DECIMAL FRACTIONS

(DECIMALS)

5.1 DEFINITION

A decimal fraction or a decimal number is a fraction whose denominator can be expressed as 10 or some higher power of 10.

- (i) $\frac{7}{10}$, $\frac{13}{100}$, $\frac{851}{1000}$, $\frac{79}{10^4}$, $\frac{2547}{10^7}$ etc., are all decimal fractions.
- (ii) Since, $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$ i.e., $\frac{3}{5}$ can be expressed as a fraction with denominator 10, therefore $\frac{3}{5}$ is a decimal fraction.
- (iii) $\frac{7}{8} = \frac{7 \times 125}{8 \times 125} = \frac{875}{1000}$ *i.e.*, $\frac{7}{8}$ can be expressed as a fraction with denominator 1000 (*i.e.* 10³, which is higher power of 10), therefore $\frac{7}{8}$ is also a decimal fraction.

For the same reason, each of $\frac{7}{20}$, $\frac{16}{25}$, $\frac{33}{50}$, $\frac{64}{125}$, etc., is also a decimal fraction.

In order to express a given decimal fraction in shorter form, the denominator is not written, but its absence is shown by a dot called a decimal point, inserted in a proper place.

e.g.,
$$\frac{3}{10} = 0.3$$
; $\frac{213}{100} = 2.13$; $\frac{7}{100} = 0.07$; $\frac{59}{10^4} = \frac{59}{10000} = 0.0059$, etc.

- 1. When there is no number to the left of the decimal point, generally, a zero is written.
 - i.e. (i) ·72 is written as 0·72
 - (ii) .004 is written as 0.004 and so on.
- 2. 2.4 means, 2 + 0.4. Here, 2 is the integral part and 0.4 is the decimal part of the number 2.4.
- 3. Any extra zero or zeroes written after the decimal part of a number does not change its value.

 e.g., value of 3.5 is the same as 3.50 or 3.500 or 3.5000 and so on.

5.2 READING DECIMAL NUMBERS

The integral part is read according to its value and decimal part is read by naming each digit, in order, separately.

- e.g., (i) 21.45 will be read as: Twenty one point four-five.
 - (ii) 152.639 will be read as: One fifty two point six-three-nine.
 - (iii) 0.08 will be read as : Point zero-eight or zero-point zero-eight.

In decimal system, the first place on the right of the decimal point is called tenths' place, second place to the right of decimal is called hundredths' place and so on.

Similarly, the first place on the left of decimal is the units' place, the second place on the left of decimal is the tens' place, and so on.

e.g., in number 5.46; 5 is at units' place, 4 is at tenths' place and 6 is at hundredths' place. The following table shows the place values of different digits in a decimal number:

	Number	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten thousandths
.,	45.986			4	5		9	8	6	
	936-4527		9	3	6		4	5	2	7
	7042-93	7	0	4	2		9	3		

5.3 CONVERTING A DECIMAL NUMBER INTO A VULGAR FRACTION

Remove decimal point from the given decimal number. And, in its denominator write as many zeroes, as the number of digits in the decimal part, to the right of 1. Then simplify, if possible, to get the fraction obtained to its lowest terms.

Thus,
$$0.47 = \frac{47}{100}$$
; $2.739 = \frac{2739}{1000}$; $0.0244 = \frac{244}{10000} = \frac{61}{2500}$ and so on.

5.4 CONVERTING A GIVEN FRACTION INTO A DECIMAL FRACTION

1. When the denominator of the given fraction is 10, 100, 1000, etc.:

Counting from extreme right to left, mark the decimal point after as many digits of the numerator as there are zeroes in the denominator.

Thus, (i)
$$\frac{259}{10} = 25.9$$
, (ii) $\frac{259}{100} = 2.59$, (iii) $\frac{259}{1000} = 0.259$, (iv) $\frac{259}{10000} = 0.0259$ and so on.

2. When the denominator of the given fraction is other than 10, 100, 1000, etc.:

Divide in an ordinary way and mark the decimal point in the quotient just after the division of unit digit is completed. After this, any number of zeroes (one by one) can be placed to complete the division.

