CHAPTER 14

PRODUCTS

14.1 SOME SPECIAL PRODUCTS AS IDENTITY

1. $(x + a)(x + b) = x(x + b) + a(x + b)$
$= x^2 + bx + ax + ab$
$= x^2 + ax + bx + ab = \mathbf{x^2 + (a + b)\,x + ab}$

Consider, $(x + 5)(x + 2) = x^2 + (5 + 2)\,x + (5 \times 2)$
$= \mathbf{x^2 + 7x + 10}$

If $x = 3$, it becomes :

$(3 + 5)(3 + 2) = 3^2 + 7 \times 3 + 10$

i.e. $8 \times 5 = 9 + 21 + 10$

$\Rightarrow$ $40 = 40$; ***which is true.***

For $x = -8$, we get :

$(-8 + 5)(-8 + 2) = (-8)^2 + 7 \times -8 + 10$

i.e. $-3 \times -6 = 64 - 56 + 10$

$\Rightarrow$ $18 = 18$; ***which is true.***

Similarly, for each and every value of x, we shall find
$(x + 5)(x + 3) = x^2 + 7x + 10$ is true.

As such a statement, which is true for each and every value of its variable, is called an **identity**.

In the same way, each of the following is also an identity.

2. $(x + a)(x - b) = x(x - b) + a(x - b)$
$= x^2 - bx + ax - ab$
$= x^2 + ax - bx - ab = \mathbf{x^2 + (a - b)\,x - ab}$
3. $(x - a)(x + b) = x(x + b) - a(x + b)$
$= x^2 + bx - ax - ab$
$= x^2 - ax + bx - ab = \mathbf{x^2 - (a - b)\,x - ab}$
4. $(x - a)(x - b) = x(x - b) - a(x - b)$
$= x^2 - bx - ax + ab$
$= x^2 - ax - bx + ab = \mathbf{x^2 - (a + b)\,x + ab}$

Hence,

1. $(x + a)(x + b) = x^2 + (a + b)\,x + ab$
2. $(x + a)(x - b) = x^2 + (a - b)\,x - ab$
3. $(x - a)(x + b) = x^2 - (a - b)\,x - ab$
4. $(x - a)(x - b) = x^2 - (a + b)\,x + ab$

It can be combined as :

$(x \pm a)(x \pm b) = x^2 +$ (sum of second terms with their given signs) $x +$ (product of second terms).

For example :

1. $(x + 10)(x + 7) = x^2 + (10 + 7)x + (10 \times 7)$
$= \mathbf{x^2 + 17x + 70}$

2. $(x + 10)(x - 7) = x^2 + (10 - 7)x + (10 \times -7)$
$= \mathbf{x^2 + 3x - 70}$

3. $(x - 10)(x + 7) = x^2 + (-10 + 7)x + (-10 \times 7)$
$= \mathbf{x^2 - 3x - 70}$

4. $(x - 10)(x - 7) = x^2 + (-10 - 7)x + (-10 \times -7)$
$= \mathbf{x^2 - 17x + 70}$

In the same way :

(i) $(y + 15)(y - 3) = y^2 + (15 - 3)y + (15 \times -3)$
$= \mathbf{y^2 + 12y - 45}$

(ii) $(m - 9)(m + 4) = m^2 + (-9 + 4)m + (-9 \times 4)$
$= \mathbf{m^2 - 5m - 36}$

(iii) $(p - \frac{1}{2})(p + \frac{3}{4}) = p^2 + (-\frac{1}{2} + \frac{3}{4})p + (-\frac{1}{2} \times \frac{3}{4})$
$= \mathbf{p^2 + \frac{1}{4}p - \frac{3}{8}}$

(iv) $(n + 0{\cdot}6)(n + 0{\cdot}8) = n^2 + (0{\cdot}6 + 0{\cdot}8)n + (0{\cdot}6 \times 0{\cdot}8)$
$= \mathbf{n^2 + 1{\cdot}4n + 0{\cdot}48}$

EXERCISE 14(A)

1. Find :

(i) $(x + 5)(x + 12)$ (ii) $(x + 8)(x + 2)$ (iii) $(y + 3)(y + 15)$

(iv) $(n + 12)(n + 15)$ (v) $(x + 0{\cdot}4)(x + 0{\cdot}5)$ (vi) $(y + 0{\cdot}7)(y + 0{\cdot}1)$

(vii) $(x + \frac{1}{3})(x + \frac{2}{3})$ (viii) $(y + \frac{4}{5})(y + \frac{2}{5})$

