CHAPTER 4

FRACTION

[INCLUDING PROBLEMS BASED ON FRACTIONS]

4.1 BASIC CONCEPT

If an apple is divided into five equal parts; each part is said to be one-fifth $\left(\frac{1}{5}\right)$ of the whole apple. And, if out of these five equal parts, 2 parts are eaten; we say two-fifth $\left(\frac{2}{5}\right)$ of the apple is eaten or three-fifth $\left(\frac{3}{5}\right)$ of the apple is left.

The numbers $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{3}{5}$ used in the statement, given above, are called **fractions**. *Each* of these fractions *indicates a part of the whole*.

In fraction $\frac{a}{b}$, ***a*** is called the **numerator** and ***b*** is called the **denominator** of the fraction.

$$\therefore \text{ FRACTION} = \frac{\text{Numerator}}{\text{Denominator}}$$

Every fraction can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$ *i.e.* denominator is not equal to zero.

4.2 CLASSIFICATION OF FRACTIONS

Types of fractions	Condition	Examples
1. ***Decimal fraction***	denominator is 10 or higher power of 10.	$\frac{1}{10}, \frac{3}{100}, \frac{15}{1000}, \frac{8}{10^5}, \ldots\ldots$
2. ***Vulgar fraction***	denominator is other than 10, 100, 1000, etc.	$\frac{2}{5}, \frac{4}{7}, \frac{8}{19}, \frac{23}{107}, \ldots\ldots$
3. ***Proper fraction***	denominator is greater than its numerator.	$\frac{4}{5}, \frac{3}{7}, \frac{101}{235}, \ldots\ldots$
4. ***Improper fraction***	denominator is less than its numerator.	$\frac{7}{5}, \frac{18}{13}, \frac{181}{60}, \ldots\ldots$
5. ***Mixed fraction***	consists of an integer and a proper fraction.	$2\frac{5}{7}, 1\frac{3}{5}, 10\frac{1}{9}, \ldots\ldots$

If the numerator is equal to the denominator, the fraction is equal to **unity** (one).

e.g. $\frac{4}{4} = 1$, $\frac{-3}{-3} = 1$, $\frac{49}{49} = 1$ and so on.

Important : (a) $\frac{\mathbf{7}}{\mathbf{20}} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100}$, **a decimal fraction.**

(b) $\frac{\mathbf{81}}{\mathbf{500}} = \frac{81 \times 2}{500 \times 2} = \frac{162}{1000}$, **a decimal fraction.**

$\therefore$ If the denominator of a fraction can be expressed as 10 or as some higher power of 10, it is a decimal fraction.

Example 1 :

(a) Convert : (i) $3\frac{2}{7}$ (ii) $2\frac{5}{8}$ into improper fractions.

(b) Convert : (i) $\frac{11}{4}$ (ii) $\frac{19}{5}$ into mixed fractions.

Solution :

(a) (i) $3\frac{2}{7} = \frac{3\times7+2}{7} = \frac{23}{7}$ **(Ans.)**

$$\text{Given mixed fraction} = \frac{\text{Integral part} \times \text{Denominator} + \text{Numerator}}{\text{Denominator}}$$

(ii) $2\frac{5}{8} = \frac{2\times8+5}{8} = \frac{16+5}{8} = \frac{21}{8}$ **(Ans.)**

(b) (i) $\frac{11}{4} = \frac{2\times4+3}{4}$ $\because$ $\begin{array}{r|l} 4\,) & 11\,(2 \\ & \underline{8} \\ & 3 \end{array}$

$= 2 + \frac{3}{4} = 2\frac{3}{4}$ **(Ans.)**

(ii) $\frac{19}{5} = \frac{3\times5+4}{5}$ $\because$ $\begin{array}{r|l} 5\,) & 19\,(3 \\ & \underline{15} \\ & 4 \end{array}$

$= 3 + \frac{4}{5} = 3\frac{4}{5}$ **(Ans.)**

1. The value of a fraction remains the same if both its numerator and denominator are (i) multiplied or (ii) divided by the same non-zero number.

   e.g. (i) $\frac{5}{8} = \frac{5\times2}{8\times2} = \frac{10}{16}$; $\frac{3}{7} = \frac{3\times5}{7\times5} = \frac{15}{35}$ and so on.

   (ii) $\frac{10}{16} = \frac{10\div2}{16\div2} = \frac{5}{8}$; $\frac{15}{35} = \frac{15\div5}{35\div5} = \frac{3}{7}$ and so on.

2. *A fraction must always be expressed in its lowest term.*

4.3 REDUCING A GIVEN FRACTION TO ITS LOWEST TERM

Steps : First of all find H.C.F. of both the terms (numerator and denominator) of the given fraction. Then divide each term by this H.C.F.

Example 2 :

Reduce : (i) $\frac{48}{60}$ (ii) $\frac{18}{27}$ to their lowest terms.

Solution :

(i) Since, H.C.F. of terms 48 and 60 = 12.

$\therefore \frac{48}{60} = \frac{48\div12}{60\div12}$ [Dividing each term by 12]

$= \frac{4}{5}$ **(Ans.)**

(ii) Since, H.C.F. of 18 and 27 is 9 $\therefore \frac{\mathbf{18}}{\mathbf{27}} = \frac{18 \div 9}{27 \div 9} = \frac{\mathbf{2}}{\mathbf{3}}$ **(Ans.)**

Alternative Method :

Resolve both the numerator and the denominator into prime factors, then cancel out the common factors among both.

