CHAPTER 25

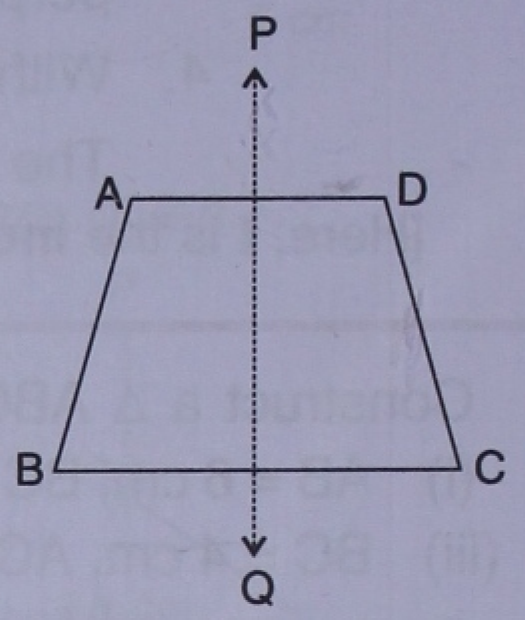
SYMMETRY

(INCLUDING REFLECTION AND ROTATION)

25.1 REVIEW

Concept of symmetry is already done in Class VI.

Revision : A geometrical figure is said to be symmetric about a line in it, if on folding the figure about this line, the two parts of the figure exactly coincide.



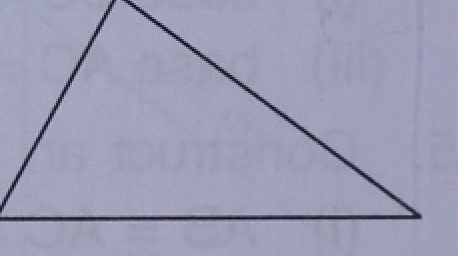
The adjoining figure shows a quadrilateral ABCD and a line PQ in it. If the figure is folded about the line PQ and the two parts of the figure coincide, *i.e.,* A and D coincide, B and C coincide, AB and DC coincide and so on, then the whole figure is said to be *symmetric* about the line PQ.

If the figure is symmetric about a line in it, the line is said to be a **line of symmetry** or an **axis of symmetry.** Thus, in the figure ABCD, discussed above, line PQ is the line of symmetry of ABCD.

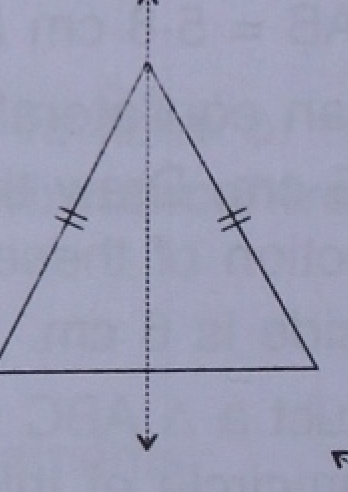
25.2 LINES OF SYMMETRY OF GIVEN GEOMETRICAL FIGURES

It is not necessary that every figure under consideration will definitely have a line symmetry. If we consider different types of triangle, we find :

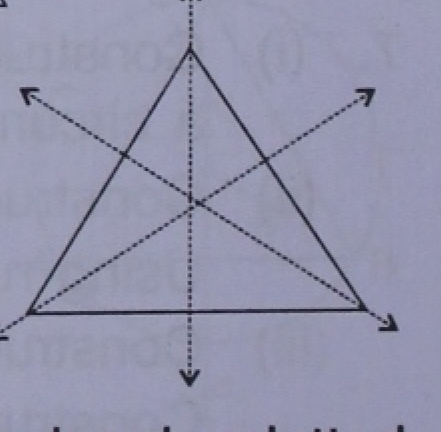
1. A **scalene triangle** has **no line of symmetry :**
   We can not have a line in a scalene triangle about which if the figure (triangle) is folded, the two parts of the figure will coincide.



2. An **isosceles triangle** has only **one line of symmetry.**
   The bisector of angle of vertex which is also the perpendicular bisector of its base.

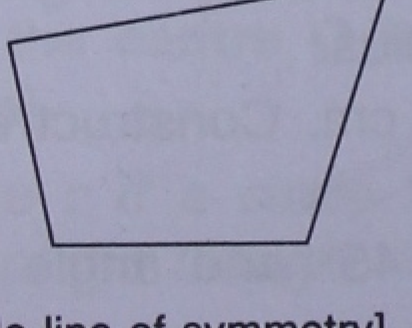


3. An **equilateral triangle** has **three lines of symmetry.**
   The ***bisectors of the angle of vertices*** which are also the ***perpendicular bisectors of*** its ***sides.***



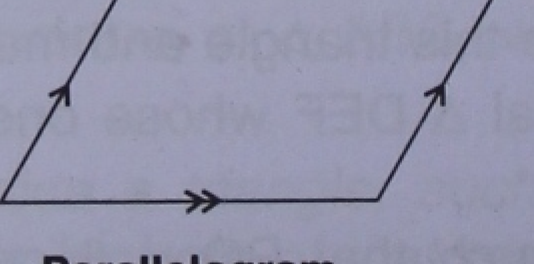
4. Line(s) of symmetry of different types of quadrilaterals are shown below by dotted lines :

(i)



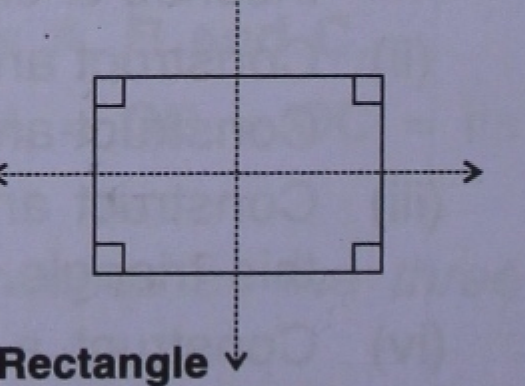
[No line of symmetry]

