CHAPTER 27

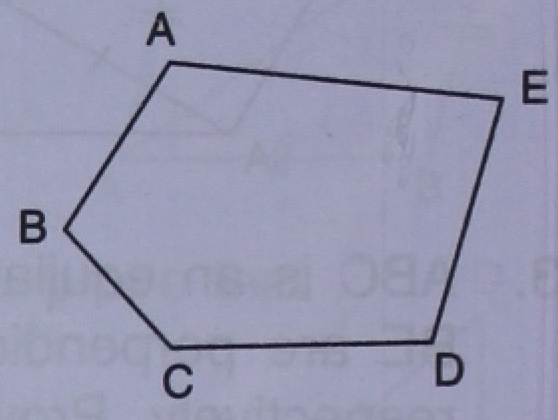
POLYGONS

[INCLUDING QUADRILATERALS AND THEIR TYPES]

27.1 INTRODUCTION

A closed plane geometrical figure, bounded by atleast three line segments, is called a polygon.

The adjoining figure is a polygon as it is :



(i) closed

(ii) bounded by five line segments AB, BC, CD, DE and AE.

Also, it is clear from the given polygon that :

(i) the line segments AB, BC, CD, DE and AE intersect at their end points.

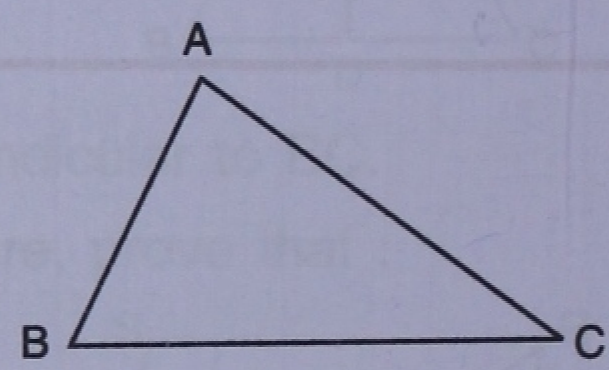
(ii) two line segments, with a common vertex, are not collinear, *i.e.,* the angle at any vertex is not 180°.

A polygon is named according to the number of sides (line-segments) in it :

No. of sides :	3	4	5	6
Name of polygon :	Triangle	Quadrilateral	Pentagon	Hexagon

27.2 SUM OF INTERIOR ANGLES OF A POLYGON

1. *Triangle :*

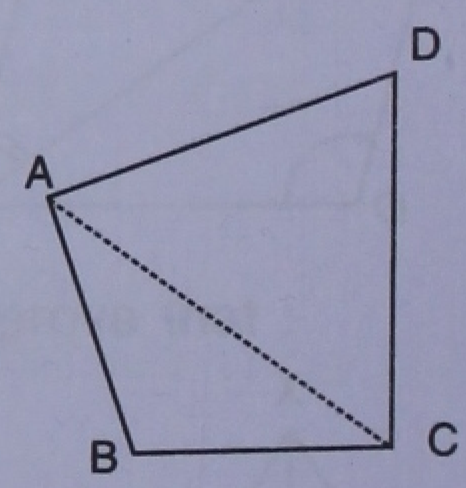


Students already know that the sum of interior angles of a triangle is always 180°.

∴ In Δ ABC, ∠BAC + ∠ABC + ∠ACB = 180°

⇒ **∠A + ∠B + ∠C = 180°**

2. *Quadrilateral :*



Consider a quadrilateral ABCD as shown alongside.

If diagonal AC of the quadrilateral is drawn, the quadrilateral will be divided into two triangles ABC and ADC.

Since, the sum of interior angles of a triangle is 180°.