Since, $48 = 2 \times 2 \times 2 \times 2 \times 3$ and $60 = 2 \times 2 \times 3 \times 5$

$$\therefore \quad \frac{\mathbf{48}}{\mathbf{60}} = \frac{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times 5} \quad \text{[Cancelling out the common factors]}$$

$$= \frac{2 \times 2}{5} = \frac{\mathbf{4}}{\mathbf{5}}$$ **(Ans.)**

4.4 EQUIVALENT (EQUAL) FRACTIONS

Fractions having the same value are called equivalent fractions.

e.g., Since, $\frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5}$ and $\frac{28}{35} = \frac{28 \div 7}{35 \div 7} = \frac{4}{5}$

$\therefore$ Fractions $\frac{20}{25}$ and $\frac{28}{35}$ are equivalent, *i.e.*, $\frac{\mathbf{20}}{\mathbf{25}} = \frac{\mathbf{28}}{\mathbf{35}} = \frac{4}{5}$.

4.5 SIMPLE AND COMPLEX FRACTIONS

A fraction, whose *numerator and denominator both are integers,* is called a *simple fraction;* whereas a fraction, whose *numerator or denominator or both are not integers,* is called a *complex fraction.*

e.g. (i) Each of $\frac{3}{8}, \frac{-10}{17}, \frac{8}{-15}$, etc., is a simple fraction.

(ii) Each of $\frac{5}{2/3}, \frac{1{\cdot}4}{8}, \frac{9/14}{2\frac{3}{7}}$, etc., is a complex fraction.

EXERCISE 4(A)

1. Classify, each fraction given below, as decimal or vulgar fraction, proper or improper fraction and mixed fraction :

(i) $\frac{3}{5}$ (ii) $\frac{11}{10}$ (iii) $\frac{13}{20}$ (iv) $\frac{18}{7}$ (v) $3\frac{2}{9}$

2. Express the following improper fractions as mixed fractions :

(i) $\frac{18}{5}$ (ii) $\frac{7}{4}$ (iii) $\frac{25}{6}$ (iv) $\frac{38}{5}$ (v) $\frac{22}{5}$

3. Express the following mixed fractions as improper fractions :

(i) $2\frac{4}{9}$ (ii) $7\frac{5}{13}$ (iii) $3\frac{1}{4}$ (iv) $2\frac{5}{48}$ (v) $12\frac{7}{11}$

4. Reduce the given fractions to lowest terms :

(i) $\frac{8}{18}$ (ii) $\frac{27}{36}$ (iii) $\frac{18}{42}$ (iv) $\frac{35}{75}$ (v) $\frac{18}{45}$

5. State *true* or *false* :

   (i) $\frac{30}{40}$ and $\frac{12}{16}$ are equivalent fractions.

   (ii) $\frac{10}{25}$ and $\frac{25}{10}$ are equivalent fractions.

   (iii) $\frac{35}{49}, \frac{20}{28}, \frac{45}{63}$ and $\frac{100}{140}$ are equivalent fractions.

6. Distinguish each of the following fractions, given below, as *a simple fraction* or a *complex fraction* :

   (i) $\frac{0}{8}$ (ii) $\frac{-3}{-8}$ (iii) $\frac{5}{-7}$ (iv) $\frac{3\frac{3}{5}}{18}$

   (v) $\frac{-6}{2\frac{2}{5}}$ (vi) $\frac{3\frac{1}{3}}{7\frac{2}{7}}$ (vii) $\frac{-5\frac{2}{9}}{5}$ (viii) $\frac{-8}{0}$

> ***Remember :*** Each of the numbers of the form $\frac{5}{0}, \frac{-7}{0}, \frac{8}{0}$, etc., is neither a simple fraction nor a complex fraction; as the division by '0' is not defined.

4.6 LIKE AND UNLIKE FRACTIONS

Fractions having the *same denominators* are called *like fractions;* whereas the fractions *with different denominators* are called *unlike fractions.*

e.g. (i) $\frac{3}{8}, \frac{5}{8}, \frac{9}{8}$, etc., are *like fractions.*

(ii) $\frac{2}{7}, \frac{5}{9}, \frac{15}{23}, \frac{24}{37}$, etc., are *unlike fractions.*

4.7 CONVERTING UNLIKE FRACTIONS INTO LIKE FRACTIONS

Steps :
1. Find the L.C.M of the denominators of all given fractions.
2. For each given fraction, multiply its denominator by a suitable number so that the product obtained is equal to the L.C.M. obtained in Step 1.
3. Multiply the numerator also by the same number.

Example 3 :

Change $\frac{3}{4}, \frac{3}{5}, \frac{7}{8}$ and $\frac{9}{16}$ to like fractions.

Solution :

Since, L.C.M. of the denominators 4, 5, 8 and 16 is 80.

$\therefore \quad \frac{3}{4} = \frac{3\times 20}{4\times 20} = \frac{60}{80}; \qquad \frac{3}{5} = \frac{3\times 16}{5\times 16} = \frac{48}{80}$

$$\frac{7}{8} = \frac{7\times10}{8\times10} = \frac{70}{80}; \qquad \frac{9}{16} = \frac{9\times5}{16\times5} = \frac{45}{80}$$

$\therefore$ **Required like fractions are :** $\frac{60}{80}, \frac{48}{80}, \frac{70}{80}$ and $\frac{45}{80}$ **(Ans.)**

4.8 COMPARING FRACTIONS

Steps : Convert all the given fractions *into like fractions,* then *the* ***fraction with the greater numerator is greater.***

Example 4 :

Compare the fractions : $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$.