(ii)



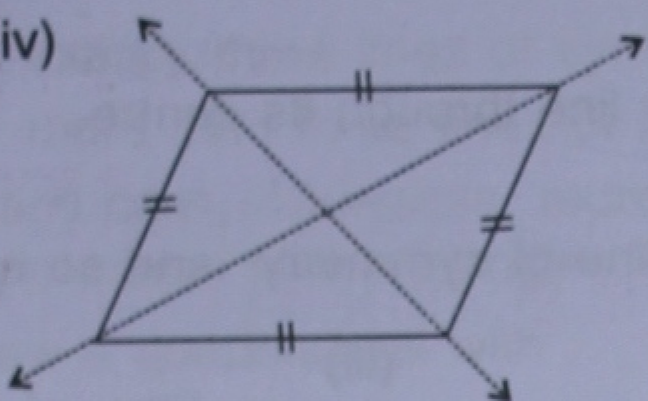
Parallelogram
[No line of symmetry]

(iii)



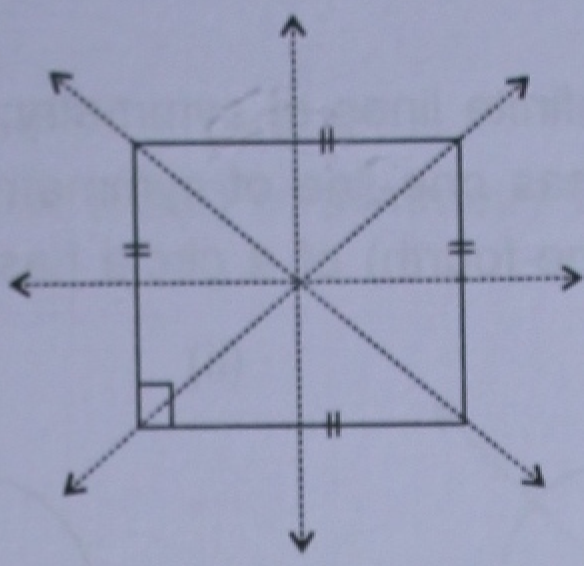
Rectangle
[Two lines of symmetry]

(iv)



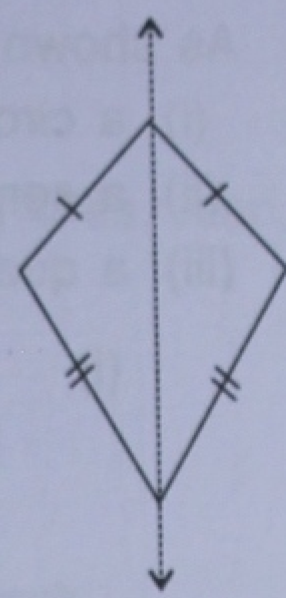
Rhombus
[Two lines of symmetry]

(v)



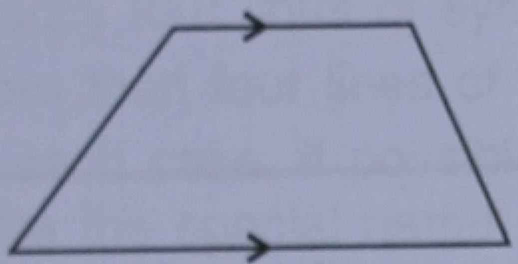
Square
[Four lines of symmetry]

(vi)



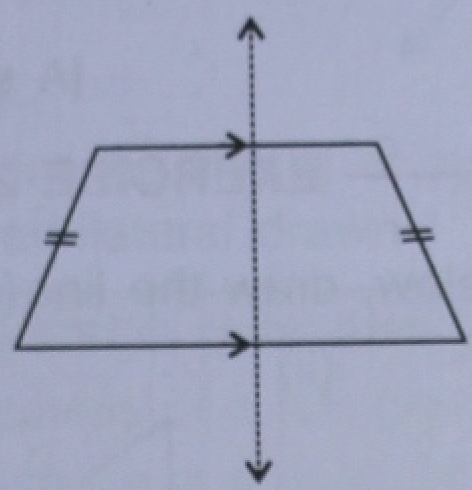
Kite-shaped figure
[One line of symmetry]

(vii)



Trapezium
[No line of symmetry]

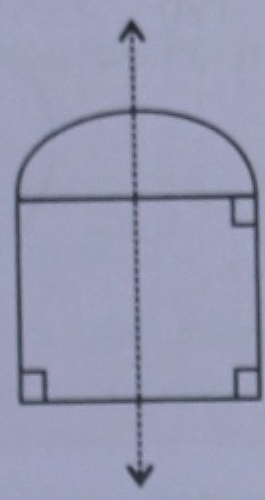
(viii)



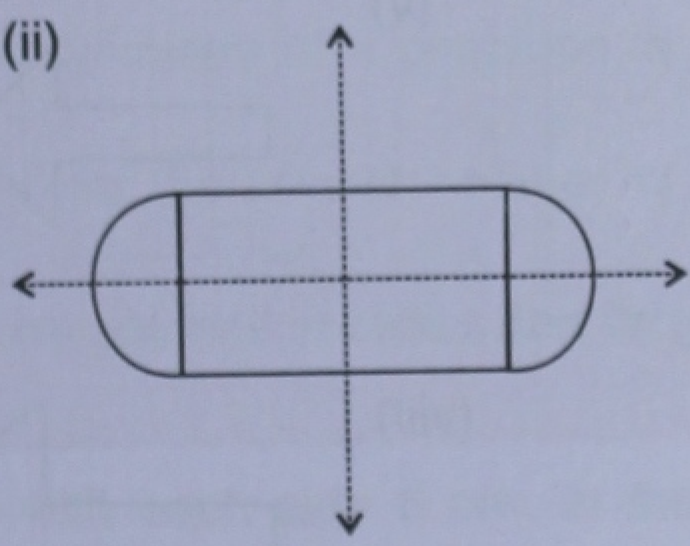
Isosceles trapezium
[One line of symmetry]

