UNIT – I
PURE ARITHMETIC

CHAPTER 1
NUMBER SYSTEM

1.1 REVIEW

1. Natural numbers (N)	(i) Each of 1, 2, 3, 4, etc., is a *natural number*. (ii) **The smallest** (*i.e.*, the first) natural number is **one** (1); whereas the **largest** natural number **can not be obtained.** (iii) Consecutive natural numbers **differ by one** (1). (iv) Let x be any natural number, then the natural numbers that come just after x are $x + 1$, $x + 2$, $x + 3$, etc.
2. Even natural numbers (E)	(i) Each of 2, 4, 6, etc., is an **even natural number**. (ii) Every even natural number is always divisible by 2. (iii) The **smallest** (*i.e.*, the first) even natural number is **two** (2); whereas the **largest** of them **can not be obtained.** (iv) Consecutive even natural numbers **differ by two** (2). (v) Let x be any even natural number; then the even natural numbers just after x are $x + 2$, $x + 4$, $x + 6$, etc. (vi) An even number can be represented by $\mathbf{2n}$; where $n \in$ N. **Reason :** When n = 1, 2, 3, 4, ... $2n = 2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4,$ .... = 2, 4, 6, 8, ..., *i.e.*, $2n \in$ E
3. Odd natural numbers (O)	(i) Each of 1, 3, 5, 7, etc., is an **odd natural number**. (ii) No odd natural number is divisible by **two** (2). (iii) **The smallest** (*i.e.*, the first) odd natural number is **one** (1); whereas the **largest** of them **can not be obtained.** (iv) Consecutive odd natural numbers **differ by two** (2). (v) Let x be any odd natural number; then the odd natural numbers just after x are $x + 2$, $x + 4$, $x + 6$, etc. (vi) An odd number can be represented by $\mathbf{2n - 1}$; where $n \in$ N. **Reason :** When n = 1, 2, 3, ... $2n - 1 = 2 \times 1 - 1, 2 \times 2 - 1, 2 \times 3 - 1,$ .... = 2 – 1, 4 – 1, 6 – 1, ... = 1, 3, 5, ..., *i.e.*, **odd natural numbers.**
4. Whole numbers (W)	(i) 0, 1, 2, 3, 4, etc., are whole numbers. (ii) **The smallest** whole number is **zero** (0); whereas the **largest** whole number **can not be obtained.** (iii) Consecutive whole numbers **differ by one** (1). (iv) If x be any whole number; then whole numbers just after x are $x + 1$, $x + 2$, $x + 3$, etc. (v) Except zero (0), every whole number is a natural number and because of this : a) *Every even natural number is an even whole number.* b) *Every odd natural number is an odd whole number.*

1.2 MORE ABOUT NUMBERS

1. Prime Numbers (P) :

1. **Prime numbers** are whole numbers greater than 1 and each of which is divisible by unity (1) and by itself only.
2. Except 2, *all other **prime numbers** are odd.*

∴ Prime numbers, **P** = 2, 3, 5, 7, 11, 13, ..., etc.

2. Composite Numbers (C) :

Composite numbers are whole numbers greater than 1 and none of these numbers is a prime number.

e.g., 6 is a whole number greater than 1 and 6 is not a prime number, therefore ***6 is a composite number***.

∴ Composite numbers, **C** = 4, 6, 8, 9, ..., etc.

> The whole number 1 (unity) is neither a prime number nor a composite number.

Example 1 :

State, with reason, which of the numbers 13 and 18 is a prime number and which is a composite number.

Solution :

13 is a prime number **(Ans.)**

Reason : 13 is a whole number greater than 1(one) and is divisible by 1 and by itself only.

18 is a composite number **(Ans.)**

Reason : 18 is a whole number greater than 1(one) and is divisible by each of 1, 2, 3, 6, 9 and 18.

> Every composite number is divisible by more than two natural numbers.

3. Integers (Z or I) :

1. *The integers consist of natural numbers, zero and negative of natural numbers.*

   Thus, Z or I = ..., –4, –3, –2, –1, 0 1, 2, 3, 4, 5, ...

