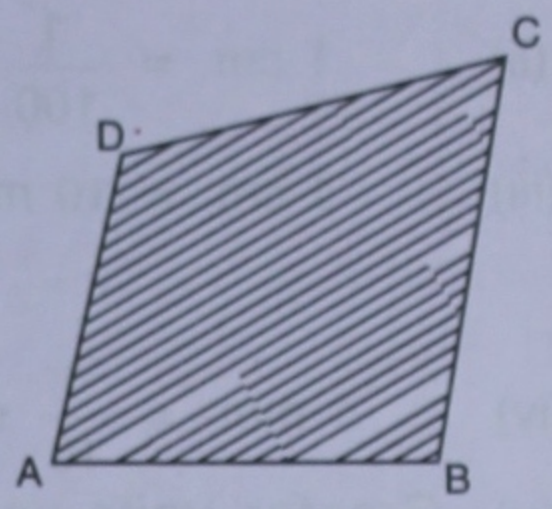
UNIT – 5
MENSURATION

CHAPTER 31
PERIMETER AND AREA
[For Plane Figure]

31.1 PERIMETER

The perimeter of a plane figure is the length of its boundary.



Thus, the perimeter of the given figure (quadrilateral)

$$= AB + BC + CD + DA$$

31.2 AREA

The area of a plane figure is the amount of surface enclosed by its sides.

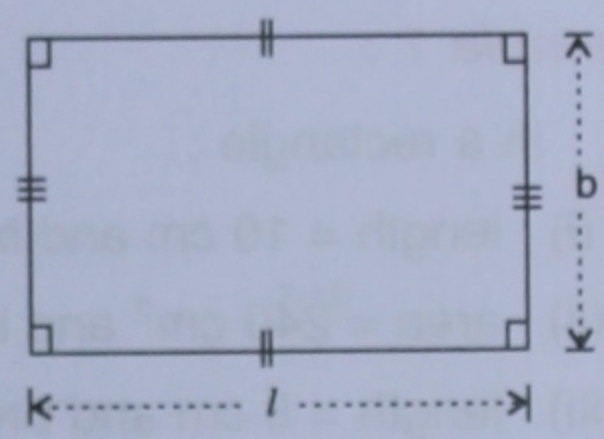
In the figure, given above, the shaded portion shows its area.

31.3 PERIMETER AND AREA OF SOME SPECIAL FIGURES

1. Rectangle :

A rectangle is a four sided closed figure with opposite sides equal and each angle 90°.

In general, the *longer side* of a rectangle is called its *length* and is denoted by letter '*l* ' whereas, the *shorter side* is called its *breadth* and is denoted by letter '*b*'.



$\therefore$ Perimeter, P = Length of its boundary

$= l + b + l + b$

$= 2l + 2b \Rightarrow$ **$P = 2(l + b)$**

And, area, A = its length $\times$ its breadth $\Rightarrow$ **$A = l \times b$**

2. Square :

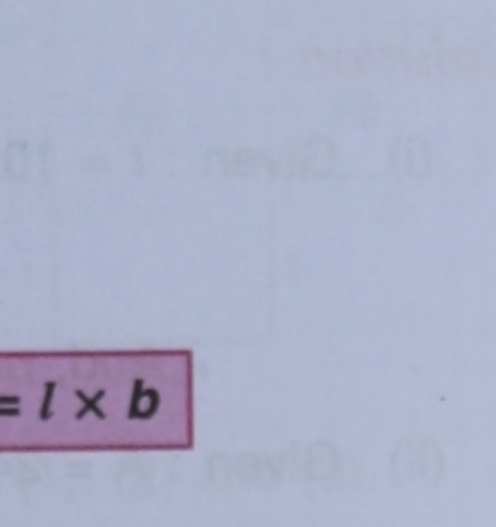
A square is a four sided closed figure with all its sides equal and each angle of 90°.

Clearly, its perimeter P = Length of its boundary

$= l + l + l + l \Rightarrow$ **$P = 4l$**

And, its area, A = length $\times$ breadth

$= l \times l \Rightarrow$ **$A = l^2$**



31.4 UNITS OF PERIMETER AND AREA

If the *sides* are in *centimetre* (cm), the unit of *perimeter* is also in *centimetre* and the unit of *area is square centimetre* (cm^2).

Similarly, if *the sides* are *in metre* (m), the unit of *perimeter* is also in *metre* and the unit of *area is square metre* (m^2).

