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UNIT – 4 GEOMETRY

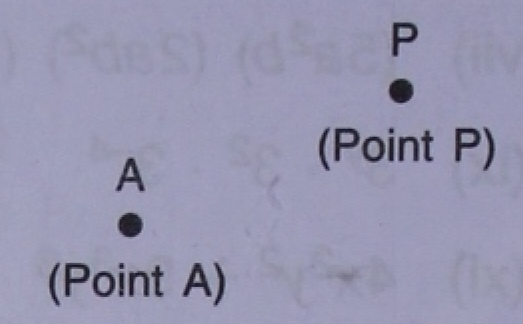
CHAPTER 23

LINES AND ANGLES

(INCLUDING CONSTRUCTION OF ANGLES)

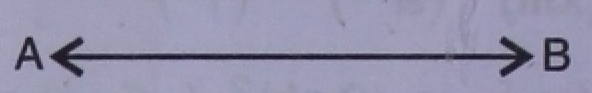
23.1 REVIEW

1. **POINT :** *A point is a mark of position, which has no length, no breadth and no thickness.* In general, it is represented by a capital letter as shown alongside.



2. **LINE :** *A line has length*, but *no breadth* or *thickness.*

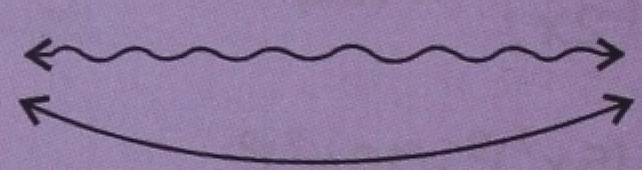
The given figure shows a line $\overleftrightarrow{AB}$ in which two arrow-heads in opposite directions show that it can be extended infinitely in both the directions.



1. *A line may be straight or curved but when we say 'a line' it means a straight line only.*

(i) 

(A line or a straight line)

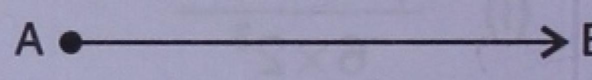
(ii) 

(Curved lines)

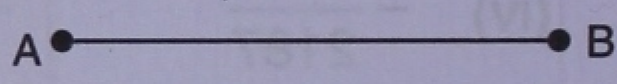
2. *Each line, whatever be its length, has an infinite number of points in it.*

3. **RAY :** *It is a straight line which starts from a fixed point and moves in the same direction.*

The given figure shows a ray $\overrightarrow{AB}$ with fixed initial point A and moving in the direction AB.

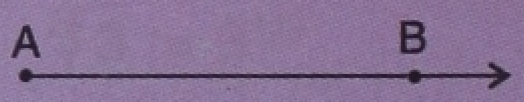


4. **LINE SEGMENT :** *It is a straight line with its both ends fixed.* The given figure shows a line segment, whose both the ends A and B are fixed.



1. The adjoining figure shows a line AB which can be extended upto infinity on its both sides.

2. The adjoining figure shows a ray AB with fixed end as point A and which can be extended upto infinity through point B. It is clear from the figure, that a ray is a part of a line.



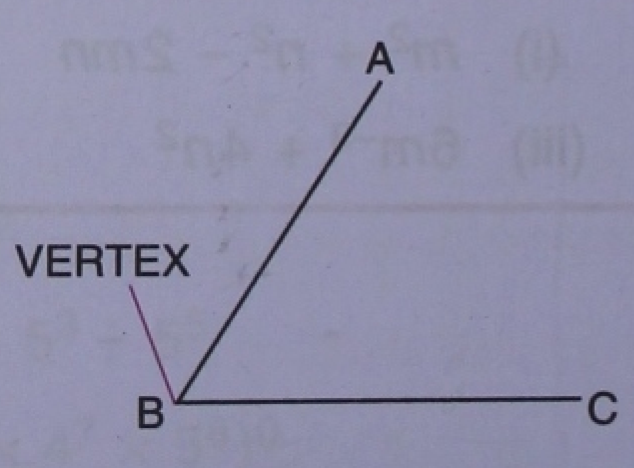
3. The adjoining figure shows a line-segment AB with fixed ends A and B.

It is clear from the different figures, that a line-segment is a part of a ray as well as of a line. Also, a line segment is the shortest distance between two fixed points.

5. **ANGLE :** *An angle is formed when two line segments or two rays have a common end-point.*

The *two line segments, forming an angle,* are called the **arms** of the angle whereas their *common end-point* is called the **vertex** of the angle.

The adjacent figure represents an angle ABC or ∠ABC or simply ∠B. **AB** and **BC** are the **arms of the angle** and their **common point B is the vertex.**



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23.2 MEASUREMENT OF AN ANGLE

The unit of measuring an angle is ***degree***. The symbol for degree is °.

Thus : 60 degree = 60°, 87 degree = 87° and so on.

If *one degree is divided into 60 equal parts,* each part is called a *minute* (′) and if *one minute is further divided into 60 equal parts, each part* is called *a second* (″).

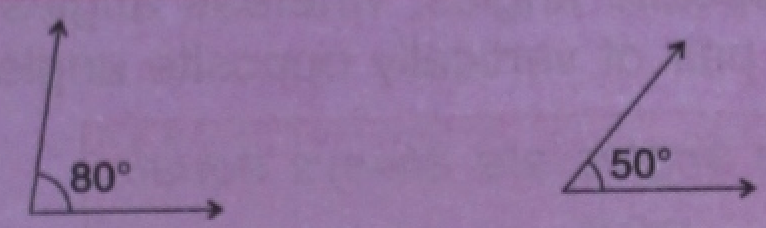
Thus, (i) 1° = 60′ and 1′ = 60″

(ii) 8 minutes 45 seconds = 8′ 45″

(iii) 25 degrees 30 minutes 15 seconds = 25° 30′ 15″ and so on.

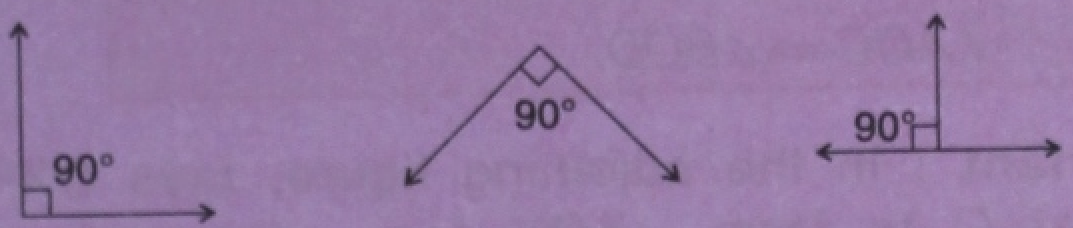
23.3 TYPES OF ANGLES

1. ***Acute angle :*** measures less than 90°



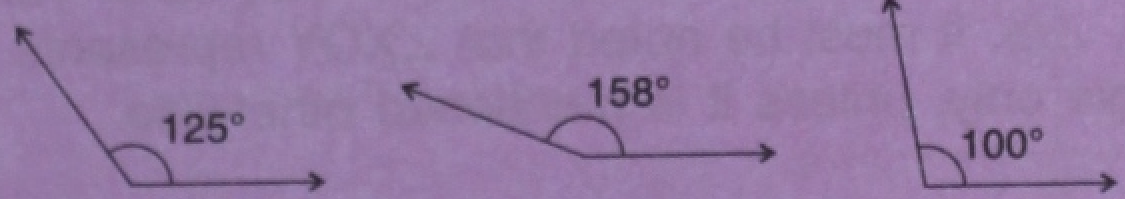
etc.

2. ***Right angle :*** measures 90°



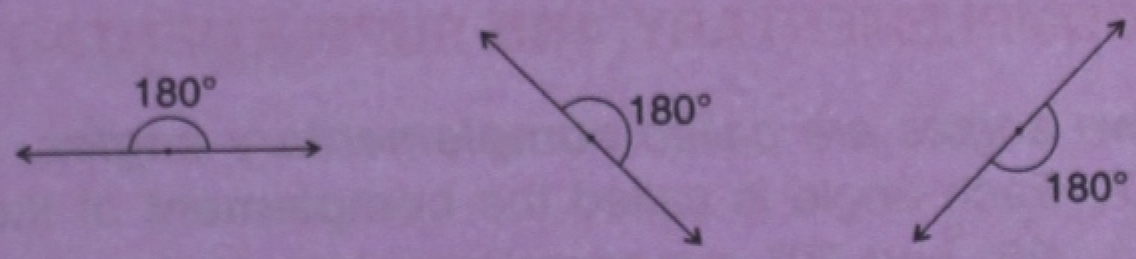
etc.

3. ***Obtuse angle :*** measures between 90° and 180°



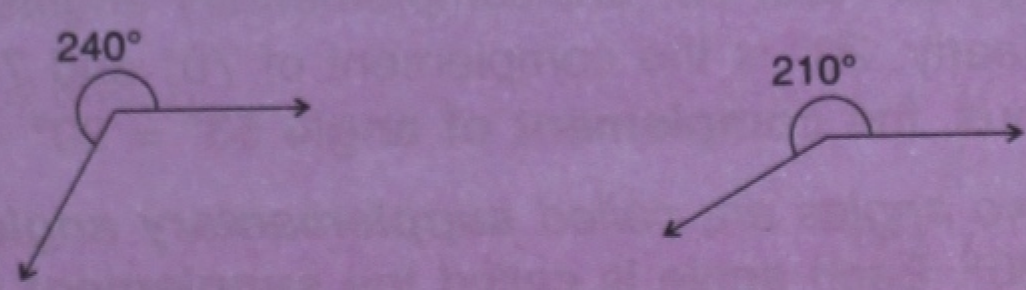
etc.

4. ***Straight angle :*** measures 180°



etc.

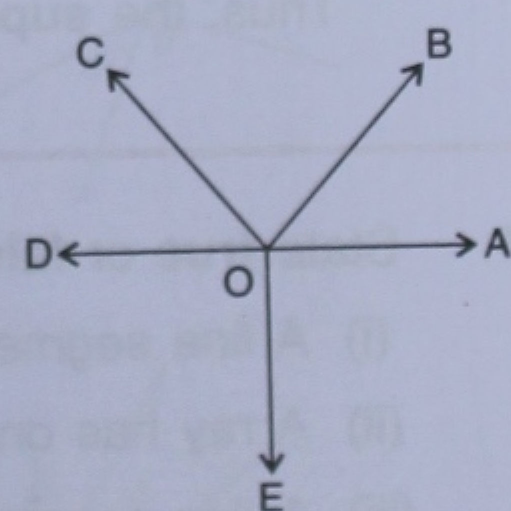
5. ***Reflex angle :*** measures between 180° and 360°



etc.

