SIMPLE (LINEAR) EQUATIONS
[Including Word Problems]

15.1 BASIC CONCEPT :

A mathematical statement which shows that two expressions are equal is called an equation.

For example :

If the expressions $3x - 5$ and $x + 8$ are equal, we write $3x - 5 = x + 8$, which is an equation.

Similarly :

(i) If the expressions $x - 3$ and $7 - x$ are equal,
It is written as $x - 3 = 7 - x$ [Which is an equation]

(ii) Seven subtracted from a number (x) equals 4 [A statement]
$\Rightarrow$ $x - 7 = 4$ [An equation]

(iii) A certain number (x) multiplied by 4 equals 20 [A statement]
$\Rightarrow$ $4x = 20$ [An equation]

(iv) A number (x) divided by 7 equals 2 [A statement]
$\Rightarrow$ $\frac{x}{7} = 2$ [An equation]

An equation is said to be a **linear equation** if it contains only **one variable** (literal) with **highest power 1** (one).

Since each of the equations discussed above has only one variable (which is x) with highest power 1, each of these equations is a linear equation.

15.2 SOLVING A LINEAR EQUATION

Solving linear equation means finding the value of an unknown algebraic quantity (variable) used in the equation.

For example :

(i) To solve the equation $x + 5 = 7$ means to find the value of x.

(ii) To solve the equation $3y + 2 = 9$ means to find the value of y.

(iii) To solve the equation $\frac{2a}{3} + 4a = 10$ means to find the value of a and so on.

15.3 RULES FOR SOLVING A LINEAR EQUATION

Rule 1 : The given equation does not change if the same quantity is added on both sides.

e.g. $x + 5 = 2 \Rightarrow x + 5 + \mathbf{7} = 2 + \mathbf{7}$,

$3x - 2 = 8 \Rightarrow 3x - 2 + \mathbf{4} = 8 + \mathbf{4}$, etc.

Rule 2 : The given equation does not change if the same quantity is subtracted from both the sides of it.

e.g. $x + 5 = 2 \Rightarrow x + 5 - \mathbf{7} = 2 - \mathbf{7}$,

$3x - 2 = 8 \Rightarrow 3x - 2 - \mathbf{4} = 8 - \mathbf{4}$, etc.

Rule 3 : The given equation does not change if each of its terms is multiplied by the same quantity.

e.g. $5x = 2 \Rightarrow 5x \times \mathbf{3} = 2 \times \mathbf{3}$,

$\frac{3x}{2} = 7 \Rightarrow \frac{3x}{2} \times 2 = 7 \times 2$, etc.

Rule 4 : The given equation does not change if each of its term is divided by the same non-zero quantity.

e.g. $3x = 5 \Rightarrow \frac{3x}{3} = \frac{5}{3}$,

$7x = 8 \Rightarrow \frac{7x}{4} = \frac{8}{4}$, etc.

15.4 SOLVING AN EQUATION OF THE FORM $x + a = b$.

Example 1 : Solve : $x + 3 = 10$

Solution :

$x + 3 = 10 \Rightarrow x + 3 - 3 = 10 - 3$ [**Rule 2 :** Subtracting 3 from both the sides]

$\Rightarrow x = 7$ **(Ans.)**

15.5 SOLVING AN EQUATION OF THE FORM $x - a = b$

Example 2 : Solve : $x - 5 = 2$

Solution :

$x - 5 = 2 \Rightarrow x - 5 + 5 = 2 + 5$ [**Rule 1 :** Adding 5 on both the sides]

$\Rightarrow x = 7$ **(Ans.)**

15.6 SOLVING AN EQUATION OF THE FORM $ax = b$

Example 3 : Solve : $2x = 6$

Solution :

$2x = 6 \Rightarrow \frac{2x}{2} = \frac{6}{2}$ [**Rule 4 :** Dividing each term by 2]

$\Rightarrow x = 3$ **(Ans.)**

15.7 SOLVING AN EQUATION OF THE FORM $\frac{x}{a} = b$

Example 4 : Solve : $\frac{b}{2} = 5$

Solution :

$$\frac{b}{2} = 5 \Rightarrow \frac{b}{2} \times 2 = 5 \times 2$$ [**Rule 3 :** Multiplying each term by 2]

$$\Rightarrow b = 10$$ **(Ans.)**

Example 5 : Solve : (i) $p - 2{\cdot}5 = 7{\cdot}3$ (ii) $x + 3\frac{1}{3} = 6$

Solution :