(i) $1\ m = 100\ cm$ and $1\ m^2 = 100\ cm \times 100\ cm = 10{,}000\ cm^2$

(ii) $1\ cm = \frac{1}{100}\ m$ and $1\ cm^2 = \frac{1}{10{,}000}\ m^2$

(iii) $1\ cm = 10\ mm$ and $1\ cm^2 = 10\ mm \times 10\ mm = 100\ mm^2$

(iv) $1\ mm = \frac{1}{10}\ cm$ and $1\ mm^2 = \frac{1}{100}\ cm^2$

(v) *Greater units* used *for area* (usually, for the area of land) are *Are* and *Hectare,* such that :

$1\ Are = 100\ m^2$

and $1\ Hectare = 100\ Are = 100 \times 100\ m^2 = 10{,}000\ m^2$.

Example 1 :

In a rectangle :

(i) length = 10 cm and breadth = 6 cm, find its area and its perimeter.

(ii) area = 240 cm^2 and length = 20 cm, find its breadth and perimeter.

(iii) length = 8 cm and breadth = 8 cm, find its area and perimeter.

Solution :

(i) Given : $l = 10$ cm and $b = 6$ cm

Area $= l \times b = 10\ cm \times 6\ cm =$ **60 cm^2** **(Ans.)**

And, **perimeter** $= 2(l + b) = 2(10 + 6)\ cm = 2 \times 16\ cm =$ **32 cm** **(Ans.)**

(ii) Given : $A = 240\ cm^2$ and $l = 20$ cm

$\therefore$ $A = l \times b \Rightarrow$ **breadth**, $b = \frac{A}{l}$

$= \frac{240}{20}\ cm =$ **12 cm** **(Ans.)**

and, **perimeter** $= 2(l + b)$

$= 2(20 + 12)\ cm = 2 \times 32\ cm =$ **64 cm** **(Ans.)**

(iii) Here $l = 8$ cm and $b = 8$ cm [Since, $l = b$, its a square]

$\therefore$ **Area** $= l^2$

$= 8\ cm \times 8\ cm =$ **64 cm^2** **(Ans.)**

and, **perimeter** $= 4l$

$= 4 \times 8\ cm =$ **32 cm** **(Ans.)**

Example 2 :

The length of a rectangular field is 200 m and its width is 100 m.

Find : (i) the cost of ploughing it at the rate of ₹ 10 per m^2.

(ii) the cost of fencing it with wire at the rate of ₹ 15 per metre.

Solution :

(i) For ploughing, we need to calculate the area (A),

Since, l = 200 m and b = 100 m

∴ Area of the field = 200 m × 100 m = 20,000 m^2

And, **cost of ploughing the field**

= Area × Rate

= 20,000 × ₹ 10 = **₹ 2,00,000** **(Ans.)**

(ii) Length of fence = Perimeter

= 2 (l + b)

= 2(200 + 100) m = 600 m

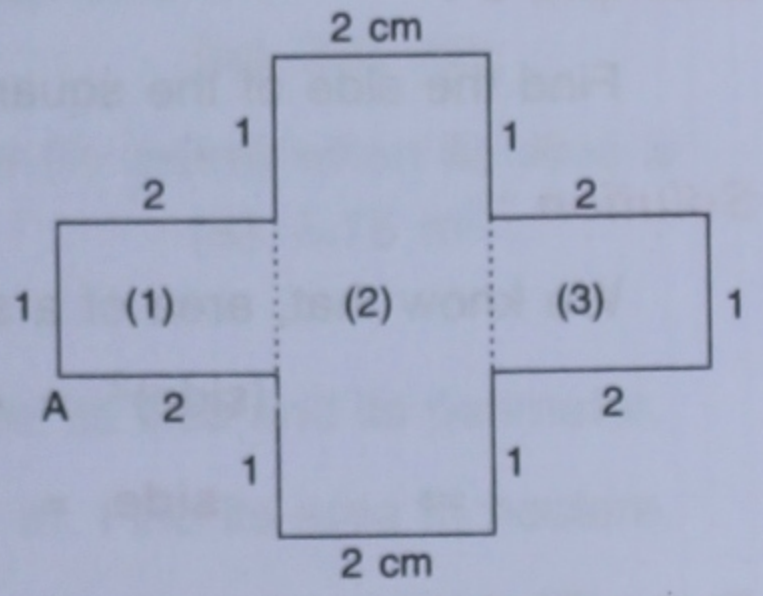
∴ **Cost of fencing** = Length of fence × Rate

= 600 × ₹ 15

= **₹ 9,000** **(Ans.)**

Example 3 :

Find the area and the perimeter of the given figure. All measurements are in cm and the angle at each vertex is 90°.



