CHAPTER 32

VOLUME AND SURFACE AREA

[FOR A REGULAR SHAPED BODY]

32.1 VOLUME AND SURFACE AREA

1. Volume :

Volume of a body is the space occupied by it.

Infact :

1. For a solid body :

Its volume = The volume of the material in the body
= Amount of space occupied by the body

2. For a hollow body :

(i) Its external volume = Amount of space occupied by the body.

(ii) Its internal volume = Amount of space inside the body.

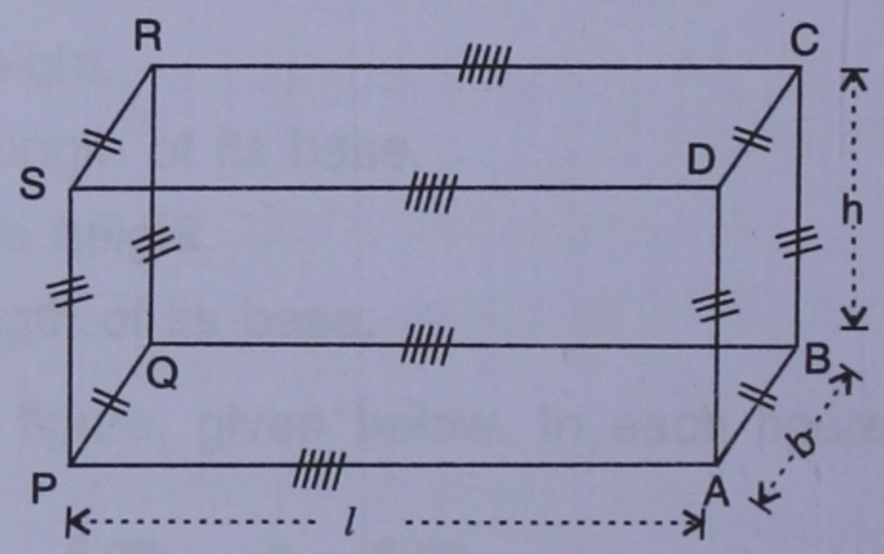
(iii) Volume of material used to make the body
= External volume of the body – Its internal volume

2. Surface area :

Surface area of a body is the sum of the areas of all its faces.

32.2 CUBOID

In everyday life, we come across many objects like, a match box, a brick, etc. Each of these objects is a cuboid in shape.



A body which has six faces, all rectangles, is called a ***cuboid*** *or a* ***rectangular solid.***

As shown in the adjoining fiugre :

(i) *The cuboid has 12 edges,* namely :

AB, BC, CD, DA, PQ, QR, RS, SP, PA, QB, SD and RC such that the opposite edges are equal in length.

$\Rightarrow$ PA = QB = SD = RC = length (l) of the cuboid,

AB = DC = PQ = SR = breadth (b) of the cuboid,

and, AD = BC = PS = QR = height (h) of the cuboid.

(ii) *The cuboid has 6 faces,* namely :

ABCD, PQRS, ABQP, DCRS, ADSP and BCRQ such that the *opposite faces are equal in area.*

$\Rightarrow$ area of face ABQP = area of face DCRS = $l \times b$,

area of face ABCD = area of face PQRS = $b \times h$,

and, area of face ADSP = area of face BCRQ = $h \times l$

∴ Total surface area of the cuboid

= sum of the areas of its six faces

= area of (ABQP + DCRS + ABCD + PQRS + ADSP + BCRQ)

= $l \times b + l \times b + b \times h + b \times h + h \times l + h \times l$

= $\mathbf{2(l \times b + b \times h + h \times l)}$.

Volume of the cuboid = Its length × breadth × height

= $\mathbf{l \times b \times h}$.

Example 1 :

Find the total surface area and the volume of a cuboid, whose :

(i) length = 25 cm, breadth = 20 cm and height = 12 cm.

(ii) length = 1.8 m, breadth = 1.2 m and height = 60 cm.

Solution :

(i) Given : l = 25 cm, b = 20 cm and h = 12 cm.

