# CHAPTER 22

# INDICES

## 22.1 INDEX OR EXPONENT

In $3 \times 3 \times 3 \times 3 \times 3$, the factor 3 is being multiplied 5 times by itself and can also be written as $3^5$, *i.e.*, $3 \times 3 \times 3 \times 3 \times 3 = 3^5$.

**In $3^5$, the repeated factor 3** is called the **base** and the **number 5**, written slightly raised at the right of the factor 3, is called the **index** or **exponent**.

Thus, an index or an exponent is a number which indicates how many times the base is used as a repeated factor.

***The plural of index is indices.***

1. If ***n*** is a **whole number** and **a** is **any number**, then :

   $a^n = \underbrace{a \times a \times a \times a \times \ldots\ldots \times a}_{n \text{ factors}}$ ***(base = a*** and ***index = n.)***

2. ***In $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$, base = 5*** *and* ***index = 7.***
3. *If base = –3 and index (exponent) = 8, the number = $(-3)^8$.*

## 22.2 LAWS OF INDICES

**First Law** (Product Law) :

$$a^m \times a^n = a^{m+n}$$

*When numbers are in the exponent form with the same base, to get their product (multiplication), add their powers (indices) keeping the base same.*

*For example :*

(i) $a^3 \times a^7 = a^{3+7} = a^{10}$

(ii) $x^2y^3 \times x^4y^2 = (x^2 \times x^4) \times (y^3 \times y^2)$
$= x^{2+4} \times y^{3+2} = x^6y^5$

(iii) $4a^2b^3c^2 \times 8a^9b^6c \times 3ab^{10}c^5 = 4 \times 8 \times 3 \times a^{2+9+1} \times b^{3+6+10} \times c^{2+1+5}$
$= 96a^{12}b^{19}c^8$

**Second Law** (Quotient Law) :

$$\frac{a^m}{a^n} = a^{m-n}; \text{ if } m > n \quad \text{and} \quad \frac{a^m}{a^n} = \frac{1}{a^{n-m}}; \text{ if } m < n.$$

*When a number in exponent form is divided by another number in the exponent form (both the numbers having the same base), the smaller index* (power) *is subtracted from the bigger index* (power) *and the base is kept the same.*

*For example :*

(i) $x^5 \div x^3 = \frac{x^5}{x^3} = x^{5-3} = x^2$ (ii) $15a^2 \div 5a^{10} = \frac{15a^2}{5a^{10}} = \frac{3}{a^{10-2}} = \frac{3}{a^8}$

**Third Law** (Power Law) :

$$(a^m)^n = a^{mn}$$

*when a number in the index form is raised to another index, the base is raised to the product of these two indices.*

*For example :*

(i) $(a^3)^6 = a^{3 \times 6} = a^{18}$ (ii) $(x^6)^{3/2} = x^{6 \times 3/2} = x^9$

## 22.3 MORE ABOUT INDICES

1. $(ab)^m = a^m b^m$ And $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

e.g. (i) $(xy)^3 = x^3y^3$ (ii) $(2a^3)^2 = 2^2(a^3)^2 = 4a^6$

(iii) $\left(\frac{3x}{4y^2}\right)^4 = \frac{3^4 \cdot x^4}{4^4 \cdot (y^2)^4} = \frac{81x^4}{256\,y^8}$ and so on.

2. Any non-zero base raised to the power **zero** is equal to unity (*i.e.*, 1).

*i.e.* $a^0 = 1$, if $a \neq 0$

e.g. (i) $5^0 = 1$ (ii) $(-3)^0 = 1$ (iii) $\left(\frac{x^2}{2y}\right)^0 = 1$ and so on.

3. **Negative Index :** If $a \neq 0$ then : $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = a^m$

e.g. (i) $a^{-4} = \frac{1}{a^4}$ (ii) $\frac{1}{x^{-7}} = x^7$ and so on.