2. Find :

(i) $(x + 4)(x - 1)$ (ii) $(x + 15)(x - 8)$ (iii) $(m + 8)(m - 8)$

(iv) $(n + 15)(n - 12)$ (v) $(x + 0{\cdot}8)(x - 0{\cdot}5)$ (vi) $(y + 0{\cdot}7)(y - 0{\cdot}1)$

(vii) $(x + \frac{3}{4})(x - \frac{1}{4})$ (viii) $(y + \frac{4}{5})(y - \frac{2}{5})$

3. Find :

(i) $(x - 8)(x + 5)$ (ii) $(x - 12)(x + 7)$ (iii) $(m - 5)(m + 5)$

(iv) $(x - 18)(x + 3)$ (v) $(x - 1{\cdot}6)(x + 0{\cdot}9)$ (vi) $(y - 0{\cdot}8)(y + 0{\cdot}1)$

(vii) $(x - \frac{7}{15})(x + \frac{2}{15})$ (viii) $(y - \frac{4}{5})(y + \frac{3}{5})$

4. Find :

(i) $(x - 7)(x - 4)$ (ii) $(x - 5)(x - 20)$ (iii) $(m - 9)(m - 9)$

(iv) $(n - 18)\ (n - 8)$ (v) $(x - 0{\cdot}7)\ (x - 1{\cdot}4)$ (vi) $(y - 2{\cdot}4)\ (y - 0{\cdot}6)$

(vii) $(x - \frac{4}{9})\ (x - \frac{5}{9})$ (viii) $(y - \frac{7}{13})\ (y - \frac{3}{13})$

5. Find :

(i) $(x + 15)\ (x + 9)$ (ii) $(x + 15)\ (x - 9)$ (iii) $(x - 15)\ (x + 9)$

(iv) $(x - 15)\ (x - 9)$ (v) $(y + 0{\cdot}8)\ (y + 0{\cdot}4)$ (vi) $(y + 0{\cdot}8)\ (y - 0{\cdot}4)$

(vii) $(y - 0{\cdot}8)\ (y + 0{\cdot}4)$ (viii) $(y - 0{\cdot}8)\ (y - 0{\cdot}4)$ (ix) $(z + \frac{2}{5})\ (z - \frac{1}{5})$

(x) $(z - \frac{2}{5})\ (z - \frac{1}{5})$ (xi) $(z - \frac{2}{5})\ (z + \frac{1}{5})$

14.2 SQUARE OF THE SUM OF TWO TERMS

$$(a + b)^2 = (a + b)\ (a + b)$$
$$= a(a + b) + b(a + b)$$
$$= a^2 + ab + ba + b^2 = \mathbf{a^2 + 2ab + b^2}$$

Thus : $\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$

i.e. $(\text{Sum of two terms})^2 = (\text{1st term})^2 + 2 \times \text{1st term} \times \text{2nd term} + (\text{2nd term})^2$

Using the same property (formula), we get :

(i) $\mathbf{(2x + y)^2} = (\text{1st term})^2 + 2 \times \text{1st term} \times \text{2nd term} + (\text{2nd term})^2$

$$= (2x)^2 + 2 \times 2x \times y + (y)^2$$
$$= \mathbf{4x^2 + 4xy + y^2}$$

(ii) $\mathbf{(3m + 5n)^2} = (3m)^2 + 2 \times 3m \times 5n + (5n)^2$

$$= \mathbf{9m^2 + 30mn + 25n^2}$$

(iii) Square of $\mathbf{5a + \frac{2}{3}b} = \left(5a + \frac{2}{3}b\right)^2$

$$= (5a)^2 + 2 \times 5a \times \frac{2}{3}b + \left(\frac{2}{3}b\right)^2$$
$$= \mathbf{25a^2 + \frac{20}{3}ab + \frac{4}{9}b^2}$$

14.3 SQUARE OF THE DIFFERENCE OF TWO TERMS

$$(a - b)^2 = (a - b)\ (a - b)$$
$$= a(a - b) - b(a - b)$$
$$= a^2 - ab - ba + b^2 = \mathbf{a^2 - 2ab + b^2}$$

Thus : $\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$

i.e., $(\text{Difference of two terms})^2 = (\text{1st term})^2 - 2 \times \text{1st term} \times \text{2nd term} + (\text{2nd term})^2$.