Solution :

$\because$ L.C.M. of the denominators 3, 4, 12 and 16 = 48.

$$\left.\begin{array}{llll} \therefore & \frac{2}{3} = \frac{2\times16}{3\times16} = \frac{32}{48} & ; & \frac{3}{4} = \frac{3\times12}{4\times12} = \frac{36}{48} \\ & \frac{5}{12} = \frac{5\times4}{12\times4} = \frac{20}{48} & \text{and} & \frac{9}{16} = \frac{9\times3}{16\times3} = \frac{27}{48} \end{array}\right] \text{Converting into like fractions}$$

Since, the biggest numerator is 36, thus the biggest fraction is $\frac{36}{48}$ (*i.e.,* $\frac{3}{4}$).

Next one is $\frac{32}{48}$ (*i.e.,* $\frac{2}{3}$) and the smallest fraction is $\frac{20}{48}$ (*i.e.,* $\frac{5}{12}$)

$\therefore$ **Fractions in ascending order** of values are : $\frac{5}{12}, \frac{9}{16}, \frac{2}{3}$ and $\frac{3}{4}$. **(Ans.)**

i.e. $\frac{5}{12} < \frac{9}{16} < \frac{2}{3} < \frac{3}{4}$

And, **fractions in descending order** of values are : $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$. **(Ans.)**

i.e. $\frac{3}{4} > \frac{2}{3} > \frac{9}{16} > \frac{5}{12}$

Ascending means **smaller to greater** and **descending** means **greater to smaller**.

Alternate Method (By making numerators equal) **:**

Steps :
1. Convert all the given fractions into fractions of equal numerators.
2. The fraction which has a smaller denominator is greater.

Example 5 :

Compare : $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$ by making their numerators equal.

Solution :

Step 1 : Since, L.C.M. of numerators 2, 3, 5 and 9 is 90

$$\begin{array}{llll} \therefore & \frac{2}{3} = \frac{2\times45}{3\times45} = \frac{90}{135} & ; & \frac{3}{4} = \frac{3\times30}{4\times30} = \frac{90}{120} \\ & \frac{5}{12} = \frac{5\times18}{12\times18} = \frac{90}{216} & \text{and} & \frac{9}{16} = \frac{9\times10}{16\times10} = \frac{90}{160} \end{array}$$

Step 2 : Since, $\frac{90}{120}$ has the smallest denominator, the biggest fraction is $\frac{90}{120}$ (*i.e.,* $\frac{3}{4}$).

As, $\frac{90}{216}$ has the biggest denominator, the smallest fraction is $\frac{90}{216}$ (*i.e.,* $\frac{5}{12}$).

$\therefore$ **Fractions in ascending order** are : $\frac{5}{12}, \frac{9}{16}, \frac{2}{3}$ and $\frac{3}{4}$. **(Ans.)**

And, **in descending order** they are : $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$. **(Ans.)**

In order to compare two fractions, say : $\frac{a}{b}$ *and* $\frac{c}{d}$*, find their cross-product, i.e., find* $a \times d$ *and* $b \times c$*. Then, if :*

(i) $a \times d$ *is greater than* $b \times c \Rightarrow \frac{a}{b} > \frac{c}{d}$, *(ii)* $a \times d$ *is less than* $b \times c \Rightarrow \frac{a}{b} < \frac{c}{d}$,

(iii) $a \times d$ *is equal to* $b \times c \Rightarrow \frac{a}{b} = \frac{c}{d}$.

Example 6 :

Compare the fractions : $\frac{3}{13}$ and $\frac{7}{18}$.

Solution :

Taking the cross multiplication we get : $3 \times 18 = 54$ and $7 \times 13 = 91$

Since, 3×18 (*i.e.,* 54) is *smaller* than 7×13 (*i.e.,* 91) $\therefore \frac{3}{13} < \frac{7}{18}$ **(Ans.)**

4.9 TO INSERT A FRACTION BETWEEN TWO GIVEN FRACTIONS

Steps : Add numerators of the given fractions to get the numerator of required fraction. Similarly, add their denominators to get denominator of the required fraction. Then simplify, if required.

Example 7 :

Insert one fraction between :

(i) $\frac{1}{2}$ and $\frac{3}{5}$ (ii) 2 and $3\frac{1}{2}$

Solution :

(i) $\frac{1}{2}, \frac{3}{5} = \frac{1}{2}, \frac{1+3}{2+5}, \frac{3}{5}$ [Adding numerators and denominators]

$= \frac{1}{2}, \frac{4}{7}, \frac{3}{5}$ **(Ans.)**

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions then fraction $\frac{a+c}{b+d}$ lies between $\frac{a}{b}$ and $\frac{c}{d}$.

Also, 1. If $\frac{a}{b} > \frac{c}{d}$, then $\frac{a}{b} > \frac{a+c}{b+d} > \frac{c}{d}$. **2.** If $\frac{a}{b} < \frac{c}{d}$, then $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$.

(ii) $2, 3\frac{1}{2} = \frac{2}{1}, \frac{7}{2} = \frac{2}{1}, \frac{2+7}{1+2}, \frac{7}{2} = 2, \frac{9}{3}, \frac{7}{2} = 2, 3, 3\frac{1}{2}$ **(Ans.)**

Example 8 :

Insert three fractions between $\frac{1}{2}$ and $\frac{3}{5}$.