5. In each of the following, the dotted line/lines are the line(s) of symmetry of the given figure :

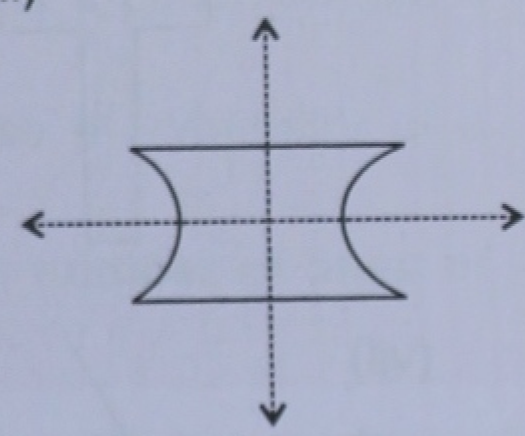
(i)



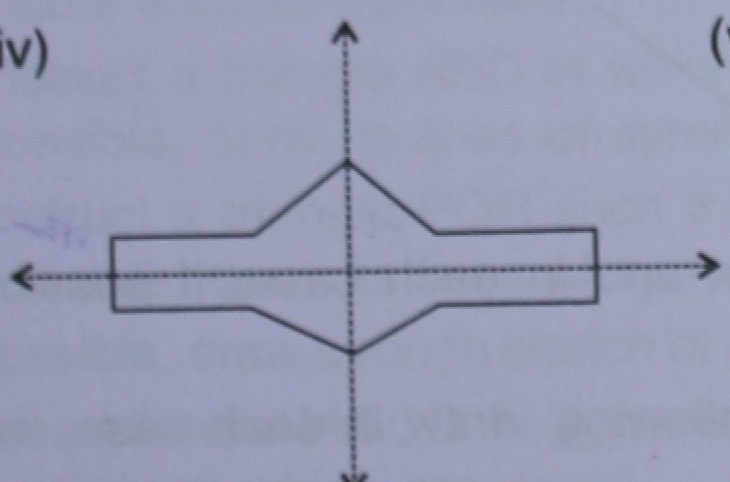
(ii)



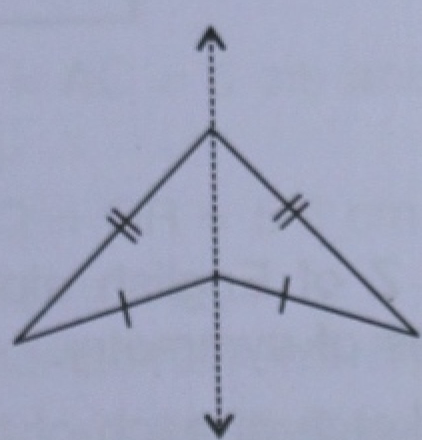
(iii)



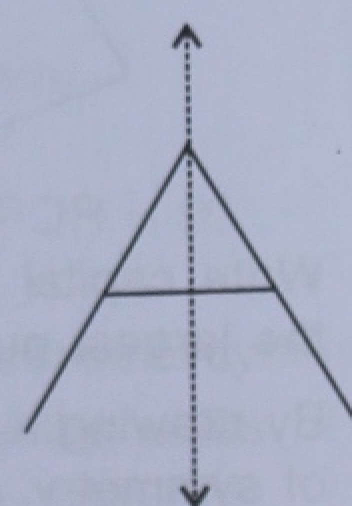
(iv)



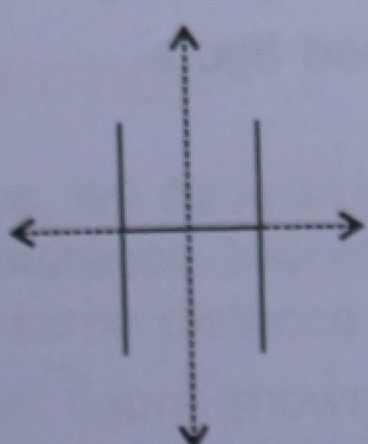
(v)



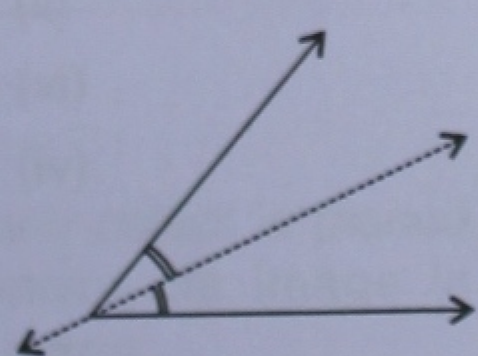
(vi)



(vii)

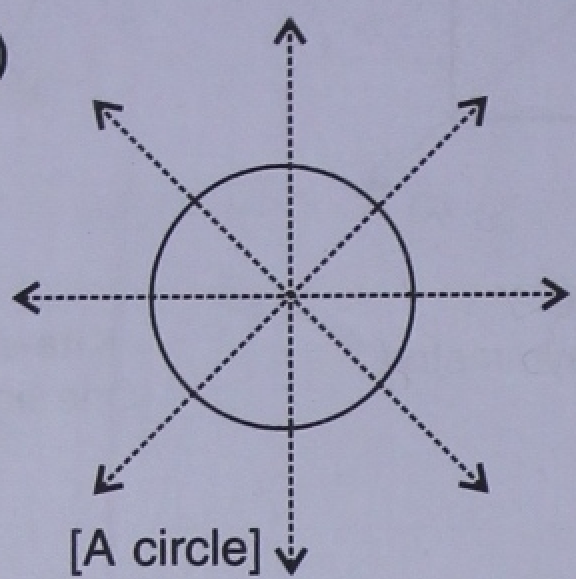


(viii)



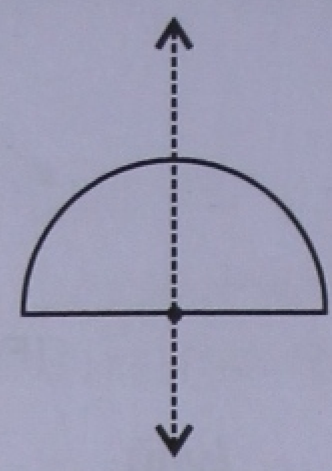
6. As shown below,
   (i) a circle has infinite lines of symmetry; every line through its centre.
   (ii) a semi-circle has one line of symmetry.
   (iii) a quadrant (one-fourth) of a circle has one line of symmetry and so on.