∴ In Δ ABC, ∠ABC + ∠BAC + ∠ACB = 180°

And, in Δ ADC ∠DAC + ∠ADC + ∠ACD = 180°

Adding we get :

∠ABC + ∠BAC + ∠ACB + ∠DAC + ∠ADC + ∠ACD = 180° + 180°

⇒ (∠BAC + ∠DAC) + ∠ABC + (∠ACB + ∠ACD) + ∠ADC = 360°

⇒ ∠BAD + ∠ABC + ∠BCD + ∠ADC = 360°

⇒ **∠A + ∠B + ∠C + ∠D = 360°**

Alternative method :

On drawing the diagonal AC, the given quadrilateral is divided into two triangles. Since, the sum of the interior angles of a triangle is 180°.

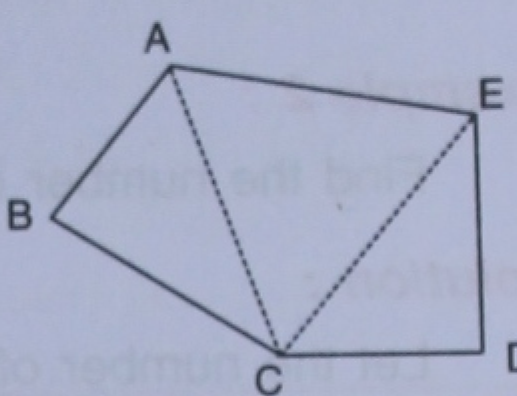
∴ ***Sum of interior angles of the quadrilateral ABCD***

= Sum of interior angles of Δ ABC + sum of interior angles of Δ ADC

= 180° + 180° = **360°**

3. *Pentagon :*

Consider a pentagon ABCDE as shown alongside.



On joining CA and CE, the given pentagon is divided into three triangles ABC, CDE and ACE.

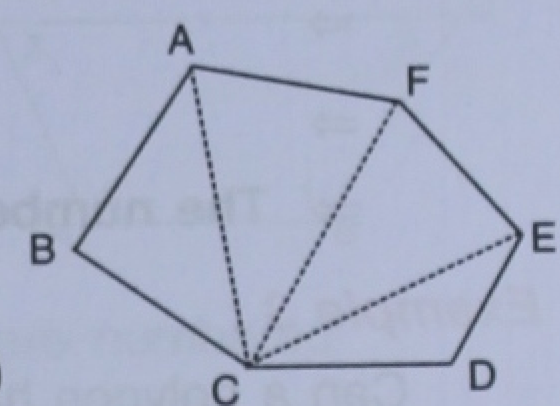
Since, the sum of the interior angles of a triangles is 180°

∴ Sum of the interior angles of the pentagon ABCDE

= Sum of interior angles of (Δ ABC + Δ CDE + Δ ACE)

= 180° + 180° + 180° = **540°**

4. *Hexagon :*



It is clear from the given figure that there are four triangles and as such the sum of the interior angles of the hexagon ABCDEF

= Sum of interior angles of (Δ ABC + Δ ACF + Δ FCE + Δ ECD)

= 180° + 180° + 180° + 180° = **720°**

27.3 USING FORMULA

The sum of interior angles of a polygon can also be obtained by using the following formula :

> Sum of interior angles of a polygon = $(n - 2) \times 180°$
> where, n = number of sides of the polygon.

∴ (i) **For a triangle :**

$n = 3$ (a triangle has 3 sides)

and, **sum of interior angles** = $(n - 2) \times 180°$

= $(3 - 2) \times 180°$ = **180°**

(ii) **For a quadrilateral :**

$n = 4$

and, **sum of interior angles** = $(n - 2) \times 180°$

= $(4 - 2) \times 180°$ = **360°**

(iii) **For a pentagon :**

$n = 5$

and, **sum of interior angles** = $(n - 2) \times 180°$

= $(5 - 2) \times 180° = 3 \times 180°$ = **540°**

(iv) **For a hexagon :**

$n = 6$

and, **sum of interior angles** = $(n - 2) \times 180°$

= $(6 - 2) \times 180° = 4 \times 180°$ = **720°**

Example 1 :

Find the sum of the interior angles of a polygon with 8 sides.