   ← Negative of natural numbers | Zero | Natural numbers →

2. There are infinite integers towards positive side and infinite integers towards negative side.
3. Positive Integers, (Z^+) = 1, 2, 3, 4, ..., etc. = N

   Negative integers, (Z^-) = –1, –2, –3, ..., etc.

4. Rational Numbers (Q) :

The numbers, each of which can be expressed in the form $\frac{a}{b}$, where *a* and *b* both are integers and *b* is not equal to zero, are called **rational numbers**.

i.e. $\frac{a}{b}$ is a rational number when; $a, b \in Z$ and $b \neq 0$.

e.g. (i) $\frac{2}{5}$ is a rational number, since 2, 5, ∈ Z and 5 ≠ 0.

(ii) 5 is a rational number, since, $5 = \frac{5}{1}$, where 5, 1 ∈ Z and 1 ≠ 0.

(iii) 0 is a rational number, as 0 can be expressed as $\frac{0}{5}$, $\frac{0}{-7}$, etc.

(iv) $\frac{5}{0}$ is *not a rational number*, since 5, 0 $\in$ Z but denominator = 0.

(v) $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc., are *not rational numbers*, since these numbers can not be expressed as $\frac{a}{b}$, where a, $b \in$ Z and $b \neq 0$.

(vi) $-\frac{3}{4}$ is *a rational number*, since –3, 4 $\in$ Z and 4 $\neq$ 0.

Remember : $-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4}$

(vii) 0·3 is *a rational number*, since $0{\cdot}3 = \frac{3}{10}$; where 3, 10 $\in$ Z and 10 $\neq$ 0.

1. *Each natural number, each whole number, each integer and each fraction (including decimal fraction) is a rational number.*
2. *There are infinite number of rational numbers.*

5. Irrational Numbers ($\overline{Q}$)

The numbers, which are not rational, are called **irrational numbers**.

Each of $-\sqrt{7}$, $-\sqrt{2}$, $3\sqrt{4}$, $\sqrt{5}$, etc., is an **irrational number**.

Note : The number $\frac{a}{b}$ is neither rational nor irrational, if $b = 0$; e.g., $\frac{5}{0}$, $\frac{7}{0}$, $\frac{-3}{0}$, etc.

Infact, if $b = 0$, $\frac{a}{b}$ is said to be **not defined** or **infinity**.

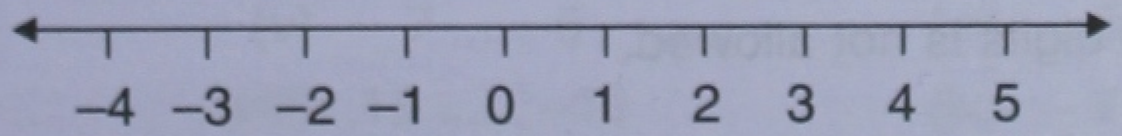
6. Real Numbers (R) :

Numbers which are either *rational or irrational* are called ***real numbers***.

(i) Each natural number is a real number,
(ii) Each whole number is a real number,
(iii) Each integer is a real number,
(iv) Each rational number is a real number,
(v) Each irrational number is also a real number.

1.3 NUMBER LINE

A number line is shown below :



1. Arrow-heads are drawn at both the ends of the line to show that the line continues to be of infinite length and so the numbers are also upto infinity on both the sides.
2. To the left of zero, on this line, are negative and to its right are positive numbers.
3. For any two numbers on the number line, the one which is on the right of the other, is greater. And, the number, which is on the left, is smaller.

   e.g. (i) –1 *is to the right* of –2 $\quad\therefore$ –1 *is greater than* –2

   (ii) –1 *is to the left* of 3 $\quad\therefore$ –1 *is smaller than* 3 and so on.

1. *If a* **is greater than** *b; then –a* **is smaller than** *–b, i.e., if a > b; then –a < –b.*
   And*, if a* **is smaller than** *b; then –a* **is greater than** *–b. i.e., if a < b; then –a > –b.*
   e.g. (i) 8 > 5; *but* –8 < –5. (iii) –3 > –6; *but* 3 < 6.
   (ii) 2 < 6; *but* –2 > –6. (iv) –15 < –8; *but* 15 > 8.
2. (i) ***Every negative number is smaller than zero*** (0).
   (ii) ***Every positive number is greater than zero*** (0).
   (iii) ***Every positive number is greater than every negative number.***

1.4 THE FACE AND THE PLACE VALUES OF A DIGIT

Each and every number (whether it is very small or very-very large) is made up of one or more of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Each of these ten symbols is called a **digit**.