23.4 MORE ABOUT ANGLES

1. *Angles about a point* : If a number of angles are formed about a point, their sum is always 360°.



In the adjoining figure :

$\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$.

2. *Adjacent angles* : Two angles are said to be adjacent angles, if :

(i) they have a common vertex,

(ii) they have a common arm and

(iii) the other arms of the two angles lie on opposite sides of the common arm.

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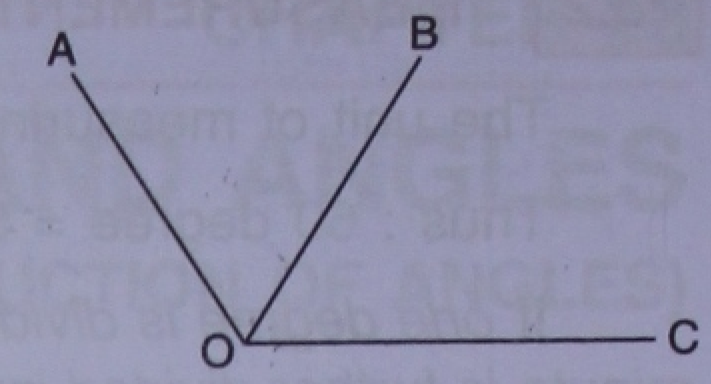
The adjoining figure shows a pair of adjacent angles :

because (i) they have a common vertex (O),

(ii) they have a common arm (OB) and

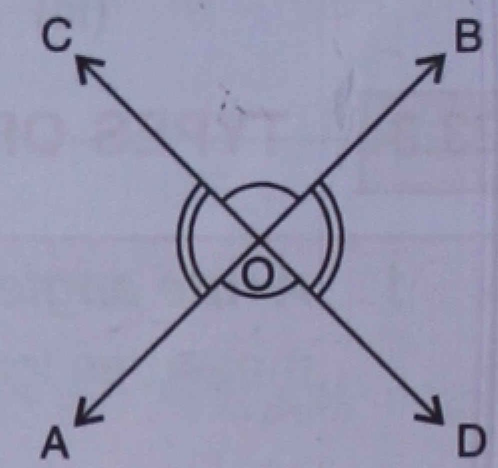
(iii) the other arms OA and OC of the two angles are on opposite sides of the common arm OB.



3. *Vertically opposite angles* : When two straight lines intersect each other four angles are formed.

The pair of angles which lie on the opposite sides of the point of intersection are called **vertically opposite angles**.

In the adjoining figure, two straight lines AB and CD intersect each other at point O. Angles AOD and BOC form one pair of vertically opposite angles, whereas angles AOC and BOD form another pair of vertically opposite angles.

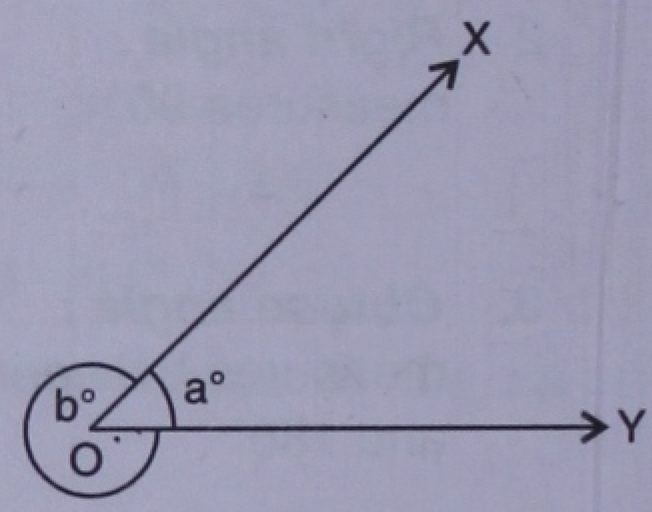


Vertically opposite angles are always equal.

i.e. $\angle AOD = \angle BOC$

and $\angle AOC = \angle BOD$.

Important : In the adjoining figure, rays OX and OY meet at O to form $\angle XOY$ (*i.e.*, $\angle a$) and reflex $\angle XOY$ (*i.e.*, $\angle b$). It must be noted that $\angle XOY$ represents the smaller angle only, unless it is mentioned otherwise.



23.5 COMPLEMENTARY AND SUPPLEMENTARY ANGLES

1 *Two angles are called* ***complementary angles,*** *if their sum is one right angle, i.e.,* 90°. Each angle is called the **complement** of the other.
e.g., 20° and 70° are complementary angles, because 20° + 70° = 90°.
Clearly, 20° is the complement of 70° and 70° is the complement of 20°.
Thus, the **complement of angle 53°** = 90° – 53° = **37°**.

2. *Two angles are called* ***supplementary angles,*** *if their sum is two right angles, i.e.,* 180°. Each angle is called the **supplement of the other.**
e.g., 30° and 150° are supplementary angles because 30° + 150° = 180°.
Clearly, 30° is the supplement of 150° and vice-versa.
Thus, the **supplement of 105°** = 180° – 105° = **75°**.

EXERCISE 23 (A)

1. State **true** or **false** :

(i) A line segment 4 cm long can have only 2000 points in it.

(ii) A ray has one end point and a line segment has two end-points.

(iii) A line segment is the shortest distance between any two given points.

(iv) An infinite number of straight lines can be drawn through a given point.

(v) 40° is the complement of 60°.

(vi) 45° is the supplement of 45°.

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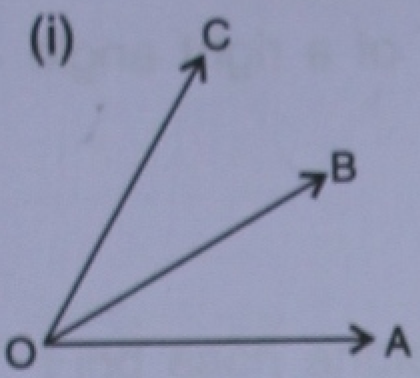
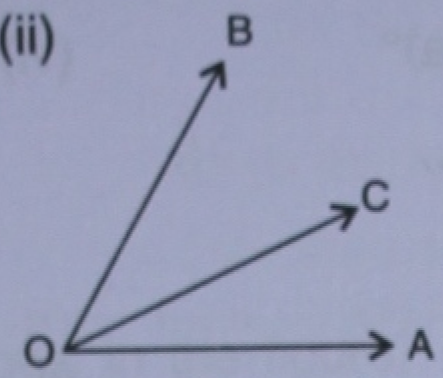
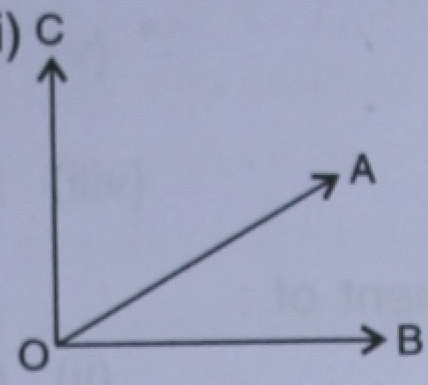
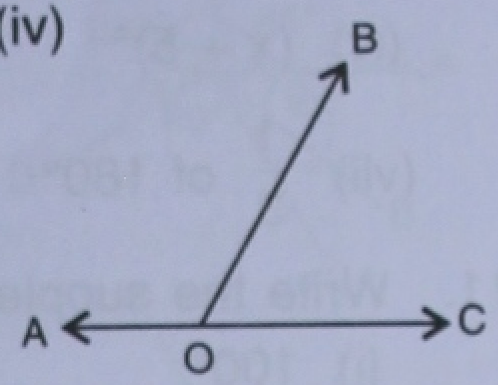
2. In which of the following figures, are ∠AOB and ∠AOC adjacent angles ? Give, in each case, reason for your answer.

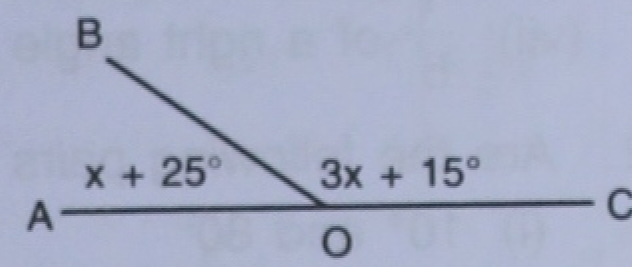
(i)


(ii)


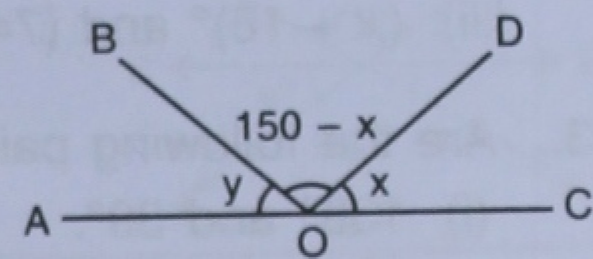
(iii)


(iv)


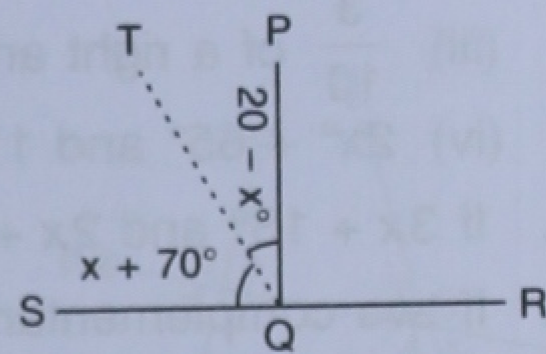
3. In the given figure, AC is a straight line. Find :
   (i) x,
   (ii) ∠AOB,
   (iii) ∠BOC.



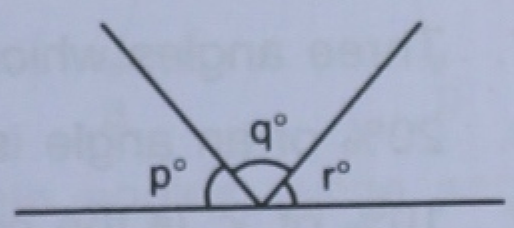
4. Find y in the given figure.



