EQUATIONS AND INEQUATIONS

DEFINITION OF AN EQUATION

An equation is a statement which states that the two expressions are equal.

For example:

- (i) If the expressions 3x + 2 and x 7 are equal, then 3x + 2 = x 7 forms an equation.
- (ii) The expressions 8y 15 and $\frac{y}{2}$ are equal $\Rightarrow 8y 15 = \frac{y}{2}$ is an equation.

To solve an equation means to find the value of the variable used in it.

e.g., in equation
$$x + 2 = 6$$
 [where x is a variable]

$$\Rightarrow x = 6 - 2 \Rightarrow x = 4$$

:. x = 4 is the solution (root) of the equation x + 2 = 6

Important note: An equation remains unchanged if:

The same number is added to each side of the equation.

i.e.
$$x-5=6 \Rightarrow x-5+5=6+5$$
 [Adding 5 on both the sides] $\Rightarrow x=11$

2. The same number is subtracted from each side of the equation.

i.e.
$$x+5=6 \Rightarrow x+5-5=6-5$$
 [Subtracting 5 from both the sides] $\Rightarrow x=1$

3. The same number is multiplied to each side of the equation.

i.e.
$$\frac{x}{3} = 2 \Rightarrow \frac{x}{3} \times 3 = 2 \times 3$$
 [Multiplying both the sides by 3] $\Rightarrow x = 6$

4. Each side of the equation is divided by the same non-zero number.

i.e.
$$4x = 12 \Rightarrow \frac{4x}{4} = \frac{12}{4}$$
 [Dividing both the sides by 4] $\Rightarrow x = 3$

Example 1:

Solve the equation x + 3 = 9.

Solution:

$$x + 3 = 9 \Rightarrow x + 3 - 3 = 9 - 3$$
$$\Rightarrow x = 6$$

x + 3 = 9 \Rightarrow x + 3 - 3 = 9 - 3 [Subtracting 3 from both the sides] (Ans.)

Example 2:

Solve: (i)
$$2x + 5 = 11$$

(ii)
$$\frac{x}{3} - 8 = 10$$

Solution:

(i)
$$2x + 5 = 11$$
 \Rightarrow $2x + 5 - 5 = 11 - 5$ [Subtracting 5 from both the sides] \Rightarrow $2x = 6$

$$\Rightarrow \frac{2x}{2} = \frac{6}{2}$$
 [Dividing each side by 2]
$$\Rightarrow x = 3$$
 (Ans.)

(ii)
$$\frac{x}{3} - 8 = 10$$
 $\Rightarrow \frac{x}{3} - 8 + 8 = 10 + 8$ [Adding 8 on both the sides] $\Rightarrow \frac{x}{3} = 18$ $\Rightarrow \frac{x}{3} \times 3 = 18 \times 3$ [Multiplying both the sides by 3] $\Rightarrow x = 54$ (Ans.)

16.2 SHORT-CUT METHOD (SOLVING AN EQUATION BY TRANSPOSING TERMS)

1. In an equation, if a + ve (positive) term is transposed (taken) from one side to the other, its sign is reversed, *i.e.*, it becomes – ve (negative).

i.e.,
$$x + 3 = 6$$
 $\Rightarrow x = 6 - 3$ [Transposing + 3] $\Rightarrow x = 3$

2. In an equation, if a – ve (negative) term is transposed from one side to the other, its sign becomes + ve (positive).

i.e.,
$$x-3=6$$
 \Rightarrow $x=6+3$ [Transposing -3] \Rightarrow $x=9$

3. In an equation, if a term in multiplication is transposed to the other side, its sign is reversed, *i.e.*, it goes in division.

i.e.,
$$3x = 6 \Rightarrow x = \frac{6}{3}$$
 [Transposing 3, which is in multiplication with x] $\Rightarrow x = 2$

4. In an equation, if a term is in division, it is transposed to the other side in multiplication.

i.e.,
$$\frac{x}{3} = 6$$
 $\Rightarrow x = 6 \times 3$ [Transposing 3, which is in division with x] $\Rightarrow x = 18$

Although it is not a rule, the variable (x, y or z, etc.) is prefered to be kept on the left hand side of the equation.

