

FUNDAMENTAL OPERATIONS

[Related to Algebraic Expressions]

13.1 BASIC CONCEPT

In Mathematics, the operations **addition** (+), **subtraction** (-), **multiplication** (×) and **division** (÷) are the **four fundamental operations**.

Students are familiar with these operations as they have already studied about these operations in Arithmetic in their lower classes.

(i) ADDITION OF LIKE TERMS :

The addition of like terms is a single term (like to the given terms) whose coefficient is equal to the sum of the coefficients of the given (like) terms.

Thus :

$$(i) \quad \text{Addition of } 3x \text{ and } 8x = 3x + 8x = (3 + 8)x = \mathbf{11x}$$

$$(ii) \quad \text{Addition of } 8x^2y \text{ and } -5x^2y = 8x^2y + (-5x^2y) \\ = 8x^2y - 5x^2y = (8 - 3)x^2y = \mathbf{5x^2y}$$

$$(iii) \quad 2xy + 3xy + 5xy = (2 + 3 + 5)xy = \mathbf{10xy}$$

$$(iv) \quad 7y^2 - 4y^2 + 3y^2 = (7 - 4 + 3)y^2 \\ = (10 - 4)y^2 = \mathbf{6y^2}$$

For addition, the terms are taken with their given signs, e.g.

$$(i) \quad \text{addition of } 7xy \text{ and } -3xy = 7xy - 3xy = (7 - 3)xy = \mathbf{4xy}$$

$$(ii) \quad \text{addition of } -7xy \text{ and } 3xy = -7xy + 3xy = (-7 + 3)xy = \mathbf{-4xy}$$

$$\text{and (iii)} \quad -7x - 3x = (-7 - 3)x = \mathbf{-10x}$$

In the same way :

$$(i) \quad \text{addition of } -3xy^2, -5xy^2 \text{ and } -xy^2 \\ = (-3xy^2) + (-5xy^2) + (-xy^2) \\ = -3xy^2 - 5xy^2 - xy^2 \\ = (-3 - 5 - 1)xy^2 = \mathbf{-9xy^2}$$

$$(ii) \quad \text{addition of } 7ab, -2ab, -5ab, 6ab \text{ and } -ab \\ = 7ab - 2ab - 5ab + 6ab - ab \\ = (7 - 2 - 5 + 6 - 1)ab \\ = (13 - 8)ab = \mathbf{5ab}$$

(ii) ADDITION OF UNLIKE TERMS :

As shown above, the sum of two or more like terms is a single like term, but two unlike terms cannot be added together to get a single term.

For example : the unlike terms $2ab$ and $4bc$ cannot be added together to form a single term. All that can be done is to connect them by the sign of addition and leave the result in the form $2ab + 4bc$.

In the same way,

(i) addition of $5x^2$ and $8xy$

$$= 5x^2 + 8xy$$

(ii) addition of $2y^3$, $-5xy$ and $3x^3$

$$= 2y^3 - 5xy + 3x^3 \quad \text{and so on}$$

(iii) SUBTRACTION OF LIKE TERMS :

For subtraction of like terms, the rules are the same as those for subtraction of integers.

For example :

Since $-4 + 2 = -2$,

$$-4 + 2 = -2$$

$\therefore -4x + 2x = -2x$

Since $3 - 7 = -4$,

$$3 - 7 = -4$$

$\therefore 3x - 7x = -4x$

Example 1 :

Subtract : (i) $4x$ from $-8x$

(ii) $-3x$ from $-7x$

Solution :

In each subtraction, change the sign of the term to be subtracted.

(i) $-8x - (4x) = -8x - 4x = -12x$

(Ans.)

(ii) $-7x - (-3x) = -7x + 3x = -4x$

(Ans.)

The result of subtraction of two like terms is also a like term.

(iv) SUBTRACTION OF UNLIKE TERMS :

Just as it is with addition of unlike terms, we cannot get a single term by the subtraction of unlike terms. For example, $2ab$ and $4bc$ are two unlike terms; the subtraction of $2ab$ from $4bc$ is $4bc - 2ab$, which cannot be simplified further to get a single term.

Similarly, the subtraction of $4bc$ from $2ab$ is $2ab - 4bc$, which cannot be simplified further to get a single term.