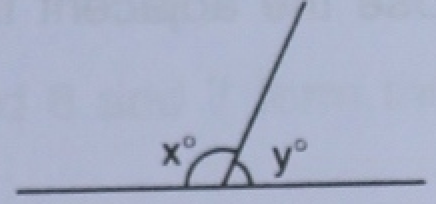
5. In the given figure, find ∠PQR.



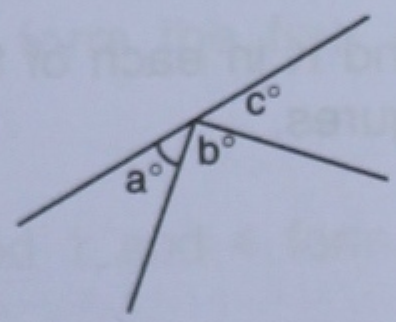
6. In the given figure, $p° = q° = r°$, find each.



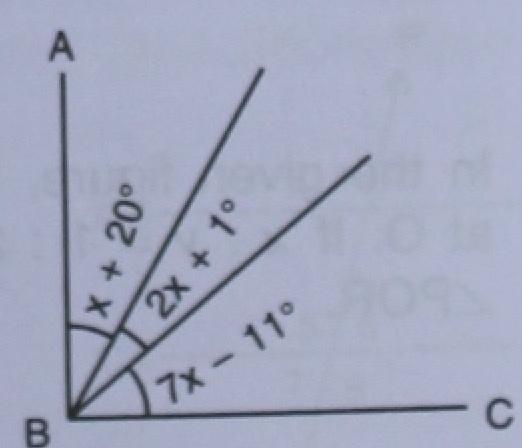
7. In the given figure, if $x = 2y$, find x and y.



8. In the adjoining figure, if $b° = a° + c°$, find b.



9. In the given figure, AB is perpendicular to BC at B.
   Find : (i) the value of x,
   (ii) the complement of angle x.



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10. Write the complement of :
   (i) 25°   (ii) 90°   (iii) $a°$
   (iv) $(x + 5)°$ (v) $(30 - a)°$ (vi) $\frac{1}{2}$ of a right angle
   (vii) $\frac{1}{3}$ of 180°   (viii) 21° 17′

11. Write the supplement of :
   (i) 100°   (ii) 0°   (iii) $x°$
   (iv) $(x + 35)°$ (v) $(90 + a + b)°$ (vi) $(110 - x - 2y)°$
   (vii) $\frac{1}{5}$ of a right angle   (viii) 80° 49′ 25″

12. Are the following pairs of angles complementary ?
   (i) 10° and 80°   (ii) 37° 28′ and 52° 33′
   (iii) $(x + 16)°$ and $(74 - x)°$ (iv) 54° and $\frac{2}{5}$ of a right angle.

13. Are the following pairs of angles supplementary ?
   (i) 139° and 39°.   (ii) 26° 59′ and 153° 1′.
   (iii) $\frac{3}{10}$ of a right angle and $\frac{4}{15}$ of two right angles.
   (iv) $2x° + 65°$ and $115° - 2x°$.

14. If $3x + 18°$ and $2x + 25°$ are supplementary, find the value of x.

15. If two complementary angles are in the ratio 1 : 5, find them.

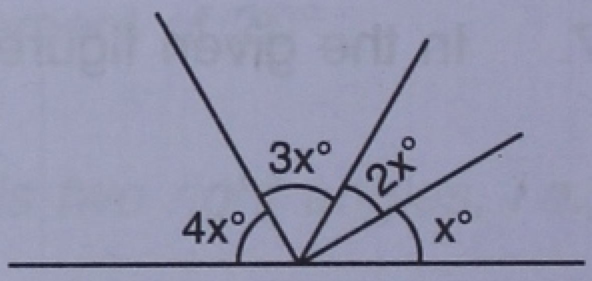
16. If two supplementary angles are in the ratio 2 : 7, find them.

17. Three angles which add upto 180° are in the ratio 2 : 3 : 7. Find them.

18. 20% of an angle is the supplement of 60°. Find the angle.

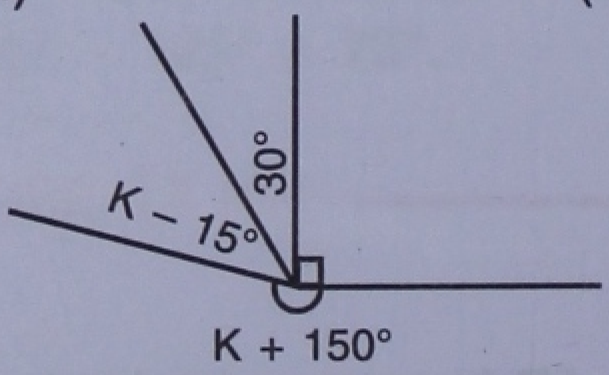
19. 10% of $x°$ is the complement of 40% of $2x°$. Find x.

20. Use the adjacent figure, to find angle x and its supplement.

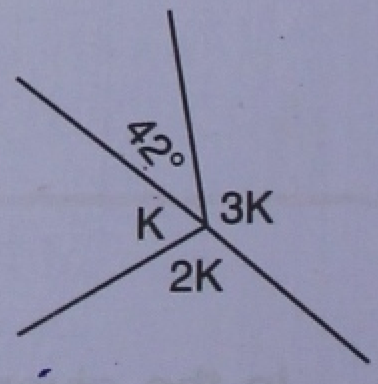


21. Find K in each of the given figures.

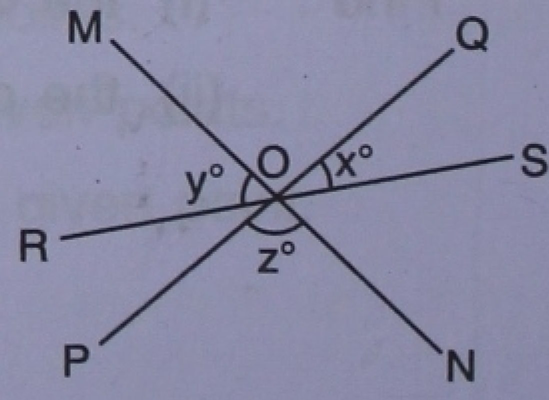
(i)



(ii)



22. In the given figure, lines PQ, MN and RS intersect at O. If $x : y = 1 : 2$ and $z = 90°$, find ∠ROM and ∠POR.



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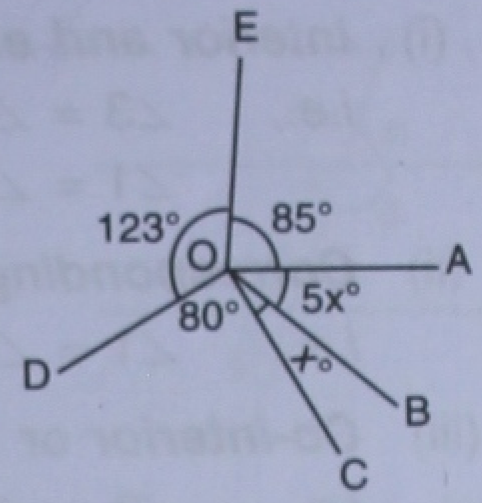
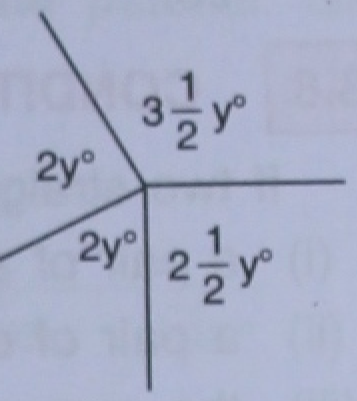
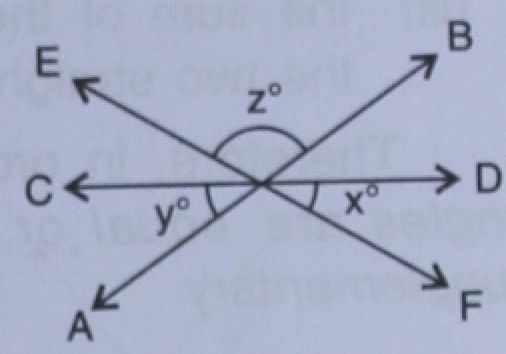
23. In the given figure, find ∠AOB and ∠BOC.



24. Find each angle shown in the figure.



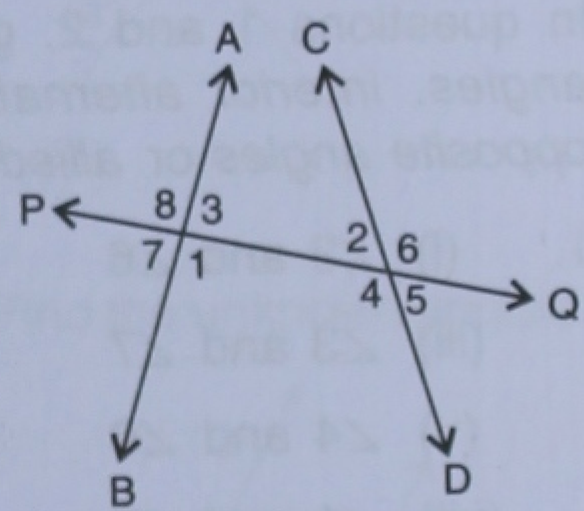
25. *AB, CD* and *EF* are three lines intersecting at the same point.
   (i) Find x, if $y = 45°$ and $z = 90°$.
   (ii) Find a, if $x = 3a$, $y = 5x$ and $z = 6x$.



23.6 THE CONCEPT OF A TRANSVERSAL

A straight line which cuts two or more given straight lines is called a **transveral**.

In the adjoining figure, PQ cuts straight lines AB and CD, and so it is a transversal.



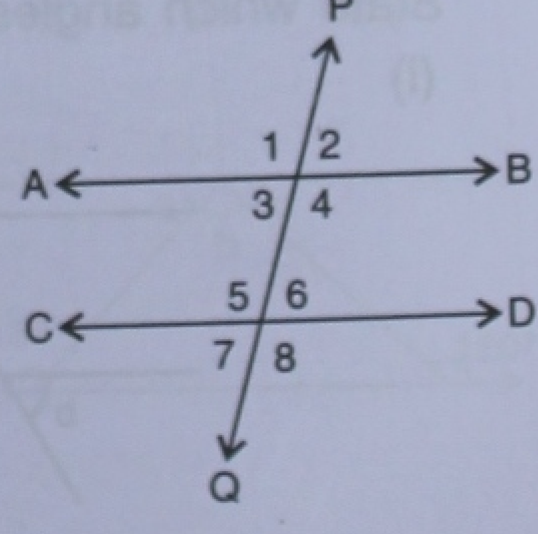
When a transversal cuts two given straight lines (refer the adjoining figure), the following pairs of angles are formed.