Example 3:

Solve: (i)
$$\frac{2}{3}x = 16$$
 (ii) $\frac{3}{4}x + 5 = 8$ (ii) $5x - \frac{1}{2} = \frac{3}{4}$

Solution:

(i)
$$\frac{2}{3}x = 16$$
 $\Rightarrow 2x = 16 \times 3$ [Transposing 3] $\Rightarrow x = \frac{48}{2}$ [Transposing 2] $\Rightarrow x = 24$ (Ans.)

(ii)
$$\frac{3}{4}x + 5 = 8$$
 $\Rightarrow \frac{3}{4}x = 8 - 5$
 $\Rightarrow \frac{3}{4}x = 3$ $\Rightarrow 3x = 3 \times 4$
 $\Rightarrow x = \frac{12}{3}$ $\Rightarrow x = 4$ (Ans.)

(iii)
$$5x - \frac{1}{2} = \frac{3}{4}$$
 \Rightarrow $5x = \frac{3}{4} + \frac{1}{2}$
 \Rightarrow $5x = \frac{5}{4}$
$$\left[\frac{3}{4} + \frac{1}{2} = \frac{3+2}{4} = \frac{5}{4}\right]$$

$$\Rightarrow x = \frac{5}{4 \times 5} \Rightarrow x = \frac{1}{4}$$
 (Ans.)

EXERCISE16 (A)

Solve the following equations:

1.
$$x + 5 = 10$$

2. $2 + y = 7$
3. $a - 2 = 6$
4. $x - 5 = 8$
5. $5 - d = 12$
6. $3p = 12$
7. $14 = 7m$
8. $2x = 0$
9. $\frac{x}{9} = 2$

10.
$$\frac{y}{-12} = -4$$

11. $8x - 2 = 38$
12. $2x + 5 = 5$
13. $5x - 1 = 74$
14. $14 = 27 - x$
15. $10 + 6a = 40$

16.
$$c - \frac{1}{2} = \frac{1}{3}$$

17. $\frac{a}{15} - 2 = 0$
18. $12 = c - 2$
19. $4 = x - 2.5$
20. $y + 5 = 8\frac{1}{4}$
21. $x + \frac{1}{4} = -\frac{3}{8}$

22.
$$p + 0.02 = 0.08$$
 23. $p - 12 = 2\frac{2}{3}$ 24. $-3x = 15$

28.
$$2a-3=5$$
 29. $3p-1=8$ 30. $9y-7=20$

31.
$$2b - 14 = 8$$
 32. $\frac{7}{10}x + 6 = 41$ 33. $\frac{5}{12}m - 12 = 48$.

16.3 SOLVING AN EQUATION WITH THE VARIABLE ON BOTH THE SIDES

Transpose the terms, containing the variable, to one side and the constants (i.e., terms without the variable) on the other side.

Example 4:

Solve :
$$8x - 3 = 5x + 9$$

Solution:

$$8x - 5x = 9 + 3$$

$$\Rightarrow 3x = 12$$

$$\Rightarrow x = \frac{12}{2} \Rightarrow x = 4$$
[Transposing + 5x to left and -3 to the right side]
[Transposing + 5x to left and -3 to the right side]

Example 5:

Solve: 7 + 4x = 9x - 13

Solution:

$$7 + 4x = 9x - 13$$

$$\Rightarrow 7 + 13 = 9x - 4x$$

$$\Rightarrow 4x - 9x = -13 - 7$$

$$\Rightarrow 20 = 5x$$

$$\Rightarrow -5x = -20$$

$$\Rightarrow \frac{20}{5} = x$$

$$\Rightarrow 4 = x \quad (Ans.)$$

$$\Rightarrow x = 4 \quad (Ans.)$$

Example 6:

Solve: 2(x-5) + 3(x-2) = 8 + 7(x-4)

Solution:

$$2x - 10 + 3x - 6 = 8 + 7x - 28$$

$$\Rightarrow 5x - 16 = 7x - 20$$

$$\Rightarrow 5x - 7x = -20 + 16$$

$$\Rightarrow -2x = -4 \Rightarrow x = \frac{-4}{-2} \Rightarrow x = 2$$
(Ans.)