Example 2 :

Evaluate : (i) $3x - 4x + 7x$

(ii) $6ab + 3ab - 4ab$

(iii) $5ax + ax - 8ax$

(iv) $8a + 3a - 5a - 2a$

Solution :

1. Add the positive terms together and separately add the negative terms together as well.
2. Find the result of the two terms obtained.

(i) $3x - 4x + 7x = 10x - 4x$
 $= 6x$

$$3x + 7x = 10x$$

(Ans.)

$$(ii) \quad 6ab + 3ab - 4ab = 9ab - 4ab = 5ab \quad (\text{Ans.})$$

$$(iii) \quad 5ax + ax - 8ax = 6ax - 8ax = -2ax \quad (\text{Ans.})$$

$$(iv) \quad 8a + 3a - 5a - 2a = 11a - 7a = 4a \quad (\text{Ans.})$$

Example 3 :

Evaluate :

$$(i) \quad 3x + 1\frac{2}{5}x$$

$$(ii) \quad 5a - 2\frac{1}{2}a + 1\frac{1}{2}a$$

$$(iii) \quad 4\frac{5}{6}xy - 2\frac{1}{3}xy - 1\frac{1}{2}xy$$

Solution :

$$(i) \quad 3x + 1\frac{2}{5}x = \frac{3x}{1} + \frac{7x}{5}$$

$$= \frac{15x + 7x}{5} = \frac{22x}{5} = 4\frac{2}{5}x \quad (\text{Ans.})$$

$1\frac{2}{5} = \frac{1 \times 5 + 2}{5} = \frac{7}{5}$

$$(ii) \quad 5a - 2\frac{1}{2}a + 1\frac{1}{2}a = \frac{5a}{1} - \frac{5a}{2} + \frac{3a}{2}$$

$$= \frac{10a - 5a + 3a}{2} = \frac{13a - 5a}{2} = \frac{8a}{2} = 4a \quad (\text{Ans.})$$

$$(iii) \quad 4\frac{5}{6}xy - 2\frac{1}{3}xy - 1\frac{1}{2}xy = \frac{29}{6}xy - \frac{7}{3}xy - \frac{3}{2}xy$$

$$= \frac{29xy - 14xy - 9xy}{6}$$

L.C.M. of 6, 3 and 2 = 6

$$= \frac{29xy - 23xy}{6} = \frac{6xy}{6} = xy \quad (\text{Ans.})$$

EXERCISE 13(A)

1. Fill in the blanks :

$$(i) \quad 5 + 4 = \dots\dots\dots \quad \text{and} \quad 5x + 4x = \dots\dots\dots$$

$$(ii) \quad 12 + 18 = \dots\dots\dots \quad \text{and} \quad 12x^2y + 18x^2y = \dots\dots\dots$$

$$(iii) \quad 7 + 16 = \dots\dots\dots \quad \text{and} \quad 7a + 16b = \dots\dots\dots$$

$$(iv) \quad 1 + 3 = \dots\dots\dots \quad \text{and} \quad x^2y + 3xy^2 = \dots\dots\dots$$

$$(v) \quad 7 - 4 = \dots\dots\dots \quad \text{and} \quad 7ab - 4ab = \dots\dots\dots$$

$$(vi) \quad 12 - 5 = \dots\dots\dots \quad \text{and} \quad 12x - 5y = \dots\dots\dots$$

$$(vii) \quad 35 - 16 = \dots\dots\dots \quad \text{and} \quad 35ab - 16ba = \dots\dots\dots$$

$$(viii) \quad 28 - 13 = \dots\dots\dots \quad \text{and} \quad 28ax^2 - 13a^2x = \dots\dots\dots$$

2. Fill in the blanks :

$$(i) \quad \text{The sum of } -2 \text{ and } -5 = \dots\dots\dots \quad \text{and the sum of } -2x \text{ and } -5x = \dots\dots\dots$$

$$(ii) \quad \text{The sum of } 8 \text{ and } -3 = \dots\dots\dots \quad \text{and the sum of } 8ab \text{ and } -3ab = \dots\dots\dots$$

$$(iii) \quad \text{The sum of } -15 \text{ and } -4 = \dots\dots\dots \quad \text{and the sum of } -15x \text{ and } -4y = \dots\dots\dots$$

$$(iv) \quad 15 + 8 + 3 = \dots\dots\dots \quad \text{and } 15x + 8y + 3x = \dots\dots\dots$$