1. **Two pairs of *interior alternate angles* :**
   Angles marked 1 and 2 form one pair of interior alternate angles, while angles marked 3 and 4 form another pair of interior alternate angles.
2. **Two pairs of *exterior alternate angles* :**
   Angles marked 5 and 8 form one pair, while angles marked 6 and 7 form the other pair of exterior alternate angles.
3. **Four pairs of *corresponding angles* :**
   Angles marked 3 and 6, 1 and 5, 8 and 2, 7 and 4 form the four pairs of corresponding angles.
4. **Two pairs of *allied* or *co-interior or conjoined angles* :**
   Angles marked 3 and 2 form one pair and angles marked 1 and 4 form another pair of allied angles.

23.7 PARALLEL LINES

Two straight lines are said to be parallel, if they do not meet anywhere, no matter how long are they produced in any direction.

The adjacent figure shows two parallel lines AB and CD.



When two parallel lines AB and CD are cut by a transversal PQ :

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(i) ***Interior and exterior alternate angles are equal :***

i.e., ∠3 = ∠6 and ∠4 = ∠5 [Interior alternate angles]

∠1 = ∠8 and ∠2 = ∠7 [Exterior alternate angles]

(ii) ***Corresponding angles are equal :***

i.e., ∠1 = ∠5; ∠2 = ∠6; ∠3 = ∠7 and ∠4 = ∠8

(iii) ***Co-interior or allied angles are supplementary :***

i.e., ∠3 + ∠5 = 180° and ∠4 + ∠6 = 180°.

23.8 CONDITIONS OF PARALLELISM

If two straight lines are cut by a transversal such that :

(i) a pair of alternate angles are equal, or

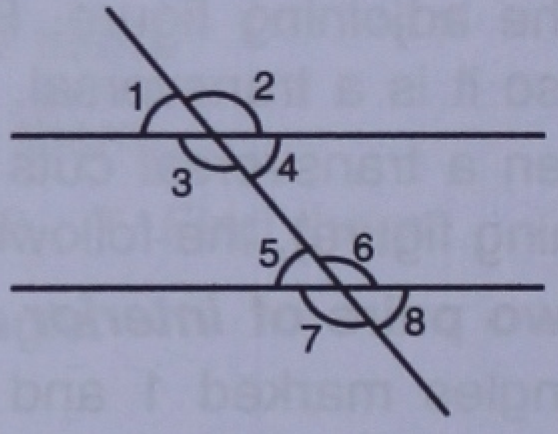
(ii) a pair of corresponding angles are equal, or

(iii) the sum of the interior angles on the same side of the transversal is 180°, then the two straight lines are parallel to each other.

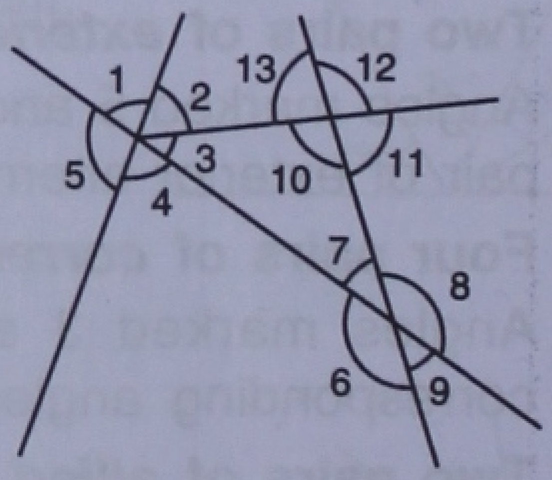
Therefore, in order to prove that the given lines are parallel; show either *alternate angles* are *equal* or, *corresponding angles are equal* or, the *co-interior angles are supplementary.*

EXERCISE 23 (B)

In questions 1 and 2, given below, identify the given pairs of angles as *corresponding angles, interior alternate angles, exterior alternate angles, adjacent angles, vertically opposite angles* or *allied angles :*



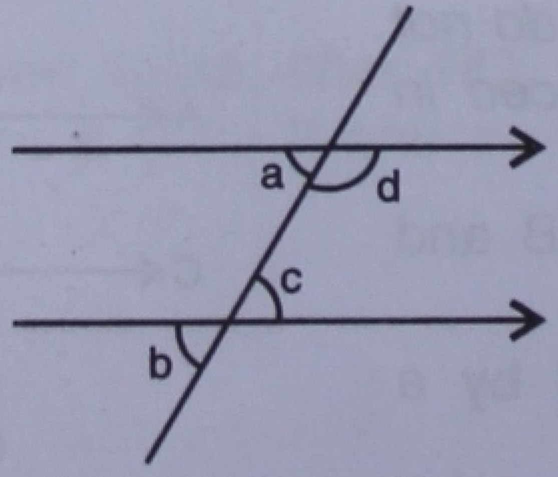
1. (i) ∠3 and ∠6 (ii) ∠2 and ∠4
(iii) ∠3 and ∠7 (iv) ∠2 and ∠7
(v) ∠4 and ∠6 (vi) ∠1 and ∠8
(vii) ∠1 and ∠5 (viii) ∠1 and ∠4
(ix) ∠5 and ∠7



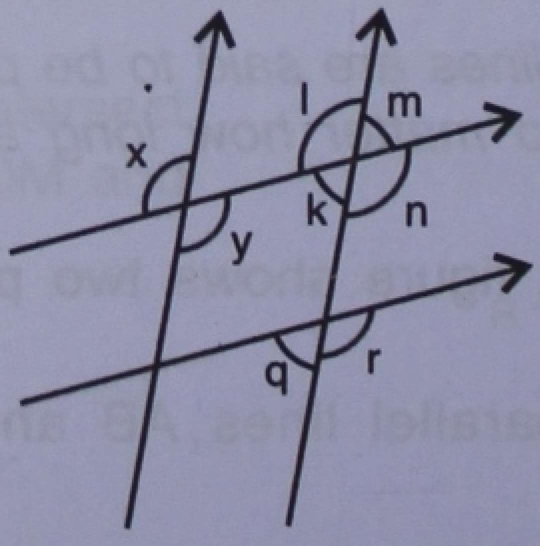
2. (i) ∠1 and ∠4 (ii) ∠4 and ∠7
(iii) ∠10 and ∠12 (iv) ∠7 and ∠13
(v) ∠6 and ∠8 (vi) ∠11 and ∠8
(vii) ∠7 and ∠9 (viii) ∠4 and ∠5
(ix) ∠4 and ∠6 (x) ∠6 and ∠7
(xi) ∠2 and ∠13.

3. In the following figures, the arrows indicate parallel lines.
State which angles are equal. Give reasons.

(i)



(ii)



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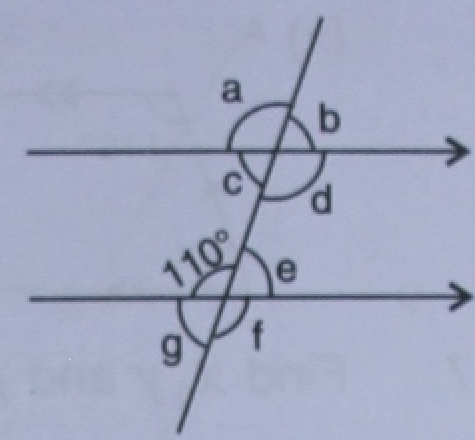
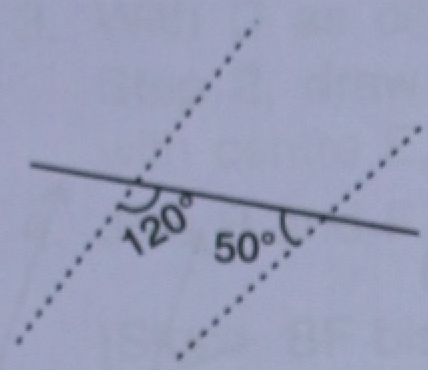
4. In the given figure, find the measure of the unknown angles :

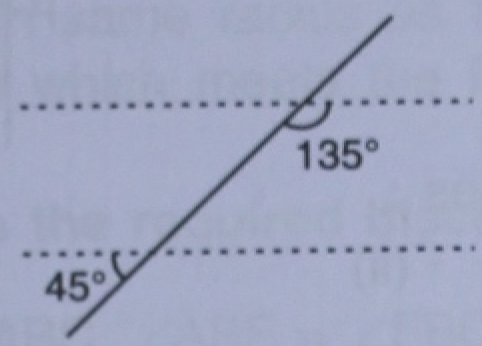


5. Which pair of the dotted line segments, in the following figures, are parallel. Give reason :

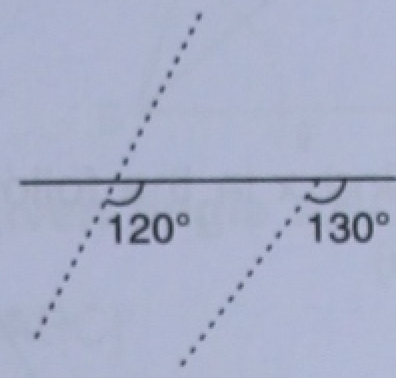
(i)



(ii)



(iii)



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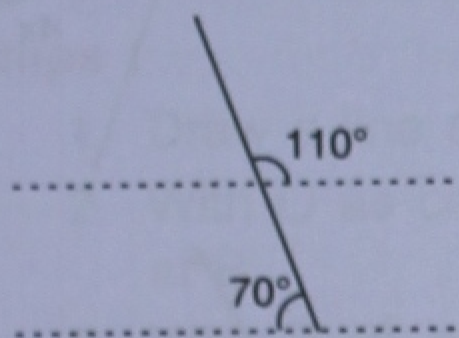