Thus, (i)
$$\frac{15}{4} = 3.75$$
 (ii) $\frac{27}{5} = 5.4$ $5\sqrt{27}$ 25 20 20 20 \times Ans. = 3.75 Ans. = 5.4

5.5 DECIMAL PLACES

The number of figures that follow the decimal point is called the number of decimal places.

Thus, 28-497 has 3 decimal places, 153-46 has 2 decimal places, 0-5497 has 4 decimal places and so on.

Example 1:

Convert each of the following decimal fractions into vulgar fraction in lowest terms :

Solution:

(i)
$$0.125 = \frac{125}{1000} = \frac{1}{8}$$
 (Ans.)

(ii)
$$5.08 = \frac{508}{100} = \frac{127}{25} = 5\frac{2}{25}$$
 (Ans.)

OR,
$$5.08 = 5 + 0.08$$

= $5 + \frac{8}{100} = 5 + \frac{2}{25} = 5\frac{2}{25}$ (Ans.)

(iii)
$$26.25 = \frac{2625}{100} = \frac{105}{4} = 26\frac{1}{4}$$
 (Ans.) OR,
$$26.25 = 26 + 0.25$$

$$= 26 + \frac{25}{100} = 26 + \frac{1}{4} = 26 \frac{1}{4}$$
 (Ans.)

Example 2:

Convert each of the following into decimal fraction:

(i)
$$5\frac{3}{8}$$
 (ii) $\frac{2}{25}$ (iii) $2\frac{7}{100}$

Solution:

(i)
$$5\frac{3}{8} = \frac{5 \times 8 + 3}{8} = \frac{43}{8} = 5.375$$
 (Ans.)

OR,
$$5\frac{3}{8} = 5 + \frac{3}{8} = 5 + 0.375 = 5.375$$
 (Ans.)

(ii)
$$\frac{2}{25} = 0.08$$
 (Ans.)

OR,
$$\frac{2}{25} = \frac{2 \times 4}{25 \times 4} = \frac{8}{100} = 0.08$$
 (Ans.)

(iii)
$$2\frac{7}{100} = \frac{2 \times 100 + 7}{100} = \frac{207}{100} = 2.07$$
 (Ans.)

OR,
$$2\frac{7}{100} = 2 + \frac{7}{100} = 2 + 0.07 = 2.07$$
 (Ans.)

EXERCISE 5(A)

- 1. Convert the following into fractions in their lowest terms :
 - (i) 3.75

(ii) 0.5

- (iii) 2·04
- (iv) 0.65

(v) 2.405

- (vi) 0.085
- (vii) 8.025

- 2. Convert into decimal fractions :
 - (i) $2\frac{4}{5}$
- (ii) $\frac{79}{100}$
- (iii) $\frac{37}{10,000}$
- (iv) $\frac{7543}{10^4}$

- (v) $\frac{3}{4}$
- (vi) $9\frac{3}{5}$
- (vii) $8\frac{5}{8}$
- (viii) $5\frac{7}{8}$

- 3. Write the number of decimal places in :
 - (i) 0·4762

- (ii) 7.00349
- (iii) 8235·403

(iv) 35·4

- (v) 2.608
- (vi) 0.000879

- 4. Write the following decimals as word statements:
 - (i) 0.4, 0.9, 0.1
- (ii) 1.9, 4.4, 7.5
- (iii) 0.02, 0.56, 13.06

- 0.005, 0.207, 111.519 (v) 0.8, 0.08, 0.008, 0.0008
- 256.1, 10.22, 0.634 (vi)

ADDITION OF DECIMALS

Write the given decimal numbers in such a way, that the decimal points of all the numbers fall in the same vertical line. Digits with the same place value are placed one below the other, i.e., units are written below units, tens below tens and so on.

Addition is started from the right side, as done in the usual addition (empty places may be filled up by zeroes). In the result (total), the decimal point is placed under decimal points of the numbers added.

