CHAPTER 14

SUBSTITUTION

(Including Use of Brackets as Grouping Symbols)

14.1 BASIC CONCEPT

The value of an expression depends on the value(s) of its variable(s).

Consider the algebraic expression : $3x + 2$

In the expression $3x + 2$, the variable used is x, and so the value of the expression $3x + 2$ depends on the value of its variable x. That is :

(i) if $x = 2$, the value of the expression $3x + 2 = 3 \times 2 + 2 = 6 + 2 = 8$

(ii) if $x = 0$, the value of the expression $3x + 2 = 3 \times 0 + 2 = 0 + 2 = 2$

(iii) if $x = -2$, the value of the expression $3x + 2 = 3 \times -2 + 2 = -6 + 2 = -4$ and so on.

Now consider the algebraic expression $5x - 2y$.

This expression consists of variables x and y.

Now, if :

(i) $x = 3$ and $y = 2$, $5x - 2y = 5 \times 3 - 2 \times 2 = 15 - 4 = 11$

(ii) $x = 8$ and $y = 5$, $5x - 2y = 5 \times 8 - 2 \times 5 = 40 - 10 = 30$.

The process of finding the value of an algebraic expression by substituting the given value (values) of its variable (variables) is called **substitution**.

Example 1 :

If $x = 6$ and $y = 3$, find the value of :

(i) $4x + y$ (ii) $3x - 4y$ (iii) $3xy$ (iv) $\frac{5x}{4y}$

Solution :

(i) $4x + y = 4 \times 6 + 3 = 24 + 3 = 27$ **(Ans.)**

(ii) $3x - 4y = 3 \times 6 - 4 \times 3 = 18 - 12 = 6$ **(Ans.)**

(iii) $3xy = 3 \times 6 \times 3 = 54$ **(Ans.)**

(iv) $\frac{5x}{4y} = \frac{5 \times 6}{4 \times 3} = \frac{30}{12} = \frac{5}{2} = 2\frac{1}{2}$ **(Ans.)**

Example 2 :

If $a = 2$, $b = 5$ and $c = 8$, find the value of : $3ab + 10bc - 2abc$

Solution :

$$\begin{aligned} 3ab + 10bc - 2abc &= 3 \times 2 \times 5 + 10 \times 5 \times 8 - 2 \times 2 \times 5 \times 8 \\ &= 30 + 400 - 160 \\ &= 430 - 160 = 270 \end{aligned}$$ **(Ans.)**

Example 3 :

If p = 8, q = 1 and r = 2, find the value of : $\dfrac{10pq - 3qr}{4pqr - 2p}$

Solution :

$$\frac{10pq - 3qr}{4pqr - 2p} = \frac{10 \times 8 \times 1 - 3 \times 1 \times 2}{4 \times 8 \times 1 \times 2 - 2 \times 8} = \frac{80 - 6}{64 - 16} = \frac{74}{48} = \frac{37}{24} = 1\frac{13}{24} \quad \text{(Ans.)}$$

Example 4 :

If x = 2; find the value of $3x^2 + x$.

Solution :

$$3x^2 + x = 3(2)^2 + 2$$
$$= 3 \times 2 \times 2 + 2 = 12 + 2 = 14 \quad \text{(Ans.)}$$

Example 5 :

If x = 5, y = 6 and z = 10, find the value of :

(i) $\dfrac{3x^2}{x}$ (ii) $\dfrac{xy}{xz}$ (iii) $\dfrac{x^2y}{z}$

Solution :

(i) $\dfrac{3x^2}{x} = \dfrac{3 \times 5^2}{5} = \dfrac{3 \times 5 \times 5}{5} = 3 \times 5 = 15$ (Ans.)

Alternative method :

$\dfrac{3x^2}{x} = \dfrac{3 \times x \times x}{x} = 3 \times x = 3 \times 5 = 15$ (Ans.)

(ii) $\dfrac{xy}{xz} = \dfrac{5 \times 6}{5 \times 10} = \dfrac{30}{50} = \dfrac{3}{5}$ (Ans.)

