CHAPTER 29

INTRODUCTION OF THEOREM

29.1 BASIC TREATMENT

Some of the properties of geometrical figures are self-evident and are accepted without any proof. Such self-evident truths are called **axioms** and the statements, stating these self-evident truths, are called **postulates**.

e.g., if $\angle a = \angle b$ and $\angle b = \angle c$, then $\angle a = \angle c$.

With the help of these axioms and postulates, some other important properties, in geometry, can easily be proved. The course of proving such properties is called a **theorem**.

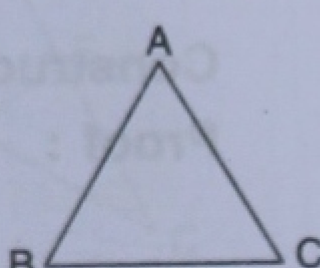
*A **theorem** is a statement about geometrical figures for which a proof is required.*

To prove a theorem following steps are required :

1. *General statement or general enunciation* : This is a combined statement of the facts which are given and those which are to be proved.
   (i) *Figure* : A figure helps in making the proof more understandable.
   (ii) *Particular statement or enunciation* : This is the description of the theorem which is explained with the help of a labelled figure.
   This part is referred to as *given*.
2. *To prove* : The proposition to be proved is written in brief.
3. *Construction* : Sometimes a line is to be drawn or extended or some points are to be joined in the given figure, to make the proof possible. This step is mentioned here but is not always necessary.
4. (i) *Proof* : This gives the step by step statements with reasons, so that the proof appears to be logical.
   It is preferable to write the statements and reasons separately.
   (ii) *Q.E.D.* : The letters Q.E.D. may be written at the end of a theorem and stand for "Quod Erat Demonstrandum", that means :– "which was to be proved". Usually we write "Proved or Hence Proved" in place of Q.E.D.

Theorem 1

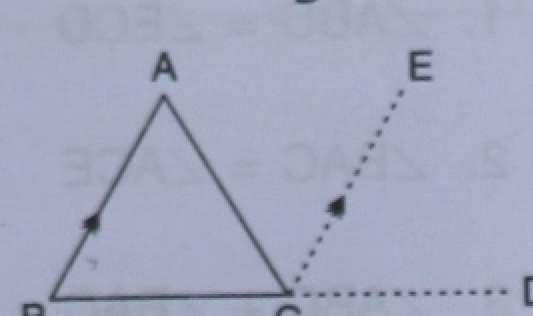
The sum of the angles of a triangle is equal to two right angles.



Given : Triangle ABC

To Prove : $\angle ABC + \angle ACB + \angle BAC$

$= 2$ right angles, *i.e.*, $180°$.



Construction : At C, draw CE parallel to BA. Produce BC to any point D.

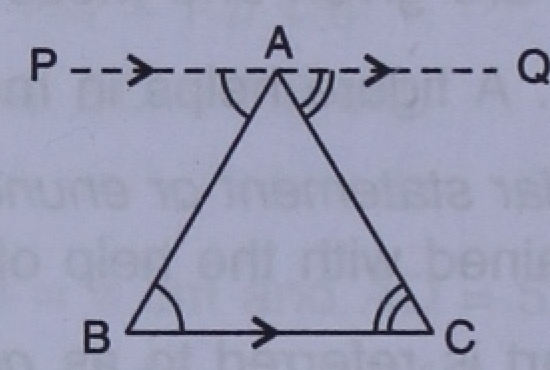
Proof :

Statement :	**Reason :**
1. $\angle ABC = \angle ECD$	Corresponding angles, as AB // EC and BD is transversal.
2. $\angle BAC = \angle ACE$	Alternate angles, as AB // EC and AC is transversal.
3. $\angle ABC + \angle BAC = \angle ECD + \angle ACE$	Adding 1 and 2.
4. $\angle ABC + \angle BAC + \angle ACB$ $= \angle ECD + \angle ACE + \angle ACB$	Adding $\angle ACB$ on both the sides.
5. $\angle ECD + \angle ACE + \angle ACB = 180°$	$\angle ECD + \angle ACE + \angle ACB$ = straight line angle BCD. = 180°
6. **$\angle ABC + \angle BAC + \angle ACB = 180°$**	From 4 and 5. **Hence Proved.**

Alternative Method :

Construction : Through vertex A draw PQ parallel to BC.