Example 3:

Add:

(i) 3.92, 11.057 and 236.84

(ii) 2.8, 104.91 and 37.056

Solution:

Note: A whole number can also be expressed as a decimal number by putting a decimal after its last (unit) digit and after it as many zeroes as are required.

e.g. 15 = 15.0 = 15.00 = 15.0000, etc.

SUBTRACTION OF DECIMALS

In subtraction also, the numbers are written in such a way that their decimals are in the same vertical line. Digits with the same place value are placed one below the other (empty places may be filled by zeroes).

Subtraction is started from the right side, as in the case of normal subtraction.

In the result, decimal point is placed just under the other decimal points.

Example 4:

Subtract:

- 5.27 from 13.89
- 0.283 from 2
- 0.45 from 4.5 (iii)
- 0.5 from 0.84 (iv)

Solution:

EXERCISE 5(B)

(ii)

(iv)

(vi)

(viii)

(x)

1. Add:

- 0.5 and 0.37
- 0.02, 0.008 and 0.309 (iii)
- (V) 9.25, 3.4 and 6.666
- 0.2, 0.02 and 2.0002 (vii)
- 29.03, 0.0003, 0.3 and 7.2 (ix)
- 2. Subtract the first number from the second:
 - (ii) 0.16, 4.3
 - 0.07, 8.43 (v) 2.237, 9.425
 - (vii) 3.92, 26.86 (viii) 4.73, 8.5
 - 0.845, 3.71 (X)

(i) 5.4, 9.8

0.82, 8.6 (iii)

3.8 and 8.7

(vi) 41.03, 59.46

0.4136, 0.3195 and 0.52

3.007, 0.587 and 18.341

6.08, 60.8, 0.608 and 0.0608

3.4, 2.025, 9.36 and 3.6221

12.63, 36.2 (ix)

3. Simplify:

(iv)

- 28.796 13.42 2.555
- 36 18.59 3.2(iii)
- 32.8 13 10.725 + 3.517
- 0.1835 + 163.2005 25.9 100(vii)
- (ix) 555.555 + 55.555 5.55 0.555
- 93.354 62.82 13.045(ii)
- 86 + 16.95 3.0042(iv)
- 4000 30.51 753.101 69.43 (vi)
- (viii) 38.00 - 30 + 200.200 - 0.230
- 4. Find the difference between 6.85 and 0.685.
- Take out the sum of 19.38 and 56.025 then subtract it from 200.111.
- Add 13.95 and 1.003, and from the result, subtract the sum of 2.794 and 6.2.
- What should be added to 39.587 to give 80.375 ?
- 8. What should be subtracted from 100 to give 19.29 ?
- What is the excess of 584-29 over 213-95 ?

10. Evaluate:

(i) (5.4 - 0.8) + (2.97 - 1.462)

(ii) (6.25 + 0.36) - (17.2 - 8.97)

(iii) 9.004 + (3 - 2.462)

(iv) 879.4 - (87.94 - 8.794)

MULTIPLICATION OF DECIMAL NUMBERS

1. Multiplication by 10, 100, 1000, etc. :

Shift the decimal point, in the multiplicand, to the right by as many digits as there are zeroes in the multiplier.

- Thus,
- $3.2985 \times 10 = 32.985$ (ii) $3.2985 \times 100 = 329.85$
- (iii) $3.2985 \times 1000 = 3298.5$ and so on.

[Here, multiplicand is 3.2985 and multipliers are 10, 100, 1000, etc.]

2. Multiplication by a whole number :

Multiply in an ordinary way, without considering the decimal point. In the product, the decimal point should be fixed by counting as many digits from the right as there are decimal places in the multiplicand.

- Thus, (i) $0.3 \times 6 = 1.8$

(ii) $0.26 \times 18 = 4.68$ and so on

3. Multiplication of a decimal number by another decimal number :

Multiply the two numbers in a normal way, ignoring their decimals.

In the product, decimal point is fixed counting from right, the digits equal to the sum of decimal places in the multiplicand and the multiplier.