(i) $p - 2{\cdot}5 = 7{\cdot}3 \Rightarrow p - 2{\cdot}5 + 2{\cdot}5 = 7{\cdot}3 + 2{\cdot}5$

$\Rightarrow p = 9{\cdot}8$ **(Ans.)**

(ii) $x + 3\frac{1}{3} = 6 \Rightarrow x + \frac{10}{3} - \frac{10}{3} = 6 - \frac{10}{3}$

$\Rightarrow x = \frac{18 - 10}{3} = \frac{8}{3} = 2\frac{2}{3}$ **(Ans.)**

EXERCISE 15(A)

1. Solve :

(i) $x + 2 = 6$ (ii) $x + 6 = 2$ (iii) $y + 8 = 5$

(iv) $x + 4 = -3$ (v) $y + 2 = -8$ (vi) $b + 2{\cdot}5 = 4{\cdot}2$

(vii) $p + 4{\cdot}6 = 8{\cdot}5$ (viii) $y + 3{\cdot}2 = -6{\cdot}5$ (ix) $a + 8{\cdot}9 = -12{\cdot}6$

(x) $x + 2\frac{1}{3} = 5$ (xi) $z + 2 = 4\frac{1}{5}$ (xii) $m + 3\frac{1}{2} = 4\frac{1}{4}$

(xiii) $x + 2 = 1\frac{1}{4}$ (xiv) $y + 5\frac{1}{3} = 4$ (xv) $a + 3\frac{1}{5} = 1\frac{1}{2}$

2. Solve :

(i) $x - 3 = 2$ (ii) $m - 2 = -5$ (iii) $b - 5 = 7$

(iv) $a - 2{\cdot}5 = -4$ (v) $y - 3\frac{1}{2} = 6$ (vi) $z - 2\frac{1}{3} = -6$

(vii) $p - 5{\cdot}4 = 2{\cdot}7$ (viii) $x - 1{\cdot}5 = -4{\cdot}9$ (ix) $n - 4 = -4\frac{1}{5}$

3. Solve :

(i) $3x = 12$ (ii) $2y = 9$ (iii) $5z = 8{\cdot}5$

(iv) $2{\cdot}5m = 7{\cdot}5$ (v) $3{\cdot}2p = 16$ (vi) $2a = 4{\cdot}6$

4. Solve :

(i) $\frac{x}{2} = 5$ (ii) $\frac{y}{3} = -2$ (iii) $\frac{a}{5} = -15$

(iv) $\frac{z}{4} = 3\frac{1}{4}$ (v) $\frac{m}{6} = 2\frac{1}{2}$ (vi) $\frac{n}{7} = -2{\cdot}8$

5. Solve :

(i) $-2x = 8$ (ii) $-3{\cdot}5y = 14$ (iii) $-5z = 4$

(iv) $-5 = a + 3$ (v) $2 = p + 5$ (vi) $4{\cdot}5 = m - 2{\cdot}7$

(vii) $3\frac{2}{5} = x - 2\frac{1}{3}$ (viii) $5 = m + 3\frac{4}{7}$ (ix) $-2\frac{1}{5} = y - 4$

15.8 SOLVING EQUATIONS USING MORE THAN ONE PROPERTY

Example 6 :

Solve : (i) $3x + 8 = 14$ (ii) $\frac{m}{3} + 7 = 11$ (iii) $2 + \frac{5x}{3} = x + 6$

Solution :

(i) $3x + 8 = 14 \Rightarrow 3x + 8 - 8 = 14 - 8$ [Subtracting 8 from both the sides]

$\Rightarrow 3x = 6$

$\Rightarrow \frac{3x}{3} = \frac{6}{3}$ [Dividing each side by 3]

$\Rightarrow x = 2$ **(Ans.)**

(ii) $\frac{m}{3} + 7 = 11 \Rightarrow \frac{m}{3} + 7 - 7 = 11 - 7$ [Subtracting 7 from each side]