Solution :

$$\mathbf{\frac{1}{2}, \frac{3}{5}} = \frac{1}{2}, \mathbf{\frac{1+3}{2+5}}, \frac{3}{5} \qquad \text{[Inserting one fraction between } \frac{1}{2} \text{ and } \frac{3}{5}\text{]}$$

$$= \frac{1}{2}, \frac{4}{7}, \frac{3}{5}$$

$$= \frac{1}{2}, \mathbf{\frac{1+4}{2+7}}, \frac{4}{7}, \mathbf{\frac{4+3}{7+5}}, \frac{3}{5} \qquad \text{[Inserting one fraction between } \frac{1}{2} \text{ and } \frac{4}{7}\text{; and one between } \frac{4}{7} \text{ and } \frac{3}{5}\text{]}$$

$$= \frac{1}{2}, \frac{5}{9}, \frac{4}{7}, \frac{7}{12}, \frac{3}{5}$$

(Ans.)

EXERCISE 4(B)

1. For each pair, given below, state whether it forms *like fractions* or *unlike fractions* :

   (i) $\frac{5}{8}$ and $\frac{7}{8}$ (ii) $\frac{8}{15}$ and $\frac{8}{21}$ (iii) $\frac{4}{9}$ and $\frac{9}{4}$

2. Convert given fractions into fractions with *equal denominators* :

   (i) $\frac{5}{6}$ and $\frac{7}{9}$ (ii) $\frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12}$ (iii) $\frac{4}{5}, \frac{17}{20}, \frac{23}{40}$ and $\frac{11}{16}$

3. Convert given fractions into fractions with *equal numerators* :

   (i) $\frac{8}{9}$ and $\frac{12}{17}$ (ii) $\frac{6}{13}, \frac{15}{23}$ and $\frac{12}{17}$ (iii) $\frac{15}{19}, \frac{25}{28}, \frac{9}{11}$ and $\frac{45}{47}$

4. Put the given fractions in ascending order by making denominators equal :

   (i) $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}$ and $\frac{1}{6}$ (ii) $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}$ and $\frac{3}{10}$ (iii) $\frac{5}{7}, \frac{3}{8}, \frac{9}{14}$ and $\frac{20}{21}$

5. Arrange the given fractions in descending order by making numerators equal :

   (i) $\frac{5}{6}, \frac{4}{15}, \frac{8}{9}$ and $\frac{1}{3}$ (ii) $\frac{3}{7}, \frac{4}{9}, \frac{5}{7}$ and $\frac{8}{11}$ (iii) $\frac{1}{10}, \frac{6}{11}, \frac{8}{11}$ and $\frac{3}{5}$

6. Find the greater fraction :

   (i) $\frac{3}{5}$ and $\frac{11}{15}$ (ii) $\frac{4}{5}$ and $\frac{3}{10}$ (iii) $\frac{6}{7}$ and $\frac{5}{9}$

7. Insert one fraction between :

   (i) $\frac{3}{7}$ and $\frac{4}{9}$ (ii) 2 and $\frac{8}{3}$ (iii) $\frac{9}{17}$ and $\frac{6}{13}$

8. Insert three fractions between :

   (i) $\frac{2}{5}$ and $\frac{4}{9}$ (ii) $\frac{1}{2}$ and $\frac{5}{7}$ (iii) $\frac{3}{8}$ and $\frac{6}{11}$

9. Insert two fractions between :

   (i) 1 and $\frac{3}{11}$ (ii) $\frac{5}{9}$ and $\frac{1}{4}$ (iii) $\frac{5}{6}$ and $1\frac{1}{5}$

4.10 OPERATIONS ON FRACTIONS

1. *Addition and Subtraction :*

(i) *For like fractions, add or subtract* (as required) *their numerators, keeping the denominator same :*

$$\therefore \ \frac{1}{8} + \frac{5}{8} = \frac{1+5}{8} = \frac{6}{8} = \frac{3}{4} \quad \text{and} \quad \frac{9}{10} - \frac{3}{10} = \frac{9-3}{10} = \frac{6}{10} = \frac{3}{5}$$

(ii) *For unlike fractions,* first of all change given fractions into like fractions and then do the addition or subtraction as above :

$$\therefore \ \frac{5}{7} - \frac{1}{4} = \frac{5 \times 4}{7 \times 4} - \frac{1 \times 7}{4 \times 7} \qquad \text{[L.C.M. of 7 and 4 is 28]}$$

$$= \frac{20}{28} - \frac{7}{28} = \frac{20-7}{28} = \frac{13}{28}$$

or, simply : $\frac{5}{7} - \frac{1}{4} = \frac{5 \times 4 - 1 \times 7}{28} = \frac{20-7}{28} = \frac{13}{28}$

And, $\frac{3}{4} + \frac{2}{5} - \frac{1}{3} = \frac{3 \times 15}{4 \times 15} + \frac{2 \times 12}{5 \times 12} - \frac{1 \times 20}{3 \times 20}$ [L.C.M. of 4, 5 and 3 = 60]

$$= \frac{45}{60} + \frac{24}{60} - \frac{20}{60} = \frac{45+24-20}{60} = \frac{49}{60}$$

or, simply : $\frac{3}{4} + \frac{2}{5} - \frac{1}{3} = \frac{3 \times 15 + 2 \times 12 - 1 \times 20}{60}$

$$= \frac{45 + 24 - 20}{60} = \frac{49}{60}.$$

2. *Multiplication :*

(i) *To multiply a fraction with an integer, multiply its numerator with the integer.*

$$\therefore \ 5 \times \frac{3}{8} = \frac{5 \times 3}{8} = \frac{15}{8} = 1\frac{7}{8} \quad \text{and} \quad \frac{4}{15} \times -7 = \frac{4 \times -7}{15} = \frac{-28}{15} = -1\frac{13}{15}.$$