Solution :

For such figures, first of all draw dotted lines to divide the figure in convenient parts of squares and rectangles.

As shown in the figure, the three parts obtained are marked as (1), (2) and (3).

Now, find the area of each part.

Area of rectangle shown by part (1) = 2 cm × 1 cm = 2 cm^2

Area of rectangle shown by part (2) = 2 cm × 3 cm = 6 cm^2

Area of rectangle shown by part (3) = 2 cm × 1 cm = 2 cm^2

∴ **Total required area** = 2 cm^2 + 6 cm^2 + 2 cm^2 = **10 cm^2** **(Ans.)**

For finding the perimeter, we have to add the outer boundary lines (not the dotted lines).

For this, the simplest way is to start adding the sides, starting from any point of its boundary and then reach to the same point again.

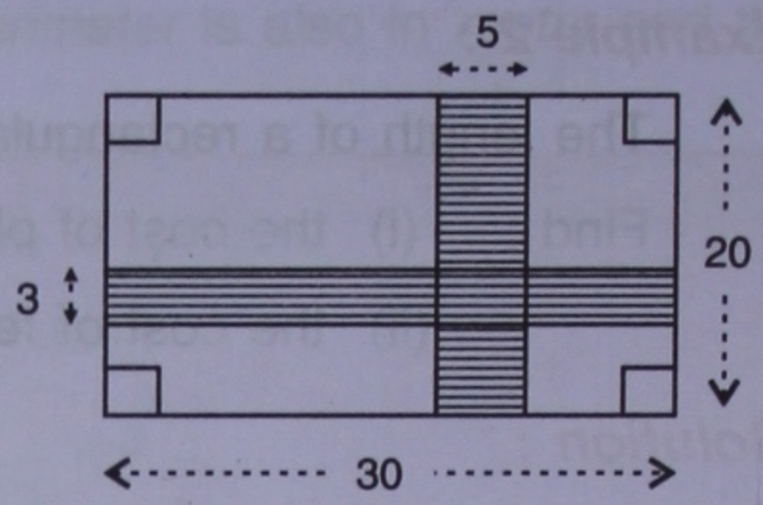
Here, if we start from A and move to right (in the anticlockwise direction), we get :

Perimeter = (2 + 1 + 2 + 1 + 2 + 1 + 2 + 1 + 2 + 1 + 2 + 1) cm = **18 cm** **(Ans.)**

Example 4 :

Use the informations given in the adjoining figure to find the area of the shaded portion.

[Every measurement, given in the figure, is in metre].



Solution :

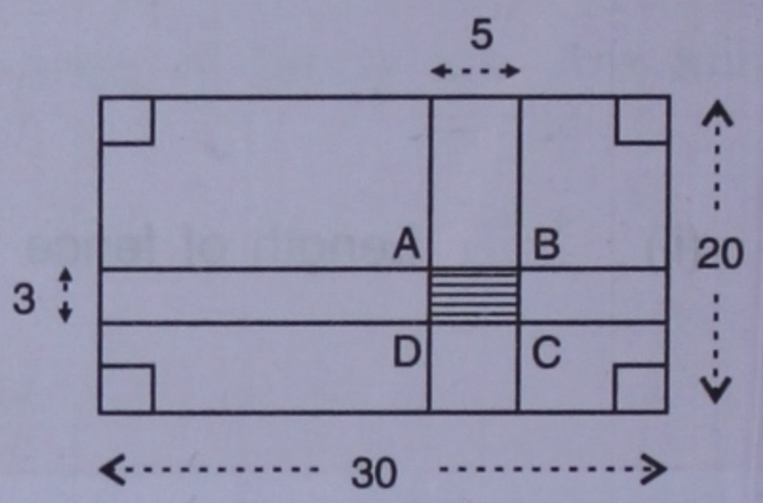
Area of the shaded portion along the length

$= 3 \text{ m} \times 30 \text{ m} = 90 \text{ m}^2$

Area of the shaded portion along the width

$= 5 \text{ m} \times 20 \text{ m} = 100 \text{ m}^2$

The portion ABCD with AB = 5 m and BC = 3 m is common to both the shaded portions, one along the length and the other along the width. So, this portion has been taken twice. Since area of this portion ABCD = 5 m × 3 m = 15 m^2.