$\Rightarrow \frac{m}{3} = 4$

$\Rightarrow \frac{m}{3} \times 3 = 4 \times 3$ [Multiplying each side by 3]

$\Rightarrow m = 12$ **(Ans.)**

(iii) $2 + \frac{5x}{3} = x + 6 \Rightarrow 2 + \frac{5x}{3} - 2 = x + 6 - 2$ [Subtracting 2]

$\Rightarrow \frac{5x}{3} = x + 4$

$\Rightarrow \frac{5x}{3} - x = x + 4 - x$ [Subtracting x]

$\Rightarrow \frac{5x}{3} - x = 4$

$\Rightarrow \frac{5x}{3} \times 3 - x \times 3 = 4 \times 3$ [Multiplying each term by 3]

$\Rightarrow 5x - 3x = 12$

$\Rightarrow 2x = 12 \Rightarrow x = 6$ **(Ans.)**

Multiplying each term of an equation by the same number does not change the equation

15.9 SOLVING A LINEAR EQUATION USING TRANSPOSITION

Transposition of a positive or a negative term.

In transposition (shifting), a positive term becomes negative and a negative term becomes positive :

positive term becomes negative

e.g. $x + 3 = 7 \Rightarrow x = 7 - 3$

and $2x - 5 = 8 \Rightarrow 2x = 8 + 5$

negative term becomes positive

Example 7 : Solve : (i) $x + 5 = 32$ (ii) $y - 4 = 3$

(iii) $3z - 1 = 8$ (iv) $\frac{x}{3} + 4 = 12$

Solution :

(i) $x + 5 = 32 \Rightarrow x = 32 - 5$ [Transposing + 5]

$\Rightarrow x = 27$ **(Ans.)**

(ii) $y - 4 = 3 \Rightarrow y = 3 + 4$ [Transposing – 4]

$\Rightarrow y = 7$ **(Ans.)**

(iii) $3z - 1 = 8 \Rightarrow 3z = 8 + 1 = 9$ [Transposing – 1]

$\Rightarrow \frac{3z}{3} = \frac{9}{3}$ [Dividing by 3]

$\Rightarrow z = 3$ **(Ans.)**

(iv) $\frac{x}{5} + 4 = 2 \Rightarrow \frac{x}{5} = 2 - 4$ [Transposing + 4]

$\Rightarrow \frac{x}{5} = -2$

$\Rightarrow \frac{x}{5} \times 5 = -2 \times 5$ [Multiplying by 5]

$\Rightarrow x = -10$ **(Ans.)**

EXERCISE 15(B)

1. Solve :

(i) $2x + 5 = 17$ (ii) $3y - 2 = 1$ (iii) $5p + 4 = 29$

(iv) $4a - 3 = -27$ (v) $2z + 3 = -19$ (vi) $7m - 1 = 20$

(vii) $2{\cdot}4x - 3 = 4{\cdot}2$ (viii) $4m + 9{\cdot}4 = 5$ (ix) $6y + 4 = -4{\cdot}4$

2. Solve :

(i) $\frac{x}{3} - 5 = 2$ (ii) $\frac{y}{2} - 3 = 8$ (iii) $\frac{z}{7} + 1 = 2\frac{1}{2}$

(iv) $\frac{a}{2{\cdot}4} - 5 = 2{\cdot}4$ (v) $\frac{b}{1{\cdot}6} + 3 = -2{\cdot}5$ (vi) $\frac{m}{4} - 4{\cdot}6 = -3{\cdot}1$

3. Solve :

(i) $-8m - 2 = -10$ (ii) $4x + 2x = 3 + 5$

(iii) $4x - x + 5 = 8$ (iv) $6x + 2 = 2x + 10$

(v) $18 - (2a - 12) = 8a$ (vi) $3x + 5 + 2x + 6 + x = 4x + 21$

(vii) $3{\cdot}5x - 9 - 3 = x + 1$ (viii) $8x + 6 + 2x - 4 = 4x + 8$

(ix) $-m + (3m - 6m) = -8 - 14$ (x) $5x - 14 = x - (24 + 4x)$

EXERCISE 15(C)

Solve :

