UNIT - 2 COMMERCIAL ARITHMETIC

CHAPTER 6

UNITARY METHOD

(INCLUDING TIME AND WORK)

6.1 BASIC CONCEPT

Consider the following examples :

1. If the cost of 15 m cloth = ₹ 300

⇒ The cost of 1 m cloth =
$$\frac{₹ 300}{15}$$
 = ₹ 20

2. If 15 men can do some work in 300 days

 \Rightarrow 1 man will do the same work in 300 \times 15 days = 4500 days

and, 10 men will do it in
$$\frac{4500}{10}$$
 days = 450 days

In part 1, given above, the cost of 1 m cloth is first obtained from the given cost of 15 m and then the cost of 10 m is found.

In the same way; in part 2, given above, the number of days taken by 1 man is first calculated from the number of days by 15 men and then the number of days taken by 10 men is obtained.

The method in which the value of a unit quantity is first calculated to get the value of any quantity is called the unitary method.

In unitary method, we come across two types of variations:

Type of variations	Condition	Examples
1. Direct variation	Increase in one quantity causes increase in the other and decrease in one quantity causes decrease in the other.	 (i) With more money, more articles can be bought. (ii) With a greater speed a larger distance can be covered in the same time. (iii) With less number of men, less work can be done in same time, etc.
2. Inverse variation	Increase in one quantity causes decrease in the other and decrease in one quantity causes increase in the other.	 (i) With a greater speed, less time will be taken to cover the same distance. (ii) With less number of men, more days are required for the same work, etc.

6.2 EXAMPLES OF DIRECT VARIATION

Example 1:

A man earns ₹ 400 in 10 days. How much will he earn in 28 days ?

Solution:

∴ In 1 day, he will earn =
$$₹ \frac{400}{10} = ₹ 40$$

[Less money is earned in 1 day, so divide]

Note: For solving problems (using unitary method), the sentences (statements) should be framed in such a way that, whatever is to be found is written at the end of the statement.

Arrow method:

Steps:		No. of days	Earnings
1.	Form two columns, one heading earnings and the other heading no. of days (as shown alongside). [The quantity to be obtained must be written at the extreme right column. Here, quantity to be obtained is earnings of 28 days].		
2.	Let earnings of 28 days be ₹ x. Write the no. of days and corresponding earnings as shown alongside.	No. of days 10 28	Earnings ₹ 400 ₹ x
3.	For the column on the extreme right, mark an arrow in the downward direction.	No. of days 10 28	Earnings ₹ 400 ₹ x ↓
4.	If it is the case of direct variation, the arrow for the column (headed: no. of days) must be in the same direction as that for earnings.	No. of days	Earnings ₹ 400 ₹ x

In the case of inverse variation, the arrow for this column must be in the direction opposite to the direction of the first arrow.

Since, here we have the case of direct variation, therefore, for both the columns arrows must be in the same direction.

5. Now according to the arrows marked, take :

$$\frac{\text{value on the head}}{\text{value on the tail}} \text{ for one arrow} = \frac{\text{value on the head}}{\text{value on the tail}} \text{ for the other arrow.}$$

Thus, we get:

$$\frac{₹ x}{₹ 400} = \frac{28}{10}$$
 ⇒ $x = \frac{28}{10} \times 400 = 1,120$

.: 28 men will earn = ₹ 1,120

(Ans.)

Example 2:

0.75 metre cloth costs ₹ 45. What will be the cost of 0.6 metre of same cloth ?

Solution:

Given: the cost of 0.75 m cloth = ₹45

∴ The cost of 1 m cloth =
$$₹ \frac{45}{0.75}$$

= ₹ 60

And, the cost of 0.6 m cloth =
$$0.6 \times ₹60$$
 = ₹36 (Ans.)

Arrow method:

Cloth (m) Cost (₹)

$$0.75 \downarrow 0.6 \downarrow 0.6$$

 $0.6 \downarrow 0.6$
 $0.6 \downarrow 0.6$
 $0.6 \downarrow 0.75$
 $0.6 \downarrow 0.75$
∴ Cost of 0.6 m cloth = ₹ 36 (Ans.)

6.3 EXAMPLES OF INVERSE VARIATION

Example 3:

4 men can do a piece of work in 5 days. How many men will do it in 4 days?