(v) $12 - 9 + 15 = \dots\dots\dots$ and $12ab - 9ab + 15ba = \dots\dots\dots$

(vi) $25 - 7 - 9 = \dots\dots\dots$ and $25xy - 7xy - 9yx = \dots\dots\dots$

(vii) $-4 - 6 - 5 = \dots\dots\dots$ and $-4ax - 6ax - 5ay = \dots\dots\dots$

3. Add :

(i) $8xy$ and $3xy$

(ii) $2xyz$, xyz and $6xyz$

(iii) $2a$, $3a$ and $4b$

(iv) $3x$ and $2y$

(v) $5m$, $3n$ and $4p$

(vi) $6a$, $3a$ and $9ab$

(vii) $3p$, $4q$ and $9q$

(viii) $5ab$, $4ba$ and $6b$

(ix) $50pq$, $30pq$ and $10pr$

(x) $-2y$, $-y$ and $-3y$

(xi) $-3b$ and $-b$

(xii) $5b$, $-4b$ and $-10b$

(xiii) $-2c$, $-c$ and $-5c$

4. Evaluate :

(i) $6a - a - 5a - 2a$

(ii) $2b - 3b - b + 4b$

(iii) $3x - 2x - 4x + 7x$

(iv) $5ab + 2ab - 6ab + ab$

(v) $8x - 5y - 3x + 10y$

5. Evaluate :

(i) $-7x + 9x + 2x - 2x$

(ii) $5ab - 2ab - 8ab + 6ab$

(iii) $-8a - 3a + 12a + 13a - 6a$

(iv) $19abc - 11abc - 12abc + 14abc$

6. Evaluate :

(i) $3\frac{1}{2}x + 4x - 5\frac{1}{2}x$

(ii) $7b - 2\frac{1}{3}b - 1\frac{1}{3}b$

(iii) $5ax + 3\frac{1}{5}ax + \frac{2}{5}ax$

(iv) $9y - 7\frac{3}{4}y + 2\frac{1}{4}y$

(v) $3xy - 2\frac{1}{2}xy + 1\frac{2}{3}xy$

7. Subtract the first term from the second :

(i) $4ab$, $6ba$

(ii) $4\cdot 8b$, $6\cdot 8b$

(iii) $3\cdot 5abc$, $10\cdot 5abc$

(iv) $3\frac{1}{2}mn$, $8\frac{1}{2}nm$

8. Simplify :

(i) $2a^2b^2 + 5ab^2 + 8a^2b^2 - 3ab^2$

(ii) $4a + 3b - 2a - b$

(iii) $2xy + 4yz + 5xy + 3yz - 6xy$

(iv) $ab + 15ab - 11ab - 2ab$

(v) $6a^2 - 3b^2 + 2a^2 + 5b^2 - 4a^2$

(vi) $8abc + 2ab - 4abc + ab$

(vii) $9xyz + 15yxz - 10zyx - 2zxy$

(viii) $13pqr + 2p + 4q - 6pqr + 5pqr$

(ix) $4ab + 0 - 2ba$

(x) $6x^2y - 2xy^2 + 5x^2y - xy^2$

(xi) $6\cdot 4a + 5\cdot 3b - 2\cdot 4a - 2\cdot 2b$

(xii) $2\cdot 5a + 4\cdot 6b + 1\cdot 2a - 3\cdot 6b$

(xiii) $22m - 12\frac{1}{2}n - 15p + 16n$

(xiv) $6p + \frac{2}{3}q - 1\frac{1}{2}p + \frac{1}{3}q + 2q$

(xv) $2\frac{2}{3}xy - 3\frac{1}{2}xy + 3\frac{1}{3}xy - 2\frac{1}{2}xy$

13.2 MORE ABOUT ADDITION AND SUBTRACTION

(i) Addition of Polynomials :

Example 4 :

Add : $4a + 2b$, $3a - 3b + c$ and $-2a + 4b + 2c$.

Solution :

First method (Row method) :

Steps :

1. Write all the given polynomials in a row.
2. Group the like terms.
3. Add the like terms.

The required addition

$$= (4a + 2b) + (3a - 3b + c) + (-2a + 4b + 2c) \quad \text{[Step 1]}$$

$$= 4a + 2b + 3a - 3b + c - 2a + 4b + 2c \quad \text{[Step 2]}$$

$$= 4a + 3a - 2a + 2b - 3b + 4b + c + 2c \quad \text{[Step 3]}$$

$$= \mathbf{5a + 3b + 3c} \quad \text{(Ans.)}$$

Second method (Column method) :

Arrange the given polynomials so that the like terms of the polynomials are one below the other in a vertical column; then add.

$$\begin{array}{r} \therefore 4a + 2b \\ 3a - 3b + c \\ -2a + 4b + 2c \\ \hline 5a + 3b + 3c \end{array} \quad \text{(Ans.)}$$

In general, the column method is preferred.