EXERCISE 16 (B)-

Solve:

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1.	8y - 4y = 20	2.	9b - 4b + 3b = 16
3.	5y + 8 = 8y - 18	4.	6 = 7 + 2p - 5
5.	8 - 7x = 13x + 8	6.	4x - 5x + 2x = 28 + 3x
7.	9 + m = 6m + 8 - m	8.	24 = y + 2y + 3 + 4y
9.	19x + 13 - 12x + 3 = 23	10.	6b + 40 = -100 - b
11.	6 - 5m - 1 + 3m = 0	12.	0.4x - 1.2 = 0.3x + 0.6
13.	6(x+4)=36	14.	9(a+5)+2=11
15.	4(x-2)=12	16.	-3(a-6)=24
17.	7(x-2) = 2(2x-4)	18.	$(x-4)(2x+3)=2x^2$

16.4 EQUATIONS INVOLVING FRACTIONS

19. 21 - 3(b - 7) = b + 20

Example 7:

Solve: (i)
$$\frac{x}{5} + x = 12$$
 (ii) $\frac{x-1}{3} - \frac{2x-3}{5} = 1$

Solution:

(i)
$$\frac{x}{5} + x = 12 \implies \frac{x}{5} + \frac{x}{1} = 12$$

$$\Rightarrow \frac{x + 5x}{5} = 12$$

$$\Rightarrow \frac{6x}{5} = 12$$

$$\Rightarrow 6x = 12 \times 5 \implies x = \frac{60}{6} \implies x = 10$$
 (Ans.)

20. $x(x + 5) = x^2 + x + 32$

(ii)
$$\frac{x-1}{3} - \frac{2x-3}{5} = 1$$

$$\Rightarrow \frac{5(x-1)-3(2x-3)}{15} = 1 \qquad \text{[L.C.M. of 3 and 5 = 15]}$$

$$\Rightarrow \frac{5x-5-6x+9}{15} = 1$$

$$\Rightarrow -x+4 = 15 \times 1$$

$$\Rightarrow -x = 15-4$$

$$\Rightarrow -x = 11 \Rightarrow x = -11 \qquad (Ans.)$$

Example 8:

Solve: y + 15% of y = 46

Solution:

$$y + \frac{15}{100} \times y = 46 \implies y + \frac{3}{20}y = 46$$

$$\Rightarrow \frac{20y + 3y}{20} = 46$$

$$\Rightarrow \frac{23y}{20} = 46$$

$$\Rightarrow 23y = 46 \times 20 \implies y = \frac{46 \times 20}{23} \implies y = 40 \text{ (Ans.)}$$

EXERCISE 16 (C)-

Solve:

$$1. \quad \frac{x}{2} + x = 9$$

3.
$$\frac{3x}{4} + 4x = 38$$

5.
$$\frac{x}{3} - \frac{x}{4} = 2$$

7.
$$\frac{4x}{3} - \frac{7x}{3} = 1$$

9.
$$\frac{2x}{3} + \frac{x}{2} - \frac{3x}{4} = 1$$

11.
$$\frac{2p}{3} - \frac{p}{5} = 35$$

13.
$$p + 1.4p = 48$$

15.
$$y + 20\%$$
 of $y = 18$

17.
$$\frac{x+4}{2} + \frac{x}{3} = 7$$

$$19. \ \frac{3a-2}{7} - \frac{a-2}{4} = 2$$

$$21. \ \frac{x-1}{2} - \frac{x-2}{3} - \frac{x-3}{4} = 0$$

2.
$$\frac{x}{5} + 2x = 33$$

4.
$$\frac{x}{2} + \frac{x}{5} = 14$$

6.
$$y + \frac{y}{2} = \frac{7}{4} - \frac{y}{4}$$

8.
$$\frac{1}{2}m + \frac{3}{4}m - m = 2.5$$

10.
$$\frac{3a}{4} + \frac{a}{6} = 66$$

12.
$$0.6a + 0.2a = 0.4a + 8$$

14.
$$10\%$$
 of $x = 20$

16.
$$x - 30\%$$
 of $x = 35$

18.
$$\frac{y+2}{3} + \frac{y+5}{4} = 6$$

20.
$$\frac{1}{2}(x+5) - \frac{1}{3}(x-2) = 4$$

22.
$$\frac{x+1}{3} + \frac{x+4}{5} = \frac{x-4}{7}$$

23.
$$15 - 2(5 - 3x) = 4(x - 3) + 13$$

$$24. \quad \frac{2x+1}{3x-2} = 1\frac{1}{4}$$

25.
$$21 - 3(x - 7) = x + 20$$

$$26. \quad \frac{3x-2}{7} - \frac{x-2}{4} = 2$$

27.
$$\frac{2x-3}{3} - (x-5) = \frac{x}{3}$$

28.
$$\frac{x-4}{7} = \frac{x+3}{7} + \frac{x+4}{5}$$

29.
$$\frac{x-1}{5} - \frac{x}{3} = 1 - \frac{x-2}{2}$$

30.
$$2x + 20\%$$
 of $x = 12.1$

16.5 INEQUATION (LINEAR INEQUATION)

It is a statement of inequality between two expressions involving a single variable with the highest power one (1).