Alternative method :

$\dfrac{xy}{xz} = \dfrac{y}{z} = \dfrac{6}{10} = \dfrac{3}{5}$ (Ans.)

(iii) $\dfrac{x^2y}{z} = \dfrac{5^2 \times 6}{10} = \dfrac{5 \times 5 \times 6}{10} = \dfrac{150}{10} = 15$ (Ans.)

Example 6 :

If a = 2, b = 3 and c = 4, find the value of :

(i) a^b (ii) b^a (iii) c^b (iv) $a^2 - b^2 + c^2$

Solution :

(i) $a^b = 2^3 = 2 \times 2 \times 2 = 8$ (Ans.)

(ii) $b^a = 3^2 = 3 \times 3 = 9$ (Ans.)

(iii) $c^b = 4^3 = 4 \times 4 \times 4 = 64$ (Ans.)

(iv) $a^2 - b^2 + c^2 = (2)^2 - (3)^2 + (4)^2$
$= 4 - 9 + 16 = 20 - 9 = 11$ (Ans.)

EXERCISE 14(A)

1. Fill in the following blanks, when : $x = 3$, $y = 6$, $z = 18$, $a = 2$, $b = 8$, $c = 32$ and $d = 0$.

(i)	$x + y = 3 + 6 = 9$	(ix)	$a + b + x =$ ....
(ii)	$y - x =$	(x)	$b + z - d =$ ....
(iii)	$\frac{y}{x} =$	(xi)	$a - b + y =$ ....
(iv)	$c \div b =$	(xii)	$z - a - b =$ ....
(v)	$z \div x =$	(xiii)	$d - a + x =$ ....
(vi)	$y \times d =$	(xiv)	$xy - bd =$ ....
(vii)	$d \div x =$	(xv)	$xz + cd =$ ....
(viii)	$ab + y =$		

2. Find the value of :
   (i) $p + 2q + 3r$, when $p = 1$, $q = 5$ and $r = 2$
   (ii) $2a + 4b + 5c$, when $a = 5$, $b = 10$ and $c = 20$
   (iii) $3a - 2b$, when $a = 8$ and $b = 10$
   (iv) $5x + 3y - 6z$, when $x = 3$, $y = 5$ and $z = 4$
   (v) $2p - 3q + 4r - 8s$, when $p = 10$, $q = 8$, $r = 6$, and $s = 2$
   (vi) $6m - 2n - 5p - 3q$, when $m = 20$, $n = 10$, $p = 2$ and $q = 9$
3. Find the value of :
   (i) $4pq \times 2r$, when $p = 5$, $q = 3$ and $r = 1/2$ (ii) $\frac{yx}{z}$, when $x = 8$, $y = 4$ and $z = 16$
   (iii) $\frac{a+b-c}{2a}$, when $a = 5$, $b = 7$ and $c = 2$
4. If $a = 3$, $b = 0$, $c = 2$ and $d = 1$, find the value of :
   (i) $3a + 2b - 6c + 4d$ (ii) $6a - 3b - 4c - 2d$ (iii) $ab - bc + cd - da$
   (iv) $abc - bcd + cda$ (v) $a^2 + 2b^2 - 3c^2$ (vi) $a^2 + b^2 - c^2 + d^2$
   (vii) $2a^2 - 3b^2 + 4c^2 - 5d^2$
5. Find the value of : $5x^2 - 3x + 2$, when $x = 2$.
6. Find the value of : $3x^3 - 4x^2 + 5x - 6$, when $x = -1$.
7. Show that the value of : $x^3 - 8x^2 + 12x - 5$ is zero, when $x = 1$.
8. State **true** and **false** :
   (i) The value of $x + 5 = 6$, when $x = 1$ (ii) The value of $2x - 3 = 1$, when $x = 0$
   (iii) $\frac{2x - 4}{x + 1} = -1$, when $x = 1$.
9. If $x = 2$, $y = 5$ and $z = 4$, find the value of each of the following :
   (i) $\frac{x}{2x^2}$ (ii) $\frac{xz}{yz}$ (iii) z^x (iv) y^x
   (v) $\frac{x^2\ y^2\ z^2}{xz}$ (vi) $\frac{5x^4\ y^2\ z^2}{2x^2}$ (vii) $xy \div y^2z$ (viii) $\frac{x^2\ y^x}{x}$
10. If $a = 3$, find the values of a^2 and 2^a.
11. If $m = 2$, find the difference between the values of $4m^3$ and $3m^4$.