Example 5 :

- (i) Add : $3x^3 - 5x^2 + 8x + 10$, $15x^3 - 6x - 23$ and $9x^2 - 4x + 15$.
- (ii) Add : $3ab^2 - 2b^2 + a^2$, $5a^2b - 2ab^2 - 3a^2$ and $8a^2 - 5b^2$.

Solution :

Using the row method :

$$(i) \quad (3x^3 - 5x^2 + 8x + 10) + (15x^3 - 6x - 23) + (9x^2 - 4x + 15)$$

$$= 3x^3 - 5x^2 + 8x + 10 + 15x^3 - 6x - 23 + 9x^2 - 4x + 15$$

$$= 3x^3 + 15x^3 - 5x^2 + 9x^2 + 8x - 6x - 4x + 10 - 23 + 15$$

$$= 18x^3 + 4x^2 + 8x - 10x + 25 - 23$$

$$= \mathbf{18x^3 + 4x^2 - 2x + 2} \quad \text{(Ans.)}$$

$$(ii) \quad (3ab^2 - 2b^2 + a^2) + (5a^2b - 2ab^2 - 3a^2) + (8a^2 - 5b^2)$$

$$= 3ab^2 - 2b^2 + a^2 + 5a^2b - 2ab^2 - 3a^2 + 8a^2 - 5b^2$$

$$= 5a^2b + 3ab^2 - 2ab^2 + a^2 - 3a^2 + 8a^2 - 2b^2 - 5b^2$$

$$= \mathbf{5a^2b + ab^2 + 6a^2 - 7b^2} \quad \text{(Ans.)}$$

Using the column method :

$$\begin{array}{r} \text{(i)} \quad 3x^3 - 5x^2 + 8x + 10 \\ 15x^3 \quad \quad - 6x - 23 \\ + 9x^2 - 4x + 15 \\ \hline \end{array}$$

$$\underline{18x^3 + 4x^2 - 2x + 2} \quad \text{(Ans.)}$$

$$\begin{array}{r} \text{(ii)} \quad 3ab^2 - 2b^2 + a^2 \\ 5a^2b - 2ab^2 \quad - 3a^2 \\ \quad \quad \quad - 5b^2 + 8a^2 \\ \hline \end{array}$$

$$\underline{5a^2b + ab^2 - 7b^2 + 6a^2} \quad \text{(Ans.)}$$

(ii) Subtraction in Polynomials :**Steps** (for the row method) :

1. Enclose the expression to be subtracted in brackets with a minus sign prefixed.
2. Remove the bracket by changing the sign of each term kept in the bracket.

Examples :

$$\text{(i)} \quad (2x - y) - (x + 5y) = 2x - y - x - 5y$$

$$\text{(ii)} \quad (3a + b - c) - (2a - 3b + c) = 3a + b - c - 2a + 3b - c, \text{ etc.}$$

3. Combine the like terms and add.

Example 6 :

Subtract : $3a - 4b + 5c$ from $4a - b + 6c$.

Solution :

$$4a - b + 6c - (3a - 4b + 5c) \quad \text{[Step 1]}$$

$$= 4a - b + 6c - 3a + 4b - 5c \quad \text{[Step 2]}$$

$$= 4a - 3a - b + 4b + 6c - 5c \quad \text{[Step 3]}$$

$$= a + 3b + c \quad \text{(Ans.)}$$

Row method is used

Whenever there is a negative sign before a bracket, open (remove) the bracket and, at the same time, change the sign of each term inside the bracket.

$$\text{e.g. } (x + y) - (x - y + z) = x + y - x + y - z = 2y - z$$

Alternative method (column method) :**Steps** (for the column method) :

1. Rewrite the given expressions in two lines (rows) such that the lower line is the expression to be subtracted and like terms of both the expressions are one below the other.
2. Change the sign of each term in the lower line, *i.e.* change the sign of each term of the expression to be subtracted.
3. Add column-wise.