1. $5 - x = 3$ **2.** $2 - y = 8$ **3.** $8{\cdot}4 - x = -2$

4. $x + 2\frac{1}{5} = 3$ **5.** $y - 3\frac{1}{2} = 2\frac{1}{3}$ **6.** $5\frac{2}{3} - z = 2\frac{1}{2}$

7. $1{\cdot}6z = 8$ **8.** $3a = -2{\cdot}1$ **9.** $\frac{z}{4} = -1{\cdot}5$

10. $\frac{z}{6} = -1\frac{2}{3}$ 11. $-5x = 10$ 12. $2\cdot4z = -4\cdot8$

13. $2y - 5 = -11$ 14. $2x + 4\cdot6 = 8$ 15. $5y - 3\cdot5 = 10$

16. $3x + 2 = -2\cdot2$ 17. $\frac{y}{2} - 5 = 1$ 18. $\frac{z}{3} - 1 = -5$

19. $\frac{x}{4} + 3\cdot6 = -1\cdot1$ 20. $-3y - 2 = 10$ 21. $4z - 5 = 3 - z$

22. $7x - 3x + 2 = 22$ 23. $6y + 3 = 2y + 11$ 24. $3(x + 5) = 18$

25. $5(x - 2) - 2(x + 1) = 3$ 26. $(5x - 3) \div 4 = 3$ 27. $3(2x + 1) - 2(x - 5) - 5(5 - 2x) = 16$

15.10 SOLVING WORD PROBLEMS

Steps :

1. Read the given statement carefully to know what is given and what is required to be found.
2. Take the unknown quantity required to be found as x or y or z, etc.
3. Form an equation according to the given relationship between the knowns and unknowns.
4. Solve the equation obtained in step 3 to get the required unknown quantity.

Example 8 :

A number increased by 13 is equal to 31. Find the number.

Solution :

Step 1 : On reading the given statement carefully, we conclude that we have to find a number that satisfies the given condition.

Step 2 : Let the required number be x.

Step 3 : The given relationship is :

The required number increased by 13 = 31.

$\Rightarrow \quad x + 13 = 31$

Step 4 : $x = 31 - 13 = 18$

$\therefore$ **The required number = 18** **(Ans.)**

Example 9 :

One-third of a number added to one-fifth of it gives 32. Find the number.

Solution :

Let the required number be x.

Given : $\frac{1}{3}x + \frac{1}{5}x = 32$

$\Rightarrow \quad \frac{5x + 3x}{15} = 32$

$\Rightarrow \quad \frac{8x}{15} = 32$ and $x = 32 \times \frac{15}{8} = 60$

$\therefore$ **The required number = 60** **(Ans.)**

Example 10 :

A number is decreased by 15 and the new number so obtained is multiplied by 3; the result is 81. Find the number.

Solution :

Let the number be x.

The number decreased by 15 = x – 15

The new number (x – 15) multiplied by 3 = 3(x – 15)

Given : $3(x - 15) = 81 \Rightarrow 3x - 45 = 81$

$\Rightarrow 3x = 81 + 45 = 126$

$\Rightarrow x = \frac{126}{3} = 42$

∴ **The required number = 42** **(Ans.)**

Example 11 :

The age of a man is 38 years more than the age of his son. If the sum of their ages is 82 years, find the age of the son and his father.

Solution :

Let the age of the son = x years

∵ The age of the man is 38 years more than the age of his son

∴ The age of the man = (x + 38) years

Given : The sum of the ages of the man and his son = 82 years

∴ $(x + 38) + x = 82$

$\Rightarrow x + 38 + x = 82$ and $2x + 38 = 82$

$\Rightarrow 2x = 82 - 38 = 44$ and $x = \frac{44}{2} = 22$

∴ **The age of the son** = x years = **22 years**

And **the age of his father** = (x + 38) years = (22 + 38) years

= **60 years** **(Ans.)**

EXERCISE 15(D)

1. A number increased by 17 ibecomes 54. Find the number.
2. A number decreased by 8 equals 26; find the number.
3. One-fourth of a number added to two-sevenths of it gives 135; find the number.
4. Two-fifths of a number subtracted from three-fourths of it gives 56; find the number.
5. A number is increased by 12 and the new number obtained is multiplied by 5. If the resulting number is 95, find the original number.
6. A number is increased by 26 and the new number obtained is divided by 3. If the resulting number is 18, find the original number.
7. The age of a man is 27 years more than the age of his son. If the sum of their ages is 47 years, find the age of the son and his father.
8. The difference between the ages of Gopal and his father is 26 years. If the sum of their ages is 56 years, find the ages of Gopal and his father.