For example :

(a) Each of 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 is a single digit number.
(b) Each of 10, 22, 53, 78, 87, 90, etc., is a two digit number.
(c) Each of 3297, 5228, 4444, etc., is a four digit number and so on.

1. The face value of a digit :

In a given number, the **face** (true) **value** of a digit **is** the **digit itself**.

For example :

In the number 837;
the face value of 8 is 8, the face value of 3 is 3
and the face value of 7 is 7.

2. The place value of a digit :

In a given number, the place (local) value of a non-zero digit depends on the place it occupies in the given number.

For example :

In the number 837;
the digit 8 occupies hundred's place, so its place value is 8 × 100 = 800;
the digit 3 occupies ten's place, so its place value is 3 × 10 = 30
and the digit 7 occupies unit's place, so its place value is 7 × 1 = 7.

Example 2 :

Write all possible 2-digit numbers formed out of the digits 4, 2 and 9, if :
(i) repetition of digits is allowed.
(ii) repetition of digits is not allowed.

Solution :

(i) **Required 2-digit numbers = 44, 42, 49, 24, 22, 29, 94, 92 and 99.** **(Ans.)**
(ii) **Required numbers = 42, 49, 24, 29, 94 and 92** **(Ans.)**

Example 3 :

Write all possible 3-digit numbers formed out of the digits 4, 2 and 9, if :
Repetition of digits is not allowed.

Solution :

Required numbers = 429, 492, 249, 294, 924 and 942. **(Ans.)**

EXERCISE 1(A)

1. From the numbers, given below,

   $-7,\ -6.5,\ 0,\ \sqrt{2},\ \frac{5}{3},\ \sqrt{8},\ \frac{3}{-4},\ 3.06,\ 0.05,\ \sqrt{4},\ 5 \text{ and } 8$

   find : (i) natural numbers (ii) integers
   (iii) negative integers (iv) rational numbers
   (v) irrational numbers (vi) real numbers

2. State **true** or **false** :
   (i) All positive integers are whole numbers.
   (ii) All whole numbers are integers.
   (iii) All real numbers are rational numbers.
   (iv) All rational numbers are real numbers.
   (v) All irrational numbers are real numbers.
   (vi) All integers are rational numbers.
   (vii) Some real numbers are rational numbers.

3. State to which system of numbers (integers, real numbers, rational numbers and irrational numbers) the following numbers belong to (some numbers may belong to more than one system) :
   (i) 7·8 (ii) 0·08 (iii) –3·95
   (iv) – 84 (v) $\sqrt{7}$ (vi) $\frac{-2}{7}$
   (vii) 0 (viii) $\frac{3}{1}$ (ix) $\frac{8}{-4}$
   (x) $\frac{2}{2}$ (xi) 3×5 (xii) $\frac{0}{8}$
   (xiii) $\frac{\sqrt{3}}{1}$ (xiv) $5 \times \frac{\sqrt{2}}{3}$ (xv) $\frac{-80}{16}$

4. From the numbers, given below, write which numbers are :
   (i) prime numbers (ii) composite numbers.
   15, 19, 27, 16, 7, 1, 53, 68, 31, 89, 2, 22, 73, 45 and 96.

5. Using a number line, write the sign '>' or '<' in order to make each of the following true :
   (i) – 2 ..... – 8 (ii) 3 ...... 0 (iii) 0 .... – 5
   (iv) 3 ...... – 3 (v) – 7 ...... 5 (vi) – 8 0
   (vii) 0 ...... – 4 (viii) 12 ...... –10 (ix) – 7 2

6. Write all possible 2-digit numbers using the digits 2, 4 and 6, if repetition of digits is :
   (i) not allowed. (ii) allowed.

