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CHAPTER 19

FACTORISATION

19.1 BASIC CONCEPT

Since, the product of 5, x and y is $5xy$, $5xy$ is divisible by 5, x and y and so we say : the factors of term $5xy$ are 5, x and y.

Similarly, the product of x^2 and $(x + 3y)$

$= x^2(x + 3y) = x^3 + 3x^2y$

$\Rightarrow$ the factors of $x^3 + 3x^2y$ are x^2 and $x + 3y$.

And, we write : $x^3 + 3x^2y = x^2 (x + 3y)$

19.2 FACTORS AND FACTORISATION

To find the factors of a given expression means to determine two or more smaller expressions whose product is equal to the given expression.

The process of finding the factors of a given expression is called factorisation.

For example :

Product	Factors
1. $3a(3a + 4b) = 9a^2 + 12ab$	$9a^2 + 12ab = 3a(3a + 4b)$
2. $(x + 3)(x + 2) = x^2 + 5x + 6$	$x^2 + 5x + 6 = (x + 3)(x + 2)$
3. $(3x - 4y)(3x + 4y) = 9x^2 - 16y^2$	$9x^2 - 16y^2 = (3x - 4y)(3x + 4y)$

19.3 DIFFERENT TYPES OF FACTORISATIONS

Type 1 : Taking out the common factor

Steps : 1. Find, by inspection, the greatest monomial by which each term of the given expression can be divided completely.

2. Divide each term by this monomial and enclose the quotient within a bracket, keeping the common monomial outside the bracket.

Example 1 :

Factorise : $4xy - 8y^2$.

Solution :

Step 1 : The given expression $4xy - 8y^2$ has two terms $4xy$ and $8y^2$. The greatest monomial by which these terms can completely be divided is $4y$.

Step 2 : $4xy - 8y^2 = 4y\left(\frac{4xy}{4y} - \frac{8y^2}{4y}\right)$ [Dividing each term by 4y and keeping 4y outside the bracket]

$= 4y(x - 2y)$ **(Ans.)**

Alternative method :

Step 1 : Express each term of the given expression as a product of all its factors.

Step 2 : From the factors, obtained in step 1, find out the highest common factor.

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Step 3 : Divide each term of the given expression by the highest common factor, obtained in step 2, and enclose the quotient within a bracket, keeping the highest common factor, obtained in step 2, outside the bracket.

Since, $4xy = 2 \times 2 \times x \times y$
and $8y^2 = 2 \times 2 \times 2 \times y \times y$ } Step 1

Highest common factor $= 2 \times 2 \times y = 4y$ } Step 2

$$\therefore \quad 4xy - 8y^2 = 4y\left(\frac{4xy}{4y} - \frac{8y^2}{4y}\right)$$
$$= 4y(x - 2y) \quad \textbf{(Ans.)}$$

Example 2 :

Factorise : $6a^2 + 9ab + 12a$

Solution :

The greatest monomial by which all the terms of the given expression can be divided completely is $3a$.

Since,
$6a^2 = 2 \times 3 \times a \times a$
$9ab = 3 \times 3 \times a \times b$ and
$12a = 2 \times 2 \times 3 \times a$
H.C.F. $= 3 \times a = 3a$

$$\therefore 6a^2 + 9ab + 12a = 3a\left(\frac{6a^2}{3a} + \frac{9ab}{3a} + \frac{12a}{3a}\right)$$
$$= 3a(2a + 3b + 4) \quad \textbf{(Ans.)}$$

Example 3 :

Factorise : (i) $4x^2 + 5xy - 6xy^2$ (ii) $-3a^2 - 6ab + 12ab^2$

Solution :

(i) $$4x^2 + 5xy - 6xy^2 = x\left(\frac{4x^2}{x} + \frac{5xy}{x} - \frac{6xy^2}{x}\right)$$
$$= x(4x + 5y - 6y^2) \quad \textbf{(Ans.)}$$

(ii) $$-3a^2 - 6ab + 12ab^2 = -3a\left(\frac{-3a^2}{-3a} - \frac{6ab}{-3a} + \frac{12ab^2}{-3a}\right)$$
$$= -3a(a + 2b - 4b^2) \quad \textbf{(Ans.)}$$

EXERCISE 19(A)

Factorise :

1. $8x + 24y$
2. $ab - 2ax$
3. $2xy + 3xz$
4. $12a^2 - 18ab$
5. $6x + 9x^2$
6. $15a^2 - 18a$
7. $2x^2y - 5xy^2$
8. $a^2 + 8a^2b$
9. $a^2b^3 - a^3b^2$
10. $9xy - 21xz$
11. $3y^2 - yz$
12. $10ab^2 + 15a^2b$
13. $-9x^3 - 12x^2$
14. $-8x^2y + 12xy$
15. $abc - bcd$
16. $-55a^2b + 165ab^2$
17. $27x^3 - 9x^3y$
18. $-6xy^2 - 24x^2y^2$
19. $ax - ay + az$
20. $a^3 + a^2 - a$
21. $5x^2 - 6xy - xz$
22. $15a - 20b + 30c$
23. $6x^2 + 3x - 12x^2y$
24. $-8 - 12m - 20m^2$

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Type 2 : Grouping

Consider the expression : $ax + bx + ay + by$.