14.2 BRACKETS

In general, the symbols ( ), { }, [ ], etc., are called **brackets**.

1. ( ) *is called a* ***small bracket*** *or* ***parenthesis.***
2. { } *is called a* ***middle bracket*** *or* ***curly bracket.***
3. [ ] *is* ***called a big bracket*** *or* ***square bracket.***

Note : *If one more bracket is needed, we use the bar bracket, i.e. a line* —— *is drawn over a group of terms.*

Thus, in $2x + \overline{3y - 4z}$, *the line over 3y – 4z serves as the* ***bar bracket*** *and is* ***called vinculum.***

If an expression is enclosed within a bracket, it is considered a single quantity even if it is made up of many terms.

For example :

Each of (x – y) and (2a + 3b – 2) will be treated as a single quantity.

While simplifying an expression containing a bracket, first of all, the terms inside brackets are operated (combined).

Example 7 :

Evaluate : (i) 15 – (8 – 6) (ii) (15 – 8) – 6

Solution :

(i) 15 – (8 – 6) = 15 – 2 Simplifying the terms in the bracket first

= 13 **(Ans.)**

(ii) (15 – 8) – 6 = 7 – 6 = 1 **(Ans.)**

Similarly,

15 – (8 + 6) = 15 – 14 = 1; (15 – 8) + 6 = 7 + 6 = 13 and so on.

14.3 OPENING OR REMOVING A BRACKET

Removing a bracket

(i) ***When there is + ve*** (positive) ***sign before a bracket :***

The bracket is removed without changing the signs of the terms in the bracket.

For example :

(i) 10 + (7 – 3)
= 10 + 7 – 3 = **14**

(ii) a + (b – c + d)
= **a + b – c + d**

(ii) ***When there is – ve*** (negative) ***sign before a bracket :***

The bracket is removed and, at the same time, the sign of each term in side the bracket is also changed,

i.e. + sign will change to – sign, and – sign will change to + sign.

For example :

(i) $12 - (8 - 5)$
$= 12 - 8 + 5 = 17 - 8 = \mathbf{9}$

(ii) $a - (b - c + d)$
$= \mathbf{a - b + c - d}$

14.4 REMOVAL OF BRACKETS

In a combined operation, the brackets should be removed in the following order :

(i) $\overline{\quad\quad}$ (ii) ( ) (iii) { } (iv) [ ]

For example :

(i) $3x - (2y - \overline{x - y})$

$= 3x - (2y - x + y)$ [On removing the bar bracket]

$= 3x - (3y - x)$

$= 3x - 3y + x$ [On removing the small bracket]

$= \mathbf{4x - 3y}$

(ii) $6a - \{a + (2a - \overline{4 - a})\}$

$= 6a - \{a + (2a - 4 + a)\}$

$= 6a - \{a + (3a - 4)\}$

$= 6a - \{a + 3a - 4\}$

$= 6a - \{4a - 4\} = 6a - 4a + 4 = \mathbf{2a + 4}$

First of all remove the bar bracket and simplify; then remove the small bracket ( ) and simplify. Finally remove the curly bracket { } and simplify.