$\therefore$ **The area of the shaded portion**

$= 90 \text{ m}^2 + 100 \text{ m}^2 - 15 \text{ m}^2$

$= \mathbf{175 \text{ m}^2}$ **(Ans.)**

Example 5 :

Find the side of the square, whose area is 441 sq. cm.

Solution :

We know that, area of a square = $(\text{side})^2$

$\therefore \quad (\text{side})^2 = 441 \text{ cm}^2$

$\Rightarrow \quad \textbf{side} = \sqrt{441} \text{ cm} = \mathbf{21 \text{ cm}}$ **(Ans.)**

Example 6 :

Area of a square is 100 Hectare. Find its each side and perimeter.

Solution :

If the side of a square is l m, its area = l^2 sq. m. = (m^2)

Given, area of the square = 100 Hectare

$\Rightarrow \quad l^2 = 100 \times 10{,}000 \text{ m}^2 \quad [\because 1 \text{ Hectare} = 10{,}000 \text{ m}^2]$

$\therefore \quad l = \sqrt{100 \times 10{,}000} \text{ m} = 1{,}000 \text{ m}$

$\Rightarrow$ **Side of the given square = 1,000 m** **(Ans.)**

Also, **perimeter,** P = $4l$

$= 4 \times 1{,}000 \text{ m} = \mathbf{4{,}000 \text{ m}}$ **(Ans.)**

Example 7 :

If length of a rectangle is 40 cm and its perimeter is 130 cm; find its breadth and area.

Solution :

Given, perimeter = 130 cm

$\Rightarrow$ $2l + 2b = 130$ cm [Since, $P = 2l + 2b$]

$\Rightarrow$ 2×40 cm $+ 2b = 130$ cm

$\Rightarrow$ $2b = 130$ cm $- 80$ cm $= 50$ cm

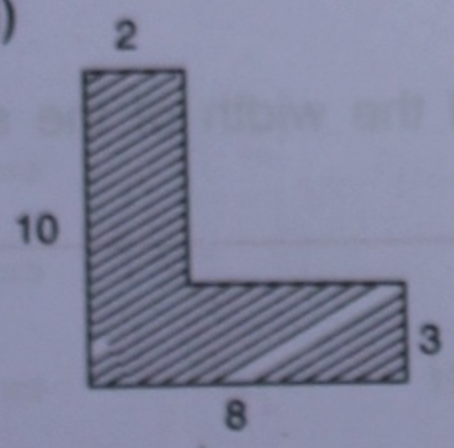
$\therefore$ **Breadth,** $b = \frac{50}{2}$ cm = **25 cm** **(Ans.)**

And, **area** $= l \times b = 40$ cm $\times 25$ cm = **1,000 cm^2** **(Ans.)**

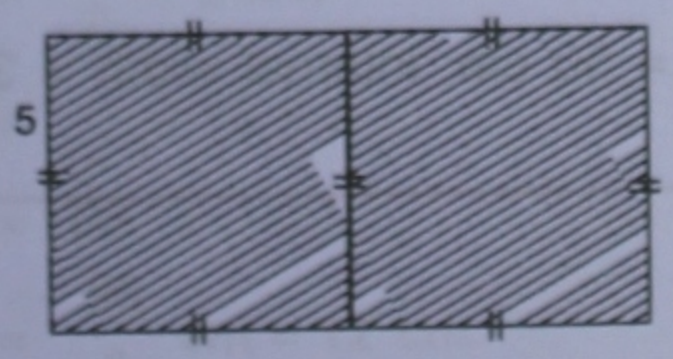
EXERCISE 31(A)