Thus, for Example 6 given above, we have :

$$\text{Step 1 :} \quad \begin{array}{r} 4a - b + 6c \\ 3a - 4b + 5c \\ \hline \end{array}$$

$$\text{Step 2 :} \quad \begin{array}{r} - \quad + \quad - \\ \hline \end{array}$$

$$\text{Step 3 :} \quad \underline{a + 3b + c} \quad \text{(Ans.)}$$

Example 7 :

From the sum of $5x^2 - 7x + 4$ and $-3x^2 + 5x + 2$ subtract $x^2 + x + 1$.

Solution :**Row method :**

$$\begin{aligned} & (5x^2 - 7x + 4) + (-3x^2 + 5x + 2) - (x^2 + x + 1) \\ &= 5x^2 - 7x + 4 - 3x^2 + 5x + 2 - x^2 - x - 1 \\ &= 5x^2 - 3x^2 - x^2 - 7x + 5x - x + 4 + 2 - 1 \\ &= 5x^2 - 4x^2 - 8x + 5x + 6 - 1 \\ &= x^2 - 3x + 5 \end{aligned}$$

Column method :

$$\begin{array}{r} 5x^2 - 7x + 4 \\ - 3x^2 + 5x + 2 \\ \hline 2x^2 - 2x + 6 \\ x^2 + x + 1 \\ \hline \text{Subtract} \\ \hline x^2 - 3x + 5 \quad (\text{Ans.}) \end{array}$$

Example 8 :

What should be added to :

(i) $8a$ to get $15a$?

(ii) $6m - 3n$ to make $4m + 2n$?

Solution :

(i) $15a - 8a = 7a$ (Ans.)

(ii) $4m + 2n - (6m - 3n) = 4m + 2n - 6m + 3n$
 $= 4m - 6m + 2n + 3n = -2m + 5n = 5n - 2m$ (Ans.)

Example 9 :

(i) By how much is $3a + 2b$ greater than $5a + b$?

(ii) Take out $3x + 2y - 6z$ from $8x - 4y - 2z$.

(iii) By how much should $2m + 7n - 4p$ be increased to get $6p$?

Solution :

(i) $3a + 2b - (5a + b) = 3a + 2b - 5a - b$
 $= 3a - 5a + 2b - b = -2a + b$ (Ans.)

(ii) $8x - 4y - 2z - (3x + 2y - 6z) = 8x - 4y - 2z - 3x - 2y + 6z$
 $= 8x - 3x - 4y - 2y - 2z + 6z$
 $= 5x - 6y + 4z$ (Ans.)

(iii) $6p - (2m + 7n - 4p) = 6p - 2m - 7n + 4p$
 $= 6p + 4p - 2m - 7n = 10p - 2m - 7n$ (Ans.)

EXERCISE 13(B)

1. Find the sum of :

- | | | | | |
|-------|--------------------|---------------------|-----|-----------------------|
| (i) | $3a + 4b + 7c,$ | $-5a + 3b - 6c$ | and | $4a - 2b - 4c.$ |
| (ii) | $2x^2 + xy - y^2,$ | $-x^2 + 2xy + 3y^2$ | and | $3x^2 - 10xy + 4y^2.$ |
| (iii) | $x^2 - x + 1,$ | $-5x^2 + 2x - 2$ | and | $3x^2 - 3x + 1.$ |
| (iv) | $a^2 - ab + bc,$ | $2ab + bc - 2a^2$ | and | $-3bc + 3a^2 + ab.$ |
| (v) | $4x^2 + 7 - 3x,$ | $4x - x^2 + 8$ | and | $-10 + 5x - 2x^2.$ |
| (vi) | $3x + 4xy - y^2,$ | $xy - 4x + 2y^2$ | and | $3y^2 - xy + 6x.$ |

2. Add the following expressions :

- (i) $-17x^2 - 2xy + 23y^2$, $-9y^2 + 15x^2 + 7xy$ and $13x^2 + 3y^2 - 4xy$.
 (ii) $-x^2 - 3xy + 3y^2 + 8$, $3x^2 - 5y^2 - 3 + 4xy$ and $-6xy + 2x^2 - 2 + y^2$.
 (iii) $a^3 - 2b^3 + a$, $b^3 - 2a^3 + b$ and $-2b + 2b^3 - 5a + 4a^3$.