(i)



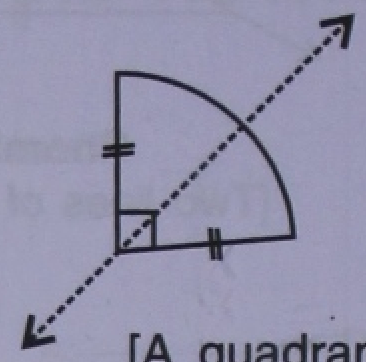
[A circle]

(ii)



[A semi-circle]

(iii)

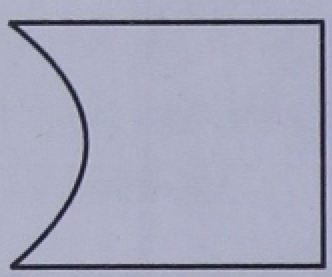


[A quadrant]

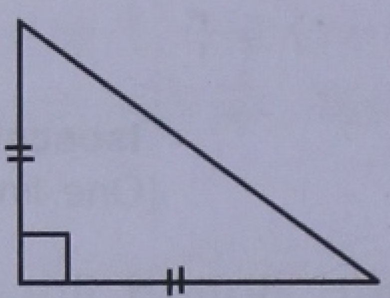
EXERCISE 25 (A)

1. For each figure, given below, draw the line(s) of symmetry, if possible :

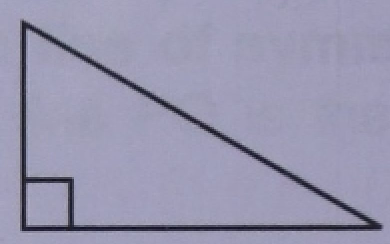
(i)



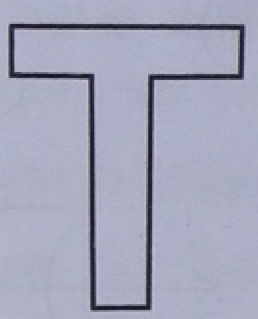
(ii)



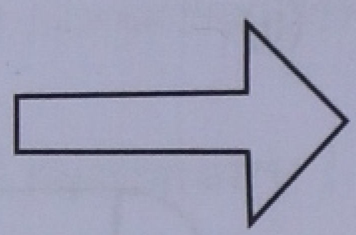
(iii)



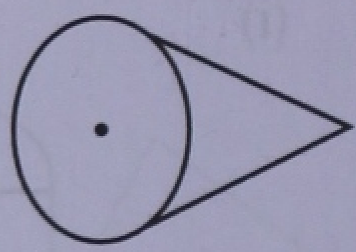
(iv)



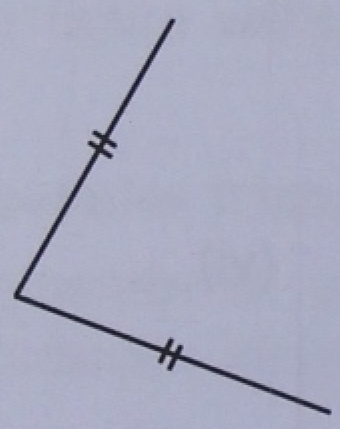
(v)



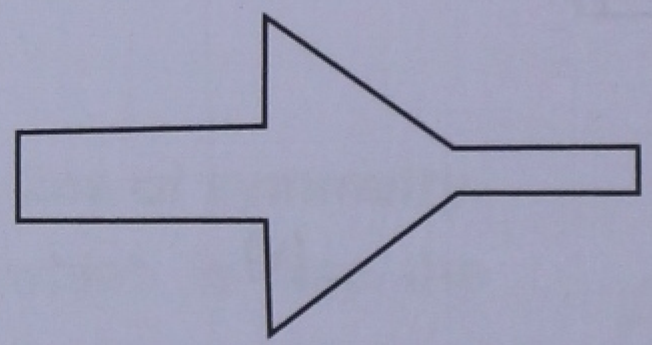
(vi)



(vii)



(viii)



2. Write capital letters A to Z of English alphabet and in each case, if possible, draw the largest number of lines of symmetry.

3. By drawing a free hand sketch of each of the following, draw in each case, the line(s) of symmetry, if any :
   (i) a scalene triangle
   (ii) an isosceles right angled triangle
   (iii) a rhombus
   (iv) a kite shaped figure
   (v) a rectangle
   (vi) a square
   (vii) an isosceles triangle.

4. Draw a triangle with :
   (i) no line of symmetry,
   (ii) only one line of symmetry,
   (iii) exactly two lines of symmetry,

(iv) exactly three lines of symmetry,

(v) more than three lines of symmetry.

In each case, if possible, represent the line(s) of symmetry by dotted lines. Also, write the special name of the triangle drawn.

5. Draw a quadrilateral with :

(i) no line of symmetry.

(ii) only one line of symmetry.

(iii) exactly two lines of symmetry.

(iv) exactly three lines of symmetry.

(v) exactly four lines of symmetry.

(vi) more than four lines of symmetry.