7. Write all possible 2-digit numbers using the digits 5, 0 and 3, if repetition of digits is :
   (i) allowed. (ii) not allowed.

8. Write all possible 3-digit numbers using the digits 8, 0 and 5, if repetition of digits is not allowed.

1.5 FOUR FUNDAMENTAL OPERATIONS

Addition, Subtraction, Multiplication and ***Division*** are called the ***four fundamental operations.***

1. Addition :

It is the process of finding a single number whose value is equal to the sum of two or more given numbers taken together.

The sum (addition) of 5 and 3 = 5 + 3; which is read as : **five plus three.**

The sign of addition is + (read as **plus**).

Adopt the following rules for adding two numbers :

(a) *When the two numbers have like signs, i.e.,* both are positive or both are negative, **add both the numbers without considering their signs and to their sum assign the same sign.**

For example :

(i) 3 + 8 = 11 (ii) (–3) + (–8) = –11 (iii) (–15) + (–6) = –21 and so on.

When no sign is written before any number; the sign is positve, *i.e.*, "+".
∴ 3 = + 3, 8 = + 8, 11 = + 11 and so on.

(b) *When both the numbers have unlike signs, i.e.*, one is positive and the other is negative, *find the difference between the numbers without considering their signs and to the result attach the sign of greater number.*

Example 4 :

Add : (i) + 47 and – 32 (ii) – 47 and + 32
(iii) 68 and – 25 (iv) – 68 and 25

Solution :

(i) Without considering the signs of given numbers + 47 and – 32, we get : 47 and 32.
The difference between 47 and 32 = 47 – 32 = 15
On assigning the proper sign, we get : **(+47) + (–32) = +15,** *i.e.*, **15** **(Ans.)**

(ii) The difference between the two numbers without considering their signs
= 47 – 32 = 15
On assigning the proper sign, we get : **(–47) + (32) = –15** **(Ans.)**

(iii) **68 + (–25) = 43** **(Ans.)**

(iv) **(–68) + 25 = – 43** **(Ans.)**

2. Subtraction :

It is the process of finding difference when a smaller number is taken (subtracted) from a greater number.

The **greater number** is called the **minuend**, the **smaller** is called the **subtrahend** and the **result** is called the **remainder.**

The sign of subtraction is '–' (read as **minus**)

Thus : **5 – 3** is read as : **five minus three.**

Steps : *Change the sign of the number to be subtracted and then simplify.*

Example 5 :

Subtract : (i) 53 from 68 (ii) –53 from 68

(iii) 53 from –68 (iv) –53 from –68

Solution :

(i) 68 – 53 = 15 **(Ans.)**

(ii) 68 – (– 53) = 68 + 53 = 121 **(Ans.)**

(iii) – 68 – 53 = –121 **(Ans.)**

(iv) – 68 – (– 53) = – 68 + 53 = –15 **(Ans.)**

Example 6 :

Evaluate : (i) 15 – 8 + 32 – 7

(ii) 38 – 40 – 20 – 4 + 15

(iii) –56 + 82 + 32 – 24

Solution :

Add all positive numbers together and all the negative numbers separately together; then simplify.

(i) 15 – 8 + 32 – 7 = 47 – 15 [∵ 15 + 32 = 47 and – 8 – 7 = – 15]

= 32 **(Ans.)**

(ii) Since, 38 + 15 = 53 and – 40 – 20 – 4 = – 64

∴ 38 – 40 – 20 – 4 + 15 = 53 – 64 = –11 **(Ans.)**

(iii) –56 + 82 + 32 – 24 = 114 – 80 = 34 **(Ans.)**

3. Multiplication :

It is the short process of finding the sum of a given number of repetitions of the same number. The sign of multiplication is "×".

Example 7 :

(i) 8 + 8 + 8 + ... 20 times = 8 multiplied by 20 = 8 × 20.

(ii) (–3) + (–3) + (–3) + ... 16 times = (–3) multiplied by 16 = – 3 × 16.

The **repeating number** is called the **multiplicand,** and the number which indicates how often the multiplicand is to be repeated is called the **multiplier.**

The **sign of multiplication** "×" is read as **multiplied by**.

