CHAPTER 21

RELATIONS AND MAPPING

21.1 ORDERED PAIR

An *ordered pair* means, *a pair of two objects which occur in a particular order.*

For example :

If two objects x and y are written as (x, y), they form one ordered pair and if written as (y, x), they form another ordered pair.

Since, the order of writing the objects in both the pairs is different, therefore, they form two different ordered pairs.

If the ordered pairs are enclosed within curly braces (brackets), then we call it a **set of ordered pairs**.

For example :

Consider the following sets of ordered pairs :

(i) Persons and their ages :
{(Meeta, 22), (Akshay, 20), (Sonu, 27)}

(ii) Numbers and their squares :
[(4, 16), (5, 25), (6, 36), ...}

In an ordered pair (x, y), x is called the ***first component*** and y is called the ***second component***.

1. *Both the components of an ordered pair can be same i.e., ordered pairs can be of the form (x, x), (y, y), $(3, 3)$, etc.*
2. *Two ordered pairs (a, b) and (c, d) are equal, if $a = c$ and $b = d$ e.g., $(x, y) = (4, 2)$ if $x = 4$ and $y = 2$.*

Example 1 :

Given : $(a - 2, b + 1) = (1, 3)$, find a and b.

Solution :

$(a - 2, b + 1) = (1, 3)$ $\Rightarrow a - 2 = 1$ and $b + 1 = 3$

$\Rightarrow a = 1 + 2$ and $b = 3 - 1$

$\Rightarrow a = 3$ and $b = 2$ **(Ans.)**

Example 2 :

Given $(4, y - 3) = (x + 2, 7)$, find x and y.

Solution :

$(4, y - 3) = (x + 2, 7)$ $\Rightarrow 4 = x + 2$ and $y - 3 = 7$

$\Rightarrow 4 - 2 = x$ and $y = 7 + 3$

$\Rightarrow 2 = x$ and $y = 10$

$\Rightarrow x = 2$ and $y = 10$ **(Ans.)**

21.2 RELATION

The word **relation** means an association of two objects (numbers, persons, etc.) based on some property connecting them.

For example :

(i) Peter **is the son of** John.
This statement shows **a relation between two persons**.
In this statement, the relation (R) means, *"is the son of"*.

(ii) 3 **is a factor of** 12.
This statement shows **a relation between two numbers**.
The relation (R) being *"is a factor of"*.

21.3 REPRESENTATION OF A RELATION

1. Roster Form (as the set of ordered pairs) :

Example 3 :

Given A = {1, 3, 4, 5, 6, 9, 10}, B = {0, 1, 2, 3} and a relation (R) from set A to set B means "is square of". Represent the relation 'R' in roster form.

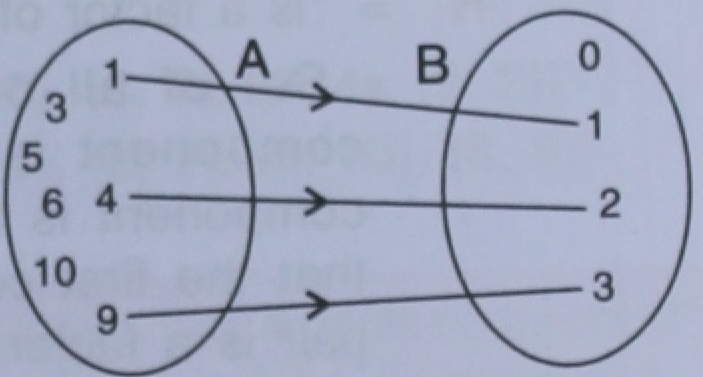
Solution :

A relation from set A to set B means, the first component of each ordered pair is to be taken from set A and the second component from set B, such that the first component **is square of** the second component.

∴ R = {(1, 1), (4, 2), (9, 3)} **(Ans.)**

2. By Arrow Diagram :

In this method, the arrows are drawn from set A to set B to indicate pairing, which satisfy the given relation.



Example 4 :

Given ordered pairs : (6, 3), (6, 4), (5, 3), (5, 4), (4, 4), (3, 3), (3, 4), (3, 5) and (5, 5).

Use these ordered pairs to find the following relations :

(i) R_1 = "is two more than" (ii) R_2 = "is greater than"
(iii) R_3 = "is equal to".