Proof :



1. Since, PQ // BC and AB is transversal

$\therefore \angle ABC = \angle BAP$ [Alternate angles]

2. Since, PQ // BC and AC is transversal

$\therefore \angle ACB = \angle CAQ$ [Alternate angles]

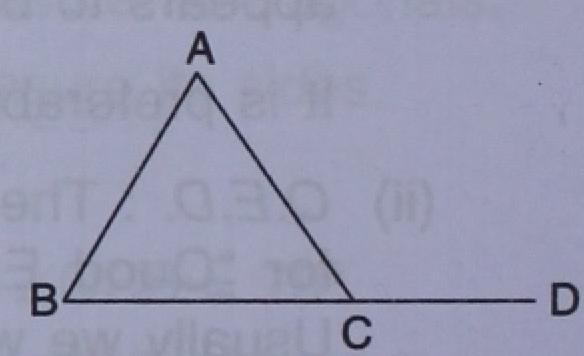
$\angle ABC + \angle ACB = \angle BAP + \angle CAQ$ [Adding results of 1 and 2]

$\Rightarrow \angle ABC + \angle ACB + \angle BAC = \angle BAP + \angle CAQ + \angle BAC$ [Adding $\angle BAC$ on both the sides]

But, $\angle BAP + \angle CAQ + \angle BAC$ = Straight line angle PAQ = 180°

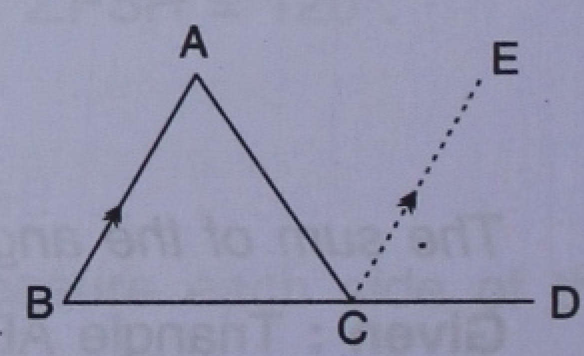
$\therefore \angle ABC + \angle ACB + \angle BAC = 180°$ **Hence Proved.**

Theorem 2



If any side of a triangle is produced, then the exterior angle so obtained is equal to the sum of the two interior opposite angles.

Given : A triangle ABC whose side BC is produced to the point D to form an exterior angle ACD.



To Prove : $\angle ACD = \angle ABC + \angle BAC$

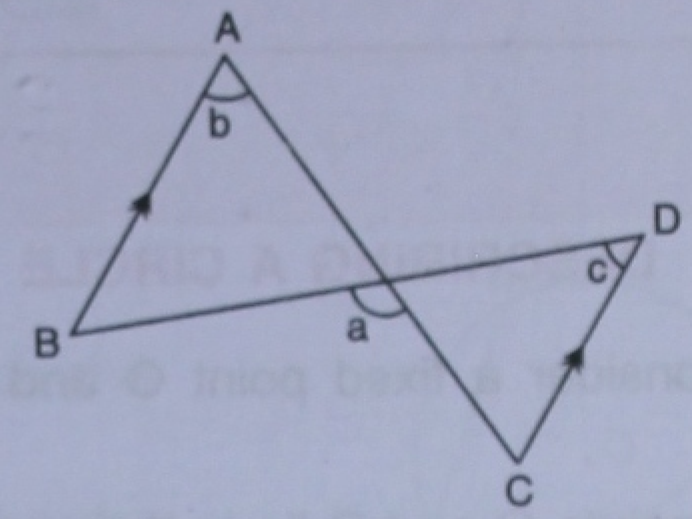
Construction : At the point C, draw CE parallel to BA.