Example 8 :

Simplify :

(i) $a - [b - \{c - (a - \overline{b - c})\}]$ (ii) $3a - [2a - \{a + (a - \overline{b - c} + c)\}]$

Solution :

(i) $\mathbf{a - [b - \{c - (a - \overline{b - c})\}]}$

$= a - [b - \{c - (a - b + c)\}]$ [On removing the bar bracket]

$= a - [b - \{c - a + b - c\}]$ [On removing the small bracket]

$= a - [b - c + a - b + c]$ [On removing the middle bracket]

$= a - b + c - a + b - c$ [On removing the square bracket]

$= \mathbf{0}$ **(Ans.)**

(i) $\mathbf{3a - [2a - \{a + (a - \overline{b - c} + c)\}]}$

$= 3a - [2a - \{a + (a - b + c + c)\}]$

$= 3a - [2a - \{a + a - b + c + c\}]$

$= 3a - [2a - a - a + b - c - c]$

$= 3a - 2a + a + a - b + c + c = \mathbf{3a - b + 2c}$ **(Ans.)**

Order of removing the brackets is : $\overline{\quad}$, ( ), { } and finally [ ].

A number placed before a bracket indicates that each term inside the bracket is to be multiplied by that number.

Thus : (i) $5(x + y) = 5x + 5y$ (ii) $2(2a - b) = 4a - 2b$

(iii) $3(m - 2n + 4a) = 3m - 6n + 12a$ *and so on.*

Example 9 :

Simplify : (i) $2x + 3(x - y)$ (ii) $5a - 2\{2b - 3(a - b)\}$

Solution :

(i) $2x + 3(x - y) = 2x + 3x - 3y$

$= 5x - 3y$ **(Ans.)**

(ii) $5a - 2\{2b - 3(a - b)\} = 5a - 2\{2b - 3a + 3b\}$

$= 5a - 4b + 6a - 6b$

$= 5a + 6a - 4b - 6b = 11a - 10b$ **(Ans.)**

EXERCISE 14(B)

1. Evaluate :
   (i) $(23 - 15) + 4$ (ii) $5x + (3x + 7x)$ (iii) $6m - (4m - m)$
   (iv) $(9a - 3a) + 4a$ (v) $35b - (16b + 9b)$ (vi) $(3y + 8y) - 5y$

2. Simplify :
   (i) $12x - (5x + 2x)$ (ii) $10m + (4n - 3n) - 5n$
   (iii) $(15b - 6b) - (8b + 4b)$ (iv) $-(-4a - 8a)$
   (v) $x - (x - y) - (-x + y)$ (vi) $p + (-q - r - s) - (p - q - r)$
   (vii) $(a + b) - (c + d) - (e - f)$ (viii) $3x + (8x - 5x) - (7x - x)$
   (ix) $a - (a - b - c)$ (x) $6a^2 + (2a^2 - a^2) - (a^2 - b^2)$
   (xi) $2m - (3m + 2n - 6n)$ (xii) $-m - n - (-m) - m$
   (xiii) $x + y - (x + \overline{y - x})$ (xiv) $25y - (5x - 10y + 6x - 3y)$
   (xv) $3x + (2x - \overline{x + 2})$ (xvi) $a - (2a - \overline{4a + 3a})$
   (xvii) $5x^2 - (3x - \overline{x^2 - 4})$ (xviii) $-(y - x) - (x + y - \overline{2x + y})$

3. Simplify :
   (i) $x - (y - z) + x + (y - z) + y - (z + x)$ (ii) $x - [y + \{x - (y + x)\}]$
   (iii) $4x + 3(2x - 5y)$ (iv) $2(3a - b) - 5(a - 3b)$
   (v) $p + 2(q - \overline{r + p})$ (vi) $a - [-\{-(a - \overline{b - c})\}]$
   (vii) $3x - [5y - \{6y + 2(10y - x)\}]$ (viii) $5\{a^2 - a(a - \overline{a - 2})\}$

14.5 INSERTING A BRACKET

1. When any part of an expression is inserted within a bracket with a *positive sign* before it, *the sign of each term kept inside the bracket remains unchanged.*

For example :

The expression $a - b + c - d$ may be written as :

$$a + (-b + c - d) \quad \text{or} \quad a - b + (c - d)$$

2. When any part of an expression is to be inserted within a bracket with a *negative sign* before it, *the sign of each term kept inside the bracket gets changed.*