Solution :

Number of sides in the polygon is 8 $\Rightarrow$ $n = 8$

$\therefore$ **The sum of the interior angles** of the 8 - sided polygon

$$= (n - 2) \times 180°$$
$$= (8 - 2) \times 180° = 6 \times 180° = \mathbf{1080°} \quad \textbf{(Ans)}$$

Example 2 :

Find the number of sides of a polygon whose sum of interior angles is 1440°.

Solution :

Let the number of sides in the polygon be n.

$$\therefore \quad (n - 2) \times 180° = 1440°$$
$$\Rightarrow \quad n - 2 = \frac{1440°}{180°}$$
$$\Rightarrow \quad n = 8 + 2 = 10$$

$\Rightarrow$ **The number of sides** in the polygon = **10** **(Ans)**

Example 3 :

Can a polygon have the sum of its interior angles as : (i) 2160° (ii) 2400°

Solution :

> The number of sides of a polygon is always a whole number which must be greater than or equal to three (3). Thus,
>
> 1. A polygon can not have its number of sides in fraction or in decimal.
> 2. The least number of sides in a polygon is 3.

(i) Let the number of sides of the polygon = n

$$\therefore (n - 2) \times 180° = 2160° \quad \Rightarrow \quad n - 2 = \frac{2160°}{180°} = 12$$
$$\Rightarrow \quad n = 12 + 2 = 14$$

Since, the number of sides is a whole number which is not less than 3, therefore **a polygon can have sum of its interior angles equal to 2160°.** **(Ans)**

(ii) Let the number of sides of the polygon = n

$$\therefore (n - 2) \times 180° = 2400° \quad \Rightarrow \quad n - 2 = \frac{2400°}{180°} = \frac{40}{3}$$
$$\Rightarrow n = \frac{40}{3} + 2 = \frac{46}{3}, \text{ which is not a whole number}$$

$\therefore$ **A polygon can not have the sum of its interior angles equal to 2400°.** **(Ans)**

EXERCISE 27 (B)

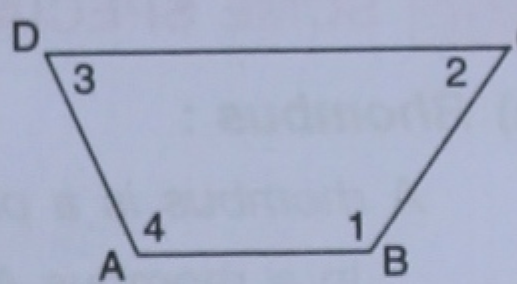
1. Find the sum of interior angles of a polygon with :
   (a) 7 sides (b) 10 sides
   (c) 11 sides (d) 15 sides
   (e) 18 sides
2. Find the number of sides of the polygon whose sum of interior angles is :
   (a) 900° (b) 4320°
   (c) 2520° (d) 3240°
   (e) 3960°
3. Can a polygon have the sum of its interior angles equal to :
   (a) 1260° (b) 2600°
   (c) 3540°

27.4 QUADRILATERAL

A quadrilateral is a plane figure enclosed by four sides.

It has four interior angles and four vertices.

In quadrilateral ABCD, shown alongside :



(i) four sides are : AB, BC, CD and DA.

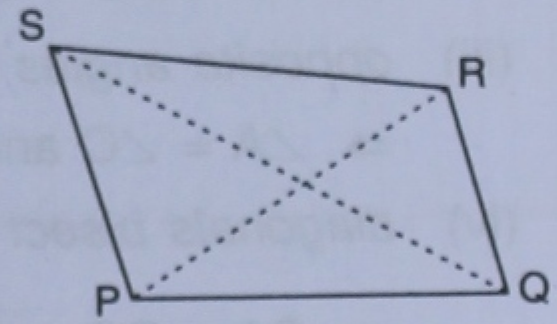
(ii) four angles are : ∠ABC, ∠BCD, ∠CDA and ∠DAB; which are numbered ∠1, ∠2, ∠3 and ∠4 respectively.

(iii) four vertices are : A, B, C and D.

