CHAPTER 15

SIMPLIFICATIONS

(INCLUDING REMOVAL OF BRACKETS)

15.1 COMBINING ALGEBRAIC EXPRESSIONS WITH INTEGRAL DENOMINATORS

In pure arithmatic, the simplification of fractions is already done.

To simplify algebraic expressions (fractions), the same rules and methods are used.

For example :

1. $\frac{x}{2}+\frac{x}{3}=\frac{3x+2x}{6}$ [L.C.M. of the denominators 2 and 3 is 6]

$=\frac{5x}{6}$

2. $\frac{y}{4}+\frac{y+2}{3}=\frac{3y+4(y+2)}{12}$ [L.C.M. of denominators 4 and 3 is 12]

$=\frac{3y+4y+8}{12}=\frac{7y+8}{12}$

3. $\frac{a}{5}-\frac{a-2}{2}+a=\frac{a}{5}-\frac{a-2}{2}+\frac{a}{1}$

$=\frac{2a-5(a-2)+10a}{10}$ [L.C.M. of 5, 2 and 1 is 10]

$=\frac{2a-5a+10+10a}{10}=\frac{7a+10}{10}$

4. $\frac{3}{5}b$ of $\left(\frac{2b+3b}{4}\right)=\frac{3}{5}b$ of $\frac{5b}{4}$ [Terms inside the bracket are simplified first]

$=\frac{3b}{5}\times\frac{5b}{4}=\frac{3b^2}{4}$

5. $\left(\frac{m}{2}\times2\frac{1}{3}\right)\div\left(4\frac{1}{2}\times\frac{p}{3}\right)=\left(\frac{m}{2}\times\frac{7}{3}\right)\div\left(\frac{9}{2}\times\frac{p}{3}\right)$

$=\frac{7m}{6}\div\frac{9p}{6}=\frac{7m}{6}\times\frac{6}{9p}=\frac{7m}{9p}$

6. $\frac{2\frac{1}{5}x}{1\frac{1}{10}}+x=\frac{\frac{11x}{5}}{\frac{11}{10}}+x=\frac{11x}{5}\times\frac{10}{11}+x=2x+x=3x$

EXERCISE 15(A)

Simplify :

1. $\frac{x}{2}+\frac{x}{4}$

2. $\frac{a}{10}+\frac{2a}{5}$

3. $\frac{y}{4}+\frac{3y}{5}$

4. $\frac{x}{2}-\frac{x}{8}$

5. $\frac{3y}{4} - \frac{y}{5}$

6. $\frac{2p}{3} - \frac{3p}{5}$

7. $\frac{k}{2} + \frac{k}{3} + \frac{2k}{5}$

8. $\frac{2x}{5} + \frac{3x}{4} - \frac{3x}{5}$

9. $\frac{4a}{7} - \frac{2a}{3} + \frac{a}{7}$

10. $\frac{2b}{5} - \frac{7b}{15} + \frac{13b}{3}$

11. $\frac{6k}{7} - \left(\frac{8k}{9} - \frac{k}{3}\right)$

12. $\frac{3a}{8} + \frac{4a}{5} - \left(\frac{a}{2} + \frac{2a}{5}\right)$

13. $x + \frac{x}{2} + \frac{x}{3}$

14. $\frac{y}{5} + y - \frac{19y}{15}$

15. $\frac{x}{5} + \frac{x+1}{2}$

16. $x + \frac{x+2}{3}$

17. $\frac{3y}{5} - \frac{y+2}{2}$

18. $\frac{2a+1}{3} + \frac{3a-1}{2}$

19. $\frac{k+1}{2} + \frac{2k-1}{3} - \frac{k+3}{4}$

20. $\frac{m}{5} - \frac{m-2}{3} + m$

21. $\frac{5(x-4)}{3} + \frac{2(5x-3)}{5} + \frac{6(x-4)}{7}$

22. $\left(p + \frac{p}{3}\right)\left(2p + \frac{p}{2}\right)\left(3p - \frac{2p}{3}\right)$

23. $\frac{7}{30}$ of $\left(\frac{p}{3} + \frac{7p}{15}\right)$

24. $\left(2p + \frac{p}{7}\right) \div \left(\frac{9p}{10} + 4p\right)$

15.2 USING BRACKETS

The brackets are used to combine the terms in different situations.

For example :

(i) For addition of $2a + 7b$ and $9a - 3b + 8$;
we write : $(2a + 7b) + (9a - 3b + 8)$

(ii) For subtraction of $2b - 7a + 5$ from $3a - 8 + 4b$;
we write : $(3a - 8 + 4b) - (2b - 7a + 5)$

(iii) For multiplication of $4x$ and $x - 8y$;
we write : $(4x) \times (x - 8y)$ or, simply : $(4x)\ (x - 8y)$ or, $4x(x - 8y)$

Similarly, $(3x^2 + 7x + 9)\ (2x^2 + 3x)$ shows the multipication of $3x^2 + 7x + 9$ and $2x^2 + 3x$.

(iv) For addition of $8x$ and 12 multiplied by $5y$;
we write : $(8x + 12) \times 5y$ or, $5y(8x + 12)$

15.3 INSERTION OF BRACKETS

Insertion of bracket means; enclosing a quantity or an expression in a bracket.