1. Find the area in m^2 and the perimeter in metre for the rectangle whose :
   (i) length = 20 m and breadth = 15 m (ii) length = 100 m and breadth = 250 cm
   (iii) length = 1.2 m and breadth = 10 cm (iv) length = 1 m 30 cm and breadth = 70 cm.
2. Find the area in m^2 of the square whose each side is :
   (i) 25 m (ii) 2000 cm (iii) 1.6 km
3. Find the perimeter and the area of a square whose each side is :
   (i) 2 m 10 cm (ii) 5 m (iii) 4.2 cm (iv) 250 mm
4. Find the side of the square (in metre) and its perimeter (in metre) when its area is :
   (i) 289 m^2 (ii) 400 cm^2 (iii) 5.76 m^2
   (iv) 1.69 cm^2 (v) 225 m^2.
5. The area of a square field is 400 hectares. Find, in metres, its side and its perimeter.
6. The sides of a rectangular ground are 300 m and 120 m. Find its area in hectare.
7. Find the other side and the area of a rectangle whose length (l) and perimeter (P) are :
   (i) $l = 10$ m and P = 34 m (ii) $l = 3.6$ m and P = 10.2 m
8. Find the breadth and the perimeter of a rectangle whose length (l)and area (A) are as given below :
   (i) A = 40 m^2 and $l = 10$ m (ii) A = 200 cm^2 and $l = 16$ cm
   (iii) A = 160 m^2 and $l = 16$ m.
9. Find the area and the perimeter of the following figures.
   All angles are 90° and all sides are in cm.

(i)



(ii)



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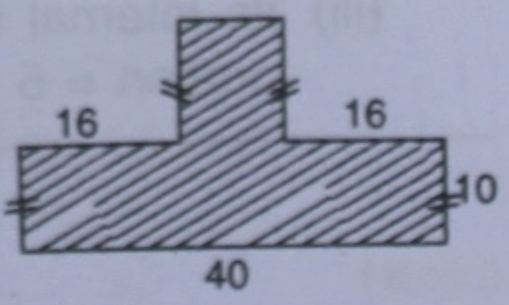
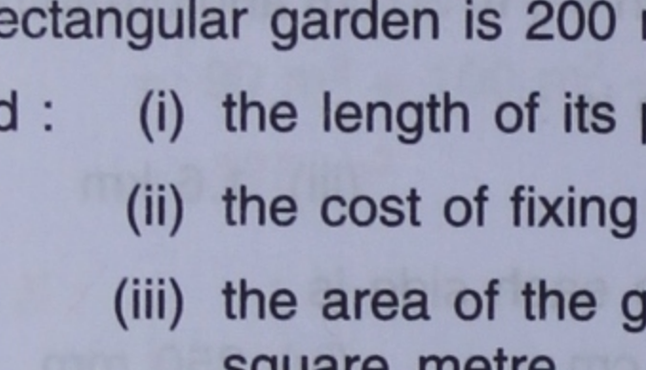


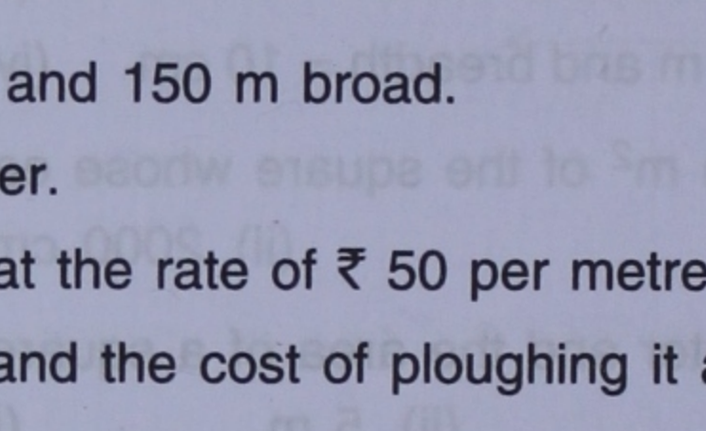
10. Find the cost of distempering four walls of a room at the rate of ₹ 20 per m^2. Each wall is a square of side 4 m.

11. Find the area of (i) shaded part, (ii) unshaded part in each figure, given below : (Each measurement is in cm)

(i)



(ii)



12. The area of a square field is 1024 m^2.

Find : (i) length of its each side.

(ii) its periemeter.

(iii) the time taken by a man to make 5 rounds of the field, walking at a speed of 4.5 km/h.

13. A rectangular garden is 200 m long and 150 m broad.

Find : (i) the length of its perimeter.

(ii) the cost of fixing fence at the rate of ₹ 50 per metre.

(iii) the area of the garden and the cost of ploughing it at the rate of ₹ 8 per square metre.

14. A hall is 20 m long and 18 m broad. A carpet is to be laid in this hall leaving a margin of 1 m all around.

Find : (i) the length and the breadth of the carpet required.