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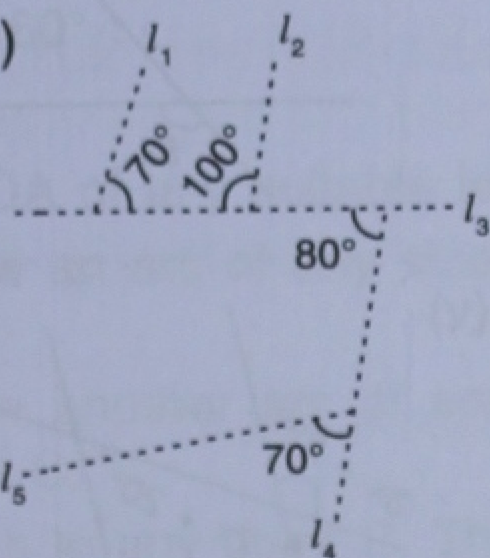

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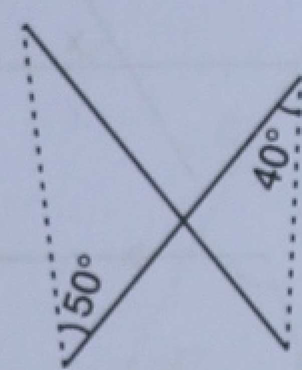
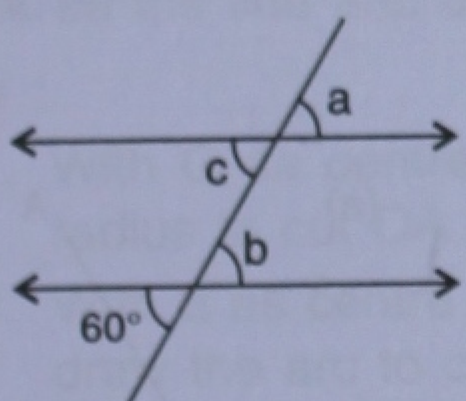
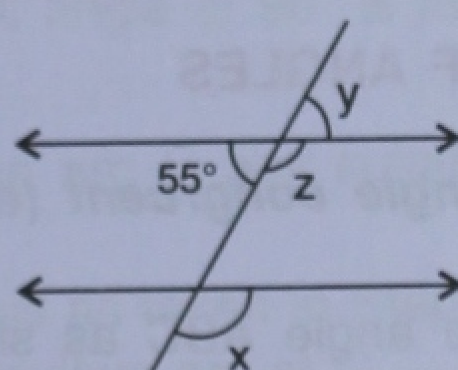


6. In the given figures, the directed lines are parallel to each other. Find the unknown angles.

(i)



(ii)



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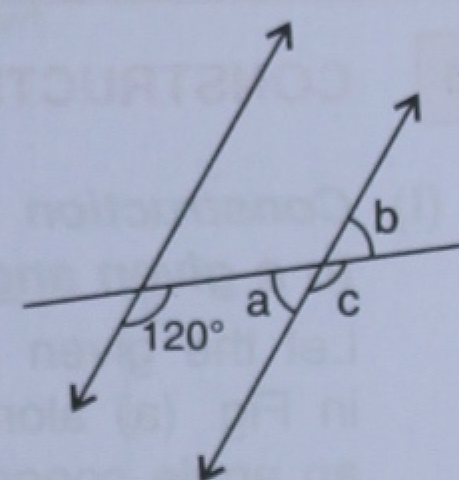

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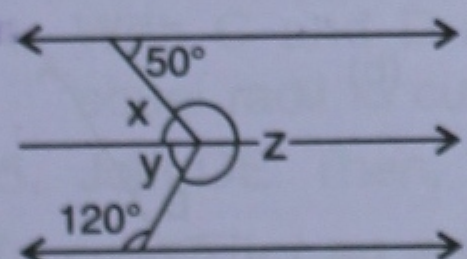

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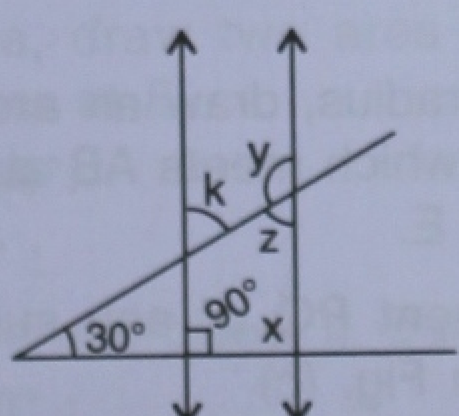

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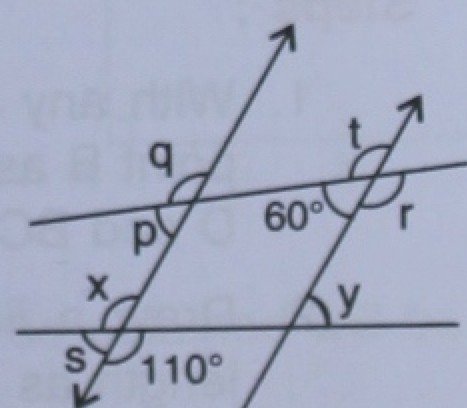

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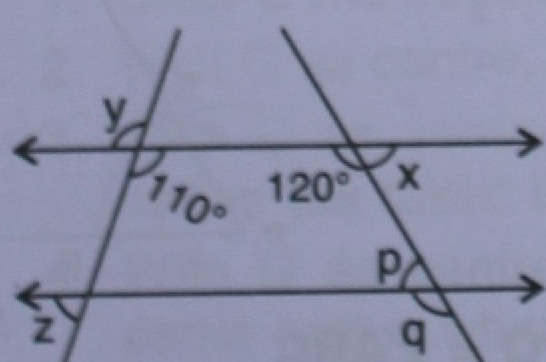
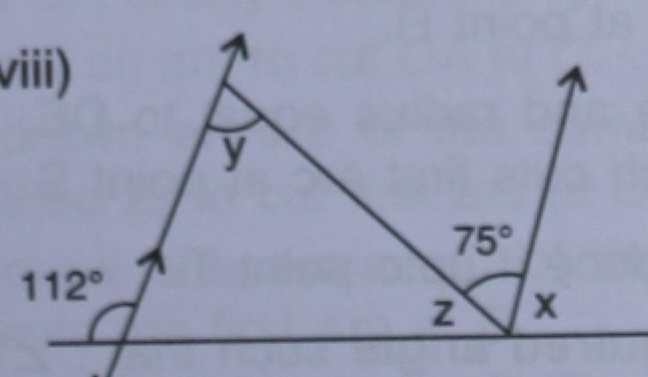


(viii)



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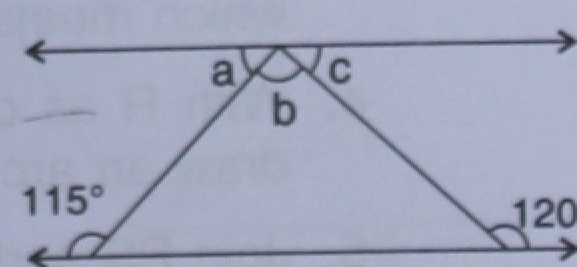

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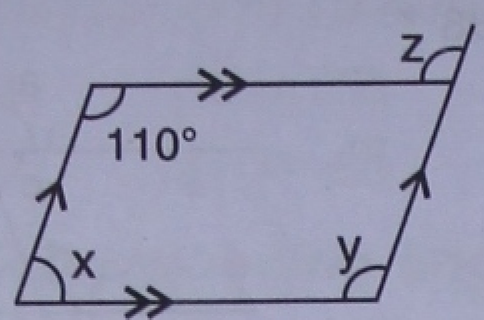
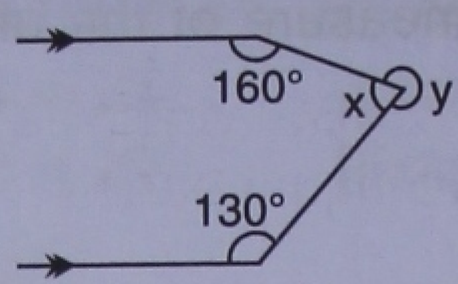
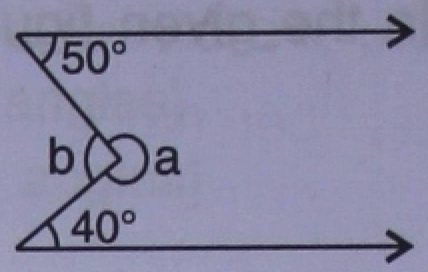
(x)



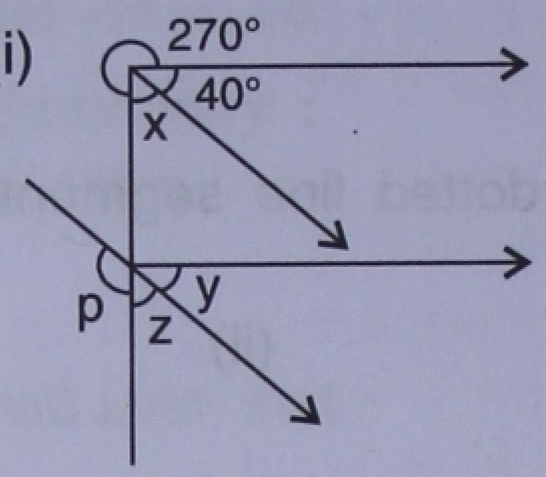
(xi)



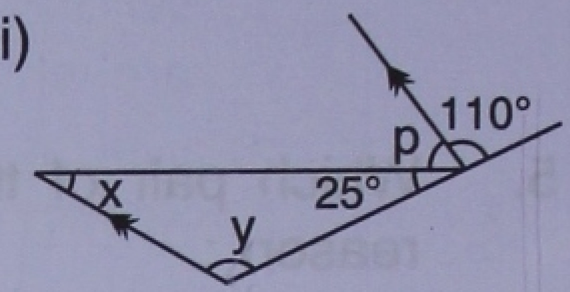
(xii)



7. Find x, y and p in the given figures : (i)

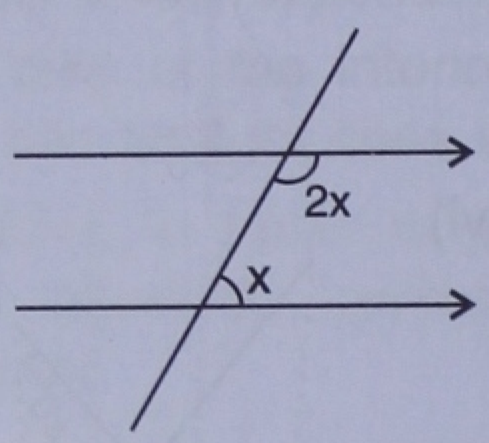


(ii)