Rules :

1. *When we have to insert a bracket such that there is a positive (+) sign before the bracket, the quantities are put in the bracket with their original signs, i.e., signs of the terms kept inside the bracket do not change.*

For example :

(i) $4x + 3y - 5z - 8 = 4x + (3y - 5z - 8)$

(ii) $4x + 3y - 5z - 8 = 4x + 3y + (-5z - 8)$

2. *When a bracket is inserted such that there is a negative (–) sign before it, the sign of each term kept inside the bracket changes.*

For example :

(i) $2a - 4b - 9c + 8 = 2a - (4b + 9c - 8)$

(ii) $3x + 5y - 7z - 4 = 3x - (-5y + 7z + 4)$

3. *When there is a common factor, put the factor before the bracket and at the same time change the values and signs (if required) of all the terms kept inside the bracket.*

For example :

(i) $6x^2 + 8x - 10 = 2(3x^2 + 4x - 5)$ [Since, 2 is a common factor]

(ii) $-10y^3 - 5y^2 + 15y = -5y(2y^2 + y - 3)$ [Since, $-5y$ is common in all the terms]

15.4 REMOVAL OF BRACKETS

For removal of brackets :

1. *When there is positive (+) sign before the bracket, remove the bracket without changing the sign of any term inside it.*

   e.g. $4x + (2y + 7z - 9) = 4x + 2y + 7z - 9.$

2. *When there is negative (–) sign before the bracket, remove the bracket and at the same time change the sign of each term inside it.*

   e.g. $3a^2 - (2a + 4b - 7) = 3a^2 - 2a - 4b + 7$

3. *When a term is written just before the bracket, it is said to be in multiplication with the bracket.*

   Remove the bracket by multiplying this term with each term inside the bracket.

   e.g. (i) $\mathbf{3x(x^2 - 5x - 7)} = 3x \times x^2 - 3x \times 5x - 3x \times 7$
   $= \mathbf{3x^3 - 15x^2 - 21x}$

   (ii) $\mathbf{-5(-8a^2 + 3ab - 1)} = \mathbf{40a^2 - 15ab + 5}$ and so on.

15.5 TYPES OF BRACKETS

The names of different types of brackets and the order in which they are removed is shown below :

(i) ——— ; bar (vinculum) bracket, (ii) (   ) ; circular bracket,

(iii) {   } ; curly bracket and then, (iv) [   ] ; square bracket.

Example 1 :

Simplify; (i) $a - [\,2b + \{c - (2a - b)\}\,]$ (ii) $4x - [\,2y - \{2x + (x - \overline{y - x})\,\}\,]$

Solution :

(i) $\boldsymbol{a - [\,2b + \{c - (2a - b)\}\,]}$

$= a - [2b + \{c - 2a + b\}\,]$ [Removing the circular bracket]

$= a - [2b + c - 2a + b\,]$ [Removing the curly bracket]

$= a - 2b - c + 2a - b$ [Removing the square bracket]

$= \mathbf{3a - 3b - c}$ **(Ans.)**

(ii) $\boldsymbol{4x - [\,2y - \{2x + (x - \overline{y - x})\,\}\,]}$

$= 4x - [\,2y - \{2x + (x - y + x)\,\}\,]$ [Removing the bar bracket]

$= 4x - [\,2y - \{2x + (2x - y)\,\}\,]$

$= 4x - [\,2y - \{2x + 2x - y\}\,]$ [Removing the circular bracket]

$= 4x - [\,2y - \{4x - y\}\,]$

$= 4x - [\,2y - 4x + y]$ [Removing the curly bracket]

$= 4x - 2y + 4x - y$ [Removing the square bracket]

$= \mathbf{8x - 3y}$ **(Ans.)**

EXERCISE 15 (B)

Enclose the given terms in brackets as required :

1. $x - y - z = x -$ (.......................)
2. $x^2 - xy^2 - 2xy - y^2 = x^2 -$ (.......................)
3. $4a - 9 + 2b - 6 = 4a -$ (.......................)
4. $x^2 - y^2 + z^2 + 3x - 2y = x^2 -$ (.......................)
5. $-2a^2 + 4ab - 6a^2b^2 + 8ab^2 = -2a$ (.......................)

Simplify :

6. $2x - (x + 2y - z)$
7. $p + q - (p - q) + (2p - 3q)$
8. $9x - (-4x + 5)$
9. $6a - (-5a - 8b) + (3a + b)$
10. $(p - 2q) - (3q - r)$
11. $9a\,(2b - 3a + 7c)$
12. $-5m\,(-2m + 3n - 7p)$
13. $-2x\,(x + y) + x^2$
14. $b\left(2b - \frac{1}{b}\right) - 2b\left(b - \frac{1}{b}\right)$
15. $8\,(2a + 3b - c) - 10\,(a + 2b + 3c)$
16. $a\left(a + \frac{1}{a}\right) - b\left(b - \frac{1}{b}\right) - c\left(c + \frac{1}{c}\right)$
17. $5x\,(2x + 3y) - 2x\,(x - 9y)$
18. $a + (b + \overline{c - d})$
19. $5 - 8x - \overline{6 - x}$
20. $2a + (b - \overline{a - b})$
21. $3x + [4x - (6x - 3)\,]$
22. $5b - \{6a + (8 - b - a)\}$
23. $2x - [5y - (3x - y) + x]$
24. $6a - 3\,(a + b - 2)$
25. $8\,[m + 2n - p - 7\,(2m - n + 3p)\,]$
26. $\{9 - (4p - 6q)\} - \{3q - (5p - 10)\}$
27. $2\,[a - 3\,\{a + 5\,(a - 2) + 7\}]$
28. $5a - [6a - \{9a - (10a - \overline{4a - 3a})\}]$
29. $9x + 5 - [4x - \{3x - 2\,(4x - 3)\}]$
30. $(x + y - z)\,x + (z + x - y)\,y - (x + y - z)z$
31. $-1\,[a - 3\,\{b - 4\,(a - \overline{b - 8}) + 4a\} + 10]$