Thus, in 8 + 8 + 8 + ... 20 times, *i.e.,* in 8 × 20; **8** is the **multiplicand** and **20** is the **multiplier.**

Adopt the following rules for the multiplication of two numbers.

(a) **When both the numbers have like** (same) **signs,** the **result of multiplication** is always **positive**.

For example :

(i) (+5) × (+9) = + 45, or simply 45. (ii) (–5) × (–9) = + 45, or simply 45.

(b) **When both the numbers have unlike** (different) **signs,** the **result of multiplication** is always **negative.**

For example :

(i) $(+5) \times (-9) = -45$ and (ii) $(-5) \times (+9) = -45$

Also, $8 \times 5 = 40$; $8 \times (-5) = -40$; $(-8) \times 5 = -40$ and $(-8) \times (-5) = 40$

4. Division :

It is the process of finding **how many times** a given number (called **divisor**) is contained in another given number (called **dividend**). The number expressing the times the divisor is contained in the dividend is called the **quotient.**

The **sign of division** is "÷" . And is read as **divided by.**

Thus, **28 ÷ 7** is **28 divided by 7.**

> ***Remember :*** $28 \div 7 = \frac{28}{7} = 4$, $72 \div 9 = \frac{72}{9} = 8$ and so on.

(i) $28 \div 7 = \frac{28}{7} = 4$ indicates that in 28, 7 is contained 4 times.

∴ **7** is **divisor, 28** is **dividend** and **4** is **quotient.**

(ii) $72 \div 9 = \frac{72}{9} = 8$, indicates that 9 is contained 8 times in 72.

∴ **9** is **divisor**, **72** is **dividend** and **8** is **quotient.**

Rules for division are same as the rules of multiplication, *i.e.*, if both the numbers have **like** (same) **signs,** the **sign of** the **quotient is positive** and if both the numbers **have unlike** (different) **signs, the sign of** the **quotient is negative.**

Thus, (i) $\frac{+8}{+2} = +4$, *i.e.*, $\frac{8}{2} = 4$ (ii) $\frac{-8}{-2} = 4$

(iii) $\frac{-8}{+2} = -4$, *i.e.*, $\frac{-8}{2} = -4$ (iv) $\frac{+8}{-2} = -4$, *i.e.*, $\frac{8}{-2} = -4$.

> ***Remember :*** $\frac{-8}{2} = \frac{8}{-2} = -\frac{8}{2} = -4$, $\frac{35}{-7} = -\frac{35}{7} = \frac{-35}{7} = -5$ and so on.

More about division :

On dividing 23 by 4, we get : quotient = 5 and remainder = 3 : here **23** is called **dividend** and **4** is called **divisor.**

$$\begin{array}{r} 5 \\ 4\,\overline{)\,23} \\ 20 \\ \hline 3 \end{array}$$

Divisor × quotient + remainder

$= 4 \times 5 + 3$

$= 20 + 3 = 23$, **the dividend.**

∴ Whenever a bigger whole number is divided by a smaller non-zero whole number, the bigger number (the number which is to be divided) is called the **dividend** and the smaller number is called the **divisor**. **(Ans.)**

> Dividend = Divisor × quotient + remainder

This fact is named as **division algorithm.**

Example 8 :

Find the number which on dividing by 15 gives quotient = 8 and remainder = 3.

Solution :

Here, divisor = 15, quotient = 8 and remainder = 3. ***To find :*** *Dividend.*

Since, dividend = divisor × quotient + remainder

$\Rightarrow$ **Dividend** = 15 × 8 + 3 = 120 + 3 = **123** **(Ans.)**

Example 9 :

On dividing 66 by a given number, the quotient is 16 and remainder is 2. Find the number.

Solution :

Here, dividend = 66, quotient = 16 and remainder = 2. ***To find :*** *Divisor*

Dividend = Divisor × Quotient + Remainder

$\Rightarrow$ 66 = Divisor × 16 + 2

$\Rightarrow$ 66 − 2 = Divisor × 16

$\Rightarrow$ $\frac{64}{16}$ = Divisor

i.e. **Divisor** = **4** **(Ans.)**

Zero (0)

1. When zero (0) is added to any number or any number is added to zero, the result is the number itself.

   i.e., **Zero + any number = the number itself**

   and, **Any number + zero = the number itself**

   Thus, 5 + 0 = 5 and 0 + 5 = 5, (–7) + 0 = –7 and 0 + (–7) = –7 and so on.