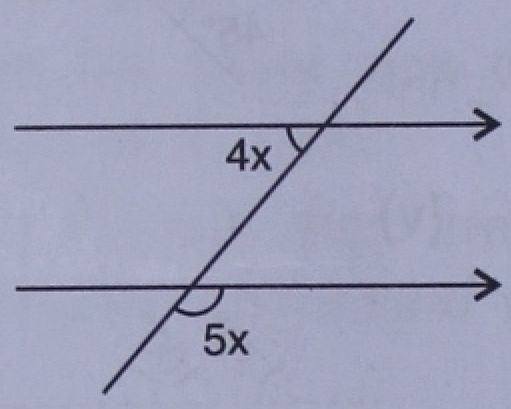


8. Find x in the following cases :

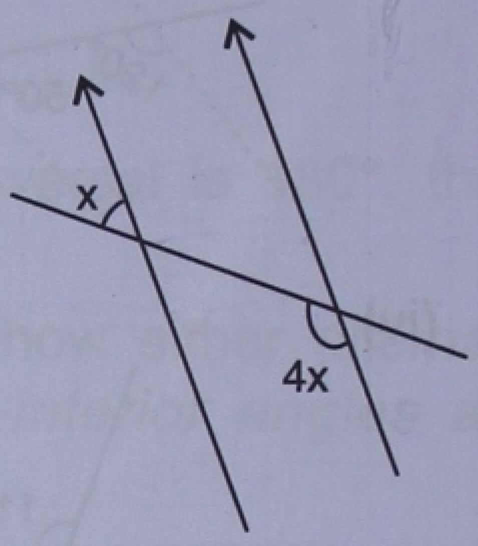
(i)



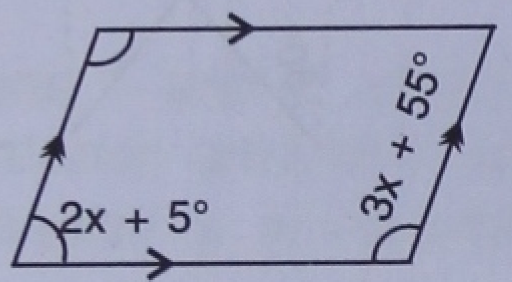
(ii)



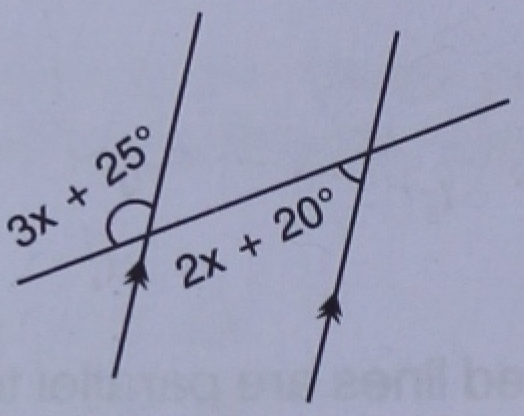
(iii)



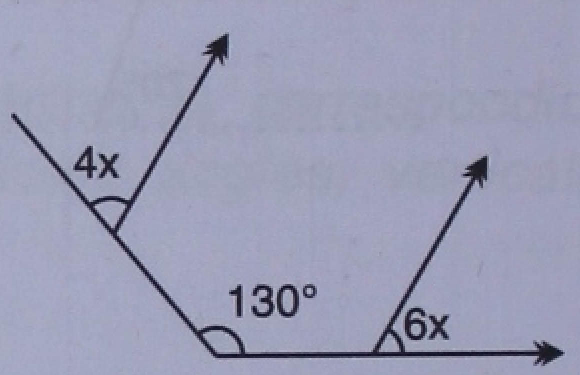
(iv)



(v)



(vi)

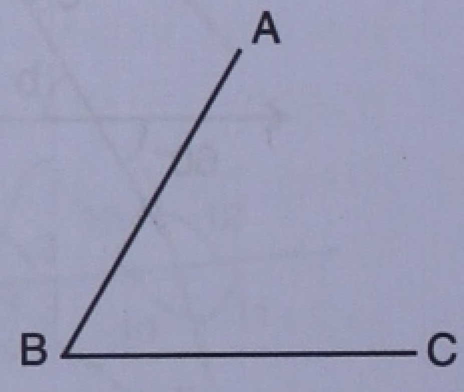


23.9 CONSTRUCTIONS OF ANGLES

(i) *Construction of an angle congruent* *(equal)* ***to a given angle :***

Let the given angle be angle ABC as shown in Fig. (a) alongside and, we are to construct an angle congruent (equal) to it.

(a)

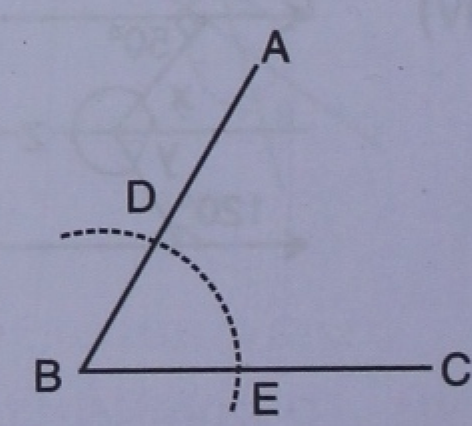


Steps :

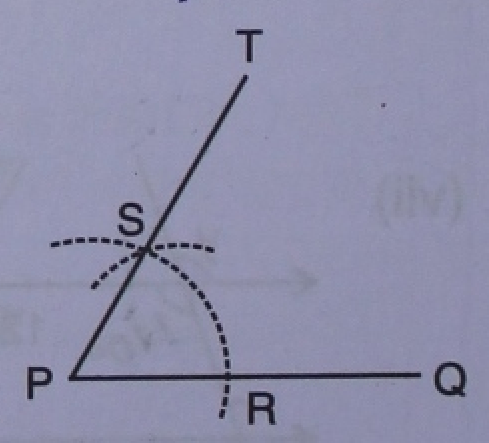
1. With any suitable radius, draw an arc with point B as centre, which meets AB at point D and BC at point E.
2. Draw a line segment PQ of any suitable length as shown in Fig. (c)
3. With the same radius as taken in Step 1, draw one more arc with point P as centre, which meets PQ at point R.
4. With R as centre and radius equal to DE, draw an arc which cuts first arc at point S.
5. Join PS and produce it upto point T.

   ∠TPQ is the required angle such that : **∠TPQ = ∠ABC**

(b)



(c)



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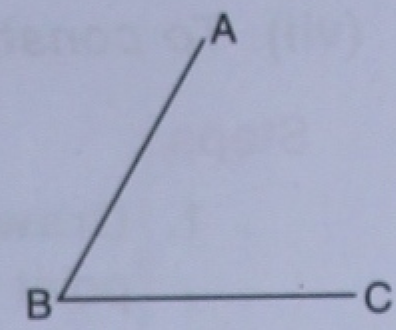
(ii) *To bisect a given angle :*

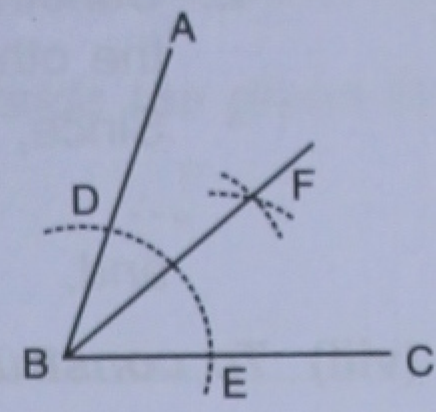
Let the given angle be ∠ABC which is to be bisected, *i.e.*, to be divided into two equal parts.



Steps :

1. With B as centre and any suitable radius, draw an arc which meets AB at point D and BC at point E.
2. With E as centre and radius equal to more than half of DE, draw an arc.
3. With D as centre and with same radius as taken in Step 2, draw another arc which meets the first arc, with centre E, at point F.
4. Join B and F. **Then BF is the required bisector of given angle ABC.**

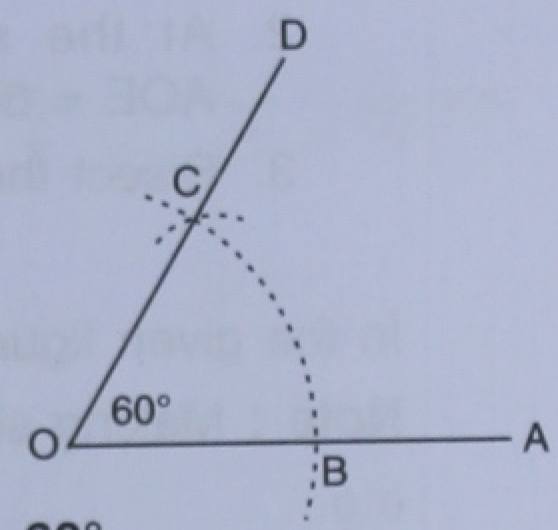
   [Since, BF bisects angle ABC, $\angle ABF = \angle FBC = \frac{1}{2} \angle ABC$]



(iii) *To construct an angle of 60° :*

Steps :

1. Draw a line segment OA of any suitable length.
2. With O as centre, draw an arc of any size to cut *OA* at B.
3. With B as centre draw another arc of same size to cut the previous arc at C.
4. Join OC and produce it to any point D. Then, **∠DOA = 60°.**

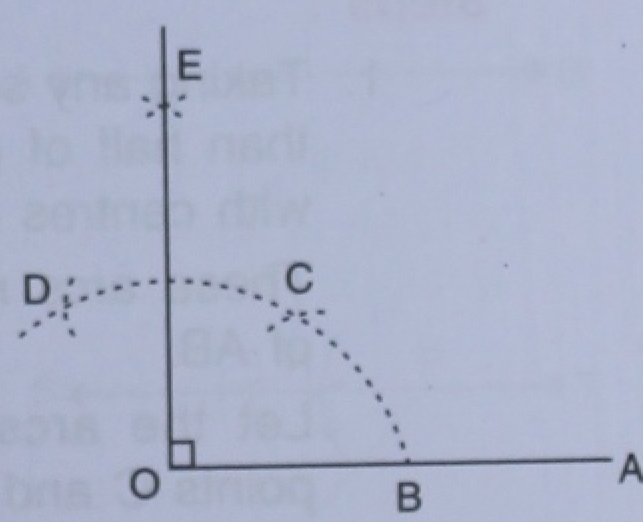


(iv) *To construct an angle of 90° :*

Let OA be the line and at point O, angle of 90° is to be drawn.

