols, etc.) *used to form a set are called elements* or *members of the set.*

In general, a set is denoted by a capital letter of English alphabet with its elements written inside curly braces and separated by commas.

e.g., Set A = {5, 10, 12, 15}

34.3 USE OF SYMBOL '∈' OR SYMBOL '∉'

The symbol '∈' stands for *'belongs to'* or *'is an element of'* or *'is a member of'*, whereas the symbol '∉' stands for *'does not belong to'* or *'is not an element of'* or *'is not a member of'*.

e.g., For set P = {3, 6, 8, 13, 18}, 3 ∈ P, 5 ∉ P and so on.

> 1. *The elements in a set can be written in any order.*
>   Thus, {*a, b, c, d*} is the same set as {*b, d, a, c*} or {*c, b, d, a*}, etc.
> 2. *The elements in a set should not be repeated, i.e., if any element occurs many times, it should be written only once.*
>   Thus, set of letters of the word 'crook' = {*c, r, o, k*}.
>   There are **two o's** in the given word "crook", but in the set, it is **written only once.**

33.4 REPRESENTATION OF A SET

A set, in general, is represented in :

(i) Description method (form)   (ii) Tabular or Roster method (form)

(iii) Set-builder or Rule method

For example :

N is the set of natural numbers	[Description method]
N = {1, 2, 3, 4, 5, ...}	[Roster or Tabular method]
N = {*x* : *x* is a natural number}, or {*x* : *x* ∈ N}	[Set-builder or Rule method]

[The symbol ' : ' stands for **such that** and the set {*x* : *x* ∈ N} is read as, "the set of *x* such that x is a natural number"].

It is clear from the example given above that :

(i) in *description method* a well-defined description about the set is given.

(ii) in *roster* or *tabular method* the elements of the set are written inside a pair of curly braces and are separated by commas.

(iii) in *set-builder* or *rule method* the actual elements of the set are not written, but *a rule* or *a statement* or *a formula is written* in the briefest possible way.

More examples :

1. Z is the set of integers   [Description method]

    Z = {...., –3, –2, –1, 0, 1, 2, 3,.....}   [Tabular or Roster method]

       = {*x* : *x* ∈ Z}   [Rule or Set-builder method]

2. A is the set of whole numbers less than 8   [Description method]

    A = {0, 1, 2, 3, 4, 5, 6, 7}   [Tabular or Roster method]

    A = {*x* : *x* ∈ W and *x* < 8}   [Rule or Set-builder method]

Example 1 :

Express each of the given set as required :

(i) The set of integers between –3 and 5, *in roster form.*

(ii) The set of even natural numbers, *in roster form.*

(iii) Set A = {1, 3, 5, 7, 9,......}, *in set-builder form.*

(iv) Set B = {0, 3, 6, 9, 12,....}, *in set-builder notation.*

(v) Set C = {*y* : *y* = 2*x* + 1 and *x* ∈ N}, *in roster form.*

Solution :

(i) **The set of integers between –3 and 5** = **{–2, –1, 0, 1, 2, 3, 4}**   (Ans.)

(ii) **The set of even natural numbers** = **{2, 4, 6, 8, 10, 12, ...}**   (Ans.)

(iii) **Set A** = **{1, 3, 5, 7, 9,......}** = **{*x* : *x* is an odd natural number}**   (Ans.)

(iv) **Set B** = **{0, 3, 6, 9, 12,....}** = **{*x* : *x* is a whole number divisible by 3}**   (Ans.)

(v) Since, *x* ∈ N (the set of natural numbers)

    ∴ *x* can be 1, 2, 3, 4, ...

    And so, **set C** = {*y* : *y* = 2*x* + 1 and *x* ∈ N}

                   = {2 × 1 + 1, 2 × 2 + 1, 2 × 3 + 1, 2 × 4 + 1, ...}

                   = **{3, 5, 7, 9, ...}**   (Ans.)

EXERCISE 34(A)

1. Find, whether or not, each of the following collections represent a set :
   (i) The collection of good students in your school.
   (ii) The collection of the numbers between 30 and 45.
   (iii) The collection of fat-people in your colony.
   (iv) The collection of interesting books in your school library.
   (v) The collection of books in the library and are of your interest.

2. State whether **true** or **false** :
   (i) Set {4, 5, 8} is same as the set {5, 4, 8} and the set {8, 4, 5}.
   (ii) Sets {a, b, m, n} and {a, a, m, b, n, n} are same.
   (iii) Set of letters in the word 'suchismita' is {s, u, c, h, i, m, t, a}.
   (iv) Set of letters in the word 'MAHMOOD' is {M, A, H, O, D}.