Proof :

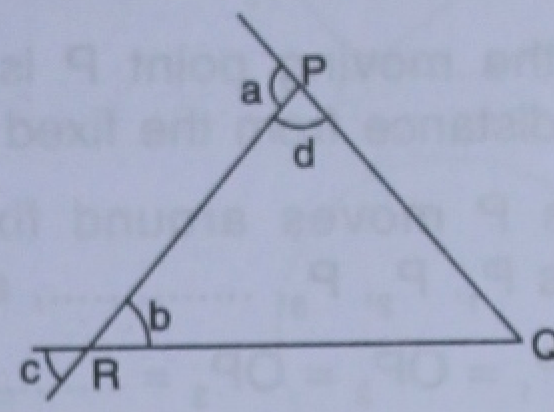
Statement :	**Reason :**
1. $\angle ABC = \angle ECD$	Corresponding angles, as AB // EC and BD is transversal.
2. $\angle BAC = \angle ACE$	Alternate angles, as AB // EC and AC is transversal.
3. $\angle ABC + \angle BAC = \angle ECD + \angle ACE$ $= \angle ACD$	Adding 1 and 2.
$\therefore$ **$\angle ACD = \angle ABC + \angle BAC$**	**Hence Proved.**

EXERCISE 29

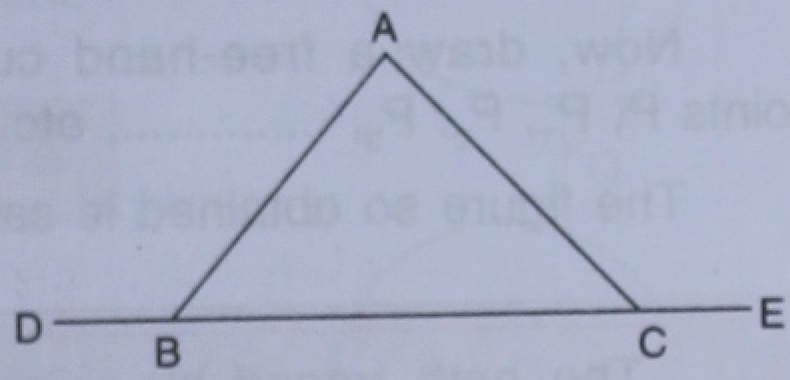
1. In the given figure, prove that :
$\angle a = \angle b + \angle c$.



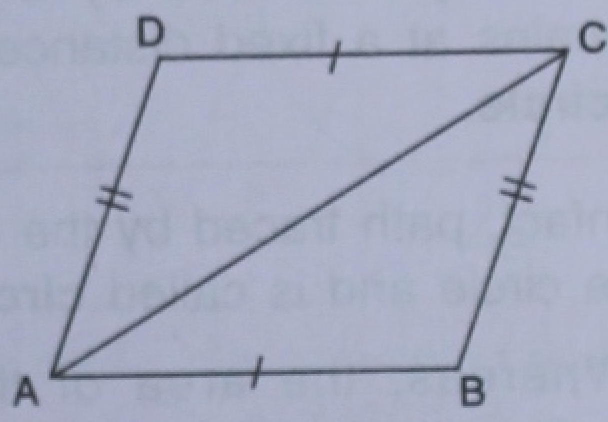
2. Given : $\angle b = \angle d$.
Prove : $\angle a + \angle c = 180°$.



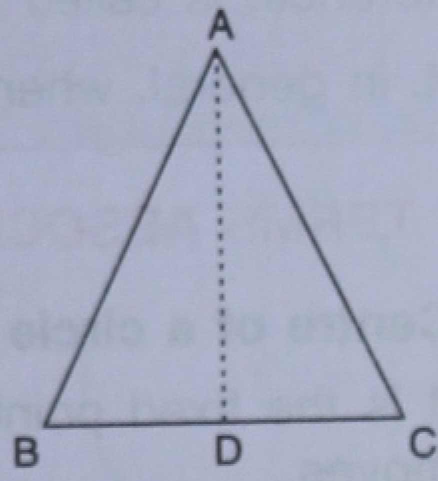
3. Prove : $\angle ABD + \angle ACE = 180° + \angle BAC$.



4. Form the given figure, prove that :
(i) Δ ABC and Δ CDA are congruent.
(ii) $\angle B = \angle D$.



5. Given : AB = AC and BD = DC.
Prove : (i) $\Delta ABD \cong \Delta ACD$,
(ii) $\angle BAD = \angle CAD$.



6. Given : AB = AC, BE $\perp$ AC and CD $\perp$ AB
Prove : (i) $\Delta ABE \cong \Delta ACD$,
(ii) $\angle B = \angle C$.