Solution :

(i) R_1 = Set of all ordered pairs, whose first component **is two more than** the second component.
= {(6, 4), (5, 3)} **(Ans.)**

(ii) R_2 = Set of all ordered pairs, whose first component **is greater than** the second component.
= {(6, 3), (6, 4), (5, 3), (5, 4)} **(Ans.)**

(iii) R_3 = Set of all ordered pairs, whose first component **is equal to** the second component.
= {(4, 4), (3, 3), (5, 5)} **(Ans.)**

Example 5 :

Given ordered pairs : (2, 5), (3, 5), (2, 4), (3, 4), (2, 3), (3, 3), (2, 2), (3, 2), (4, 2), (4, 3), (4, 4) and (4, 5).

Use these ordered pairs to find the following relations :

(i) R_1 = "is less than" (ii) R_2 = "is equal to"

(iii) R_3 = "is two less than"

Solution :

(i) R_1 = "is less than"

= Set of all ordered pairs whose first component *is less than* the second component.

= {(2, 5), (3, 5), (2, 4), (3, 4), (2, 3), (4, 5)} **(Ans.)**

(ii) R_2 = "is equal to"

= Set of all ordered pairs whose first component *is equal* to the second component.

= {(3, 3), (2, 2), (4, 4)} **(Ans.)**

(iii) R_3 = "is two less than"

= Set of all ordered pairs whose first component *is two less than* the second component.

= {(3, 5), (2, 4)}. **(Ans.)**

Example 6 :

Given A = {2, 4, 6}, B = {4, 18, 27} and a relation R from set A to set B means, "is a factor of". Find all ordered pairs which satisfy the given relation R.

Represent the given relation by an arrow diagram.

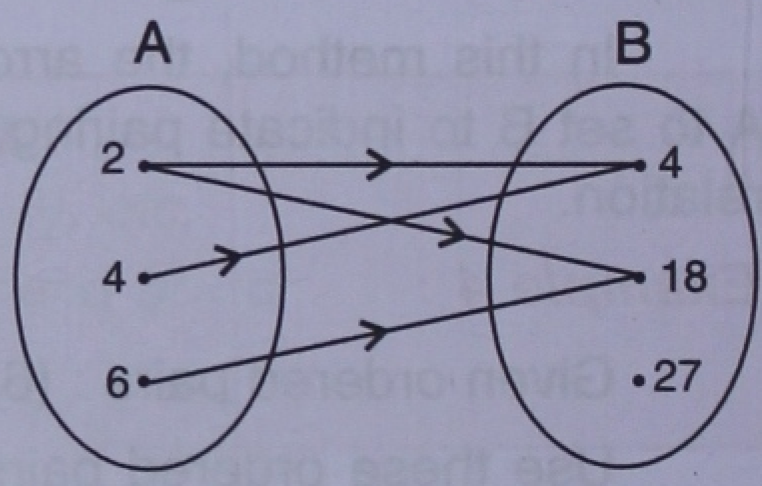
Solution :

R = "is a factor of"

= Set of all ordered pairs whose first component is from set A and second component is from set B in such a way that the first component of each ordered pair is a factor of the second component, *i.e.*, the first component completely divides the second component.

= {(2, 4), (2, 18), (4, 4), (6, 18)}. **(Ans.)**

Required arrow diagram is :



21.4 DOMAIN AND RANGE OF A RELATION

Domain : *The set of first components of all ordered pairs of a relation is called its **domain.***

In Example 4, given above :

(i) the domain of relation R_1 = {6, 5},

(ii) the domain of relation R_2 = {6, 5},

and, (iii) the domain of relation R_3 = {4, 3, 5}.

Range : *The set of second components of all ordered pairs of a relation is called its **range**.*

In Example 4, given above :

(i) the range of relation R_1 = {4, 3},

(ii) the range of relation R_2 = {3, 4},

and, (iii) the range of relation R_3 = {4, 3, 5}.

In the same way, in Example 5 given above :

(i) Domain of R_1 = {2, 3, 4} and range of R_1 = {5, 4, 3}

(ii) Domain of R_2 = {3, 2, 4} and range of R_2 = {3, 2, 4}

(iii) Domain of R_3 = {3, 2} and range of R_3 = {5, 4}

and, in Example 6, given above :

Domain = {2, 4, 6} whereas range = {4, 18}.

EXERCISE 21(A)

1. State, *true* or *false* :

(i) (*a*, *b*) is an ordered pair.

(ii) {*x*, *b*} is not an ordered pair.

(iii) (*a*, *b*) = {*a*, *b*}

(iv) (*x*, *y*) = (*y*, *x*)

(v) {*x*, *y*} = {*y*, *x*}

(vi) If (*x*, 3) = (2, *y*), then *x* = 3 and *y* = 2.