∴ **Total surface area** = $2(l \times b + b \times h + h \times l)$

= $2(25 \times 20 + 20 \times 12 + 12 \times 25)$ cm^2

= **2080 cm^2** **(Ans.)**

And, **volume** = $l \times b \times h$

= 25 cm × 20 cm × 12 cm

= **6000 cm^3** **(Ans.)**

(ii) Given : l = 1.8 m, b = 1.2 m and h = 60 cm = $\frac{60}{100}$ m = 0.6 m

∴ **Total surface area** = $2(l \times b + b \times h + h \times l)$

= $2(1.8 \times 1.2 + 1.2 \times 0.6 + 0.6 \times 1.8)$ m^2

= **7.92 m^2** **(Ans.)**

And, **volume** = $l \times b \times h$

= 1.8 m × 1.2 m × 0.6 m

= **1.296 m^3** **(Ans.)**

32.3 CUBE

A cuboid whose length, breadth and height are equal to each other is called a ***cube.***

For a cube : **length = breadth = height** and each of its six faces is a square.

Since, each face of a cube is a square : area of each face = (side)^2 = l^2

∴ Total surface area of the cube

= Sum of the areas of its six faces

= 6 × (side)^2 = $\mathbf{6 \times l^2}$

Volume of the cube = Its length × breadth × height

= $l \times l \times l$ [∵ *In a cube,* $l = b = h$]

= $\mathbf{l^3}$

Example 2 :

Find the total surface area and the volume of a cube with each side :

(i) 16 cm (ii) 1.2 m

Solution :

(i) **Total surface area of the cube** = $6 \times (\text{side})^2$

= $6 \times (16)^2$ cm^2 = **1536 cm^2** **(Ans.)**

And, **volume** = $(\text{side})^3$

= $(16)^3$ cm^3 = **4096 cm^3** **(Ans.)**

(ii) **Total surface area of the cube** = $6\ l^2$

= $6 \times (1.2)^2$ m^2 = **8.64 m^2** **(Ans.)**

And, **its volume** = $l^3 = (1.2)^3$ m^3 = **1.728 m^3** **(Ans.)**

32.4 AREA OF THE FOUR WALLS OF A ROOM

A room is in the form of a hollow cuboid with 4 walls, each rectangular in shape. The opposite walls being equal in area.

In one pair of opposite walls, the area of each wall = $l \times h$

and in the other pair of opposite walls, the area of each wall = $b \times h$.

$\therefore$ **Total area of 4 walls of a room** (including door and windows)

= $2 \times l \times h + 2 \times b \times h$

= $\mathbf{2(l + b) \times h}$

Example 3 :

(i) Find the area of the four walls of a room whose dimensions are 8 m, 4.5 m and 3 m. Find the cost of distempering the walls at the rate of ₹ 20 per m^2.

(ii) Also, find the cost of white washing its roof at the rate of ₹ 15 per m^2.

Solution :

Given : Length (l) = 8 m, breadth (b) = 4.5 m and height (h) = 3 m.

(i) **Area of four walls** = $2(l + b) \times h$

= $2(8 + 4.5) \times 3$ m^2

= $2 \times 12.5 \times 3$ m^2 = **75 m^2** **(Ans.)**

Cost of distempering 1 m^2 = ₹ 20

$\therefore$ **Cost of distempering 75 m^2** = 75 × ₹ 20 = **₹ 1,500** **(Ans.)**

(ii) Area of the roof = $l \times b$

= 8 m × 4.5 m = 36 m^2

Cost of white washing 1 m^2 = ₹ 15

$\therefore$ **Cost of white washing** 36 m^2 = 36 × ₹ 15 = **₹ 540** **(Ans.)**

Example 4 :

The volume of a tank, which is cuboid in shape, is 6.4 m^3. The dimensions of its base are 2 m × 1.6 m. Find the depth of the tank.

Solution :

Given : Length of the tank, l = 2 m, its breadth, b = 1.6 m and its volume = 6.4 m^3.

To find : depth (h) of the tank.

Since, the volume of the tank = $l \times b \times h$

$\Rightarrow$ $6.4 = 2 \times 1.6 \times h$

$\Rightarrow$ $h = \dfrac{6.4}{2 \times 1.6}$ m = 2 m

$\therefore$ **Depth of the tank** = 2 m (Ans.)

Example 5 :

Bricks of size 20 cm, 10 cm and 8 cm are used to build a wall whose length, breadth and height are 12 m, 30 cm and 3.5 m respectively. How many bricks will be required ?

Solution :

$\because$ Volume of one brick = 20 cm $\times$ 10 cm $\times$ 8 cm = 1600 cm^3

And, volume of the wall = 12 m $\times$ 30 cm $\times$ 3.5 m

= 1200 cm $\times$ 30 cm $\times$ 350 cm

= 12600000 cm^3

Number of bricks required = $\dfrac{\text{Volume of the wall}}{\text{Volume of each brick}}$

= $\dfrac{12600000}{1600}$ = 7875 (Ans.)