This expression has no common factor on the whole, but on observing carefully, we find that the first two terms have x as a common factor and the last two terms have y as a common factor.

Such an expression can be resolved into factors by adopting the following steps :

1. Arrange the terms of the given expression in groups such that each group has a common factor.
2. Factorise each group.
3. Take out the factor which is common to each group.

Thus, $ax + bx + ay + by = (ax + bx) + (ay + by)$ **[Step 1]**

$= x(a + b) + y(a + b)$ **[Step 2]**

$= (a + b)(x + y)$ **[Step 3]**

> Factorisation by grouping is possible only if the given expression has an even number of terms and the minimum number of terms in it is 4.

Example 4 :

Factorise : (i) $3x^3 - 6x^2 + ax - 2a$ (ii) $5ph - 10qk + 2rph - 4qrk$

Solution :

(i) $3x^3 - 6x^2 + ax - 2a = (3x^3 - 6x^2) + (ax - 2a)$ **[Step 1]**

$= 3x^2(x - 2) + a(x - 2)$ **[Step 2]**

$= (x - 2)(3x^2 + a)$ **(Ans.) [Step 3]**

(ii) $5ph - 10qk + 2rph - 4qrk = (5ph - 10qk) + (2rph - 4qrk)$

$= 5(ph - 2qk) + 2r(ph - 2qk)$

$= (ph - 2qk)(5 + 2r)$ **(Ans.)**

OR, $5ph - 10qk + 2rph - 4qrk = 5ph + 2rph - 10qk - 4qrk$ [On re-arranging]

$= (5ph + 2rph) - (10qk + 4qrk)$ **[Step 1]**

$= ph(5 + 2r) - 2qk(5 + 2r)$ **[Step 2]**

$= (5 + 2r)(ph - 2qk)$ **(Ans.) [Step 3]**

EXERCISE 19(B)

Factorise :

1. $x^2 + xy + x + y$
2. $2a^2 - 2ab - ac + bc$
3. $2x - 4x^2 + 5a - 10ax$
4. $a^3 - 3a^2 + a - 3$
5. $p^2 - pq + 5p - 5q$
6. $a^4 + 2a^3 + 6a + 12$
7. $ab - 2b + a^2 - 2a$
8. $x^2 - ax - bx + ab$
9. $ax^2 + a^2x + bx + ab$
10. $x^3 + x^2 + x + 1$
11. $x^3 - x^2 + x - 1$
12. $a^2 + a - 2(a + 1)$
13. $x(x + 2) - x - 2$
14. $pr - qs - qr + ps$
15. $a^3 - a^2b^2 - ab + b^3$
16. $x^2 - ax - bx + ab$
17. $ab - ac - cd + bd$
18. $2a - 4b - xa + 2xb$
19. $ax + by + bz + ay + bx + az$
20. $x^3 - x^2 + ax + x - a - 1$

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Type 3 : Difference of two squares :

Since, $(a + b)(a - b) = a(a - b) + b(a - b)$

$= a^2 - ab + ba - b^2 = a^2 - b^2$

= Difference of two squares

$\Rightarrow \quad a^2 - b^2 = (a + b)(a - b)$

∴ The difference between the squares of any two terms

= (The sum of the two terms) × (Their difference)

Example 5 :

Factorise : (i) $9x^2 - 64y^2$ (ii) $\frac{4}{9}p^2 - 1$

Solution :

(i) $9x^2 - 64y^2 = (3x)^2 - (8y)^2$ [Expressing as : $a^2 - b^2$]

$= (3x + 8y)(3x - 8y)$ [$a^2 - b^2 = (a + b)(a - b)$] **(Ans.)**

(ii) $\frac{4}{9}p^2 - 1 = \left(\frac{2}{3}p\right)^2 - (1)^2$

$= \left(\frac{2}{3}p + 1\right)\left(\frac{2}{3}p - 1\right)$ **(Ans.)**

Example 6 :

Evaluate :

(i) $(67)^2 - (33)^2$ (ii) $\left(5\frac{3}{5}\right)^2 - \left(4\frac{2}{5}\right)^2$ (iii) $(0{\cdot}83)^2 - (0{\cdot}17)^2$

Solution :

(i) $(67)^2 - (33)^2 = (67 + 33)(67 - 33)$ [Since, $a^2 - b^2 = (a + b)(a - b)$]

$= 100 \times 34 = 3400$ **(Ans.)**

(ii) $\left(5\frac{3}{5}\right)^2 - \left(4\frac{2}{5}\right)^2 = \left(5\frac{3}{5} + 4\frac{2}{5}\right)\left(5\frac{3}{5} - 4\frac{2}{5}\right)$

$= \left(\frac{28}{5} + \frac{22}{5}\right)\left(\frac{28}{5} - \frac{22}{5}\right)$