***Also note that :***

(i) $\sqrt{a} = a^{1/2}$, e.g. $\sqrt{3} = 3^{1/2}$ (ii) $\sqrt[3]{a} = a^{1/3}$, e.g. $\sqrt[3]{3} = 3^{1/3}$

(iii) $\sqrt{a^5} = a^{5/2}$, e.g. $\sqrt{3^5} = 3^{5/2}$ (iv) $\sqrt[n]{a} = a^{1/n}$, e.g. $\sqrt[n]{3} = 3^{1/n}$

### EXERCISE 22

1. Fill in the blanks :

(i) In $5^2 = 25$, base = ..... and index = .....

(ii) If index = $3x$ and base = $2y$, the number = .....

2. Evaluate :

(i) $2^8 \div 2^3$ (ii) $2^3 \div 2^8$ (iii) $(2^6)^0$

(iv) $(3^0)^6$ (v) $8^3 \times 8^{-5} \times 8^4$ (vi) $5^4 \times 5^3 \div 5^5$

(vii) $5^4 \div 5^3 \times 5^5$ (viii) $4^4 \div 4^3 \times 4^0$ (ix) $(3^5 \times 4^7 \times 5^8)^0$

3. Simplify, giving answers with positive index :

(i) $2b^6 \cdot b^3 \cdot 5b^4$ (ii) $x^2y^3 \cdot 6x^5y \cdot 9x^3y^4$

(iii) $(-a^5)\ (a^2)$ (iv) $(-y^2)\ (-y^3)$

(v) $(-3)^2\ (3)^3$ (vi) $(-4x)\ (-5x^2)$

(vii) $(5a^2b)\ (2ab^2)\ (a^3b)$ (viii) $x^{2a+7} \cdot x^{2a-8}$

(ix) $3^y \cdot 3^2 \cdot 3^{-4}$ (x) $2^{4a} \cdot 2^{3a} \cdot 2^{-a}$

(xi) $4x^2y^2 \div 9x^3y^3$ (xii) $(10^2)^3\ (x^8)^{12}$

(xiii) $(a^{10})^{10}\ (1^6)^{10}$ (xiv) $(n^2)^2\ (-n^2)^3$

(xv) $-(3ab)^2\ (-5a^2bc^4)^2$ (xvi) $(-2)^2 \times (0)^3 \times (3)^3$

(xvii) $(2a^3)^4\ (4a^2)^2$ (xviii) $(4x^2y^3)^3 \div (3x^2y^3)^3$

(xix) $\left(\frac{1}{2x}\right)^3 \times (6x)^2$ (xx) $\left(\frac{1}{4ab^2c}\right)^2 \div \left(\frac{3}{2a^2bc^2}\right)^4$

(xxi) $\frac{(5x^7)^3 \cdot (10x^2)^2}{(2x^6)^7}$ (xxii) $\frac{(7p^2q^9r^5)^2\ (4pqr)^3}{(14p^6q^{10}r^4)^2}$

4. Simplify and express the answer in the positive exponent form :

(i) $\frac{(-3)^3 \times 2^6}{6 \times 2^3}$ (ii) $\frac{(2^3)^5 \times 5^4}{4^3 \times 5^2}$ (iii) $\frac{36 \times (-6)^2 \times 3^6}{12^3 \times 3^5}$

(iv) $-\frac{128}{2187}$ (v) $\frac{a^{-7} \times b^{-7} \times c^5 \times d^4}{a^3 \times b^{-5} \times c^{-3} \times d^8}$ (vi) $(a^3b^{-5})^{-2}$

5. Evaluate :

(i) $6^{-2} \div (4^{-2} \times 3^{-2})$ (ii) $\left[\left(\frac{5}{6}\right)^2 \times \frac{9}{4}\right] \div \left[\left(-\frac{3}{2}\right)^2 \times \frac{125}{216}\right]$

(iii) $5^3 \times 3^2 + (17)^0 \times 7^3$ (iv) $2^5 \times 15^0 + (-3)^3 - \left(\frac{2}{7}\right)^{-2}$

(v) $(2^2)^0 + 2^{-4} \div 2^{-6} + \left(\frac{1}{2}\right)^{-3}$ (vi) $5^n \times 25^{n-1} \div (5^{n-1} \times 25^{n-1})$

6. If $m = -2$ and $n = 2$; find the value of :

(i) $m^2 + n^2 - 2mn$ (ii) $m^n + n^m$

(iii) $6m^{-3} + 4n^2$ (iv) $2n^3 - 3m$