9. When two consecutive natural numbers are added, the sum is 31; find the numbers.

> Let the two consecutive natural numbers be x and x + 1.
>
> $\therefore \quad x + (x + 1) = 31$
>
> Similarly, three consecutive natural numbers can be taken as : x, x + 1 and x + 2

10. When three consecutive natural numbers are added, the sum is 66, find the numbers.

Revision Exercise (Chapter 15)

1. Solve each of the following equations :

(i) $2x + 3 = 7$ (ii) $2x - 3 = 7$ (iii) $2x \div 3 = 7$

(iv) $3y - 8 = 13$ (v) $3y + 8 = 13$ (vi) $3y \div 8 = 13$

(vii) $x - 3 = 5\frac{1}{2}$ (viii) $\frac{3}{5}x + 4 = 13$ (ix) $u + 3\frac{1}{4} = 4\frac{1}{3}$

(x) $5x - 2{\cdot}4 = 4{\cdot}9$ (xi) $5y + 4{\cdot}9 = 2{\cdot}4$ (xii) $4{\cdot}8z + 3{\cdot}6 = 1{\cdot}2$

(xiii) $\frac{x}{2} - 3 = 5$ (xiv) $\frac{y}{3} + 7 = 2$ (xv) $\frac{2m}{3} = 8\frac{2}{3}$

(xvi) $-3x + 4 = 10$ (xvii) $5 = x - 3$ (xviii) $18 = 3 - 3y$

(xix) $4x + 4{\cdot}9 = 6{\cdot}5$ (xx) $3z + 2 = -4$ (xxi) $7y - 18 = 17$

(xxii) $\frac{x}{1{\cdot}2} - 6 = 1$ (xxiii) $\frac{z}{2{\cdot}4} + 3{\cdot}6 = 5{\cdot}1$ (xxiv) $\frac{y}{1{\cdot}8} - 2{\cdot}1 = -2{\cdot}8$

(xxv) $7x - 2 = 4x + 7$ (xxvi) $3y - (y + 2) = 4$ (xxvii) $3z - 18 = z - (12 - 4z)$

(xxviii) $x - 2\frac{1}{3} = 5\frac{1}{2}$ (xxix) $3\frac{2}{5} - y = 2\frac{1}{2}$ (xxx) $2z - 2\frac{1}{2} = 3\frac{1}{3}$

(xxxi) $5x - 2x + 15 = 27$ (xxxii) $5y - 15 = 27 - 2y$ (xxxiii) $7z + 15 = 3z - 13$

(xxxiv) $2(x - 3) - 3(x - 4) = 12$ (xxxv) $(7y + 8) \div 7 = 8$ (xxxvi) $2(z - 5) + 3(z + 2) - (3 - 5z) = 10$

2. A natural number decreased by 7 is 12. Find the number.

3. One-fourth of a number added to one-sixth of it is 15. Find the number.

4. A whole number is increased by 7 and the number so obtained is multipled by 5; the result is 45. Find the whole number.

5. The age of a man and the age of his daughter differ by 23 years, and the sum of their ages is 41 years. Find the age of the man.

6. The difference between the ages of a woman and her son is 19 years, and the sum of their ages is 37 years; find the age of the son.

7. Two natual numbers differ by 6 and their sum is 36. Find the larger number.

8. The difference between two numbers is 15. Taking the smaller number as x, find :
 (i) the expression for the larger number.
 (ii) the larger number if the sum of these numbers is 71.

9. The difference between two numbers is 23. Taking the larger number as x, find :
 (i) the expression for smaller number.
 (ii) the smaller number, if the sum of these two numbers is 91.

10. Find the three consecutive integers whose sum is 78.

11. The sum of three consecutive numbers is 54. Taking the middle number as x, find :
 (i) the expressions for the smallest number and the largest number.
 (ii) the three numbers.