In each case, if possible, represent the line(s) of symmetry by dotted lines. Also, write the special name of the quadrilateral drawn

It is clear from the question numbers 4 and 5, given above, that :

1. The *largest number* of *lines of symmetry* of a triangle is *three* (3).
2. The largest number of lines of symmetry of a quadrilateral is four (4).

   As the number of sides in a triangle is 3, the largest number of lines of symmetry in it is 3 and as the number of sides in a quadrilateral is 4, the largest number of lines of symmetry in it is 4.

In the same way :

1. The largest number of lines of symmetry of a pentagon is 5, as a pentagon has 5 sides.
2. The hexagon has 6 sides and so the largest number of lines of symmetry of a hexagon is 6.

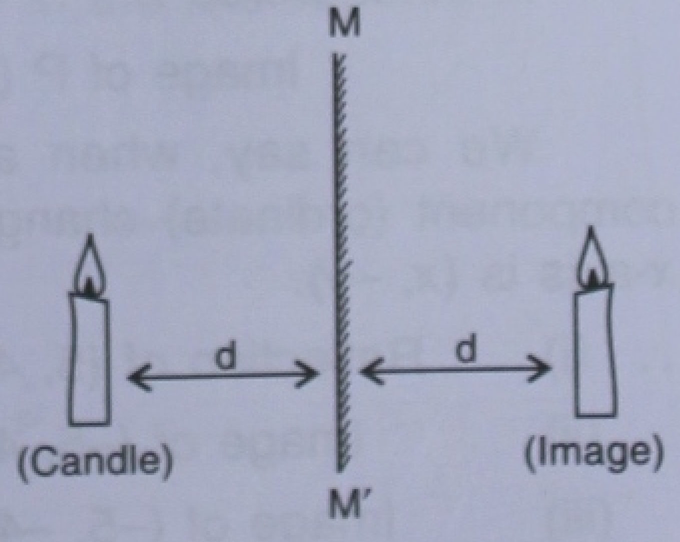
In general, we can say, that if a polygon has ***n* sides, the largest number of lines of symmetry,** it can have is ***n***.

6. Construct an equilateral triangle with each side 6 cm. In the triangle drawn, draw all possible lines of symmetry.
7. Construct a triangle ABC in which AB = AC = 5 cm and BC = 5·6 cm. If possible, draw its lines of symmetry.
8. Construct a triangle PQR such that PQ = QR = 5·5 cm and angle PQR = 90°. If possible, draw its lines of symmetry.
9. If possible, draw a rough sketch of a quadrilateral which has exactly two lines of symmetry.
10. A quadrilateral ABCD is symmetric about its diagonal AC. Name the sides of this quadrilateral which are equal.

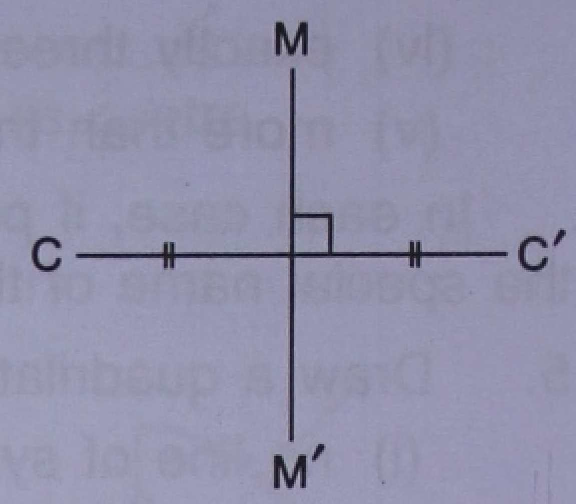
25.3 REFLECTION

In physics, we have studied that if any object is placed at a certain distance before a plane mirror, its image is formed at the same distance behind the mirror.

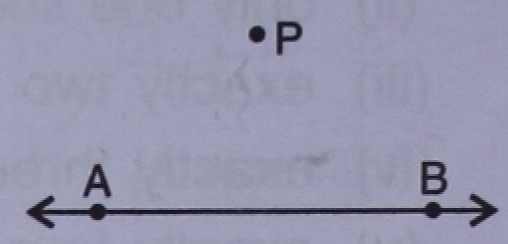
The given figure shows a candle placed at a distance *'d'* before a plane mirror MM′. The image of the candle is obtained in the mirror at the same distance *'d'* behind the mirror.



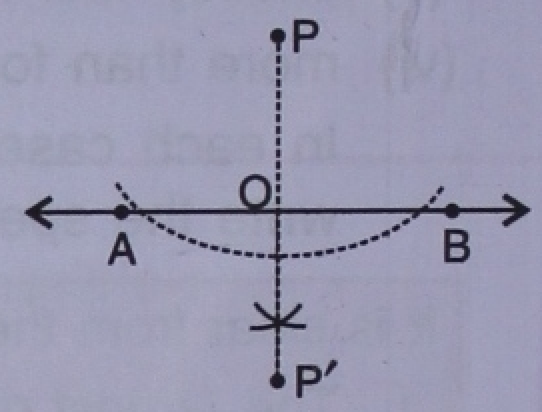
If we see, geometrically, the line joining the candle (C) and its image (C′) is perpendicularly bisected by the mirror line MM′.



Now if we want to find the image of a point P in line AB, we consider the point P as an object, the line AB as plane mirror and we find point P′ on the other side of AB so that PP′ is perpendicularly bisected by AB.



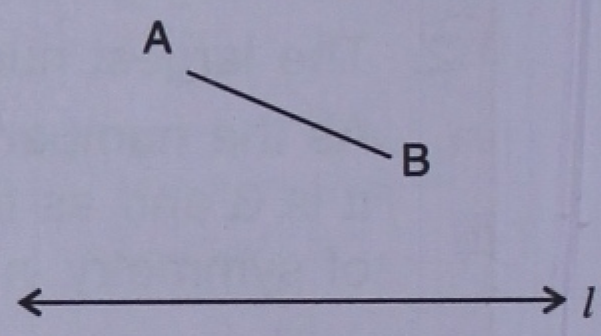
For this, from the given point P, draw perpendicular to AB which meets AB at point O. From PO produced, cut OP′ = OP.