Thus, $32.5 \times 0.07 = 2.275$

Since, the multiplicand (32.5) has one decimal place and the multiplier (0.07) has two decimal places, their product will have 1 + 2 = 3 decimal places.

Similarly: (i) $0.2 \times 0.0004 = 0.00008$ (ii) $2.895 \times 1.1 = 3.1845$ and so on.

DIVISION OF DECIMALS

1. Division by 10, 100, 1000, etc. :

Shift the decimal point to the left by as many digits as there are zeroes in the divisor.

Thus, (i) $623.42 \div 10 = 62.342$

[Shifting decimal point, one digit to the left]

 $623.42 \div 100 = 6.2342$

[Shifting decimal point, two digits to the left]

 $623.42 \div 10000 = 0.062342$ and so on.

2. Division by a whole number :

Divide in the normal manner, ignoring the decimal, and mark the decimal point in the quotient, while just crossing over the decimal point in the dividend.

Thus,

$$\therefore 0.945 \div 9 = 0.105$$
 (Ans.)

3. Division of a decimal number by another decimal number :

Shift the decimal points of the dividend and the divisor both by as many equal number of digits, so that the divisor converts into a whole number.

The division is then carried out as in Case 2 discussed above.

4.5

Thus,

i)
$$4.8 \div 0.8 = \frac{4.8}{0.8} = \frac{48}{8} = 6$$

(ii)
$$5.625 \div 1.25 = \frac{5.625}{1.25}$$

$$= \frac{562.5}{125}$$

Now consider
$$182.37 \div 2.3 = \frac{182.37}{2.3} = \frac{1823.7}{23}$$

Here, the division can not be completed, *i.e.*, all the digits in the dividend are exhausted and still there is some remainder left. So, we go on writing zeroes (one by one) with the remainder and continue the division process. We can write zeroes, because adding zero at the extreme right of a decimal number does not change the number. Therefore, division can be continued for as many decimal places as we like to.

79-291	
3 1823.7	
161	
213	
207	
67	
46	
210	
207	The same of
30	
23	and so on.
7	and so on.

5.10 TERMINATING DECIMALS

Sometimes, in a division, the dividend is exactly divisible and no remainder is left after certain number of steps. Such answers in the quotient are called terminating decimals.

Thus,
$$31.76 \div 4 = \frac{31.76}{4} = 7.94$$
 is a terminating decimal.

5.11 NON-TERMINATING DECIMALS

In a division, the remainder does not finish (terminate) no matter how long the division is continued.

In such cases the quotient is called non-terminating 7 13.78 decimal.

Thus,
$$13.78 \div 7 = \frac{13.78}{7}$$

$$= 1.9685 \dots$$
which is a non-terminating decimal.

The fact, that it is a non-terminating decimal is shown by writing the digits of the quotient till the division is carried out. After that few dots are put to show that this division continues.

5.12 RECURRING DECIMALS

On performing a division, sometimes we find that the same remainder is left, no matter how long we continue the division process.

Consider: 2 ÷ 3

Here, the remainder is always 2. For this reason, same digit 6 appears again and again in the quotient.

This fact is shown by putting a dot or a bar over the repeating digit (or digits) in the quotient, i.e., $2 \div 3 = 0.666 \dots = 0.6$ or 0.6

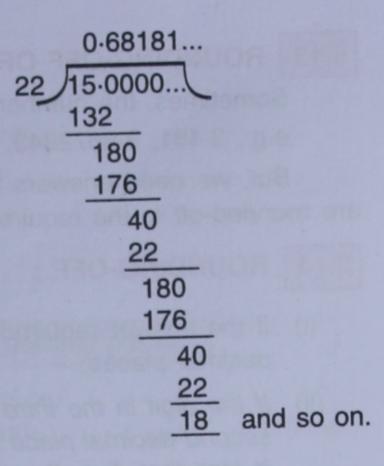
Here, 0.6 (or 0.6), which is a non-terminating repeating decimal, is called a **recurring decimal**. The dot or bar over 6 shows that 6 repeats infinitely.