(vii) If (2, *x*) = (*y*, 5), then *x* = 5 and *y* = 2.

(viii) In R = {(1, 2), (1, 3), (1, 4), ...}, R means "is less than".

(ix) If R = {(5, 4), (6, 5), (7, 6), (8, 7)}, then R means, "is one less than".

(x) A relation from set B to set A means, the first component is from set B and the second component is from set A.

2. (i) If (2*x* – 3, 3*y*) = (5, 3) ; find the values of *x* and *y*.

(ii) If (2 – 5*a*, 3) = (–3, *b* + 1) ; find the values of *a* and *b*.

(iii) If (*m* – 4, 7) = (3, *n* + 2) ; find the values of *m* and *n*.

(iv) Find the values of *x* and *y*, if : (4*x* – 5, 2 – 3*y*) = (*x* + 1, –4)

3. Let P = {2, 4, 6, 8} and Q = {9, 10}, state which of the following are relations from set P and set Q.

(i) {(2, 9), (2, 10), (4, 10), (6, 9)}

(ii) {(2, 9), (10, 6), (9, 8), (9, 4), (9, 2)}

(iii) {(2, 10)}

(iv) {(4, 9), (4, 10), (6, 9), (6, 10), (8, 9)}

In case of relation, state its domain and range.

> A relation will be from set P to set Q, if each of its ordered pairs has its first component from set P and the second component from set Q. Also, a relation will be from set Q to set P, if each of its ordered pairs has its first component from set Q and the second component from set P.

4. Let A = {*a*, *b*, *c*} and B = {1, 3, 5, 4}. State which of the following are relations from set B to set A.

(i) {(*a*, 1), (*a*, 3), (*a*, 5)}

(ii) {(1, *a*), (1, *b*), (1, *c*)}

(iii) {(1, *b*), (2, *c*), (2, *a*), (5, *b*)}

(iv) {(3, *c*), (1, *a*), (*b*, 5), (*c*, 3)}

In case of relation, state its domain and range.

5. Use under mentioned ordered pairs to find the following relations :

(4, 5), (5, 5), (3, 5), (2, 4), (2, 2), (6, 8), (6, 4), (6, 3), (2, 6), (6, 6), (5, 2), (3, 4), (3, 3), (4, 4).

(i) R_1 = "is greater than"

(ii) R_2 = "is equal to"

(iii) R_3 = "is less than"

(iv) R_4 = "is three more than"

Draw separate arrow diagrams to represent each relation given above.

6. Draw suitable arrow diagrams to represent the following relations :

(i) R_1 = {(*a*, *x*), (*a*, *y*), (*b*, *x*), (*c*, *z*)}

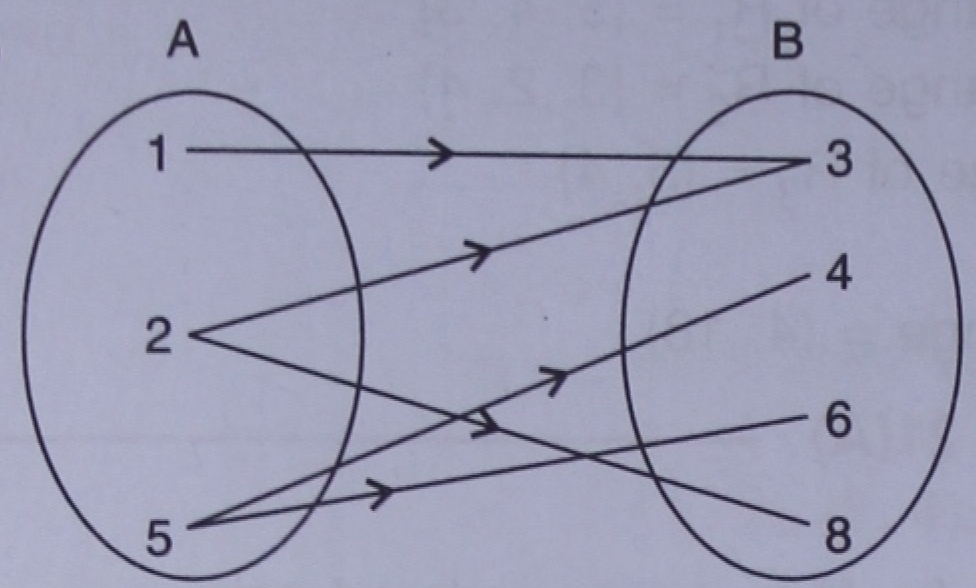
(ii) R_2 = {(3, 5), (4, 4), (3, 6), (4, 5), (5, 5)}