(ii) *To multiply two or more fractions, multiply their numerators together and their denominators separately together.*

$$\therefore \ \frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35} \quad \text{and} \quad \frac{3}{8} \times \frac{4}{5} \times \frac{2}{3} = \frac{3 \times 4 \times 2}{8 \times 5 \times 3} = \frac{1}{5}.$$

3. *Division :*

To divide one quantity (fraction or integer) *by some other quantity (fraction or integer),* multiply the first by the reciprocal of the second.

e.g. (i) $\frac{5}{8} \div 2 = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$ [Reciprocal of 2 is $\frac{1}{2}$]

(ii) $2 \div \frac{5}{8} = 2 \times \frac{8}{5} = \frac{16}{5} = 3\frac{1}{5}$ [Reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$]

(iii) $\frac{7}{10} \div \frac{3}{4} = \frac{7}{10} \times \frac{4}{3} = \frac{28}{30} = \frac{14}{15}$ and so on.

4.11 USING "OF"

The word "of" between any two fractions, is to be used as multiplication.

e.g. (i) $\frac{3}{16}$ **of 2** = $\frac{3 \times 2}{16}$ = $\frac{3}{8}$

(ii) $\frac{1}{3}$ **of 18 kg** = $\frac{1 \times 18}{3}$ kg = **6 kg**

(iii) $\frac{3}{4}$ **of ₹ 16** = ₹ $\frac{3 \times 16}{4}$ = **₹ 12** and so on.

4.12 USING "BODMAS" :

The word **'BODMAS'** is the abbreviation formed by taking the initial letters of six operations; '**B**racket', '**O**f', '**D**ivision', '**M**ultiplication', '**A**ddition' and '**S**ubtraction'.

According to the rule of BODMAS, working must be done in the order corresponding to the letters appearing in the word, *i.e.,* first of all the terms inside **B**racket must be simplified; then **O**f must be simplied and then **D**ivision, **M**ultiplication, **A**ddition and finally **S**ubtraction.

e.g. $\left(\frac{1}{3} + \frac{2}{9}\right)$ of $\frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$

$= \left(\frac{3+2}{9}\right)$ of $\frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$ **First step (B) :** Simplifying the **Bracket.**

$= \frac{5}{9}$ of $\frac{8}{15} \div \frac{4}{9} \times \frac{3}{4} - \frac{1}{2} + 1$ **Second step (O) :** Removal of **'Of'**

$= \frac{8}{27} \times \frac{9}{4} \times \frac{3}{4} - \frac{1}{2} + 1$ **Third step (D) : Division,** *i.e.,* multiply by reciprocal.

$= \frac{8 \times 9 \times 3}{27 \times 4 \times 4} - \frac{1}{2} + 1$ **Fourth step (M) : Multiplication.**

$= \frac{1}{2} - \frac{1}{2} + 1$ **Fifth step : A and S**

$= 1$

(Ans.)

Example 9 :

Evaluate :

(i) $2\frac{1}{4} \div \frac{5}{7} \times 1\frac{1}{3}$ (ii) $\frac{1}{4}$ of $2\frac{2}{7} \div \frac{4}{15}$

Solution :

If required, convert the mixed fraction / fractions into improper fraction / fractions, then apply BODMAS and simplify.

(i) $2\frac{1}{4} \div \frac{5}{7} \times 1\frac{1}{3} = \frac{9}{4} \div \frac{5}{7} \times \frac{4}{3}$

$= \frac{9}{4} \times \frac{7}{5} \times \frac{4}{3} = \frac{9 \times 7 \times 4}{4 \times 5 \times 3} = \frac{21}{5} = 4\frac{1}{5}$ **(Ans.)**

(ii) $\frac{1}{4}$ of $2\frac{2}{7} \div \frac{4}{15} = \frac{1}{4}$ of $\frac{16}{7} \div \frac{4}{15}$

$= \frac{4}{7} \div \frac{4}{15}$ $\quad\left[\because \frac{1}{4} \text{ of } \frac{16}{7} = \frac{1}{4} \times \frac{16}{7} = \frac{4}{7}\right]$

$= \frac{4}{7} \times \frac{15}{4} = \frac{15}{7} = 2\frac{1}{7}$ **(Ans.)**

Example 10 :

Evaluate :

(i) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9}$ (ii) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9}$ (iii) $\frac{5}{6}$ of $\frac{5}{13} \div \frac{15}{16} \times 1\frac{1}{2}$

Solution :

Remember : BODMAS

(i) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9} = \frac{4}{5} \div \frac{56}{135}$ $\quad\left[\frac{7}{15} \text{ of } \frac{8}{9} = \frac{56}{135}\right]$

$= \frac{4}{5} \times \frac{135}{56} = \frac{27}{14} = 1\frac{13}{14}$ **(Ans.)**

(ii) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9} = \frac{4}{5} \times \frac{15}{7} \times \frac{8}{9}$ [Division (÷) first]

$= \frac{4 \times 15 \times 8}{5 \times 7 \times 9} = \frac{32}{21} = 1\frac{11}{21}$ **(Ans.)**

(iii) $\frac{5}{6}$ of $\frac{5}{13} \div \frac{15}{16} \times 1\frac{1}{2} = \frac{25}{78} \div \frac{15}{16} \times \frac{3}{2}$