Steps :

1. With O as centre, draw an arc of any suitable radius to cut OA at B.
2. With B as centre and radius as taken in Step 1, draw the arc to cut the previous arc at C.
3. Again with C as centre and with the same radius, draw one more arc to cut the first arc at D.
4. With C and D as centres, draw two arcs of equal radii to cut each other at *E*.
5. Join OE. Then, **∠AOE = 90°.**



(v) *To construct an angle of 45° :*

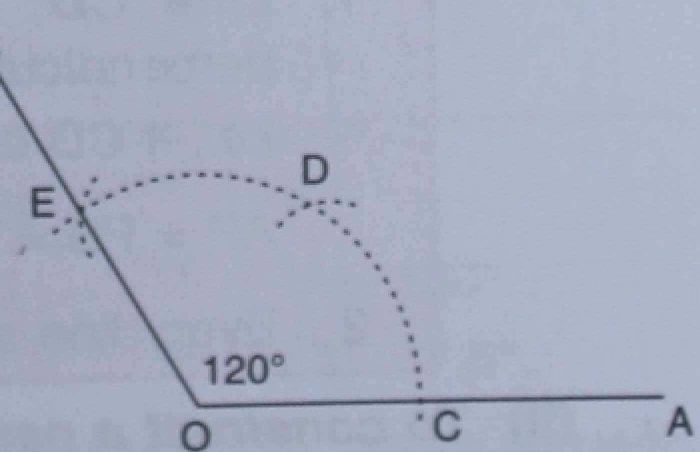
Draw an angle of 90° as above and bisect it. Each angle so obtained will be 45°.

(vi) *To construct an angle of 120° :*

Steps :

1. Draw a line segment OA of any suitable length.
2. With O as centre, draw an arc to cut OA at C.
3. With C as centre, drawn an arc of the same radius (as taken in Step 2) to cut first arc at D.
4. With D as centre, draw one more arc of the same radius which cuts the first arc at E.

   Join OE and produce it upto any point *B*. Then, **∠AOB = 120°.**



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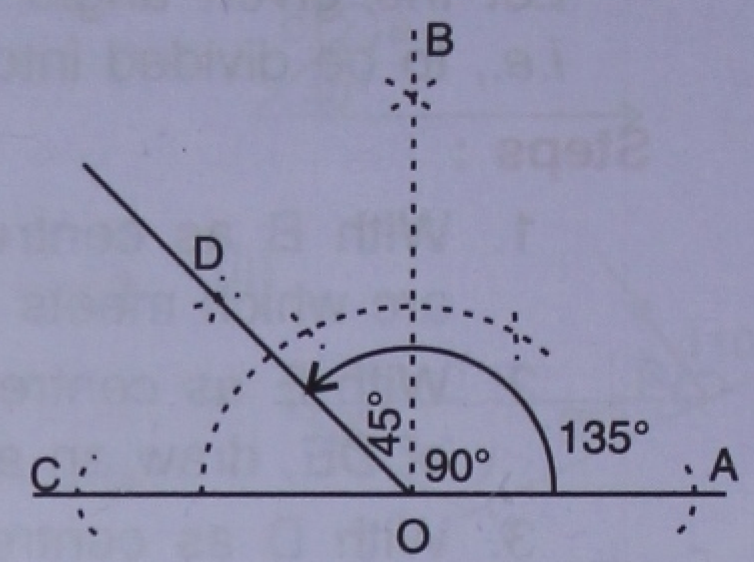
(vii) ***To construct an angle of 135° :***

Steps :



1. Draw an angle BOA = 90° at the point O on the given line segment AC.
2. Construct OD to bisect the angle BOC on the other side of OB.

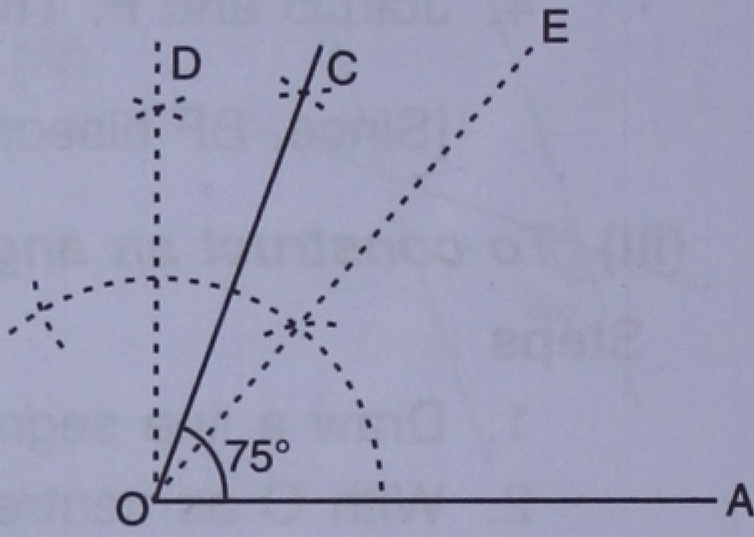
Since, $\angle BOC = \angle BOA = 90°$

$\therefore$ $\angle BOD = \angle COD = 45°$

and, $\mathbf{\angle AOD} = 90° + 45° = \mathbf{135°}$

(viii) ***To construct an angle of 75° :***

Steps :



1. Draw an angle AOD = 90° at a point O on the line segment OA.
2. At the same point O, draw an angle AOE = 60°.
3. Bisect the ∠DOE, so that

$$\angle EOC = \angle DOC = 15°.$$

In the given figure : $\mathbf{\angle AOC} = \angle AOE + \angle EOC = 60° + 15° = \mathbf{75°}$.

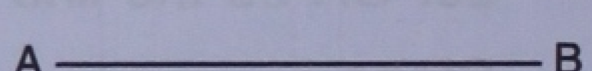
Note : Making similar combinations, many other angles can be drawn,

e.g., $105° = 90° + 15°$, $165° = 180° - 15°$, etc.

23.10 SOME SPECIAL CONSTRUCTIONS

(i) ***To construct the bisector of a given line segment :***

Let the given line segment be AB as shown alongside.

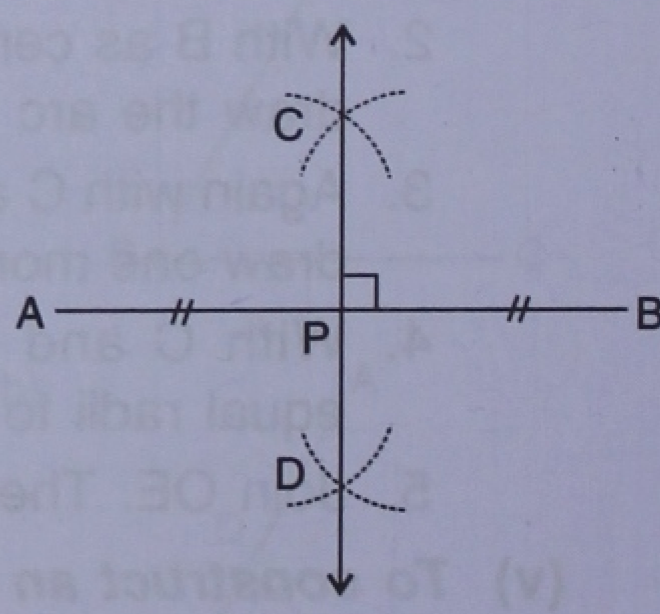


Steps :

1. Taking any suitable radius, which must be more than half of given line segment AB, draw arcs with centres at points A and B.

   These arcs must be drawn on both the sides of AB.

   Let the arcs drawn, intersect each other at points C and D as shown alongside.

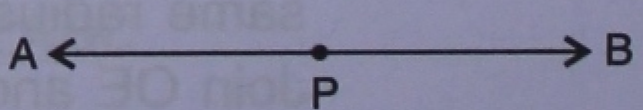


2. Draw a line through the points C and D.

> 1. Line *CD*, as drawn above, bisects the given line segment AB perpendicularly.
>
>   *i.e.,* if CD cuts AB at point P, then
>
> $AP = PB = \frac{1}{2}$ AB and $\angle APC = 90°$
>
> 2. Every line segment has one and only one perpendicular bisector of it.

(ii) ***To construct a perpendicular to a line through a point in it***

Let the given line be line AB and P be a point in it through which a perpendicular is to be drawn.



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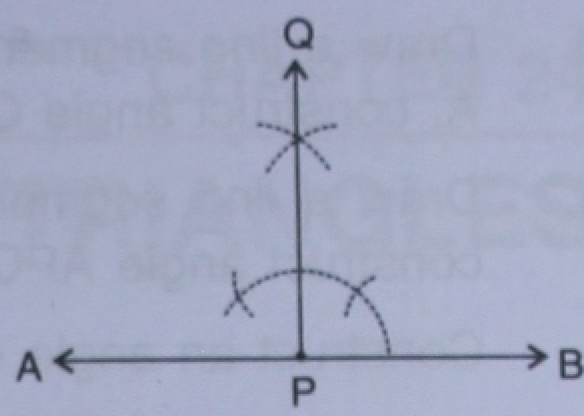
Steps :

At point P, in line AB, construct an angle of 90° *i.e.*, draw PQ so that ∠QPB = ∠QPA = 90°.

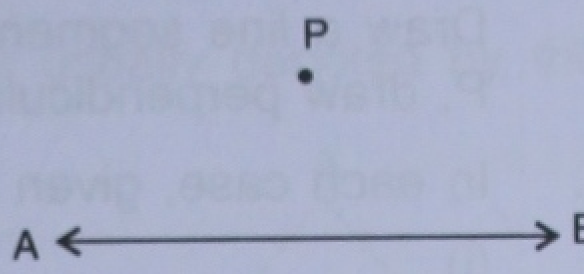
Then, **PQ is the required perpendicular to** the given line AB at point P on it.



(iii) ***To construct a perpendicular to a line through a point outside the given line.***

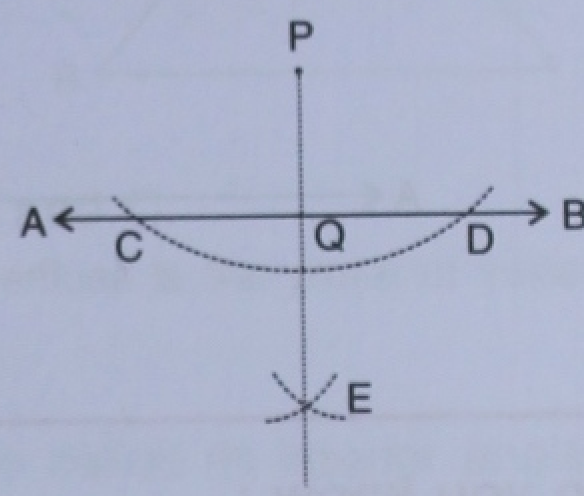
Let AB be the given line and P be the point outside AB.