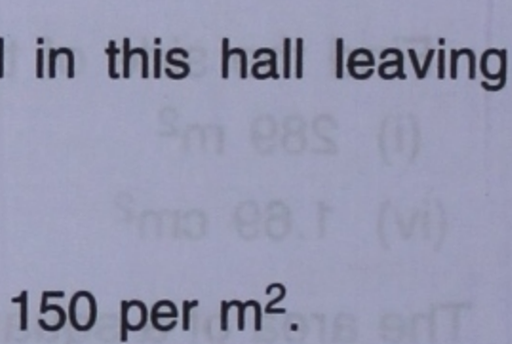
(ii) the area of the carpet and its cost at the rate of ₹ 150 per m^2.

15. The cost of fixing the fence all around a square field is ₹ 12,000. If the rate of the fence is ₹ 30 per metre, find :

(i) the perimeter of the square field.

(ii) the length of each side and the area of the field.

16. The shaded portion in the given figure, has uniform width. Find the area of the shaded portion, if :



(i) its external length = 18 cm, external breadth = 12 cm, internal length = 14 cm and internal breadth = 8 cm.

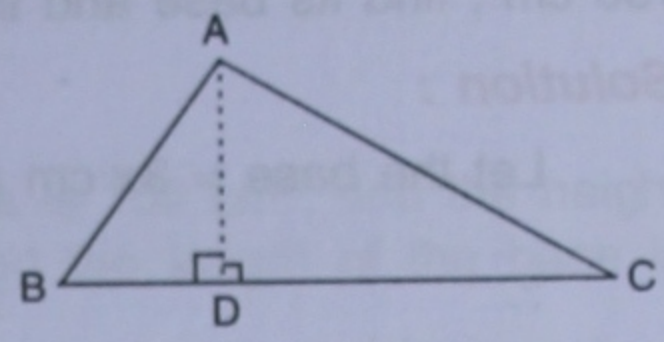
(ii) its external length = 30 cm, external breadth = 20 cm and width of the shaded portion is 4 cm.

(iii) its internal length = 50 cm, internal width = 36 cm and the width of the shaded portion = 5 cm.

31.5 AREA OF A TRIANGLE

A triangle is a three sided closed and plane figure.

And, area of a triangle $= \frac{1}{2} \times$ base $\times$ height.

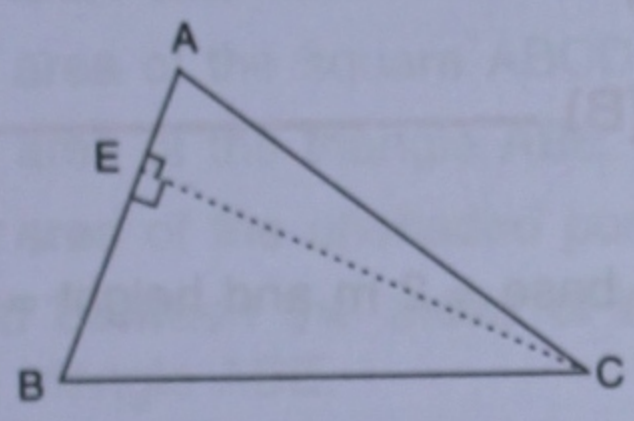


In the triangle ABC, drawn alongside, the **base** is **BC** and its **height** (*the length of the perpendicular from vertex A to the opposite side BC*) is **AD**.

$\therefore$ **Area of** Δ ABC $= \frac{1}{2} \times$ **BC** $\times$ **AD.**

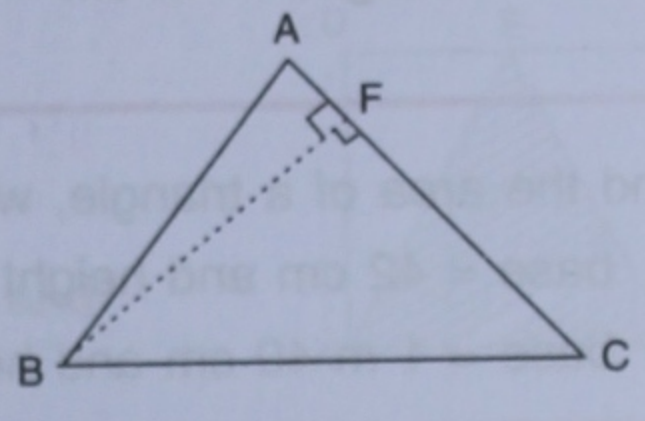
Remember : *Any side of the triangle can be taken as its base then its height will be the length of perpendicular drawn to this side (taken as base) from the opposite vertex.*

1. If side **AB** is taken as **base**, then **CE** (which is perpendicular to AB from opposite vertex C) is its **height**.