Solution:

- : In 5 days, the work is done by 4 men
- .. In 1 day, the work will be done by $4 \times 5 = 20$ men [More number of men are required to do the work in 1 day, so multiply]

In 4 days, the work will be done by $\frac{20}{4}$ men = 5 men (Ans.)

Arrow method:

Days No. of men $\begin{array}{ccc}
5 \uparrow & 4 \\
4 & x
\end{array}$ $\Rightarrow & \frac{x}{4} = \frac{5}{4} \text{ and } x = 5$ $\therefore 5 \text{ men will do the work in}$

(Ans.)

Example 4:

With a speed of 60 km/h, it takes 4 hours to cover a certain distance. What should be the speed, if the same journey is to be completed in 3 hours?

Solution:

- To cover a certain distance in 4 hours; speed required = 60 km/h
- .. To cover the same distance in 1 hour; speed required = 60 km/h × 4 = 240 km/h

And, to cover the same distance in 3 hours;

speed required =
$$\frac{240}{3}$$
 km/h = 80 km/h (Ans.)

Remember: (i) For getting more, multiply.

Arrow method:

4 days

Time (hrs) Speed (km/h)
$$\begin{vmatrix}
4 \uparrow & 60 \\
3 & x
\end{vmatrix}$$

$$\Rightarrow \frac{x}{60} = \frac{4}{3}$$

$$\Rightarrow x = \frac{4}{3} \times 60 = 80$$

$$\therefore \text{ Required speed}$$

$$= 80 \text{ km h}^{-1} \text{ (Ans.)}$$

(ii) For getting less, divide.

EXERCISE 6(A)

Fill in the blanks:

(i)	Cost of 4 articles = ₹ 72 ⇒ Cost of 1 article = =	
(ii)	8 men do a work in 16 days ⇒ 1 man does it in days = days.	
(iii)	7 pens cost ₹ 52.50 ⇒ The cost of 1 pen = ₹ = ₹	
(iv)	1 man does a work in 15 days ⇒ 3 men will do the same work in	
(v)	2.25 m cloth is required for 1 shirt ⇒ Cloth required for 3 shirts =	

- (vi) For 8 days, a family requires ₹ 320 ⇒ for 4 days, the family will require = Weight of 8 identical articles is 4.8 kg. What is the weight 11 such articles?
- 6 books weigh 1.260 kg. How many books will weigh 3.150 kg?
- 8 men complete a work in 6 hours. In how many hours will 12 men complete the same work?
- 5. If a 25 cm long candle burns for 45 minutes, how long will another candle of the same material and same thickness but 5 cm longer than the previous one burn ?
- 6. A typist takes 80 minutes to type 24 pages. How long will he take to type 87 pages ?
- 7. ₹ 750 support a family for 15 days. For how many days will ₹ 2,500 support the same family?
- 400 men have provisions for 23 weeks. They are joined by 60 men. How long will the provisions last?

Hint: For 400 men, the provisions are sufficient for 23 weeks ⇒ For 1 man, the provision will be sufficient for 23 × 400 weeks.

- 9. 200 men have provisions for 30 days. If 50 men left, the same provisions would last for the remaining men for how many days?
- 10. 8 men can finish a certain amount of provisions in 40 days. If 2 more men join with them, find in how many days will the same amount of provisions be sufficient?
- 11. If the interest on ₹ 200 be ₹ 25 in a certain time, what will be the interest on ₹ 750 for the same time ?
- 12. If 3 dozen eggs cost ₹ 90, find the cost of 3 scores of eggs. [1 score = 20]
- 13. If the fare for 48 km is ₹ 288, what will be the fare for 36 km?
- 14. What will be the cost of 3.20 kg of an item, if 3 kg of it costs ₹ 360 ?
- 15. If 9 lines of a print, in a column of a book contains 36 words. How many words will a column of 51 lines contain ?
- 16. 125 pupil have food sufficient for 18 days. If 25 more pupil join them, how long will the food last now?

What assumption have you made to come to your answer?

17. A carpenter prepares a new chair in 3 days, working 8 hours a day. Atleast how many hours per day must he work in order to make the same chair in 4 days ?

Example 5:

If cost of $\frac{8}{15}$ of a certain cargo is ₹ 2,000. Find the cost of $\frac{3}{5}$ of the same cargo.