For example:

- (i) x < 5, read : x is less than 5.
- (ii) x > 8, read: x is greater than 8.
- (iii) $x \le 5$, read : x is less than or equal to 5.
- (iv) $x \ge 8$, read : x is greater than or equal to 8.

In each of the inequation given below:

ax + b < 0, $ax \ge b$, ax + b > 0, $ax \le b$, $ax + b \le 0$, etc., x is a variable whereas a and b are constants such that $a \ne 0$.

For example:

- (i) In 3x + 2 > 0; x is variable whereas 3 and 2 are constants.
- (ii) In $5y 8 \le 0$; y is variable whereas 5 and 8 are constants and so on.

16.6 THE REPLACEMENT SET AND THE SOLUTION SET

1. Replacement set:

For a given inequation, the set from which the values of its variable are taken, is called the replacement set or domain of the variable.

For example:

- (i) If $x \in N$ and x < 5, then for the inequality x < 5, the values of its variable (x) are to be taken from N (the set of natural numbers). Hence, the **replacement set** is N.
- (ii) If $y \ge 15$ and y is an integer between -20 and +20, the **replacement set** (domain of the variable y) is the **set of integers between -20 and 20**.

2. The solution set:

It is the subset of the replacement set, consisting of those values of the variable which satisfy the given inequation.

- (i) For x < 5 and $x \in N$, the replacement set = N and, the solution set = {values of x, in the replacement set, which satisfy x < 5} = {1, 2, 3, 4}
- (ii) For $y \ge 15$ and $y \in \{\text{integers between} 20 \text{ and } 20\}$ and, the solution set = $\{\text{values of } y, \text{ in the replacement set, which satisfy } y \ge 15\}$ = $\{15, 16, 17, 18, 19\}$

16.7 MORE EXAMPLES

(i) For $y \le 8$ and $y \in W$ (whole numbers)

The replacement set = W, the set of whole numbers
and, the solution set = Set of whole numbers less than or equal to 8

 $= \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

(ii) For z > -3 and the replacement set = $\{-5, -2, 0, 3, 8\}$,

The solution set $= \{-2, 0, 3, 8\}$

(iii) For $1 < x \le 6$ (i.e., x is greater than one (1) and less than or equal to 6) and $x \in \{\text{positive integers}\}$

The replacement set = Set of positive integers = {1, 2, 3, 4, 5, 6, 7,}

and, the solution set = Set of positive integers greater than 1 and less than or equal to 6

 $= \{2, 3, 4, 5, 6\}$

(iv) For $-2 \le y \le 5$ and $y \in Z$ (integers)

The replacement set = Z, the set of integers

and, the solution set = Set of integers which are greater than or equal to -2 and also less than or equal to 5

= {-2, -1, 0, 1, 2, 3, 4, 5}

16.8 PROPERTIES OF INEQUATIONS

Property 1:

Adding the same number to each side of an inequation does not change the inequality.

For example:

(i)
$$x > 3 \Rightarrow x + 5 > 3 + 5 \Rightarrow x + 5 > 8$$

(ii)
$$x > 8 \Rightarrow x + 5 > 8 + 5 \Rightarrow x + 5 > 13$$

(iii)
$$x-3<5 \Rightarrow x-3+3<5+3 \Rightarrow x<8$$

(iv)
$$x-8 \le 3 \Rightarrow x-8+8 \le 3+8 \Rightarrow x \le 11$$

Property 2:

Subtracting the same number from each side of an inequation does not change the inequality.

For example:

(i)
$$x+5 \le 8 \Rightarrow x+5-5 \le 8-5 \Rightarrow x \le 3$$

(ii)
$$x+3 \ge 15 \Rightarrow x+3-3 \ge 15-3 \Rightarrow x \ge 12$$

(iii)
$$x+2<-6 \Rightarrow x+2-2<-6-2 \Rightarrow x<-8$$

(iv)
$$x+7>4 \Rightarrow x+7-7>4-7 \Rightarrow x>-3$$

Property 3:

Multiplying each side of an inequation by the same positive number, does not change the inequality.