[Area of Δ ABC $= \frac{1}{2} \times$ AB $\times$ CE]

2. If side **AC** is taken as **base,** the perpendicular **BF** from vertex B to side AC is its height.



[Area of Δ ABC $= \frac{1}{2} \times$ AC $\times$ BF]

Example 8 :

A triangle has a base of 4 cm and the corresponding altitude (height) is 6 cm. Find its area.

Solution :

Given : base, $b = 4$ cm and height, $h = 6$ cm

$\therefore$ **Area**, A $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \text{ cm} \times 6 \text{ cm} = 12 \text{ cm}^2$ **(Ans.)**

Example 9 :

The area of a triangle is 96 cm^2 and its base is 16 cm. Find its height.

Solution :

Given : Area, A = 96 cm^2 and base, $b = 16$ cm

$\therefore$ Area, A $= \frac{1}{2} \times b \times h$

$\Rightarrow$ $96 = \frac{1}{2} \times 16 \times h$

$\Rightarrow$ $8h = 96$

$\Rightarrow$ **Height**, $h = \frac{96}{8}$ cm $=$ **12 cm.** **(Ans.)**

Example 10 :

The base and the height of a triangle are in the ratio 3 : 4. If its area is 150 cm^2, find its base and the height.

Solution :

Let the base = $3x$ cm and the height = $4x$ cm

$$\therefore \quad \text{Area} = \frac{1}{2} \times 3x \times 4x \qquad \left[\because \text{Area} = \frac{1}{2} \times \text{base} \times \text{height}\right]$$

$$\Rightarrow \quad 150 = 6x^2$$

$$\text{or,} \quad 6x^2 = 150$$

$$\Rightarrow \quad x^2 = \frac{150}{6} = 25$$

$$\Rightarrow \quad x = \sqrt{25} \quad \Rightarrow \quad x = 5$$

Therefore, **base** = $3x$ cm = 3×5 cm = **15 cm** **(Ans.)**

and, **height** = $4x$ cm = 4×5 cm = **20 cm** **(Ans.)**

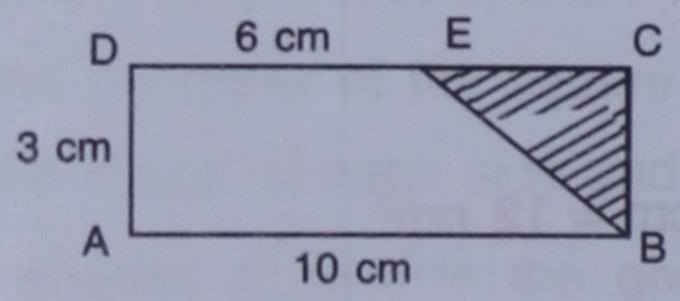
EXERCISE 31(B)

1. Find the area of a triangle, whose :
   (i) base = 42 cm and height = 9 cm
   (ii) base = 2 m and height = 30 cm
   (iii) base = 1 m 40 cm and height = 0.8 m.

2. For a triangle, if :
   (i) area = 280 cm^2 and base = 35 cm, find its height.
   (ii) area = 12.6 m^2 and height = 2.1 m, find the length of its base.
   (iii) area = 4.05 m^2 and base = 3 m 24 cm, find its height.
   (iv) area = 1 m^2 and height = 0.01 m, find the length of its base.

3. Find the area of the shaded part in each of the figure, given below. In each figure ABCD is a rectangle.

(i)



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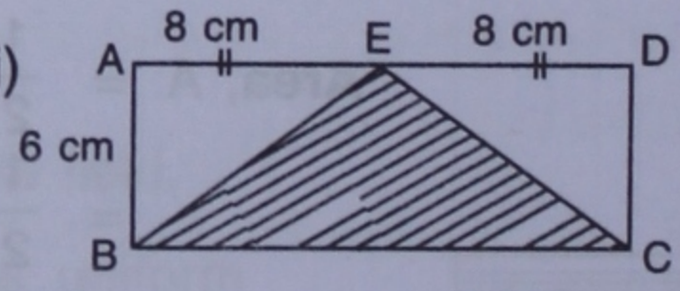