Using the same property (formula), we get :

(i) $\mathbf{(3x - 2y)^2} = (\text{1st term})^2 - 2 \times \text{1st term} \times \text{2nd term} + (\text{2nd term})^2$

$$= (3x)^2 - 2 \times 3x \times 2y + (2y)^2$$
$$= \mathbf{9x^2 - 12xy + 4y^2}$$

(ii) $(5m - 4n)^2 = (5m)^2 - 2 \times 5m \times 4n + (4n)^2$

$= \mathbf{25m^2 - 40mn + 16n^2}$

(iii) Square of $3x - \frac{5}{6}y = \left(3x - \frac{5}{6}y\right)^2$

$= (3x)^2 - 2 \times 3x \times \frac{5}{6}y + \left(\frac{5}{6}y\right)^2 = \mathbf{9x^2 - 5xy + \frac{25}{36}y^2}$

14.4 THE PRODUCT OF THE SUM AND THE DIFFERENCE OF TWO TERMS

$(a + b)(a - b) = a(a - b) + b(a - b)$

$= a^2 - ab + ba - b^2 = a^2 - b^2$

Thus : $(a + b)(a - b) = a^2 - b^2$

i.e., (sum of two terms) × (difference of the same two terms)

= (first term)^2 – (second term)^2

= Difference of the squares of the two terms

Using the same property (formula), we get :

(i) $(3x + 2y)(3x - 2y) =$ (first term)^2 – (second term)^2

$= (3x)^2 - (2y)^2$

$= 9x^2 - 4y^2$

(ii) $\left(\frac{4}{5}m + \frac{5}{4}n\right)\left(\frac{4}{5}m - \frac{5}{4}n\right) = \left(\frac{4}{5}m\right)^2 - \left(\frac{5}{4}n\right)^2 = \frac{16}{25}m^2 - \frac{25}{16}n^2$

Each of :

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $(a + b)(a - b) = a^2 - b^2$ is an identity.

14.5 PERFECT SQUARE TRINOMIAL

Since, $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow$ Trinomial $a^2 + 2ab + b^2$ is the square of $a + b$

And, we say : $a^2 + 2ab + b^2$ is a *perfect square trinomial.*

Similarly, $(a - b)^2 = a^2 - 2ab + b^2$

$\Rightarrow$ Trinomial $a^2 - 2ab + b^2$ is the square of $a - b$

And, we say : $a^2 - 2ab + b^2$ is a *perfect square trinomial.*

14.6 TO CHECK WHETHER THE GIVEN TRINOMIAL IS A PERFECT SQUARE OR NOT

If the given trinomial can be expressed as $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$, it is a perfect square trinomial.

For this :

1. Two terms of the trinomial must be squares of some monomials.
   e.g., in $4x^2 + 20xy + 25y^2$, the term $4x^2 = (2x)^2$ and the term $25y^2 = (5y)^2$

2. Now, the remaining term of the given trinomial must be equal to twice the product of monomials ($2x$ and $5y$) obtained in Step 1.

Since, twice the product of $2x$ and $5y$

$= 2 \times 2x \times 5y$

$= 20xy$, which is the remaining term of the given trinomial.

$\therefore$ Given trinomial is a perfect square

and $4x^2 + 20xy + 25y^2 = (2x + 5y)^2$.

Similarly, $4x^2 - 20xy + 25y^2 = (2x - 5y)^2$.

Example 1 :

For each trinomial, given below, check whether it is a perfact square or not :

(i) $9a^2 + 42ab + 49b^2$ (ii) $16x^2 - 24xy + 9y^2$

(iii) $4m^2 + 9n^2 - 6mn$ (iv) $x^2 + 25y^2 + 15xy$

Solution :

(i) Since, $9a^2 = (3a)^2$ and $49b^2 = (7b)^2$

Also, $2 \times 3a \times 7b = 42ab$, *which is the remaining term,*

$\therefore$ *The given trinomial* $9a^2 + 42ab + 49b^2$ *is a perfect square trinomial.* **(Ans.)**

Also, $9a^2 + 42ab + 49b^2 = (3a)^2 + 2 \times 3a \times 7b + (7b)^2$

$= \mathbf{(3a + 7b)^2}$

(ii) Since, $16x^2 = (4x)^2$ and $9y^2 = (3y)^2$

Also, $2 \times 4x \times 3y = 24xy$, *which is the remaining term*

$\therefore$ *The given trinomial* $16x^2 - 24xy + 9y^2$ *is a perfect square trinomial.* **(Ans.)**

Also, $16x^2 - 24xy + 9y^2 = (4x)^2 - 2 \times 4x \times 3y + (3y)^2$

$= \mathbf{(4x - 3y)^2}$

(iii) Since, $4m^2 = (2m)^2$ and $9n^2 = (3n)^2$

Also, $2 \times 2m \times 3n = 12mn$, *which is not equal to the remaining term*

$\therefore$ $4m^2 + 9n^2 - 6mn$ **is not a perfect square trinomial.** **(Ans.)**