2. When zero (0) is subtracted from any number, the number remains the same and when any number is subtraced from zero; the sign of the number reverses.

   i.e., **Any number – zero = the number itself**

   and, **Zero – any number = the number with sign changed**

   Thus, (i) 5 – 0 = 5, (–7) – 0 = –7, 85 – 0 = 85 and so on.

   (ii) 0 – 5 = –5, 0 – (–7) = 7, 0 – 85 = – 85 and so on.

3. The result of multiplication of any number with zero (0) is always zero.

   i.e., **Any number × 0 = 0** and **0 × any number = 0**

   Thus, 5 × 0 = 0, 0 × 5 = 0, –8 × 0 = 0, 0 × (–8) = 0 and so on.

4. When zero (0) is divided by any non-zero number, the result (quotient) is always zero and when any number is divided by zero, the result is not-defined.

   i.e., $\frac{\text{Zero}}{\text{Any non-zero number}} = \text{Zero}$ and $\frac{\text{Any number}}{\text{Zero}} = \text{Not-defined}$

   Thus, (i) $\frac{0}{5} = 0$, $\frac{0}{-7} = 0$, $\frac{0}{355} = 0$ and so on.

   (ii) $\frac{5}{0}$ = not-defined, $\frac{-7}{0}$ = not-defined, $\frac{0}{0}$ = not-defined and so on.

EXERCISE 1(B)

1. Fill in the blanks :

(i) 15 + 27 = ... (ii) 27 + 15 = ... (iii) 27 + 0 = ...

(iv) 0 + 15 = ... (v) –15 – 27 = ... (vi) 15 – 27 = ...

(vii) –15 + 27 = ... (viii) –15 – 0 = ... (ix) 0 – 27 = ...

(x) 0 – (–27) = ... (xi) (–27) + 0 = ... (xii) 0 × 8 = ...

(xiii) 8 × 0 = ... (xiv) 0 × (–8) = ... (xv) 0 ÷ 8 = ...

(xvi) 8 ÷ 0 = ... (xvii) 0 ÷ (–8) = ...

2. Evaluate :

(i) 73 – 54 – 35 + 18 + 15 (ii) 73 – 54 + 35 – 18 – 15

(iii) –73 + 54 + 35 – 18 + 15 (iv) 73 – 54 – 35 – 18 + 15

(v) 33 + 0 – 15 + 3 (vi) 33 – 0 – 18 – 3

3. Fill in the blanks :

(i) 15 × 13 = ... (ii) 13 × 15 = ... (iii) –13 × 15 ....

(iv) (–13) × (–15) = ... (v) 13 × – 15 = ... (vi) 168 ÷ 24 = ...

(vii) –168 ÷ 24 = ... (viii) 168 ÷ (–24) = ... (ix) –168 ÷ (–24) = ...

(x) $-\frac{45}{15}$ = ... (xi) $\frac{-75}{25}$ = ... (xii) $\frac{75}{-15}$ = ...

4. Add :

(i) 56 and 83 (ii) 72, –54 and –30

(iii) – 48, 56 and 9 (iv) 28, –36, –95 and 113

5. Subtract :

(i) 92 from 87 (ii) –92 from 87 (iii) –92 from –87

(iv) 92 from –87 (v) 58 from 0 (vi) –58 from 0

6. Multiply :

(i) 12 and 16 (ii) 12 and –16 (iii) –12 and –16

(iv) –12 and 16 (v) –348 and 0 (vi) 0 and 152

7. Divide :

(i) 98 by 14 (ii) –98 by 14 (iii) –98 by –14

(iv) 98 by –14 (v) 98 by 0 (vi) 0 by 98.

8. Find the number which on dividing by 23 gives quotient = 7 and remainder = 4.

9. On dividing 154 by a number, we get, quotient = 9 and remainder = 10. Find the number.

10. Find the number which on dividing by 35 gives remainder = 0 and quotient = 12.