3. Let set A = {6, 8, 10, 12} and set B = {3, 9, 15, 18}.
   Insert the symbol '$\in$' or '$\notin$' to make each of the following true :
   (i) 6 .... A   (ii) 10 .... B   (iii) 18 .... B   (iv) (6 + 3) .... B
   (v) (15 − 9) .... B   (vi) 12 .... A   (vii) (6 + 8) .... A   (viii) 6 and 8 .... A

4. Express each of the following sets in **roster form** :
   (i) Set of odd whole numbers between 15 and 27.
   (ii) A = Set of letters in the word "CHITAMBARAM"
   (iii) B = {All even numbers from 15 to 26}
   (iv) P = {$x : x$ is a vowel used in the word 'ARITHMETIC'}
   (v) S = {Squares of first eight whole numbers}
   (vi) Set of all integers between 7 and 94; which are divisible by 6.
   (vii) C = {All composite numbers between 2 and 20}
   (viii) D = Set of prime numbers from 2 to 23.
   (ix) E = Set of natural numbers below 30; which are divisible by 2 or 5.
   (x) F = Set of factors of 24.
   (xi) G = Set of names of three closed figures in Geometry.
   (xii) H = {$x : x \in$ W and $x < 10$}
   (xiii) J = {$x : x \in$ N and $2x − 3 \leq 17$}
   (xiv) K = {$x : x$ is an integer and $− 3 < x < 5$}

5. Express each of the following sets in **set-builder notation** (form) :
   (i) {3, 6, 9, 12, 15}   (ii) {2, 3, 5, 7, 11, 13, ...}
   (iii) {1, 4, 9, 16, 25, 36}   (iv) {0, 2, 4, 6, 8, 10, 12, ...}
   (v) {Monday, Tuesday, Wednesday}   (vi) {23, 25, 27, 29, ...}
   (vii) $\left\{\dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}, \dfrac{1}{6}, \dfrac{1}{7}, \dfrac{1}{8}\right\}$ (viii) {42, 49, 56, 63, 70, 77}

6. Given : A = { $x : x$ is a multiple of 2 and is less than 25 }
   B = { $x : x$ is a square of a natural number and is less than 25 }
   C = { $x : x$ is a multiple of 3 and is less than 25 }
   D = { $x : x$ is a prime number less than 25 }
   Write the sets A, B, C and D in roster form.

34.5 CARDINAL NUMBER

The cardinal number of a set is the number of elements in it.

Thus, if a set A has 5 elements; its cardinal number is 5 and we represent it by writing $n(A) = 5$.

Similarly, if set B = Set of even natural numbers less than 10

then, B = {2, 4, 6, 8} and $n(B) = 4$.

> If B = {0}, then $n(B) = 1$. Since, 0 *is an element of set B.*

34.6 TYPES OF SETS

1. Finite set :

A set is said to be a finite set, if it has a limited (countable) number of elements in it.

For example :

(i) S = Set of natural numbers between 10 and 15 = {11, 12, 13, 14}

(ii) P = {0, 1, 2, ..., 20} = {$x : x \in W$ and $x \le 20$} and so on.

2. Infinite set :

A set is said to be an infinite set, if it has an unlimited (uncountable) number of elements in it.

For example :

(i) P = Set of prime numbers = {2, 3, 5, ...}

(ii) B = {$x : x \in N$ and $x \ge 21$} = {21, 22, 23, ...} and so on.

3. Empty set or Null set :

The set, with no element in it, is called the empty set or the null set.

The empty set is represented by a pair of braces with no element in it or by the Danish letter ϕ, which is pronounced as '*oe*'.

Thus, the empty set = { } = ϕ.

> For empty set, it is wrong to call '*an empty set*' or '*a null set*' as there is one and only one empty set though it may have many descriptions.
>
> Therefore, it is always called "*the empty set*" or "*the null set*".

Some examples of the empty set :

(i) Let A = {a man of age more than 400 years}.

Since there can not be any man with the age more than 400 years, the set A will have no element in it, *i.e.*, it is the empty set. And we write : A = { } or ϕ.

(ii) If B = {Triangles with 4 sides}, it is clear that B = ϕ.

> 1. $\phi \ne \{ 0 \}$, since {0} is a set with 0 as its element whereas ϕ has no element.
>
> 2. { ϕ } $\ne$ { 0 }, since both the sets have different elements.
>
> 3. The cardinal number of the empty set is 0, i.e., $n(\phi) = 0$.

4. **Disjoint sets :**

Sets having no element in common are called disjoint sets.

For example :

Sets P = {5, 7, 9} and Q = {4, 6, 10, 12} *are disjoint,* as they do not have any element in common.

5. **Joint** *(overlapping)* **sets :**

Sets having atleast one element in common are called joint or overlapping sets.

