

# FORMULA: CHANGE OF SUBJECT OF A FORMULA

## (INCLUDING SUBSTITUTION)

### 17.1 FORMULA

A formula is an equation, which shows the relationship between two or more quantities (variables).

Infact, a formula is a translation from words to symbols.

For example :

- Formula showing the relationship between the area of rectangle (**A**), its length (**l**) and its breadth (**b**) is :

$$A = l \times b$$

- Formula showing the relationship between the distance (**d**) covered by a body in time (**t**) and with velocity (**v**) is :

$$d = v \times t$$

Example 1 :

Frame a formula for each of the following :

- Seven more than a certain number is twenty.
- After seven years, the age of father will be three times the age of his son.

Solution :

- Let the number be  $x$ .

Then the formula for the given statement is :

$$x + 7 = 20 \quad (\text{Ans.})$$

- Let the present age of father =  $x$  years and the present age of son =  $y$  years.  
After 7 years :

Father's age will be  $x + 7$  years and son's age will be  $y + 7$  years.

∴ Formula for the given statement is :  $x + 7 = 3(y + 7)$  (Ans.)

Example 2 :

On monday, Rohit worked at a shop for 11 hours, out of which 8 hours was normal work and remaining hours was overtime. If he gets ₹  $x$  for each hour of normal work and ₹  $2x$  for each hour of overtime, find (in terms of  $x$ ) the total amount of money he gets on monday.

Solution :

∴ Number of hours of normal work = 8 hours

and, for each hour of normal work, he gets ₹  $x$

∴ Money, he gets for 8 hours of normal work =  $8 \times ₹ x = ₹ 8x$

∴ Number of hours of overtime =  $(11 - 8)$  hours = 3 hours

and, for each hour of overtime, he gets ₹  $2x$

∴ Money, he gets for 3 hours of overtime =  $3 \times ₹ 2x = ₹ 6x$

⇒ The total money he gets on monday = ₹  $8x + ₹ 6x = ₹ 14x$  (Ans.)



## EXERCISE 17(A)

Frame a formula for each of the following statements :

1. D is the number of days in  $w$  weeks and  $p$  days.
2. Twelve less than thrice a certain number is twenty-four.
3. Half of a number added to  $\frac{1}{3}$  of the same number is 10.
4. When two is subtracted from twice of a certain number, the result is twenty-two.
5. If five is subtracted from a certain number and the difference is divided by fifteen, the result is three.
6. If a number is multiplied by nine and then two is subtracted from it, the result is 88.
7. The sum of three consecutive integers is seventy-eight.
8. The sum of three consecutive odd integers is fifty-seven.
9. Ajay went to a market with ₹ 500. He buys a tennis ball for ₹ 10 and spends ₹ 75 on a racket plus ₹ 5 on conveyance. He still has ₹  $x$  left.
10. A worker is paid ₹ 3 per hour for normal work and double this rate for overtime. Form a formula to find his earnings in a week (6 days) of 8 hours per day of normal work plus total overtime during this week being ten hours.
11. The final velocity ( $v$ ) of a body is the sum of its initial velocity ( $u$ ) and the product of acceleration produced ( $a$ ) and time ( $t$ ).
12. A taxi, in Delhi, charges ₹ 23 for the first kilometre and then ₹ 12 per kilometre for the remaining distance. Form an equation, if the taxi-driver charges ₹ 203 for a distance of  $x$  kilometre.
13. Eight years hence, Geeta will be twice as old as her age 5 years ago.  
Taking Geeta's present age as  $x$  years, form an equation in terms of  $x$ .
14. Mr. Verma is an officer in a Central Government office, which works for 5 days in a week. Mrs. Verma is also an officer in a State Government office which works for 6 days in a week. If per day earning of Mr. Verma is ₹ 280 more than that of Mrs. Verma; form an equation to find one week's earnings of Mrs. and Mr. Verma.  
Assume that one day earning of Mr. Verma is ₹  $x$ .

### 17.2 CHANGING THE SUBJECT OF A FORMULA

*The subject of a formula is the variable which is expressed in terms of other variables.*

*For example :*

1. In formula,  $A = l \times b$ .  $A$  is expressed in terms of  $l$  and  $b$ , so,  $A$  is the subject of the formula.
2. In  $I = \frac{P \times R \times T}{100}$ ,  $I$  is expressed in terms of  $P$ ,  $R$  and  $T$ , so,  $I$  is the subject of the formula.

**To change the subject of a given formula means** to obtain a formula for a particular (required) quantity.

*For example :*

1. The formula  $A = l \times b$  can be re-written as :
  - (i)  $l = \frac{A}{b}$ ; here  $l$  is the subject of the formula.
  - (ii)  $b = \frac{A}{l}$ ; here  $b$  is the subject of the formula.



2. The formula  $I = \frac{P \times R \times T}{100}$  can be re-written as :

(i)  $P = \frac{I \times 100}{R \times T}$ ; here P is the subject of the formula.