(iv) $\because x^2 = (x)^2$ and $25y^2 = (5y)^2$

Also, $2 \times x \times 5y = 10xy$, *which is not equal to the remaining term*

$\therefore$ $x^2 + 25y^2 + 15xy$ **is not a perfect square trinomial.** **(Ans.)**

14.7 USING FORMULAE

Example 2 :

Use formulae to evaluate : (i) 102^2 (ii) 49^2 (iii) $8{\cdot}2 \times 7{\cdot}8$

Solution :

(i) $\mathbf{102^2} = (100 + 2)^2$

$= 100^2 + 2 \times 100 \times 2 + 2^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 10000 + 400 + 4 = 10404$ **(Ans.)**

(ii) $49^2 = (50 - 1)^2$

$= 50^2 - 2 \times 50 \times 1 + 1^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 2500 - 100 + 1 = \mathbf{2401}$ **(Ans.)**

(iii) $8{\cdot}2 \times 7{\cdot}8 = (8 + 0{\cdot}2)\,(8 - 0{\cdot}2)$

$= 8^2 - (0{\cdot}2)^2$ [Since, $(a + b)\,(a - b) = a^2 - b^2$]

$= 64 - 0{\cdot}04 = \mathbf{63{\cdot}96}$ **(Ans.)**

EXERCISE 14(B)

1. Find the square of :

(i) $a + 2b$ (ii) $3b + c$ (iii) $2a - 5b$

(iv) $3x - 4$ (v) $x - \frac{1}{x}$ (vi) $a + \frac{1}{a}$

(vii) $5p - 1$ (viii) $4ab + bc$ (ix) $ab + cd$

(x) $a^2 + b^2$ (xi) $x^2 + 2x$ (xii) $abc + cd$

(xiii) $px - q$ (xiv) $3a^2 + 4b^2$

2. Use formula to find :

(i) $(x + 5)\,(x - 5)$ (ii) $(a - 2)\,(a + 2)$

(iii) $(2x - 1)\,(2x + 1)$ (iv) $(3a - 2b)\,(3a + 2b)$

(v) $(2a - 5b)\,(2a + 5b)$ (vi) $(4xy - 3pq)\,(4xy + 3pq)$

(vii) $(0{\cdot}2x + 0{\cdot}5y)\,(0{\cdot}2x - 0{\cdot}5y)$ (viii) $\left(\frac{1}{2}a + \frac{1}{3}b\right)\left(\frac{1}{2}a - \frac{1}{3}b\right)$

(ix) $(9a + 13b)\,(9a - 13b)$ (x) $(mn^2 + m^2n)\,(mn^2 - m^2n)$

3. Find which of the following are perfect square trinomials :

(i) $a^2 + 4ab + b^2$ (ii) $9x^2 + 6xy + 4y^2$

(iii) $25 + 30xy + 9x^2y^2$ (iv) $1 - 2a + a^2$

(v) $9p^2 + 30p + 100$ (vi) $x^2 - 8x + 9$

(vii) $x^2 + x + \frac{1}{4}$ (viii) $m^2 + 2m + 4$

4. Simplify the following, using formulae :

(i) 105^2 (ii) 52^2 (iii) 97^2

(iv) 68×72 (v) 101×99 (vi) $89^2 - 79^2$

(vii) $44^2 - 39^2$ (viii) $41^2 - 35^2$ (ix) $128^2 - 28^2$

5. Evaluate (using formulae) :

(i) $(3x + 2y)^2$ (ii) $(5a - 6)^2$ (iii) $(2m + 5n)^2$

(iv) $(2xy + 7)^2$ (v) $(4 - 9mn)^2$ (vi) $(2x - \frac{2}{x})^2$

(vii) $(2x + 7)\,(2x - 7)$ (viii) $(3m - 5n)\,(3m + 5n)$ (ix) $(n + 0{\cdot}6)\,(n - 0{\cdot}6)$

(x) $(x + \frac{3}{4})\,(x - \frac{3}{4})$ (xi) $(\frac{5}{7}x + 1)\,(\frac{5}{7}x - 1)$ (xii) $(y - \frac{2}{3}z)\,(y + \frac{2}{3}z)$

(xiii) $(22)^2$ (xiv) $(103)^2$ (xv) $(98)^2$

(xvi) $(207)^2 - (53)^2$ (xvii) 153×147