EXERCISE 32

1. Find the total surface area and the volume of a cuboid with :
   (i) length = 4 m, breadth = 3 m and height = 2 m
   (ii) length = 30 cm, breadth = 25 cm and height = 10 cm
   (iii) length = 4 m, breadth = 2 m 40 cm and height = 80 cm.
2. Find the total surface area and the volume of a cube whose each edge is :
   (i) 5 m (ii) 24 m (iii) 1 m 20 cm.
3. Rain water falls in a tank of base area 1.5 m $\times$ 0.8 m. Find the height to which it will fill in the tank, if the total volume of water collected in the tank is 2.88 m^3.
4. Water is to be transferred from one tank to the other tank of dimensions 2.1 m $\times$ 1.5 m $\times$ 0.6 m such that the second tank is completely filled with this water. The length and breadth of the first tank are 2 m and $\frac{1}{2}$ m respectively. Find the height of the water when it was in the first tank.
5. How many bricks, each measuring 18 cm $\times$ 12 cm $\times$ 6 cm, will be needed to build a wall of length = 6 m, width = 24 cm and height = 3.6 m ?
6. Find the area of four walls of a cube whose one edge is $2\frac{1}{2}$ m.
7. Find the area of four walls of a room whose length = 3.2 m, breadth = 2.5 m and height = 3 m.
8. Find the total surface area of a box whose length = 25 cm, breadth = 15 cm and height = 10 cm.

9. If the total surface area of a cubical box is 216 cm^2, find the length of its one side. Also, find the area of its four walls.

10. If the total surface area of a cube is 96 cm^2, find the length of its each side and its volume.

11. The volume of air in a room is 204 m^3. The height of the room is 6 m. What is the floor area of the room ?

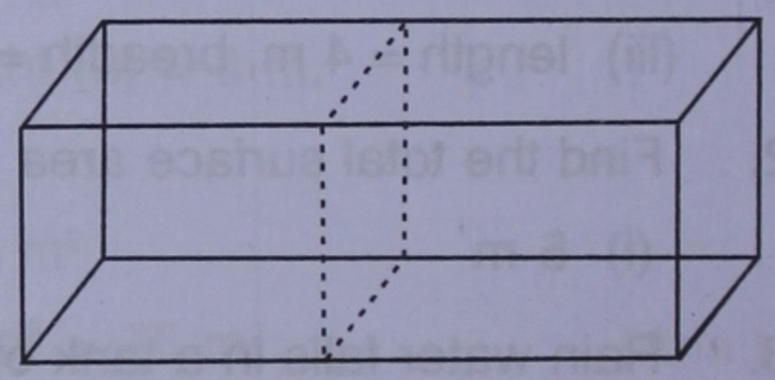
12. The dimensions of a room are 8 m, 6 m and 5 m. Find the cost of :
    (i) distempering its four walls at the rate of ₹ 30 per m^2.
    (ii) white washing its roof at the rate of ₹ 20 per m^2.

13. The dimensions of a room are, length = 4.8 m, breadth = 3.2 m and height = 3 m. The room has a window of size 1 m × 75 cm and two doors, each of size 1 m × 2 m. Find :
    (i) the area of 4 walls of the room, including doors and window.
    (ii) the area of 4 walls of the room, excluding doors and window.
    (iii) the cost of distempering the walls of the room at the rate of ₹ 40 per m^2.
    (iv) the cost of polishing the doors and window at the rate of ₹ 60 per m^2.

14. A room is 4.8 m long, 3.6 m broad and 2.4 m high. Find the cost of laying tiles on its floor and on its four walls at the rate of ₹ 80 per m^2.

15. The inside of a room has a square base of side 3·6 m. The inside height of the room is 4 m. Find :
    (i) the space (internal volume) of the room.
    (ii) how many boxes each of dimensions 0·9 m × 40 cm × 25 cm can be placed in the room.

16. The figure, given alongside, shows two identical cubes each of side 12 cm. For the resulting solid, find :
    (i) total surface area.
    (ii) volume.



17. A cubical solid of metal has length, breadth and height as 40 cm, 30 cm and 25 cm respectively.
    (i) Find the volume of the solid cuboid.
    (ii) If this solid is melted and recasted into solid cubes each of side 5 cm, find the number of cubes formed.
    (iii) Find the total surface area of all the cubes formed.

18. A rectangular tank has length = 5 m and breadth = 3 m. It is filled with water upto 60 cm depth. Find :
    (i) volume of water in the tank.
    (ii) area of walls, in the tank, in contact of water.
    (iii) the wetted surface area of the tank.