(ii)  $R = \frac{I \times 100}{P \times T}$ ; here R is the subject of the formula and so on.

For changing the subject of a given formula, we use the same steps as are used in solving the equations.

### Example 3 :

Given :  $p = 2l + 2b$ . Make  $b$  the subject.

#### Solution :

$$p = 2l + 2b \Rightarrow p - 2l = 2l + 2b - 2l \quad \text{[Subtracting } 2l \text{ from both the sides]}$$

$$\Rightarrow p - 2l = 2b$$

$$\Rightarrow \frac{p - 2l}{2} = \frac{2b}{2} \quad \text{[Dividing each side by 2]}$$

$$\Rightarrow \frac{p - 2l}{2} = b \quad \text{or,} \quad b = \frac{p - 2l}{2} \quad \text{(Ans.)}$$

### Example 4 :

Given :  $a = \frac{b + c}{m}$ . Make  $c$  the subject.

#### Solution :

$$a = \frac{b + c}{m} \Rightarrow am = b + c$$

$$\text{or} \quad b + c = am$$

$$\Rightarrow c = am - b \quad \text{(Ans.)}$$

## EXERCISE 17(B)

Change the subject for the following formulae to the indicated letter (variable) :

1.  $x + 2y = m$ ; for  $y$

2.  $v^2 = u^2 + 2as$ ; for  $s$

3.  $A = \frac{1}{2}(a + b)h$ ; for  $h$

4.  $s = \frac{n}{2}(a + l)$ ; for  $l$

5.  $C = \frac{5}{9}(F - 32)$ ; for  $F$

6.  $F = \frac{9}{5}C + 32$ ; for  $C$

7.  $A = p(a + rt)$ ; for  $p$

8.  $s = ut + \frac{1}{2}at^2$ ; for  $a$

9.  $s = \frac{n}{2}\{2a + (n - 1)d\}$ ; for  $d$

10.  $a = \frac{x - y}{x + y}$ ; for  $x$

11.  $\frac{m - a}{m + b} = \frac{2c}{3d}$ ; for  $m$

12.  $\frac{5x + 8y}{3y - x} = 2a$ ; for  $y$



### 17.3 TO EVALUATE THE UNKNOWN, USING CHANGE OF SUBJECT OF FORMULA AND SUBSTITUTION METHODS

- Steps :**
1. Change, if required, the formula to the required subject.
  2. In the new formula, substitute the values of the given quantities and simplify.

#### Example 5 :

Given :  $\frac{m + c}{m} = x$ , find  $c$ , if  $x = 5$  and  $m = 10$ .

#### Solution :

**Step 1 :**  $\frac{m + c}{m} = x \Rightarrow m + c = mx$

$$\Rightarrow c = mx - m$$

**Step 2 :** Substituting  $x = 5$  and  $m = 10$ ;

we get :  $c = 10 \times 5 - 10 = 40$  (Ans.)

Since, we are not asked to form a formula for  $c$ , it can be done directly. So,

$$\frac{m + c}{m} = x \Rightarrow \frac{10 + c}{10} = 5$$

$$\Rightarrow 10 + c = 50$$

$$\Rightarrow c = 50 - 10$$

$$= 40 \text{ (Ans.)}$$

### EXERCISE 17(C)

1.  $C = \frac{5}{9}(F - 32)$ . Find  $F$ , if  $C = 40$ .
2.  $V = \frac{1}{3}\pi r^2 h$ . Find  $h$ , if  $V = 110 \text{ cm}^3$  and  $r = 4 \text{ cm}$ .
3.  $A = \frac{1}{2}(l + b)h$ . Find  $b$ , if  $A = 60$ ,  $l = 6$  and  $h = 10$ .
4.  $t = 4\sqrt{\frac{h}{32}}$ ; express  $h$  in terms of  $t$ . Then calculate  $h$ , if  $t = 12$ .
5. In the formula  $p = \pi r + 2r$ , make  $r$  the subject. Hence, find  $r$ , if  $p = 40$  and  $\pi = 3.142$ .
6. If  $2y = \frac{x + 3}{x - 1}$  and  $y = 3$ . Find  $x$ .
7.  $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ . Find  $v$  when  $u = 15$  and  $f = 5$ .
8. If  $x = 3$  and  $y = -1$ ; find  $z$ , if :
  - (i)  $z = (x + y)^2 - 5(x - y)$
  - (ii)  $z = 8xy + x^2 - y^2$
  - (iii)  $z = x^3 - y^3 - 3x^2y + 3xy^2$
9. Given :  $A = 2\pi r(r + h)$ , find  $h$ , if  $A = 2816 \text{ cm}^2$ ,  $\pi = 3\frac{1}{7}$  and  $r = 14 \text{ cm}$ .
10. Given :  $a = 5$ ,  $b = -3$  and  $c = 2$ . Find  $m$ , if :
  - (i)  $m = abc + a^2 - b^2 + c^2$
  - (ii)  $m = a^3 + b^3 + c^3 - 5ab - 6bc$