P′ is the reflection (image) of the given point P in the line AB.

Example 1 :

The given figure shows a line segment AB and a line *l*. Find, geometrically, the reflection of AB in the line *l*.

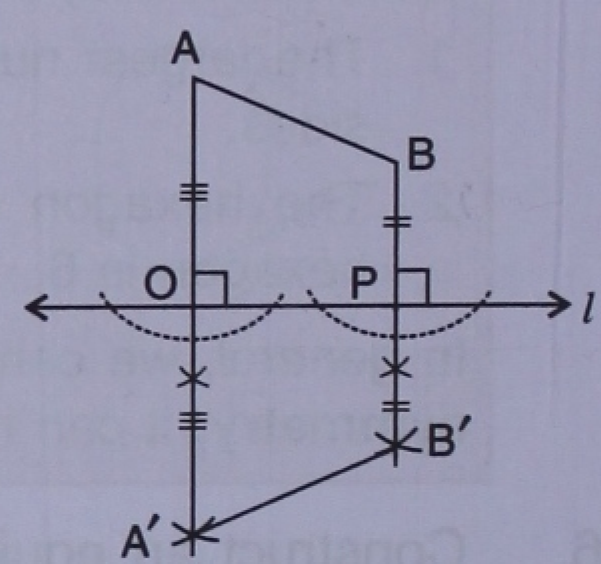


Solution :

From the point A, draw AO perpendicular to the line *l* and from AO produced cut OA′ such that OA′ = OA.

In the same way, from the point B, draw *BP* perpendicular to the line *l* and from BP produced cut PB′ such that PB′ = PB.

Join A′ and B′.

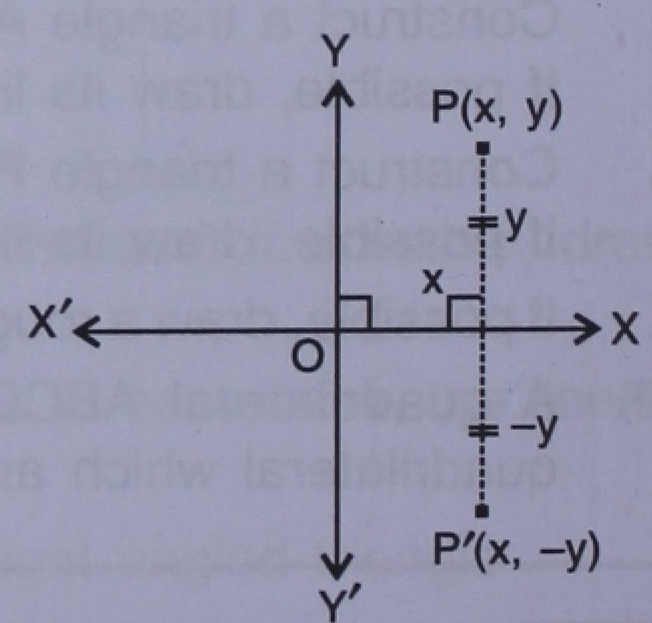


∴ A′ B′ is the required reflection of AB in line the *l*.

25.4 REFLECTON IN *X*-AXIS

Reflection in *x*-axis means the *x*-axis is considered as the plane mirror, the given point as the object and then we find (calculate) its image.

Let P (x, y) be a point as shown in the figure. When it is reflected in *x*-axis to point P′, the co-ordinates of image point P′ are $(x, -y)$.



Thus, reflection of P (x, y) in *x*-axis = P′ $(x, -y)$

In other words :

Image of P (x, y) in *x*-axis = P′ $(x, -y)$

We can say, when a point (x, y) is reflected in *x*-axis, the sign of its second component (ordinate) changes, *i.e.*, the sign of *y* changes and so the image of (x, y) in *x*-axis is $(x, -y)$.

∴ (i) Reflection of (5, 4) in *x*-axis = (5, –4)

(ii) Image of (–5, 4) in *x*-axis = (–5, –4)

(iii) Image of (–5, –4) in *x*-axis = (–5, 4)

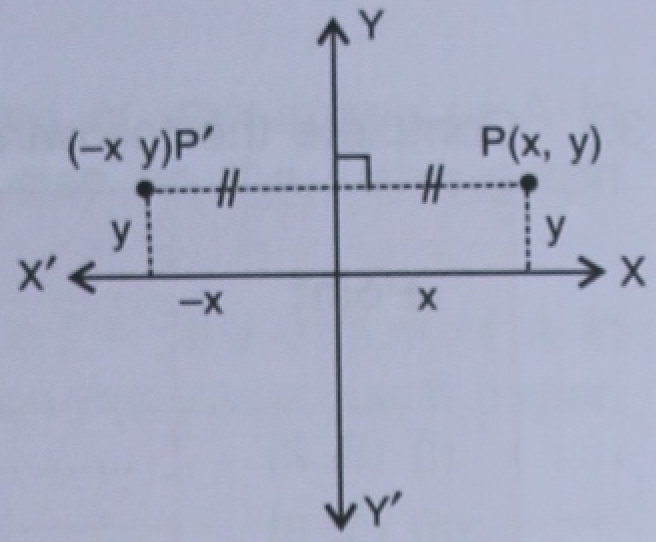
(iv) Image of (–8, 5) in x-axis = (–8, –5)

(v) Reflection of (3, 0) in x-axis = (3, 0)

(vi) Reflection of (0, –6) in x-axis = (0, 6)

(vii) Reflection of (0, 0) in x-axis = (0, 0) and so on.

25.5 REFLECTON IN Y-AXIS



As it is clear from the figure, given alongside, the reflection P (x, y) in y-axis is point P′ (–x, y).