$= 10 \times 1\frac{1}{5} = 10 \times \frac{6}{5} = 12$ **(Ans.)**

(iii) $(0{\cdot}83)^2 - (0{\cdot}17)^2 = (0{\cdot}83 + 0{\cdot}17)(0{\cdot}83 - 0{\cdot}17)$

$= 1 \times 0{\cdot}66 = 0{\cdot}66$ **(Ans.)**

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EXERCISE 19(C)

Factorise :

1. $x^2 - 9$
2. $1 - 4p^2$
3. $4a^2 - 9$
4. $16 - 49y^2$
5. $4x^2 - y^2$
6. $x^2 - 16m^2$
7. $49 - 121\ p^2$
8. $a^2 - 4x^2y^2$
9. $a^4 - x^2$
10. $m^2n^4 - 25a^4b^2$
11. $x^2 - 0{\cdot}49$
12. $16x^2 - \dfrac{9}{25}$
13. $9 - \dfrac{4x^2}{9}$
14. $0{\cdot}64 - \dfrac{x^2}{64}$
15. $m^2 - 1\dfrac{7}{9}$
16. $2\dfrac{1}{4}x^2 - \dfrac{1}{9}$
17. $\dfrac{a^2}{b^2} - \dfrac{b^2}{a^2}$
18. Evaluate :

(i) $(59)^2 - (41)^2$ (ii) $(185)^2 - (15)^2$ (iii) $\left(22\dfrac{4}{7}\right)^2 - \left(12\dfrac{3}{7}\right)^2$

(iv) $(2{\cdot}57)^2 - (2{\cdot}43)^2$ (v) $(0{\cdot}95)^2 - (0{\cdot}05)^2$

19.4 COMPLETE FACTORISATION

Example 7 :

Factorise : (i) $2x^2 - 8$ (ii) $a^4 - b^4$ (iii) $3xy - 6y - 3ax + 6a$

Solution :

(i) $2x^2 - 8 = 2(x^2 - 4)$ [Taking 2 common]

$= 2(x^2 - 2^2)$ [Expressing as : $a^2 - b^2$]

$= 2(x + 2)(x - 2)$ **(Ans.)**

(ii) $a^4 - b^4 = (a^2)^2 - (b^2)^2$ [Expressing as : $a^2 - b^2$]

$= (a^2 + b^2)(a^2 - b^2)$

$= (a^2 + b^2)(a + b)(a - b)$ **(Ans.)**

(iii) $3xy - 6y - 3ax + 6a = 3(xy - 2y - ax + 2a)$ [Taking 3 common]

$= 3[(xy - 2y) - (ax - 2a)]$ [Grouping]

$= 3[y(x - 2) - a(x - 2)] = 3(x - 2)(y - a)$ **(Ans.)**

Example 8 :

Factorise : (i) $9(a - b)^2 - 4(a + b)^2$ (ii) $50(2x + y)^2 - 18(x - 2y)^2$

Solution :

(i) $9(a - b)^2 - 4(a + b)^2 = [3(a - b)]^2 - [2(a + b)]^2$

$= (3a - 3b)^2 - (2a + 2b)^2$

$= (3a - 3b + 2a + 2b)(3a - 3b - 2a - 2b)$

$= (5a - b)(a - 5b)$ **(Ans.)**

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(ii) $50(2x + y)^2 - 18(x - 2y)^2 = 2[25(2x + y)^2 - 9(x - 2y)^2]$

$= 2[\{5(2x + y)\}^2 - \{3(x - 2y)\}^2]$

$= 2[(10x + 5y)^2 - (3x - 6y)^2]$

$= 2(10x + 5y + 3x - 6y)(10x + 5y - 3x + 6y)$

$= 2(13x - y)(7x + 11y)$ **(Ans.)**

EXERCISE 19(D)

Factorise :

1. $5x^2 - 5$
2. $18 - 50x^2$
3. $x^2y^2 - y^2$
4. $a^3 - a$
5. $a^4 - 4a^2$
6. $a^4 - 1$
7. $2x^3 - 8x$
8. $3 - 12a^2b^2$
9. $44x^2 - 99y^2$
10. $16 - 16x^2$
11. $4x^2 - 100y^2$
12. $5m^2 - 125n^2$
13. $a^4 - b^4$
14. $a^4 - 16b^4$
15. $a^4 - 16$
16. $9 - 36a^2x^2$
17. $7m^2 - 28$
18. $24 - 6p^2a^2$
19. $a^4 - \frac{1}{a^4}$
20. $1 - \frac{1}{a^4}$
21. $\frac{a^4}{b^4} - \frac{b^4}{a^4}$
22. $3x + 6y + 3mx + 6my$
23. $2a^2 + 4b - 2ab - 4a$
24. $5a^3 - 15a^2 + 5a - 15$
25. $5a^4 - 10a^3 - 20a + 40$
26. $16(a + b)^2 - (a - b)^2$
27. $25(a + 3b)^2 - 4(a - 3b)^2$
28. $49(2a - 5b)^2 - 16(a + 3b)^2$
29. $25(x + y)^2 - 9(x - y)^2$

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