For example:

(i)
$$\frac{x}{3} > 2 \implies \frac{x}{3} \times 3 > 2 \times 3 \implies x > 6$$

(ii)
$$\frac{x}{2} < 5 \implies \frac{x}{2} \times 2 < 5 \times 2 \implies x < 10$$

(iii)
$$\frac{x}{4} \ge -6 \implies \frac{x}{4} \times 4 \ge -6 \times 4 \implies x \ge -24$$

(iv)
$$\frac{x}{5} \le -3 \implies \frac{x}{5} \times 5 \le -3 \times 5 \implies x \le -15$$

Property 4:

Multiplying each side of an inequation by the same negative number, changes (reverses) the inequality.

For example:

(i)
$$x > 4 \Rightarrow x \times -3 < 4 \times -3 \Rightarrow -3x < -12$$

(ii)
$$-x < -2 \Rightarrow -x \times -5 > -2 \times -5 \Rightarrow 5x > 10$$

(iii)
$$3x \ge -5 \Rightarrow 3x \times -2 \le -5 \times -2 \Rightarrow -6x \le 10$$

(iv)
$$-x \le 4 \Rightarrow -x \times -4 \ge 4 \times -4 \Rightarrow 4x \ge -16$$

Property 5:

Dividing each side of an inequation by the same positive number, does not change the inequality.

(i)
$$3x > 6 \Rightarrow \frac{3x}{3} > \frac{6}{3} \Rightarrow x > 2$$

(ii)
$$-8x < -10 \Rightarrow \frac{-8x}{2} < \frac{-10}{2} \Rightarrow -4x < -5$$

(iii)
$$5x \ge -15 \Rightarrow \frac{5x}{5} \ge \frac{-15}{5} \Rightarrow x \ge -3$$

$$(iv) - 6x \le 36 \quad \Rightarrow \quad \frac{-6x}{6} \le \frac{36}{6} \quad \Rightarrow \quad -x \le 6$$

Property 6:

Dividing each side of an inequation by the same negative number, changes (reverses) the inequality.

For example:

(i)
$$-2x > 6 \Rightarrow \frac{-2x}{-2} < \frac{6}{-2} \Rightarrow x < -3$$

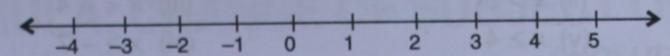
(ii)
$$3x < -15 \implies \frac{3x}{-3} > \frac{-15}{-3} \implies -x > 5$$

(iii)
$$4x \ge -12 \implies \frac{4x}{-4} \le \frac{-12}{-4} \implies -x \le 3$$

(iv)
$$-3x \le -18 \Rightarrow \frac{-3x}{-3} \ge \frac{-18}{-3} \Rightarrow x \ge 6$$

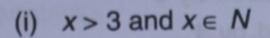
GRAPHICAL REPRESENTATION

The following figure shows a number line representing integers.

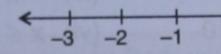


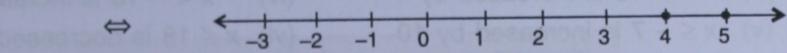
The dark arrows at both the ends represent that integers go upto infinity on the positive as well as on the negative side.

A number line can be used to represent the solution set of an inequation. The following illustrations will make the statement more clear:





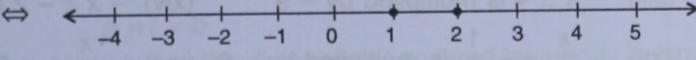




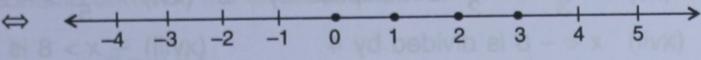
[Dark marks at 4 and 5 show the elements of the solution set and the dark arrow at the right side of number line represents that the solution set of the given inequation continues to the right side of 5].