We can say, when a point (x, y) is reflected in y-axis, the sign of its first component (abscissa) changes, *i.e.*, the sign of x changes and so the image of (x, y) in y-axis is (–x, y).

∴ (i) Image of (5, 4) in y-axis = (–5, 4)

(ii) Reflection of (5, – 4) in y-axis = (–5, – 4)

(iii) Reflection of (–5, – 4) in y-axis = (5, – 4)

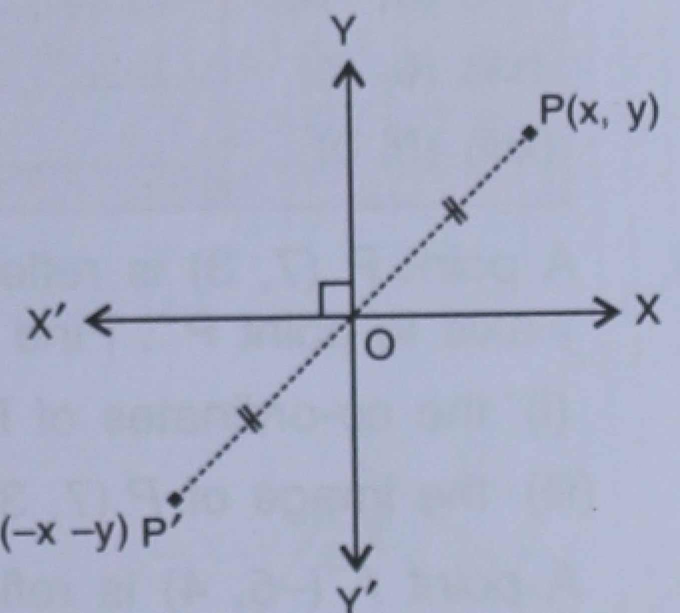
(iv) Image of (–8, 5) in y-axis = (8, 5)

(v) Image of (3, 0) in y-axis = (–3, 0)

(vi) Reflection of (0, – 6) in y-axis = (0, – 6)

(vii) Reflection of (0, 0) in y-axis = (0, 0) and so on.

25.6 REFLECTON IN ORIGIN



When point P (x, y) is reflected in origin, the signs of both of its components change, *i.e.*, the image of P (x, y) is P′ (–x, –y) as shown alongside.

∴ (i) Image of (5, 4) in origin = (–5, –4)

(ii) Reflection of (5, – 4) in origin = (–5, 4)

(iii) Image of (–5, – 4) in origin = (5, 4)

(iv) Reflection of (–8, 5) in origin = (8, –5)

(v) Image of (3, 0) in origin = (–3, 0)

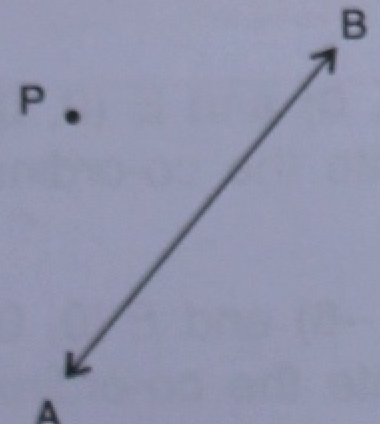
(vi) Reflection of (0, – 6) in origin = (0, 6)

(vii) Reflection of (0, 0) in origin = (0, 0) and so on.

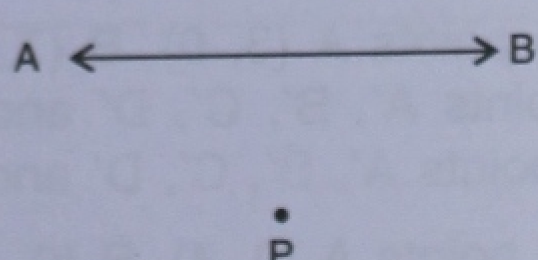
EXERCISE 25 (B)

1. In each figure, given below, find the image of the point P in the line AB :

(i)

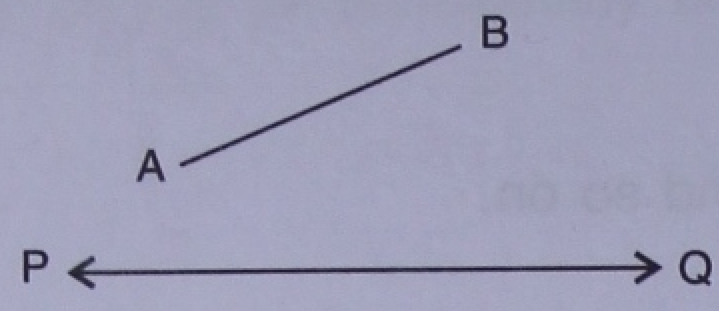


(ii)

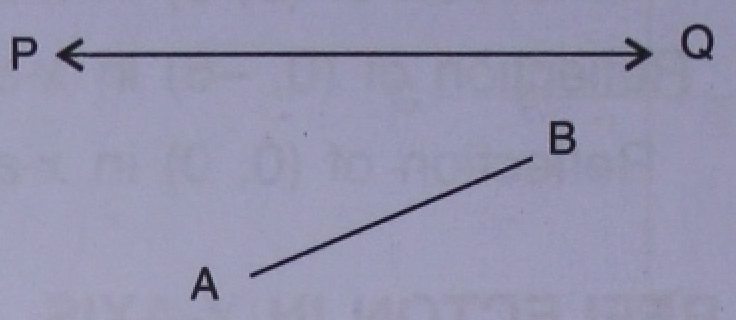


2. In each figure, given below, find the image of the line segment AB in the line PQ :

(i)



(ii)



3. Complete the following table :

Point	Reflection in		
	x-axis	*y*-axis	origin
(i) (8, 2)	....	....	....
(ii) (5, 6)	....	....	....
(iii) (4, –5)	....	....	....
(iv) (6, –2)	....	....	....
(v) (–3, 7)	....	....	....
(vi) (–4, 5)	....	....	....
(vii) (–2, –7)	....	....	....
(viii) (–6, –3)	....	....	....
(ix) (4, 0)	....	....	....
(x) (–7, 0)	....	....	....
(xi) (0, –6)	....	....	....
(xii) (0, 8)	....	....	....
(xiii) (0, 0)	....	....	....