27.5 DIAGONALS OF A QUADRILATERAL

The line segments joining the opposite vertices of a quadrilateral are called its diagonals.

The given figure shows a quadrilateral PQRS with diagonals PR and QS.

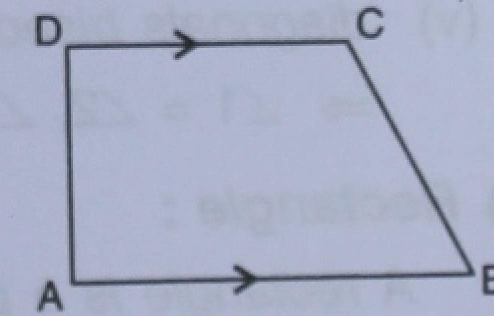


27.6 TYPES OF QUADRILATERALS

1. *Trapezium* :

A trapezium is a quadrilateral in which one pair of opposite sides are parallel.

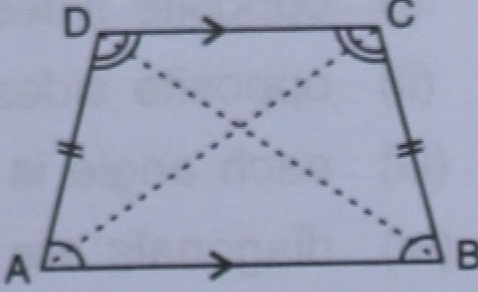
The figure, given alongside, shows a trapezium as its sides AB and DC are parallel, *i.e.*, AB // DC.



When the non-parallel sides of the trapezium are equal in length, it is called an ***isosceles trapezium.***

The given figure shows a trapezium ABCD whose non-parallel sides AD and BC are equal in length, *i.e.*, AD = BC, therefore it is an isosceles trapezium.

Also, in an isosceles trapezium :

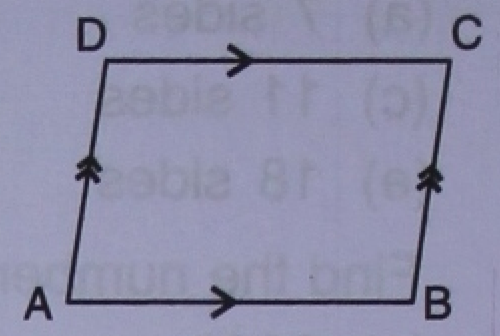


(i) *base angles are equal* :
⇒ ∠A = ∠B and ∠D = ∠C

(ii) *diagonals are equal* :
⇒ AC = BD.

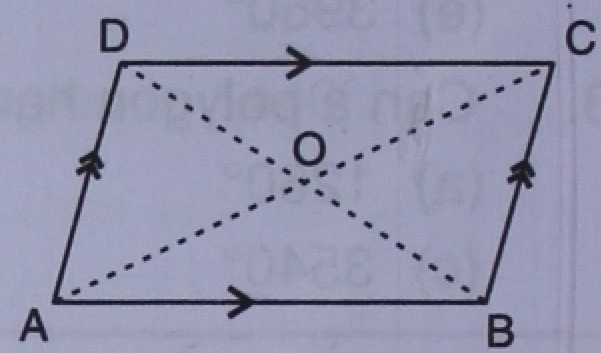
2. *Parallelogram* :

A parallelogram is a quadrilateral, in which both the pairs of opposite sides are parallel.



The quadrilateral ABCD, drawn alongside, is a parallelogram, since *AB* is parallel to DC and AD is parallel to BC, *i.e.*, AB//DC and AD//BC.

Also, in a parallelogram ABCD :



(i) *opposite sides are equal* :

$\Rightarrow$ AB = DC and AD = BC.