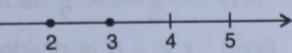
(ii)
$$x < 3$$
 and $x \in N$





(iii)
$$x \le 3$$
 and $x \in W$





(iv)
$$x \le 3$$
 and $x \in I$ (integers) \Leftrightarrow



(v)
$$-2 \le x \le 3$$
 and

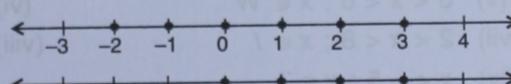
(a)
$$x \in N$$

$$\Leftrightarrow$$



(b)
$$x \in I$$

$$\Leftrightarrow$$



(c)
$$x \in W$$

$$\Leftrightarrow$$

Example 9:

Solve and in each case, represent the solution set on a number line :

(i)
$$2x-5<3$$
; $x \in W$

(ii)
$$3x + 4 > 10$$
; $x \in N$

Solution:

(i)
$$2x-5 < 3 \Rightarrow 2x < 3+5$$

$$\Rightarrow 2x < 8$$

$$\Rightarrow x < \frac{8}{2} \Rightarrow x < 4$$

$$x < 4$$
 and $x \in W \Rightarrow$ The solution set = $\{0, 1, 2, 3\}$

(Ans.)

Solution set on the number line is:

-2 -1 0 1 2 3 (Ans.)

(ii) $3x + 4 > 10 \Rightarrow 3x > 10 - 4$

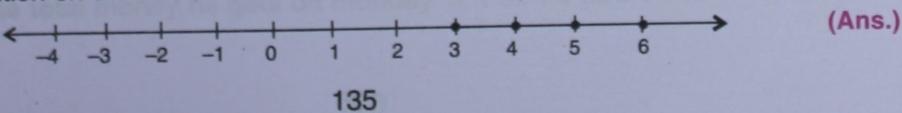
$$\Rightarrow 3x > 6$$
 $\Rightarrow x > \frac{6}{3}$ i.e. $x > 2$

Now, x > 2 and $x \in N$

(Ans.)

The solution set = $\{3, 4, 5, 6, \dots \}$

The solution on the number line is:



EXERCISE 16 (D)-

- 1. Write each of the following inequation as a word statement:
 - x > -8

(ii) x > 5

(iii) x < -4

(iv) x < 8

(v) $x \ge 4$

(vi) $x \ge -7$

- (vii) $x \le 10$
- (viii) $x \le -15$
- 2. Write the resulting inequation when each side of inequation :
 - (i) x > -5 is increased by 7
- (ii) x < 8 is increased by 15
- (iii) -x > 15 is increased by 4
- (iv) -x < -10 is increased by 8
- (v) $x \le -7$ is increased by 10
- (vi) $x \le 18$ is decreased by 4
- (vii) $-x + 4 \le 12$ is decreased by 6
- (viii) $x 4 \ge -12$ is increased by 15
- (ix) x < -15 is decreased by 4
- (x) $x \ge 4$ is decreased by 8
- (xi) $-x + 3 \le 8$ is decreased by 6
- (xii) x > 3 is multiplied by 5
- (xiii) $-x \le 4$ is multiplied by -9
- (xiv) $-x \ge -2$ is multiplied by -2
- (xv) $-\frac{x}{3} < -\frac{1}{2}$ is multiplied by -6 (xvi) $-\frac{x}{5} > -2$ is multiplied by -10
- (xvii) x < -8 is divided by 4
- (xviii) -x > 8 is divided by 4
- (xix) $3x \le -6$ is divided by -3
- $(xx) 8x \ge 12$ is divided by -4
- 3. For each inequation, given below, draw a separate number line :
 - (i) x < 5; $x \in N$

(ii) $2 \le x < 8$; $x \in N$

(iii) x > 15; $x \in N$

- (iv) $x \le 4$; $x \in W$
- (v) 0 < x < 6; $x \in W$
- (vi) $x \ge 12$; $x \in W$

(vii) 2 < x < 8; $x \in I$

(viii) $-3 < x \le 3$; $x \in I$

- (ix) x < -5; $x \in I$
- Remember: N = Set of natural numbers; W = Set of whole numbers and I = Set of integers
- For inequation $-3 \le x < 2$, draw separate number lines when:
 - (i) $X \in N$
- (ii) $x \in W$
- (iii) $X \in I$
- 5. Solve each of the inequation, given below. Then, in each case, represent the solution set on a number line:
 - (i) x-3 < 2; $x \in W$

(ii) $2x + 3 \le 15$; $x \in N$

(iii) $x + 5 \le 10$; $x \in N$

- (iv) 3x + 2 < 14; $x \in W$
- (v) $3x 7 \ge 8$; $x \in W$
- (vi) x-3 > 15; $x \in N$ (viii) $-3 \le x < 1$; $x \in I$
- (vii) -2 < x < 4; $x \in I$ (ix) $-5 < x \le 0$; $x \in I$

 $(x) - 2 \le x \le 2$; $x \in I$