Arrow method:

Solution:

Cost of
$$\frac{8}{15}$$
 of a cargo = $₹2,000$

$$\Rightarrow \text{ Cost of whole (one) cargo} = $₹2,000 \div \frac{8}{15}$

$$= ₹2,000 \times \frac{15}{8}$$

$$= ₹3,750$$

$$\Rightarrow \frac{x}{2,000} = \frac{3}{5}$$

$$\Rightarrow x = \frac{3}{5} \times \frac{15}{8} \times 2,000$$

$$= ₹2,250 \text{ (Ans.)}$$$$

Example 6:

A watch gains 42 sec in 3 days and 8 hours. How long will it take to gain 2 min 6 sec?

Solution:

Since, 3 days + 8 hours =
$$(3 \times 24 + 8)$$
 hours
= 80 hours
and, 2 min 6 sec = $(2 \times 60 + 6)$ sec
= 126 sec

Given: The watch gains 42 sec in 3 days 8 hours i.e. in 80 hours

 $\Rightarrow \qquad \text{The watch gains 1 sec in } \frac{80}{42} \text{ hours}$

and, it will gain 126 sec in $\frac{80}{42}$ × 126 hours = 240 hours.

Time taken by watch to gain 2 min 6 sec = 240 hours = 10 days (Ans.)

Example 7:

If 135 kg of corn feeds 45 horses for 8 days, for how long will the same quantity of corn feed 24 horses?

| Arrow method:

Solution:

Here, in both the cases, the quantity of corn remains the same.

Given: For 45 horses, the given quantity of corn is sufficient for 8 days.

- ⇒ For 1 horse, the same quantity of corn will be sufficient for 8 × 45 days
- \Rightarrow For 24 horses, it will be sufficient for $\frac{8 \times 45}{24}$ days = 15 days

| Arrow method : | No. of horses No. of days | $45 \uparrow 8 \downarrow x \downarrow$ | $24 \uparrow x \downarrow$ | $\Rightarrow \frac{x}{8} = \frac{45}{24} \Rightarrow x = 15$ | \therefore Required time = 15 days (Ans.)

∴ Required cost = ₹ 2,250 (Ans.)

hrs

(Ans.)

Arrow method:

 $\Rightarrow \frac{x}{80} = \frac{126}{42}$

 $x = 240 \, hrs$

∴ Required time = 240 hrs

= 10 days

Gain in time

24

(Ans.)

EXERCISE 6(B)

- 1. The cost of $\frac{3}{5}$ kg of ghee is ₹ 96, find the cost of : (i) one kg ghee. (ii) $\frac{5}{8}$ kg ghee.
- 2. $3\frac{1}{2}$ m of cloth costs ₹ 168, find the cost of $4\frac{1}{3}$ m of the same cloth.
- A wrist-watch loses 10 sec in every 8 hours, in how much time will it lose 15 sec ?
- In 2 days and 20 hours a watch gains 20 sec. Find, how much time will the watch take to gain 35 sec?
- 5. 50 men mow 32 hectares of land in 3 days. How many days will 15 men take to mow it?
- The wages of 10 workers for a six days week are ₹ 1,200. What are the one day wages: (ii) of 4 workers ? of one worker?
- 7. If 32 apples weigh 2 kg 800 g. How many apples will there be in a box, containing 35 kg of apples ?
- 8. A truck uses 20 litres of diesel for 240 km. How many litres will be needed for 1200 km?
- 9. A garrison of 1200 men has provisions for 15 days. How long will the provisions last if the garrison be increased by 600 men?
- 10. A camp has provisions for 60 pupil for 18 days. In how many days, the same provisions will finish off if the strength of the camp is increased to 72 pupil ?

TIME AND WORK

For solving problems on time and work, make the following simple facts clear:

1. If a man does a work in 50 days, his one day's work = $\frac{1}{50}$.

Conversely, if one day's work of a man is $\frac{1}{50}$, he will complete the work in 50 days.

Thus, one day's work =
$$\frac{1}{\text{No. of days required to complete the work}}$$
And, number of days required to complete a work = $\frac{1}{\text{One day's work}}$

Work to be done 2. Time required to do a certain work = Work done in unit time

Example 8:

A completes a piece of work in 4 days and B completes it in 6 days. How long will it take to complete the same work, if they both work on it together?

Solution:

Given, A does the work in 4 days,
$$\therefore$$
 A's 1 day's work = $\frac{1}{4}$

And, B does the work in 6 days, \therefore B's 1 day's work = $\frac{1}{6}$
 \Rightarrow (A + B)'s 1 day's work = $\frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12}$

Hence, A and B together will complete the work in $\frac{12}{5}$ days = $2\frac{2}{5}$ days (Ans.)