(ii) *opposite angles are equal* :

$\Rightarrow$ $\angle ABC = \angle ADC$ and $\angle BCD = \angle BAD$

(iii) *diagonals bisect each other* :

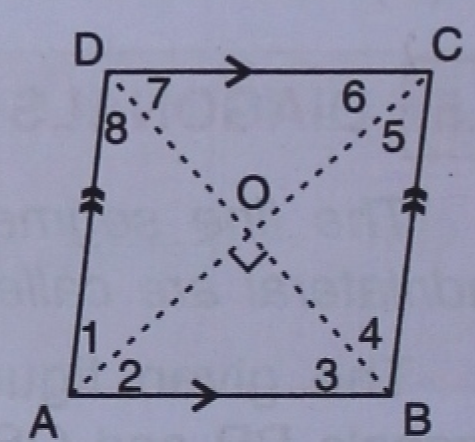
$\Rightarrow$ $OA = OC = \frac{1}{2}$ AC and $OB = OD = \frac{1}{2}$ BD.

27.7 SOME SPECIAL TYPES OF PARALLELOGRAMS

(a) *Rhombus* :

A rhombus is a parallelogram in which all its sides are equal.

$\therefore$ In a rhombus ABCD :



(i) *opposite sides are parallel* :

$\Rightarrow$ AB//DC and AD//BC.

(ii) *all the sides are equal* :

$\Rightarrow$ AB = BC = CD = DA.

(iii) *opposite angles are equal* :

$\Rightarrow$ $\angle A = \angle C$ and $\angle B = \angle D$.

(iv) *diagonals bisect each other at right angle* :

$\Rightarrow$ $OA = OC = \frac{1}{2}$ AC, $OB = OD = \frac{1}{2}$ BD.

and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90°$.

(v) *diagonals bisect the angles at the vertices* :

$\Rightarrow$ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$ and $\angle 7 = \angle 8$.

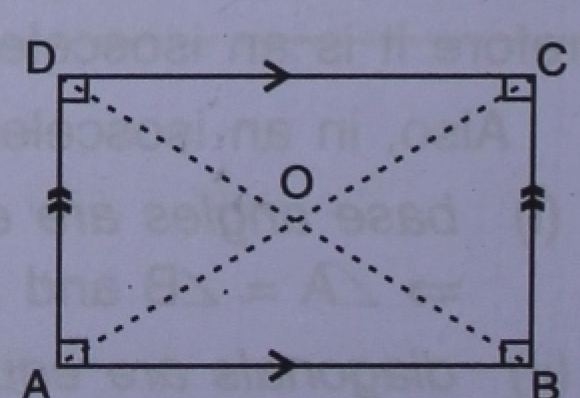
(b) *Rectangle* :

A rectangle is a parallelogram whose any angle is 90°.

A rectangle is also defined as a quadrilateral whose each angle is 90°.

If any angle of a parallelogram is 90°, automatically its each angle is 90°, since opposite angles of a parallelogram are equal.

Also, in a rectangle :



(i) opposite sides are parallel.

(ii) opposite sides are equal.

(iii) each angle is 90°.

(iv) diagonals are equal.

(v) diagonals bisect each other.

(c) *Square* :

A square is a parallelogram, whose *all sides are* equal and *each angle is 90°.*

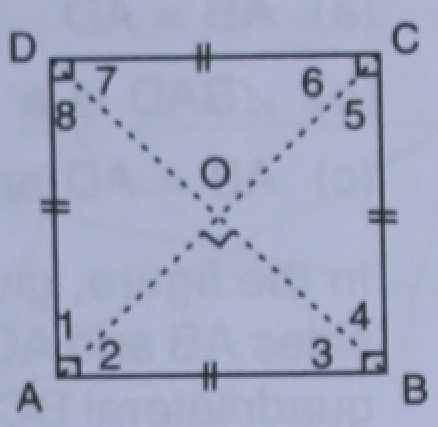
A square can also be defined as :

(i) *a rhombus whose any angle is 90°.*

(ii) *a rectangle whose all sides are equal.*

(iii) *a quadrilateral whose all sides are equal and each angle is 90°.*

∴ If ABCD is a square :



(i) *all its sides are equal.*

⇒ AB = BC = CD = DA

(ii) *each angle of it is 90°.*

⇒ ∠A = ∠B = ∠C = ∠D = 90°.