For example :

The expression $a - b + c - d$ may be written as :

$$a - (b - c + d) \quad \text{or} \quad a - b - (-c + d)$$

EXERCISE 14(C)

1. Fill in the blanks :
   (i) $2a + b - c = 2a + (\ldots\ldots\ldots\ldots)$
   (ii) $3x - z + y = 3x - (\ldots\ldots\ldots\ldots)$
   (iii) $6p - 5x + q = 6p - (\ldots\ldots\ldots\ldots)$
   (iv) $a + b - c + d = a + (\ldots\ldots\ldots\ldots)$
   (v) $5a + 4b + 4x - 2c$
   $= 4x - (\ldots\ldots \quad \ldots\ldots)$
   (vi) $7x + 2z + 4y - 3 = -3 + 4y + (\ldots\ldots\ldots\ldots)$
   (vii) $3m - 2n + 6 = 6 - (\ldots\ldots\ldots\ldots)$
   (viii) $2t + r - p - q + s = 2t + r - (\ldots\ldots\ldots\ldots)$

2. Insert the bracket as indicated :
   (i) $x - 2y = -(\ldots\ldots\ldots\ldots)$
   (ii) $m + n - p = -(\ldots\ldots\ldots\ldots)$
   (iii) $a + 4b - 4c = a + (\ldots\ldots\ldots\ldots)$
   (iv) $a - 3b + 5c = a - (\ldots\ldots\ldots\ldots)$
   (v) $x^2 - y^2 + z^2 = x^2 - (\ldots\ldots\ldots\ldots)$
   (vi) $m^2 + x^2 - p^2 = -(\ldots\ldots\ldots\ldots)$
   (vii) $2x - y + 2z = 2z - (\ldots\ldots\ldots\ldots)$
   (viii) $ab + 2bc - 3ac = 2bc - (\ldots\ldots\ldots\ldots)$

REVISION EXERCISE (Chapter 14)

1. Find the value of $3ab + 10bc - 2abc$ when $a = 2$, $b = 5$ and $c = 8$.
2. If $x = 2$, $y = 3$ and $z = 4$, find the value of $3x^2 - 4y^2 + 2z^2$.
3. If $x = 3$, $y = 2$ and $z = 1$, find the value of :
   (i) x^y
   (ii) y^x
   (iii) $3x^2 - 5y^2$
   (iv) $2x - 3y + 4z + 5$
   (v) $y^2 - x^2 + 6z^2$
   (vi) $xy + y^2z - 4zx$
4. If $P = -12x^2 - 10xy + 5y^2$, $Q = 7x^2 + 6xy + 2y^2$ and $R = 5x^2 + 2xy + 4y^2$, find :
   (i) $P - Q$
   (ii) $Q + P$
   (iii) $P - Q + R$
   (iv) $P + Q + R$
5. If $x = a^2 - bc$, $y = b^2 - ca$ and $z = c^2 - ab$, find the value of : (i) $ax + by + cz$ (ii) $ay - bx + cz$
6. Multiply and then evaluate :
   (i) $(4x + y)$ and $(x - 2y)$, when $x = 2$ and $y = 1$.
   (ii) $(x^2 - y)$ and $(xy - y^2)$, when $x = 1$ and $y = 2$.
   (iii) $(x - 2y + z)$ and $(x - 3z)$, when $x = -2$, $y = -1$ and $z = 1$.
7. Simplify :
   (i) $5(x + 3y) - 2(3x - 4y)$
   (ii) $3x - 8(5x - 10)$
   (iii) $6\{3x - 8(5x - 10)\}$
   (iv) $3x - 6\{3x - 8(5x - 10)\}$
   (v) $2(3x^2 - 4x - 8) - (3 - 5x - 2x^2)$
   (vi) $8x - (3x - \overline{2x - 3})$
   (vii) $12x^2 - (7x - \overline{3x^2 + 15})$
8. If $x = -3$, find the value of $2x^3 + 8x^2 - 15$.