Similarly, consider 15 ÷ 22.

In this division, the remainders 18 and 4 keep repeating alternately and so are the digits 8 and 1 in the quotient.

$$\therefore 15 \div 22 = \frac{15}{22} = 0.68181...$$

$$= 0.681 \text{ or } 0.681$$



EXERCISE 5(C)

- 1. Multiply:
 - (i) 0.87 by 10
- 2.948 by 100
- (iii) 6.4 by 1000
- 5.8 by 4 (iv)

- (v) 16.32 by 28
- 5.037 by 8 (vi)
- (vii) 4.6 by 2.1
- 0.568 by 6.4 (viii)

- 2. Multiply each number by 10, 100 and 1000 :
 - (i) 0.5

- (ii) 0·112
- (iii) 4.8
- 0.0359 (iv)

- (v) 16.27
- (vi) 234.8
- 3. Evaluate:
 - (i) 5.897×2.3
- 0.894×87 (ii)
- (iii) 0.01×0.001

- (iv) $0.84 \times 2.2 \times 4$
- (v) $4.75 \times 0.08 \times 3$
- (vi) $2.4 \times 3.5 \times 4.8$

- (vii) $0.8 \times 1.2 \times 0.25$
- (viii) $0.3 \times 0.03 \times 0.003$

- 4. Divide:
 - 54.9 by 10
- (ii) 7.8 by 100
- (iii) 324.76 by 1000

- 12.8 by 4 (iv)
- 27.918 by 9 (v)
- (vi) 4.672 by 8

- (vii) 4.32 by 1.2
- (viii) 7.644 by 1.4
- (ix) 4.8432 by 0.08
- 5. Divide each of the given numbers by 10, 100, 1000 and 10000:
 - (i) 2·1

(ii) 8.64

(iii) 5·01

(iv) 0.0906

- (v) 0·125
- (vi) 111.11

- Evaluate:
 - $9.75 \div 5$
- 4.4064 ÷ 4 (ii)
- (iii) 27.69 ÷ 30
- 19.25 ÷ 25 (iv) $0.14616 \div 72$ (viii)

20.64 ÷ 16 (v)

 $0.6227 \div 1300$

- (vi) 3.204 ÷ 9 $257.894 \div 0.169$ (xi) $6.3 \div (0.3)^2$ (X)
- (vii) 0·125 ÷ 25
- (ix) 7. Find, whether the given division forms a terminating decimal or a non-terminating decimal:
 - (i) 3 ÷ 8
- (ii) 8 ÷ 3
- (iii) 6 ÷ 5
- (iv) 5 ÷ 6

- (v) 12·5 ÷ 4
- (vi) 23 ÷ 0.7
- (vii) 42 ÷ 9
- (viii) 0.56 ÷ 0.11

- 8. Express as recurring decimals:

- (iii)
- (iv)

(v)

- (vi) 90
- (vii)
- (viii)

5.13 ROUNDING-OFF OF DECIMAL NUMBERS

Sometimes, the numbers are found upto larger number of decimal places e.g., 3.481, 8.6672843, 5.36592, 9.8527, etc.

But, we need answers only upto a few decimal places. In such cases, the answers are rounded-off to the required number of decimal places.

5.14 ROUNDING-OFF:

- (i) If the answer required is correct to two decimal places, we retain digits upto three decimal places.
- (ii) If the digit in the third decimal place is five or more than five, then the digit in the second decimal place is increased by one and, if the digit in the third decimal place is less than five, then the digit in the second decimal place is not altered.
- (iii) The third digit which was retained, is now omitted.

Thus, for getting 8.4813 correct to two decimal places,

write the given number upto three decimal places, i.e., 8.481.

Since, the digit in the third decimal place is 1, which is less than 5.

.. The digit in the second decimal place is not altered.

Here, 8.4813 = 8.48, correct to two decimal places.

In the same way, to get 3.946824 correct to nearest thousandth, *i.e.*, correct to three decimal places, take the given number upto four decimal places, *i.e.*, take 3.9468.

Since, the digit at the fourth place is 8, which is greater than 5.