Also,

(iii) *diagonals are equal.*

⇒ AC = BD.

(iv) *diagonals bisect each other at 90°.*

⇒ $OA = OC = \frac{1}{2}AC, \; OB = OD = \frac{1}{2}BD$

and ∠AOB = ∠BOC = ∠COD = ∠DOA = 90°.

Since, diagonals AC and BD are equal, therefore, OA = OC = OB = OD.

(v) *diagonals bisect the angles at the vertices.*

i.e. ∠1 = ∠2 = 45° [∵ ∠1 + ∠2 = 90°]

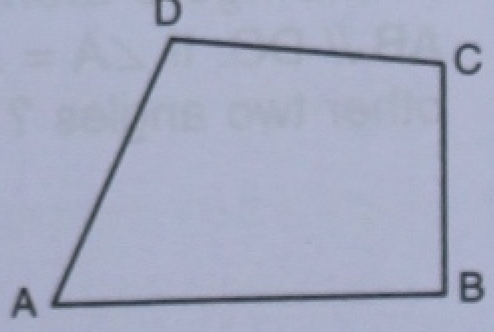
Similarly ; ∠3 = ∠4 = 45°,

∠5 = ∠6 = 45° and ∠7 = ∠8 = 45°.

EXERCISE 27 (B)

1. Complete each of the following, so as to make a true statement :
   (a) A quadrilateral has .... sides.
   (b) A quadrilateral has .... angles.
   (c) A quadrilateral has .... diagonals.
   (d) A quadrilateral has .... vertices.

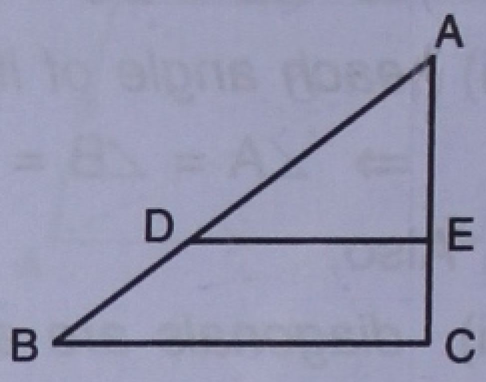
2. In the adjacent figure, ABCD is a quadrilateral.
   (a) Name a pair of adjacent sides.
   (b) Name a pair of opposite sides.
   (c) How many pairs of adjacent sides are there ?
   (d) How many pairs of opposite sides are there ?
   (e) Name a pair of adjacent angles.
   (f) Name a pair of opposite angles.
   (g) How many pairs of adjacent angles are there ?
   (h) How many pairs of opposite angles are there ?



3. For each statement, given below, indicate whether it is ***true*** or ***false*** :
   (a) A rectangle is a parallelogram. (b) A square is also a rectangle.
   (c) A parallelogram is a rhombus. (d) A square is a rhombus.
   (e) A square is a quadrilateral.

4. *A*BCD is a parallelogram. What special name (in each of the cases given below) will you give it, if :
   (a) AB = AD
   (b) ∠BAD is a right angle.
   (c) AB = AD and ∠BAD = one right angle.

5. In the figure, given alongside, D and E are the points on sides AB and AC of Δ ABC, such that DE//BC. What is the quadrilateral BCED called ? Give reason for your answer.



6. A quadrilateral has three acute angles, each measuring 80°. What is the measure of the fourth angle ?

7. How does a trapezium differ from a parallelogram ?

8. Two angles of a quadrilateral are of measure 65° each and the other two angles are equal. What is the measure of each of these two equal angles ?