$= \frac{25}{78} \times \frac{16}{15} \times \frac{3}{2} = \frac{25 \times 16 \times 3}{78 \times 15 \times 2} = \frac{20}{39}$ **(Ans.)**

EXERCISE 4(C)

1. Reduce to a single fraction :

(i) $\frac{1}{2} + \frac{2}{3}$ (ii) $\frac{3}{5} - \frac{1}{10}$ (iii) $\frac{2}{3} - \frac{1}{6}$

(iv) $1\frac{1}{3} + 2\frac{1}{4}$ (v) $\frac{1}{4} + \frac{5}{6} - \frac{1}{12}$ (vi) $\frac{2}{3} - \frac{3}{5} + 3 - \frac{1}{5}$

(vii) $\frac{2}{3} - \frac{1}{5} + \frac{1}{10}$ (viii) $2\frac{1}{2} + 2\frac{1}{3} - 1\frac{1}{4}$ (ix) $2\frac{5}{8} - 2\frac{1}{6} + 4\frac{3}{4}$

2. Simplify :

(i) $\frac{3}{4} \times 6$ (ii) $\frac{2}{3} \times 15$ (iii) $\frac{3}{4} \times \frac{1}{2}$

(iv) $\frac{9}{12} \times \frac{4}{7}$ (v) $45 \times 2\frac{1}{3}$ (vi) $36 \times 3\frac{1}{4}$

(vii) $2 \div \frac{1}{3}$ (viii) $3 \div \frac{2}{5}$ (ix) $1 \div \frac{3}{5}$

(x) $\frac{1}{3} \div \frac{1}{4}$ (xi) $-\frac{5}{8} \div \frac{3}{4}$ (xii) $3\frac{3}{7} \div 1\frac{1}{14}$

(xiii) $3\frac{3}{4} \times 1\frac{1}{5} \times \frac{20}{21}$

3. Subtract :

(i) 2 from $\frac{2}{3}$ (ii) $\frac{1}{8}$ from $\frac{5}{8}$ (iii) $-\frac{2}{5}$ from $\frac{2}{5}$

(iv) $-\frac{3}{7}$ from $\frac{3}{7}$ (v) 0 from $-\frac{4}{5}$ (vi) $\frac{2}{9}$ from $\frac{4}{5}$

(vii) $-\frac{4}{7}$ from $-\frac{6}{11}$

4. Find the value of :

(i) $\frac{1}{2}$ of 10 kg (ii) $\frac{3}{5}$ of 1 hour (iii) $\frac{4}{7}$ of $2\frac{1}{3}$ kg

(iv) $3\frac{1}{2}$ times of 2 metre (v) $\frac{1}{2}$ of $2\frac{2}{3}$ (vi) $\frac{5}{11}$ of $\frac{4}{5}$ of 22 kg

5. Simplify and reduce to a simple fraction :

(i) $\dfrac{3}{3\frac{3}{4}}$ (ii) $\dfrac{\frac{3}{5}}{7}$ (iii) $\dfrac{3}{\frac{5}{7}}$ (iv) $\dfrac{2\frac{1}{5}}{1\frac{1}{10}}$

(v) $\frac{2}{5}$ of $\frac{6}{11} \times 1\frac{1}{4}$ (vi) $2\frac{1}{4} \div \frac{1}{7} \times \frac{1}{3}$ (vii) $\frac{1}{3} \times 4\frac{2}{3} \div 3\frac{1}{2} \times \frac{1}{2}$

(viii) $\frac{2}{3} \times 1\frac{1}{4} \div \frac{3}{7}$ of $2\frac{5}{8}$ (ix) $0 \div \frac{8}{11}$ (x) $\frac{4}{5} \div \frac{7}{15}$ of $\frac{8}{9}$

(xi) $\frac{4}{5} \div \frac{7}{15} \times \frac{8}{9}$ (xii) $\frac{4}{5}$ of $\frac{7}{15} \div \frac{8}{9}$ (xiii) $\frac{1}{2}$ of $\frac{3}{4} \times \frac{1}{2} \div \frac{2}{3}$

4.13 USING BRACKETS

The types of brackets used, in general, are :

(i) (   ) is known as *Circular bracket* or *Parenthesis* or *simply bracket.*

(ii) {   } is known as *Curly bracket.*

(iii) [   ] is known as *Square bracket* or *Box bracket.*

Sometimes a **bar** is drawn above some terms which we want to treat as a single quantity.

e.g., (i) $\overline{4 + 5}$ means (4 + 5) = 9 (ii) $8 - \overline{3 + 2} = 8 - 5 = 3$

(iii) $3 + \overline{8 - 6} = 3 + 2 = 5$ and so on.

This "———" is known as **Bar bracket** or **Veniculum.**

Note : Multiplication sign is often omitted before a bracket and between the brackets.

e.g., (i) 4(9 – 3) = 4 × (9 – 3) = 4 × 6 = 24

(ii) (2 + 8) (7 – 3) = (2 + 8) × (7 – 3) = 10 × 4 = 40

4.14 REMOVAL OF BRACKETS

The brackets are removed in the order given below :

(ii) $\overline{\quad\quad}$; bar or vinculum, (ii) ( ) ; parenthesis,

(iii) { } ; curly bracket, (iv) [ ] ; square bracket.