Example 9:

Ajay and Vijay together can paint a hall in 6 days. Ajay alone can paint it in 8 days. In how many days can Vijay alone paint it?

Solution:

Given, Ajay and Vijay together paint a hall in 6 days

- .. Ajay and Vijay in 1 day can paint $\frac{1}{6}$ of the hall. Since, Ajay alone can paint the wall in 8 days
- \therefore Ajay alone in 1 day can paint $\frac{1}{8}$ of the hall.
- \Rightarrow Vijay alone in 1 day can paint $\frac{1}{6} \frac{1}{8}$ of the hall $= \frac{4-3}{24} = \frac{1}{24}$ of the hall
- :. Vijay alone can paint the hall in 24 days (Ans.)

Example 10:

A tap fills a cistern in 4 hours. Another tap empties the full tank in 6 hours. How long will it take to fill the tank, if the tank is empty and both the taps are open?

Solution:

Since, one tap in 1 hour fills $\frac{1}{4}$ of the cistern

And, the second tap in 1 hour empties $\frac{1}{6}$ of the cistern

Both the taps in 1 hour will fill = $\left(\frac{1}{4} - \frac{1}{6}\right)$ of the cistern = $\frac{3-2}{12} = \frac{1}{12}$ of the cistern.

The cistern will be full in = 12 hours (Ans.)

Example 11:

A and B can do a piece of work in 15 days and 20 days respectively.

Find: (i) the work done by A in 3 days.

- (ii) the work left after A has worked for 3 days.
- (iii) the number of days that B will take to complete the remaining work.

Solution:

(i)
$$\therefore$$
 A's 1 day's work = $\frac{1}{15}$
 \therefore Work done by A in 3 days = $\frac{1}{15} \times 3 = \frac{1}{5}$ (Ans.)

(ii) Work left after A has worked for 3 days =
$$1 - \frac{1}{5} = \frac{4}{5}$$
 (Ans.)

(iii) Remaining work =
$$\frac{4}{5}$$
 and B's 1 day's work = $\frac{1}{20}$

.. Number of days taken by B to complete the remaining work

$$= \frac{\text{work to be done}}{\text{B's 1day's work}} = \frac{\frac{4}{5}}{\frac{1}{20}} = \frac{4}{5} \times \frac{20}{1} = 16 \text{ days}$$
 (Ans.)

EXERCISE 6(C)

- 1. A can do a piece of work in 6 days and B can do it in 8 days. How long will they take to complete it together?
- 2. A and B working together can do a piece of work in 10 days. B alone can do the same work in 15 days. How long will A alone take to do the same work?
- 3. A can do a piece of work in 4 days and B can do the same work in 5 days. Find, how much work can be done by them working together in: (i) one day (ii) 2 days. What part of work will be left, after they have worked together for 2 days ?
- 4. A and B take 6 hours and 9 hours respectively to complete a work. A works for 1 hour and then B works for two hours.
 - (i) How much work is done in these 3 hours?
 - (ii) How much work is still left?
- 5. A, B and C can do a piece of work in 12, 15 and 20 days respectively. How long will they take to do it working together?
- 6. Two taps can fill a cistern in 10 hours and 8 hours respectively. A third tap can empty it in 15 hours. How long will it take to fill the empty cistern, if all of them are opened together?
- 7. Mohit can complete a work in 50 days, whereas Anuj can complete the same work in 40 days.

Find:

- (i) work done by Mohit in 20 days.
- (ii) work left after Mohit has worked on it for 20 days.
- time taken by Anuj to complete the remaining work.
- 8. Joseph and Peter can complete a work in 20 hours and 25 hours respectively.

Find:

- (i) work done by both together in 4 hrs.
- work left after both worked together for 4 hrs.
- time taken by Peter to complete the remaining work. (iii)
- A is able to complete $\frac{1}{3}$ of a certain work in 10 hrs and B is able to complete the same work in 12 hrs.

Find:

- how much work can A do in 1 hour ?
- how much work can B do in 1 hour ?
- in how much time will the work be completed, if both work together.
- 10. Shaheed can prepare one wooden chair in 3 days and Shaif can prepare the same chair in 4 days. If they work together, in how many days will they prepare :
 - one chair?
 - 14 chairs of the same kind? (ii)