(iii) R_3 = {(*x*, *a*), (*x*, *b*), (*y*, *a*), (*z*, *c*)}

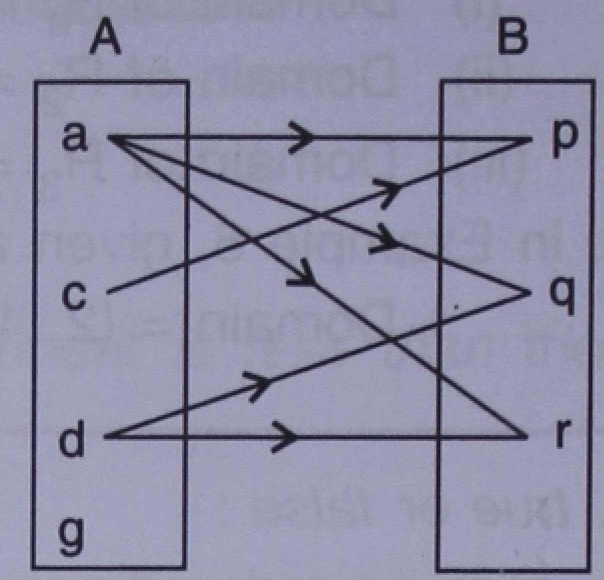
(iv) R_4 = {(0, 1), (1, 2), (2, 3), (3, 4), (4, 5)}

7. Represent the relations given by the following diagrams as the sets of ordered pairs.

(i)



(ii)



8. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 5}. Find the following relations from set A to set B:

(i) "is equal to" (ii) "is greater than"

(iii) "is less than" (iv) "is one more than"

In each case, state the domain and the range of the relation.

9. Given A = {3, 4, 5, 6}, *B* = {6, 7, 8, 9, 10} and the relation from A to B means, "is a factor of". Represent the relation :

(i) in roster form (ii) by an arrow diagram

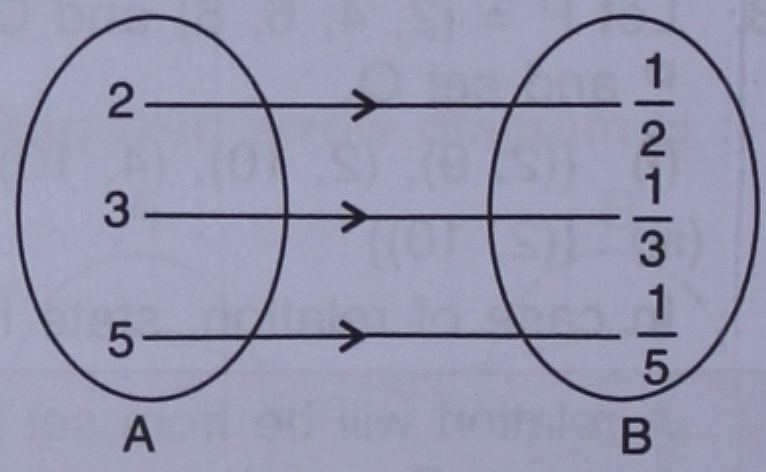
10. Given A = {12, 15, 18, 21}, *B* = {2, 3, 4, 5} and the relation from A to B means, "is divisible by".

(i) Draw an arrow diagram to represent the relation.

(ii) Also, represent the relation as the set of ordered pairs.

11. The adjacent arrow diagram represents a relation.

State, its domain and range. Also, give the relation in roster form.



21.5 MAPPING

Let A be the set of four players and B be the set of corresponding sports, such as :

A = {Kapil Dev, Dhyan Chand, R. Krishnan, Anita Sood},

B = {Tennis, Cricket, Swimming, Hockey}.

Now, if we associate each player of set A with the corresponding sport of set B, then we have :

(Kapil Dev, Cricket), (Dhyan Chand, Hockey), (R. Krishnan, Tennis) and (Anita Sood, Swimming).

In mathematics, such an association between the elements of set A and set B is called a **mapping** from set A to set B.

Necessary conditions for mapping :

1. *When the relation from set A to set B is represented in Roster form :*

(i) Every element of set A must be associated with a unique (only one) element of set B.

(ii) No two ordered pairs of the relation must have first components same.