Example 11 :

Simplify : $10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - \overline{6 - 4})\}\right]$

Solution :

$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - (7 - 2)\}\right]$ $[\because \overline{6 - 4} = 2]$

$= 10\frac{1}{2} - \left[8\frac{1}{2} + \{6 - 5\}\right]$ $[\because (7 - 2) = 5]$

$= 10\frac{1}{2} - \left[8\frac{1}{2} + 1\right]$ $[\because \{6 - 5\} = 1]$

$= 10\frac{1}{2} - 9\frac{1}{2}$ $\left[\because 8\frac{1}{2} + 1 = 9\frac{1}{2}\right]$

$= 1$ **(Ans.)**

1. *Whenever there is a positive (+) sign before a bracket, the bracket is removed without any change in the signs of its terms.*
   e.g., $8 + (3 - 1 + 5) = 8 + 3 - 1 + 5 = 16 - 1 - 15$
2. *Whenever there is a negative (–) sign before a bracket, the bracket is removed by changing the signs of all the terms inside the bracket (i.e., by changing every positive sign into negative and every negative sign into positive)*
   e.g., $8 - (3 - 1 + 5) = 8 - 3 + 1 - 5 = 9 - 8 = 1$

EXERCISE 4(D)

Simplify :

1. $6 + \left\{\frac{4}{3} + \left(\frac{3}{4} - \frac{1}{3}\right)\right\}$
2. $8 - \left\{\frac{3}{2} + \left(\frac{3}{5} - \frac{1}{2}\right)\right\}$
3. $\frac{1}{4}\left(\frac{1}{4} + \frac{1}{3}\right) - \frac{2}{5}$
4. $2\frac{3}{4} - \left[3\frac{1}{8} \div \left\{5 - \left(4\frac{2}{3} - \frac{11}{12}\right)\right\}\right]$
5. $12\frac{1}{2} - \left[8\frac{1}{2} + \left\{9 - \left(5 - \overline{3 - 2}\right)\right\}\right]$
6. $1\frac{1}{5} \div \left\{2\frac{1}{3} - \left(5 + \overline{2 - 3}\right)\right\} - 3\frac{1}{2}$
7. $\left(\frac{1}{2} + \frac{2}{3}\right) \div \left(\frac{3}{4} - \frac{2}{9}\right)$
8. $\frac{6}{5}$ of $\left(3\frac{1}{3} - 2\frac{1}{2}\right) \div \left(2\frac{5}{21} - 2\right)$
9. $10\frac{1}{8}$ of $\frac{4}{5} \div \frac{35}{36}$ of $\frac{20}{49}$
10. $5\frac{3}{4} - \frac{3}{7} \times 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{11}{25}$
11. $\frac{3}{4}$ of $7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}$

4.15 PROBLEMS INVOLVING FRACTIONS

Example 12 :

What fraction is 6 bananas of four dozen bananas ?

Solution :

Here 6 bananas are to be compared with 4 dozens *i.e.,* 4 × 12 = 48 bananas.

∴ **Required fraction** = $\frac{6}{48} = \frac{1}{8}$ **(Ans.)**

Example 13 :

Write all the natural numbers that lie between 5 and 15.

(i) How many of these natural numbers are odd ?

(ii) What fraction of these natural numbers are even ?

Solution :

Since, natural numbers between 5 and 15 are : 6, 7, 8, 9, 10, 11, 12, 13 and 14.

∴ **There are 9 natural numbers between 5 and 15.** **(Ans.)**

(i) Out of these natural numbers, odd natural numbers are : 7, 9, 11 and 13.

∴ **There are 4 odd natural numbers between 5 and 15.** **(Ans.)**

(ii) Out of all the given 9 natural numbers, 4 are odd.

∴ Remaining 9 – 4 = 5 numbers are even.

So, the required fraction = $\frac{5}{9}$ **(Ans.)**

Example 14 :

The monthly income of a man is ₹ 18,000. He gives one-third of it to his wife and one-third of the remaining he spends on his children's education. Find :

(i) the money he gave to his wife.

(ii) the money he spends on his children's education.

(iii) the money still left with him.

Solution :

(i) **The man gives to his wife** = $\frac{1}{3}$ of ₹ 18,000

= $\frac{1}{3}$ × ₹ 18,000 = **₹ 6,000** **(Ans.)**

(ii) Since, remaining money = ₹ 18,000 – ₹ 6,000 = ₹ 12,000

He spends on his children's education = $\frac{1}{3}$ × ₹ 12,000 = $\frac{1}{3}$ × ₹ 12,000 = **₹ 4,000** **(Ans.)**

(iii) **The money still left with the man**

= ₹ 12,000 – ₹ 4,000 = **₹ 8,000** **(Ans.)**

Example 15 :

Subtract the sum of $\frac{1}{4}$ and $\frac{3}{8}$ from the sum of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{7}{12}$.

Solution :

$\because$ Sum of $\frac{1}{4}$ and $\frac{3}{8}$ $= \frac{1}{4} + \frac{3}{8} = \frac{2+3}{8} = \frac{5}{8}$

And, sum of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{7}{12}$ $= \frac{2}{3} + \frac{3}{4} + \frac{7}{12} = \frac{8+9+7}{12} = \frac{24}{12} = 2$

$\therefore$ **Required number** $= 2 - \frac{5}{8} = \frac{2}{1} - \frac{5}{8} = \frac{16-5}{8} = \frac{11}{8} = 1\frac{3}{8}$ **(Ans.)**

or, directly, $\left(\frac{2}{3} + \frac{3}{4} + \frac{7}{12}\right) - \left(\frac{1}{4} + \frac{3}{8}\right) = \left(\frac{8+9+7}{12}\right) - \left(\frac{2+3}{8}\right)$

$= \frac{24}{12} - \frac{5}{8}$

$= \frac{48-15}{24} = \frac{33}{24} = \frac{11}{8} = 1\frac{3}{8}$ **(Ans.)**

Example 16 :

A man spent $\frac{2}{7}$ of his savings and still has ₹ 1,000 left with him. How much were his savings ?