4. A point P (7, 3) is reflected in *x*-axis to point P′. The point P′ is further reflected in *y*-axis to point P″. Find :
   (i) the co-ordinates of P′ (ii) the co-ordinates of P″
   (iii) the image of *P* (7, 3) in origin.

5. A point A (–5, 4) is reflected in *y*-axis to point B. The point B is further refelcted in origin to point C. Find :
   (i) the co-ordinates of B (ii) the co-ordinates of C
   (iii) the image of A (–5, 4) in *x*-axis.

6. The point P (3, –8) is reflected in origin to point Q. The point Q is further reflected in *x*-axis to point R. Find :
   (i) the co-ordinates of Q (ii) the co-ordinates of R
   (iii) the image of P (3, –8) in *y*-axis.

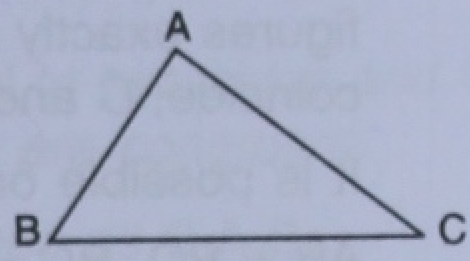
7. Each of the points A (3, 0), B (7, 0), C (–8, 0), D (–7, 0) and E (0, 0) is reflected in *x*-axis to points A′, B′, C′, D′ and E′ respectively. Write the co-ordinates of each of the image points A′, B′, C′, D′ and E′.

8. Each of the points A (0, 4), B (0, 10), C (0, –4), D (0, –6) and *E* (0, 0) is reflected in *y*-axis to points A′, B′, C′, D′ and E′ respectively. Write the co-ordinates of each of the image points A′, B′, C′, D′ and E′.

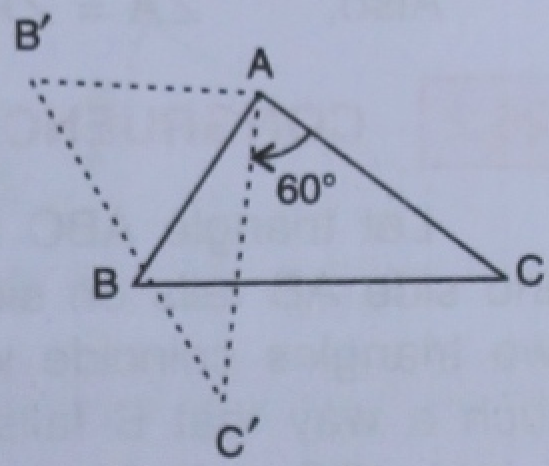
9. Each of the points A (0, 7), B (8, 0), C (0, –5), D (–7, 0) and E (0, 0) are reflected in origin to points A′, B′, C′, D′ and E′ respectively. Write the co-ordinates of each of the image points A′, B′, C′, D′ and E′.
10. Mark points A (4, 5) and B (–5, 4) on a graph paper. Find A′, the image of A in *x*-axis and B′, the image of B in *x*-axis. Mark A′ and B′ also on the same graph paper. Join AB and A′ B′.
Is AB = A′ B′ ?
11. Mark points A (6, 4) and B (4, –6) on a graph paper. Find A′, the image of A in *y*-axis and B′, the image of B in *y*-axis. Mark A′ and B′ also on the same graph paper.
12. Mark points A (–6, 5) and B (–4, –6) on a graph paper. Find A′, the image of A in origin and B′, the image of B in origin. Mark A′ and B′ also on the same graph paper. Join AB and A′ B′. Is AB = A′ B′ ?

25.6 ROTATION

Consider a triangle ABC which is free to rotate about its vertex A.



1. Let a rotation of 60° be given to this triangle about vertex A, in the clockwise direction so that the new position of the triangle is Δ AB′C′ as shown alongside:

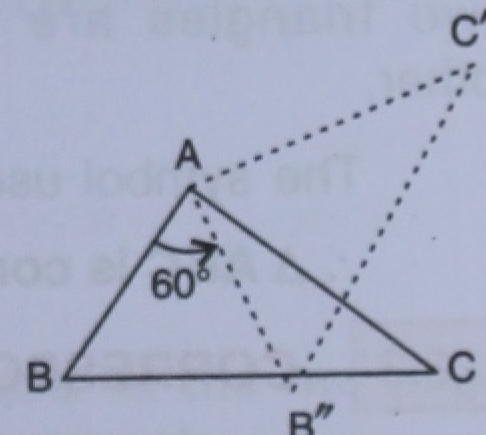


Each and every part of the given triangle will get rotated through 60° in the clockwise direction in such a way that the shapes and sizes of both the triangles are the same.

i.e., ∠B′AC′ = ∠BAC, ∠AB′C′ = ∠ABC and ∠AC′B′ = ∠ACB

And, AB′ = AB, B′C′ = BC and AC′ = AC.

2. Now, let the given triangle ABC be rotated about vertex A through an angle of 60° in the anticlockwise direction and the resulting position of it be triangle AB″C″.
Clearly, the shapes and size of the two triangles are the same and so :



∠B″AC″ = ∠BAC ; ∠AB″C″ = ∠ABC and ∠AC″B″ = ∠ACB

And, AB″ = AB, AC″ = AC and B″C″ = BC.

1. Whatever be the angle of rotation of the figure, the resulting figure and the given figure are always same in shape and size.
2. The point, about which the figure is rotated, is called the **centre of rotation.**