For example :

Set B = {4, 6, 8, 10, 12} and set C = {3, 6, 9, 12, 15} are *joint sets,* as they have elements 6 and 12 common.

6. **Equal sets :**

Two sets are said to be equal, if the elements of both the sets are the same.

For example :

If set A = {x, y, z} and set B = {last three letters of English alphabet}.

Clearly, sets A and B have the same elements and so **set A = set B.**

7. **Equivalent sets :**

Two sets are said to be equivalent, if they have equal number of elements in them, i.e., the cardinal numbers of both the sets are equal.

For example :

Let A = {3, 6, 9} and B = {a, b, c}.

Since, set A has 3 elements and set B also has 3 elements, *i.e.,* $n(A) = n(B)$. Therefore, sets A and B are equivalent and for this, we write : $A \leftrightarrow B$.

1. *Equal sets are always equivalent, but the converse is not always true (i.e., it is not necessary that equivalent sets are equal also).*

2. *In equivalent sets, the number of elements (cardinal number) are equal, whereas in equal sets the elements are the same.*

3. *Two infinite sets are always equivalent.*

─────────── EXERCISE 34(B) ───────────

1. Write the **cardinal number** of each of the following sets :

   (i) A = Set of days in a leap year.

   (ii) B = Set of numbers on a clock-face.

   (iii) C = {$x : x \in N$ and $x \leq 7$}

   (iv) D = Set of letters in the word "PANIPAT".

   (v) E = Set of prime numbers between 5 and 15.

   (vi) F = {$x : x \in Z$ and $-2 < x \leq 5$}

   (vii) G = {$x : x$ is a perfect square number, $x \in N$ and $x \leq 30$}.

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2. For each set, given below, state whether it is **finite set, infinite set** or **the null set** :
   (i)   {natural numbers more than 100}.
   (ii) A = {x : x is an integer between 1 and 2}.
   (iii) B = {x : $x \in$ W; x is less than 100}.
   (iv) Set of mountains in the world.
   (v)   {multiples of 8}.
   (vi) {even numbers not divisible by 2}.
   (vii) {squares of natural numbers}.
   (viii) {coins used in India}
   (ix) C = {x | x is a prime number between 7 and 10}.
   (x)   Planets of the Solar system.

3. State, which of the following pairs of sets are **disjoint** :
   (i)   {0, 1, 2, 6, 8} and {odd numbers less than 10}.
   (ii) { birds } and { trees }
   (iii) {x : x is a fan of cricket} and {x : x is a fan of football}.
   (iv) A = {natural numbers less than 10} and B = {x : x is a multiple of 5}.
   (v)   {people living in Calcutta} and {people living in West Bengal}

4. State whether the given pairs of sets are **equal** or **equivalent** :
   (i)   A = {first four natural numbers} and B = {first four whole numbers}.
   (ii) A = Set of letters of the word "FOLLOW" and
         B = Set of letters of the word "WOLF".
   (iii) E = {even natural numbers less than 10} and
         O = {odd natural numbers less than 9}.
   (iv) A = {days of the week starting with letter S} and
         B = {days of the week starting with letter T}.
   (v)   M = {multiples of 2 and 3 between 10 and 20} and
         N = {multiples of 2 and 5 between 10 and 20}.
   (vi) P = {prime numbers which divide 70 exactly} and
         Q = {prime numbers which divide 105 exactly}.
   (vii) A = {0^2, 1^2, 2^2, 3^2, 4^2} and B = {16, 9, 4, 1, 0}.
   (viii) E = {8, 10, 12, 14, 16} and F = {even natural numbers between 6 and 18}.
   (ix) A = {letters of the word SUPERSTITION} and
         B = {letters of the word JURISDICTION}.

5. Examine which of the following sets are the **empty sets** :
   (i)   The set of triangles having three equal sides.
   (ii) The set of lions in your class.
   (iii) {x : x + 3 = 2 and $x \in$ N}
   (iv) P = {x : 3x = 0}

6. State **true** or **false** :
   (i)   All examples of the empty set are equal.
   (ii) All examples of the empty set are equivalent.
   (iii) If two sets have the same cardinal number, they are equal sets.
   (iv) If n(A) = n(B), then A and B are equivalent sets.
   (v)   If B = {x : x + 4 = 4}, then B is the empty set.
   (vi) The set of all points in a line is a finite set.
   (vii) The set of letters in your Mathematics book is an infinite set.
   (viii) If M = {1, 2, 4, 6} and N = {x : x is a factor of 12}, then M = N.
   (ix) The set of whole numbers greater than 50 is an infinite set.
   (x)   If A and B are two different infinite sets, then n(A) = n(B).

7. Which of the following represent the **null set** ?
   ϕ, { 0 }, 0, { }, { ϕ }.