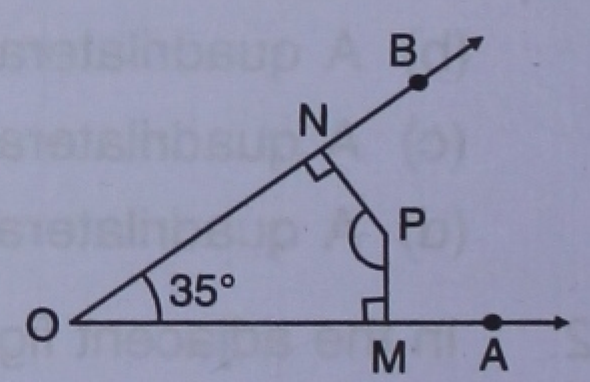
9. Three angles of a quadrilateral are equal. The fourth angle is of measure 150°. What is the measure of each equal angle ?

10. A quadrilateral has all its four angles of the same measure. What is the measure of each ?

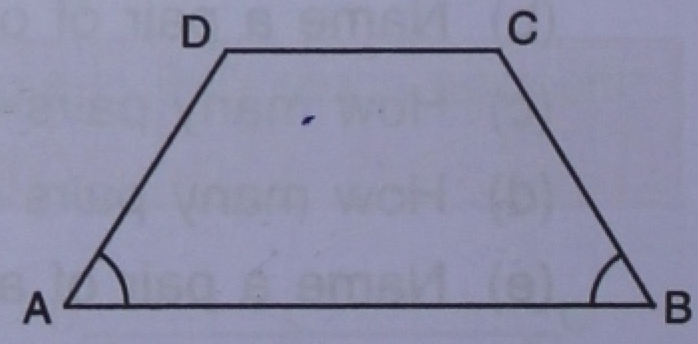
11. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4. What are the measures of the four angles separately ?

12. If the sum of two angles of a quadrilateral is 180°, what is the sum of the remaining two angles ?

13. In the figure alongside, *P* is a point in the interior of ∠AOB, PM ⊥ OA and PN ⊥ OB. If ∠AOB = 35°, what is the measure of ∠MPN ?



14. In the figure alongside, ABCD is a trapezium, in which AB // DC. If ∠A = ∠B = 40°, what are the measures of the other two angles ?



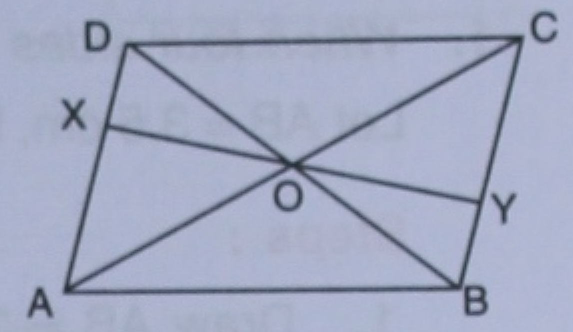
15. The measure of one angle of a parallelogram is 70°. What are the measures of the remaining angles ?

16. Two adjacent angles of a parallelogram are equal. What is the measure of each ?
    [**Hint** : Sum of adjacent angles is 180°.]

17. Two adjacent angles of a parallelogram are in the ratio 2 : 3. Find the measures of all the angles.

18. Show that a diagonal of a parallelogram divides it into two congruent triangles.
[**Hint** : Use S.S.S. congruence condition.]

19. In the given figure, diagonals of parallelogram ABCD interesect at O. XY passes through O as shown.
Give reasons for each of the following statements :



(a) OB = OD
(b) $\angle OBY = \angle ODX$
(c) $\angle BOY = \angle DOX$
(d) $\Delta BOY \cong \Delta DOX$
Now, state if XY is bisected at O.

20. Which of the following are true for a rhombus ?
(a) It has two pairs of parallel sides.
(b) It has two pairs of equal angles.
(c) It has only two pairs of equal sides.
(d) Two of its angles are right angles.
(e) Its diagonals bisect each other at right angle.
(f) Its diagonals are equal and perpendicular to each other.
(g) It has all its sides of equal lengths.

Do you know :

1. **Convex Polygon :**
If *each angle* of a polygon is *less than 180°*, it is called a **convex polygon**.

2. **Concave Polygon :**
If *atleast one angle* of a polygon is *more than 180°*, it is called a **concave polygon.**

3. Unless mentioned, a **polygon** means a **convex polygon.**