Required to draw perpendicular to AB from point P.



Steps :

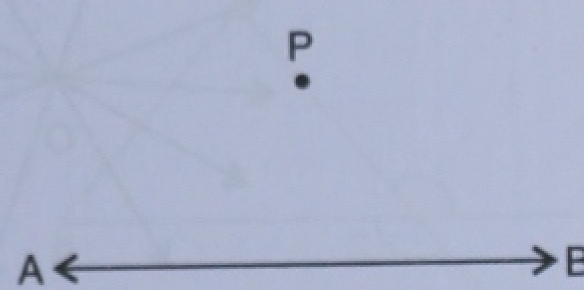
1. Taking P as centre and with a suitable radius, draw an arc which meets AB at points C and D.
2. Taking C and D as centres, draw arcs of equal radii which cut each other at point E, below the line AB.
   [The radii of arcs in this step must be of lengths more than half of CD].
3. Join P and E.
4. Let PE intersects AB at point Q.
   ∴ **PQ is the required perpendicular.**



(iv) ***To construct a line parallel to the given line and passing through the given fixed point.***

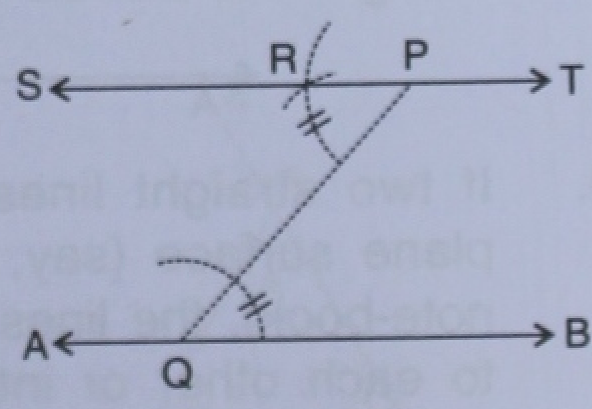
Let AB be the given line and P be the given fixed point.

Required to draw a line through the point P and parallel to the given line AB.



Steps :

1. Mark a point Q in the given line AB.
2. Join P and Q.
3. At point P, draw (copy) angle QPR equal to angle PQB.
   i.e., draw ∠QPR = ∠PQB.
4. Draw line ST through points R and P.
   ∴ **Line ST is the required line through the given point P and parallel to the given line *AB*.**



EXERCISE 23 (C)

1. Using ruler and compasses, construct the following angles :
   (i) 30° (ii) 15° (iii) 75° (iv) 180°
   (v) 165° (vi) 22.5° (vii) 37.5° (viii) 67.5°.
2. Draw ∠ABC = 120°. Bisect the angle using ruler and compasses only. Measure each angle so obtained and check whether the angles obtained on bisecting ∠ABC are equal or not.

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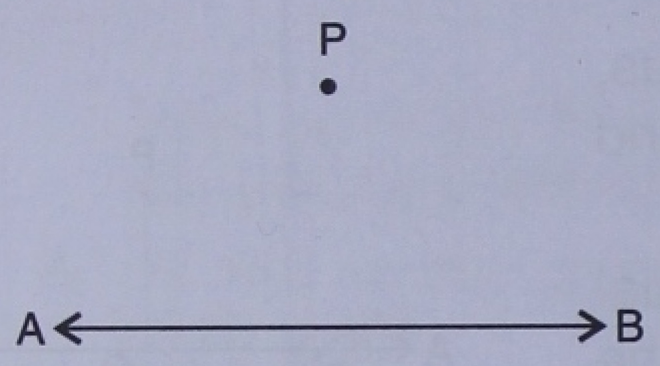
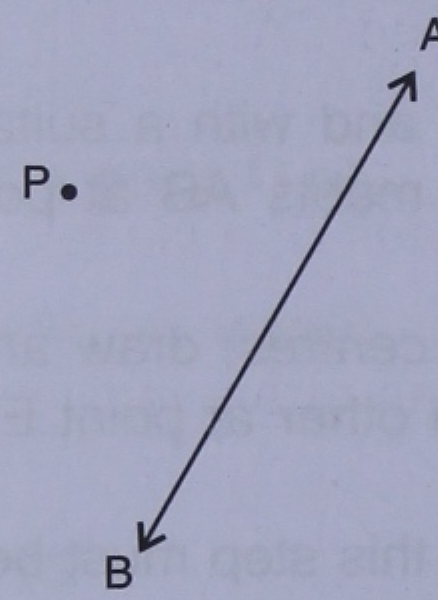
3. Draw a line segment PQ = 6 cm. Mark a point A in PQ so that AP = 2 cm. At point A, construct angle QAR = 60°.
4. Draw a line segment AB = 8 cm. Mark a point P in AB so that AP = 5 cm. At P, construct angle APQ = 30°.
5. Construct an angle of 75° and then bisect it.
6. Draw a line segment of length 6·4 cm. Draw its perpendicular bisector.
7. Draw a line segment AB = 5·8 cm. Mark a point *P* in AB such that PB = 3·6 cm. At P, draw perpendicular to AB.
8. In each case, given below, draw a line through point P and parallel to AB :

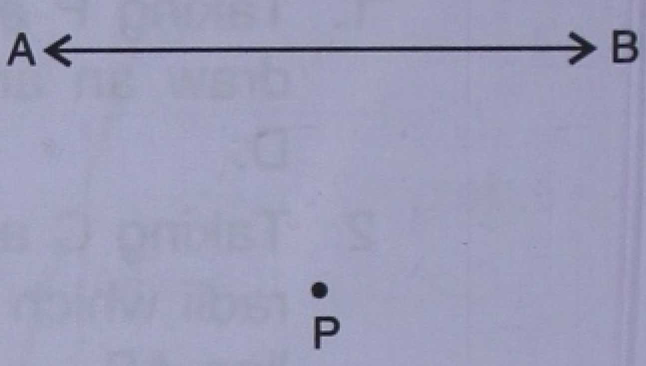
(i)



(ii)

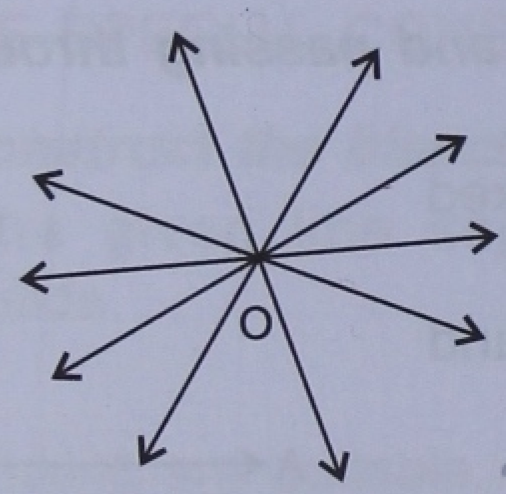


(iii)

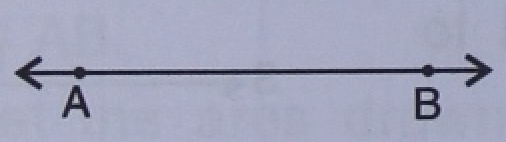


Do you know :

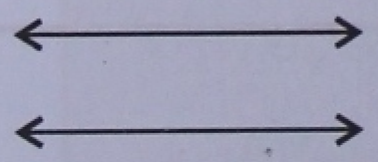
1. Through any given point (say, point O) an infinite number of lines can be drawn :



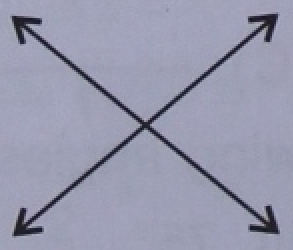
2. Through any two given points (say, points A and B) one and only one straight line can be drawn :



3. If two straight lines are drawn on a plane surface (say, the page of your note-book), the lines are either parallel to each other or intersect each other exactly at one point.



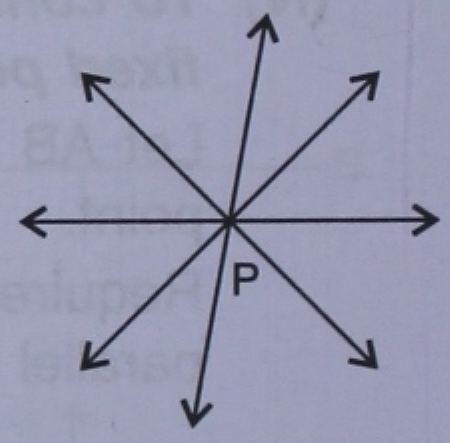
OR



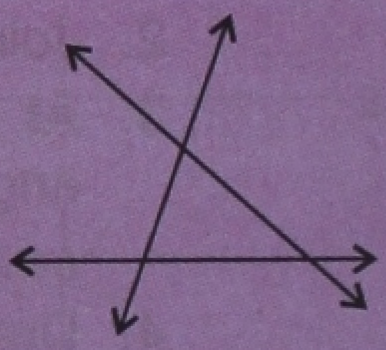
4. When several (three or more than three) lines are drawn in a plane (say, page of your note-book) such that all the lines drawn pass through the same point; the lines are called **concurrent lines**.

The point through which the lines drawn pass, is called the **point of concurrence**.

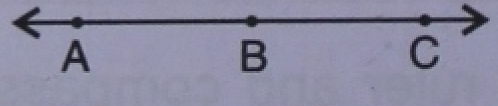
In the given figure, P is the point of concurrence.



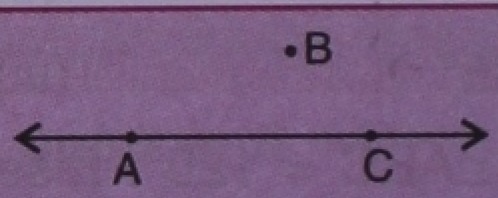
The adjoining figure shows **non-concurrent lines**.



5. If three or more points lie on the same straight line, the points are called **collinear points**.



[A, B and C are collinear points]



[A, B and C are non-collinear points]

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