Solution :

The man spent $\frac{2}{7}$ of his money.

$\therefore$ He still has $1 - \frac{2}{7} = \frac{5}{7}$ of his savings

Note : In fractions, the whole quantity is always taken as 1.

Since, $\frac{5}{7}$ of his savings = ₹ 1,000

$\therefore$ His savings = ₹ 1,000 $\div \frac{5}{7}$ = ₹ 1,000 $\times \frac{7}{5}$ = **₹ 1,400** **(Ans.)**

Example 17 :

$\frac{4}{7}$ of a pole is in the mud. When $\frac{1}{3}$ of it is pulled out, 250 cm of the pole is still in the mud. What is the full length of the pole ?

Solution :

$\frac{4}{7}$ of the pole $- \frac{1}{3}$ of the pole = 250 cm

$\Rightarrow$ $\left(\frac{4}{7} - \frac{1}{3}\right)$ of the pole = 250 cm

$\Rightarrow$ $\frac{5}{21}$ of the pole = 250 cm $\left[\frac{4}{7} - \frac{1}{3} = \frac{12-7}{21} = \frac{5}{21}\right]$

$\Rightarrow$ **Length of the pole** $= 250 \times \frac{21}{5}$ cm = **1050 cm** **(Ans.)**

EXERCISE 4(E)

1. A line AB is of length 6 cm. Another line CD is of length 15 cm. What fraction is :
   (i) the length of AB to that of CD ?
   (ii) $\frac{1}{2}$ the length of AB to that of $\frac{1}{3}$ of CD ?
   (iii) $\frac{1}{5}$ of CD to that of AB ?
2. Subtract $\left(\frac{2}{7} - \frac{5}{21}\right)$ from the sum of $\frac{3}{4}$, $\frac{5}{7}$ and $\frac{7}{12}$.
3. From a sack of potatoes weighing 120 kg, a merchant sells portions weighing 6 kg, $5\frac{1}{4}$ kg, $9\frac{1}{2}$ kg and $9\frac{3}{4}$ kg respectively.
   (i) How many kg did he sell ? (ii) How many kg are still left in the sack ?
4. If a boy works for six consecutive days for 8 hours, $7\frac{1}{2}$ hours, $8\frac{1}{4}$ hours, $6\frac{1}{4}$ hours, $6\frac{3}{4}$ hours and 7 hours respectively. How much money will he earn at the rate of ₹ 36 per hour ?
5. A student bought $4\frac{1}{3}$ m of yellow ribbon, $6\frac{1}{6}$ m of red ribbon and $3\frac{2}{9}$ m of blue ribbon for decorating a room. How many metres of ribbon did he buy ?
6. In a business, Ram and Deepak invest $\frac{3}{5}$ and $\frac{2}{5}$ of the total investment. If ₹ 40,000 is the total investment, calculate the amount invested by each.
7. Geeta had 30 problems for home work. She worked out $\frac{2}{3}$ of them. How many problems were still left to be worked out by her ?
8. A picture was marked at ₹ 90. It was sold at $\frac{3}{4}$ of its marked price. What was the sale price ?
9. Mani had sent fifteen parcels of oranges. What was the total weight of the parcels, if each weighed $10\frac{1}{2}$ kg ?
10. A rope is $25\frac{1}{2}$ m long. How many pieces each of $1\frac{1}{2}$ m length can be cut out from it ?
11. The heights of two vertical poles, above the earth's surface, are $14\frac{1}{4}$ m and $22\frac{1}{3}$ m respectively. How much higher is the second pole as compared with the height of the first pole ?
12. Vijay weighed $65\frac{1}{2}$ kg. He gained $1\frac{2}{5}$ kg during the first week, $1\frac{1}{4}$ kg during the second week, but lost $\frac{5}{16}$ kg during the third week. What was his weight after the third week ?

13. A man spends $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ on house rent, electricity, etc. What fraction of his salary is still left with him ?

14. A man spends $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ of the remaining on house rent, electricity, etc. What fraction of his salary is still left with him ?

15. Shyam bought a refrigerator for ₹ 5,000. He paid $\frac{1}{10}$ of the price in cash and the rest in 12 equal monthly instalments. How much had he to pay each month ?

16. A lamp post has half of its length in mud and $\frac{1}{3}$ of its length in water.
    (i) What fraction of its length is above the water ?
    (ii) If $3\frac{1}{3}$ m of the lamp post is above the water, find the whole length of the lamp post.

17. I spent $\frac{3}{5}$ of my savings and still have ₹ 2,000 left. What were my savings ?

18. In a school; $\frac{4}{5}$ of the children are boys. If the number of girls is 200, find the number of boys.

19. If $\frac{4}{5}$ of an estate is worth ₹ 42,000, find the worth of whole estate. Also, find the value of $\frac{3}{7}$ of it.

20. After going $\frac{3}{4}$ of my journey, I find that I have covered 16 km. How much journey is still left ?

21. When Krishna travelled 25 km, he found that $\frac{3}{5}$ of his journey was still left. What was the length of the whole journey.