For example :

(a) Let A = {*a*, *b*, *c*} and B = {*x*, *y*}, then the relation {(*a*, *x*), (*b*, *y*), (*c*, *x*)}, is a mapping since :

(i) each element of set A is associated with a unique (only one) element of set B, and

(ii) no two ordered pairs have their first components same.

(b) Let A = {1, 2, 3} and B = {4, 5, 6}, then the relation {(1, 4), (2, 5), (2, 6)}, is not a mapping as the element 3 in set A is not associated with any element of set B and also the ordered pairs (2, 5) and (2, 6) have their first components same.

(c) Let A = {*p*, *q*} and B = {*r*, *s*, *t*}, then the relation {(*p*, *r*), (*p*, *s*), (*q*, *t*)} is not a mapping as the ordered pairs (*p*, *q*) and (*p*, *s*) have their first components same.

A relation from set A to set B is a mapping if the first component of its ordered pairs is not repeated, while the second component may repeat and each element of set A must be associated with only one element of set B.

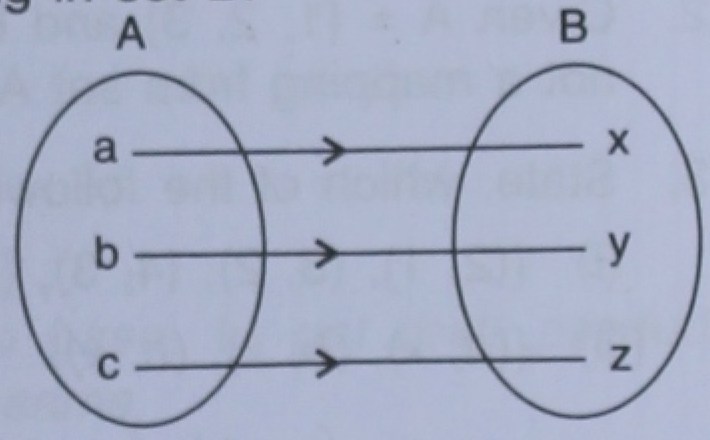
2. *When the relation from set A to set B is represented by an arrow diagram :*

(i) each element of set A should be connected with a unique element of set B. For this, there should be one and only one arrow which connects an element of set A to a particular element of set B.

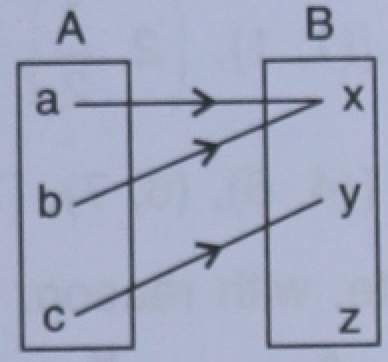
(ii) each element of set A should have a matching, *i.e.*, there should not be any element in set A which does not have its matching in set B.

For example :

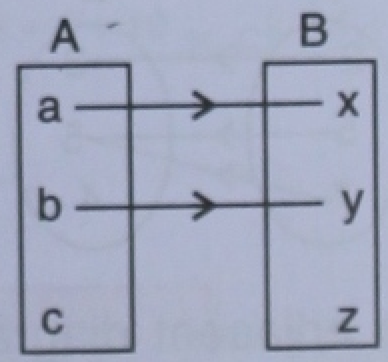
(a) The adjoining arrow diagram represents a mapping as each element of set A is matched with a unique element of set B.



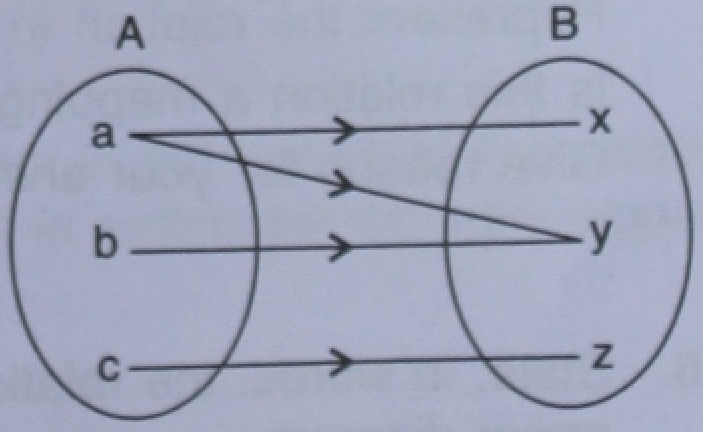
(b) The adjoining arrow diagram also represents a mapping.