.. According to the rule, the digit in the third place changes from 6 to 7.

Hence, 3.946824 = 3.947, correct to three decimal places. Similarly,

(Ans.)

- (i) 4.738 = 4.74; correct to two decimal places.
- (ii) 4.738 = 4.7; correct to one decimal place.
- (iii) 4.738 = 5; correct to nearest unit.
- (iv) 536 = 540; correct to nearest ten.
- (v) 0.0083 rounded to the nearest hundredth = 0.01

5.15 SIGNIFICANT FIGURES (DIGITS)

Significant figures are the total number of digits present in a number except the zeroes preceding the first numeral.

In counting the number of significant digits, it should be noted that :

- (i) the position of the decimal is disregarded.
- (ii) all zeroes in between the numerals are counted.
- (iii) all zeroes after the last numeral are counted.
- (iv) the zeroes preceding the first numeral are not counted.

Make the table, given below, clear:

Numbers :	502	50.2	5.02	0.502	0.0502	0.05020	502.0	50-20
No. of significant digits:	3	3	3	3	3	4	4	4

Example 5:

Round-off:

- 0.0506 and 0.36089 correct to one significant figure.
- 0.6079 and 4.083 correct to two significant figures.
- 7.04870 correct to three significant figures.
- 14.08 correct to five significant figures. (iv)

Solution:

Proceed in a way similar to that used for decimal places.

0.0506 = 0.05, correct to one significant figure. (i) (Ans.) 0.36089 = 0.4, correct to one significant figure. and,

0.6079 = 0.61, correct to two significant figures. (ii) (Ans.) 4.083 = 4.1, correct to two significant figures. and,

(Ans.) 7.04870 = 7.05, correct to three significant figures. (iii)

(Ans.) 14.08 = 14.080, correct to five significant figures. (iv)

EXERCISE 5(D)

1. Round off:

- 0.07, 0.112, 3.59, 9.489 to the nearest tenths.
- (ii) 0.627, 100.479, 0.065 and 0.024 to the nearest hundredths.
- (iii) 4.83, 0.86, 451.943 and 9.08 to the nearest whole number.
- 2. Simplify, and write your answers correct to the nearest hundredths :
 - (i) 18.35×1.2
- (ii) 62.89×0.02
- 3. Write the number of significant figures (digits) in :
 - 35.06
- 0.35 (ii)
- (iii) 7.0068
- (iv) 19.0

- 0.0062 (v)
- (vi) 4.2×0.6
- (vii) 0.08×25
- $3.6 \div 0.12$ (viii)

4. Write:

- 35-869, 0-008426, 4-952 and 382-7 correct to three significant figures.
- 60-974, 2-8753, 0-001789 and 400-04 correct to four significant figures.
- 14-29462, 19-2, 46356-82 and 69 correct to five significant figures.

EXERCISE 5(E) -

- 1. The weight of an object is 3.06 kg. Find the total weight of 48 similar objects.
- Find the cost of 17.5 m cloth at the rate of ₹ 112.50 per metre.
- One kilogramme of oil costs ₹ 73.40. Find the cost of 9.75 kilogramme of the oil.
- Total weight of 8 identical objects is 51.2 kg. Find the weight of each object.
- 18.5 m of cloth costs ₹ 666. Find the cost of 3.8 m cloth.
- Find the value of:
 - (i) 0.5 of ₹ 7.60 + 1.62 of ₹ 30
- (ii) 2.3 of 7.3 kg + 0.9 of 0.48 kg
- 0.98 of 235 0.09 of 3.2(iii) 6.25 of 8.4 - 4.7 of 3.24 (iv)
- 7. Evaluate:
 - 5.6 1.5 of 3.4
 - (iii) 0.72 of 80 ÷ 0.2
 - (v) $6.45 \div (3.9 1.75)$
- (ii) 4.8 ÷ 0.04 of 5
- (iv) 0.72 ÷ 80 of 0.2
- (vi) 0.12 of $(0.104 0.02) + 0.36 \times 0.5$