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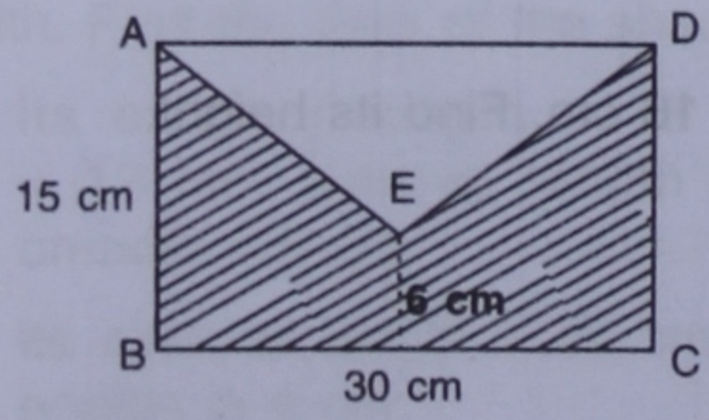

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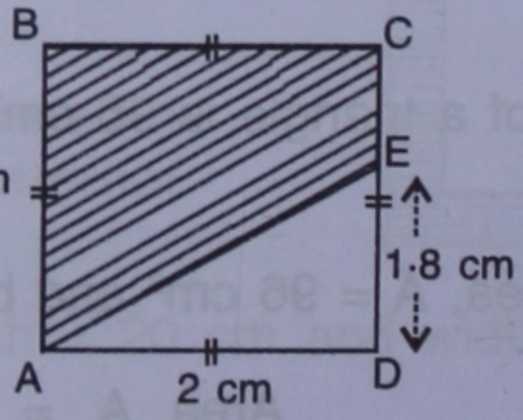

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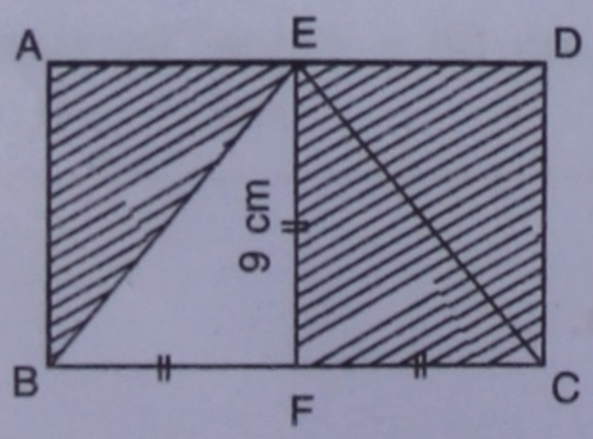

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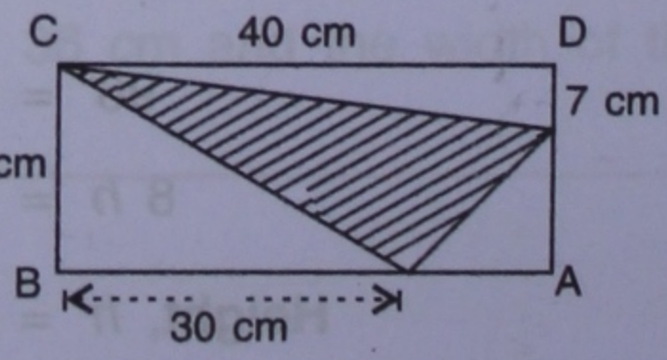

4. Find the lengths of the base and the height of the triangles whose area and ratio of base *(b)* and height *(h)* are given :

(i) area = 27 cm^2 and $b : h = 2 : 3$.

(ii) area = 360 m^2 and $b : h = 4 : 5$.

(iii) area = 216 m^2 and $b : h = \frac{1}{3} : \frac{1}{4}$.

5. The length of the base of a triangle is 12 cm and its area is 108 cm^2, find the height of the triangle. If the height of this triangle is halved and the length of the base is doubled then find :

(i) area of the new triangle,

(ii) increase or decrease in area of the triangle

6. The area of a triangular field is 324 m^2 and its base is 18 m. Find the corresponding height (length of altitude) of the triangle. This triangular field is exchanged with a rectangular field having the same area. If the length of the rectangular field is 24m, find its :

(i) breadth

(ii) perimeter

7. The adjoining figure shows a square ABCD and a triangle ABE. If each side of the square is 24 cm, find :

(i) the area of the square ABCD,

(ii) the area of the triangle ABE,

(iii) the area of the unshaded portion of the figure,

(iv) ratio between the areas of the square ABCD and the triangle ABE.