(c) The adjoining arrow diagram does not represent a mapping as the element ***c*** in set A is not matched with any element of set B, *i.e.*, no arrow connects the element ***c*** of set A with any element of set *B*.



(d) The adjoining arrow diagram does not represent a mapping as the element **a** in set A is matched with two elements ***x*** and ***y*** of set B, *i.e.*, two arrows connect the element **a** of set A with elements ***x*** and ***y*** of set B.



Every mapping is a relation but the converse is not always true.

Example 7 :

Let A = {5, 6, 7, 8}, B = {7, 8} and R is a relation from A to B such that R = 'is less than'.

(i) Express relation R in roster form.

(ii) State, with reason/reasons, whether relation R is a mapping or not.

Solution :

(i) **Required relation R**

= **{(5, 7), (5, 8), (6, 7), (6, 8), (7, 8)}.** **(Ans.)**

(ii) **It is not a mapping**

Reason 1 :

The element 8 in A is not associated with any element in B.

Reason 2 :

Ordered pairs (5, 7) and (5, 8) have the same first component and the same is true with ordered pairs (6, 7) and (6, 8). **(Ans.)**

EXERCISE 21(B)

1. Let A = {*a*, *b*, *c*} and B = {*p*, *q*, *r*, *s*}

   (i) Is relation R_1 = {(*a*, *p*), (*b*, *q*), (*c*, *r*), (*a*, *s*)} a mapping from A to B ? Give reason.

   (ii) Is relation R_2 = {(*p*, *a*), (*q*, *a*), (*r*, *a*), (*s*, *a*)} a mapping from B to A ? Give reason.

2. Given A = {1, 2, 3} and B = {6, 4, 7}. State two reasons why {(1, 6), (1, 4), (2, 7)} is not a mapping from set A to set B.

3. State, which of the following relations represent mapping :

   (i) {(2, 1), (3, 2), (4, 3), (5, 4)}

   (ii) {(3, 1), (3, 2), (3, 3)}

   (iii) {(*a*, *x*), (*b*, *x*), (*c*, *x*)}

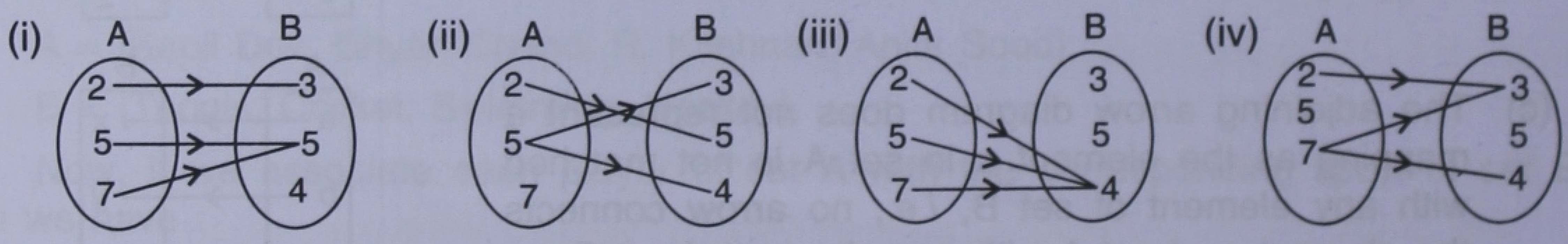
   (iv) {(2, 3), (3, 4), (4, 4), (5, 5)}

   (v) $\{(1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), ...\}$

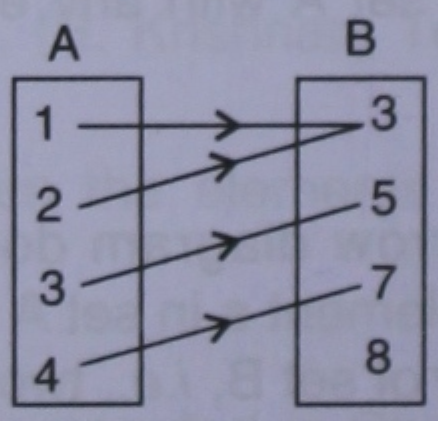
   (vi) $\{(1, 1), (2, 2^2), (3, 3^2), ...\}$

   (vii) {(4, 6), (6, 7), (7, 7), (7, 8)}

4. State, with reason, which of the following relations from A to B is a mapping :



5. The adjacent diagram represents a relation.
   Represent the relation in roster form.
   Is this relation a mapping ?
   Give reason for your answer.



6. State, in words, the relation represented by the adjoining arrow diagram.
   Is this relation a mapping ? Give reason.