3. Evaluate :

- (i) $3a - (a + 2b)$ (ii) $(5x - 3y) - (x + y)$ (iii) $(8a + 15b) - (3b - 7a)$
 (iv) $(8x + 7y) - (4y - 3x)$ (v) $7 - (4a - 5)$ (vi) $(6y - 13) - (4 - 7y)$

4. Subtract :

- (i) $5a - 3b + 2c$ from $a - 4b - 2c$. (ii) $4x - 6y + 3z$ from $12x + 7y - 21z$.
 (iii) $5 - a - 4b + 4c$ from $5a - 7b + 2c$. (iv) $-8x - 12y + 17z$ from $x - y - z$.
 (v) $2ab + cd - ac - 2bd$ from $ab - 2cd + 2ac + bd$.

5. (i) Take $-ab + bc - ca$ from $bc - ca + ab$.

(ii) Take $5x + 6y - 3z$ from $3x + 5y - 4z$.

(iii) Take $\frac{-3}{2}p + q - r$ from $\frac{1}{2}p - \frac{1}{3}q - \frac{3}{2}r$.

(iv) Take $1 - a + a^2$ from $a^2 + a + 1$.

6. (i) What should be added to $3x$ to get $5x$?

(ii) What should be added to $-x$ to get $4x$?

(iii) What should be added to $a - b$ to get $2a + b$?

(iv) By how much is $-3x$ less than $3x$?

(v) By how much is $2x + y$ greater than $x - 2y$?

(vi) What should be added to $2a - b + c$ to make it $a + b - 2c$?

7. From the sum of $x + y - 2z$ and $2x - y + z$ subtract $x + y + z$.

8. From the sum of $3a - 2b + 4c$ and $3b - 2c$ subtract $a - b - c$.

9. Subtract $x - 2y - z$ from the sum of $3x - y + z$ and $x + y - 3z$.

10. Subtract the sum of $x + y$ and $x - z$ from the sum of $x - 2z$ and $x + y + z$.

11. By how much should $x + 2y - 3z$ be increased to get $3x$?

12. The sum of two expressions is $5x^2 - 3y^2$. If one of them is $3x^2 + 4xy - y^2$, find the other.

13. The sum of two expressions is $3a^2 + 2ab - b^2$. If one of them is $2a^2 + 3b^2$, find the other.

13.3 MULTIPLICATION

(i) **Multiplication of monomials :**

We know, $a^2 = a \times a$ and $a^3 = a \times a \times a$

\therefore Multiplication of a^2 and $a^3 = a^2 \times a^3$

$$= (a \times a) \times (a \times a \times a)$$

$$= a \times a \times a \times a \times a = a^5$$

or, simply : $a^2 \times a^3 = a^{2+3} = a^5$

In multiplication, the powers of like factors are added

1. Product Law used in exponents :

Students already know : $a^m \times a^n = a^{m+n}$

$$(i) x^5 \times x^7 = x^{5+7} = x^{12}.$$

$$(ii) x^2y^3 \times x^6y^4 \\ = x^{2+6} \times y^{3+4} = x^8y^7.$$

$$(iii) xy^2z^5 \times x^4y^3z^2 \times x^7y^5z^4 \\ = x^{1+4+7} \times y^{2+3+5} \times z^{5+2+4} \\ = x^{12}y^{10}z^{11} \quad \text{and so on}$$

2. The multiplication (product) of given monomials

= (Multiplication of their coefficients) \times (multiplication of their literals).

Example 10 :

Multiples : (i) $8x^2y^3$, $6y^2z^5$ and $3xz^2$

(ii) $-5a^2xy^3$, $\frac{7}{3}ax^3y^2$ and $-9xy$

Solution :

$$(i) 8x^2y^3 \times 6y^2z^5 \times 3xz^2$$

= (Multiplication of their coefficients) \times (multiplication of their literals)

$$= (8 \times 6 \times 3) \times (x^2 \times x) \times (y^3 \times y^2) \times (z^5 \times z^2)$$

$$= 144 \times x^3 \times y^5 \times z^7 = 144x^3y^5z^7 \quad (\text{Ans.})$$

$$(ii) -5a^2xy^3 \times \frac{7}{3}ax^3y^2 \times -9xy$$

$$= \left(-5 \times \frac{7}{3} \times -9\right) \times (a^2 \times a) \times (x \times x^3 \times x) \times (y^3 \times y^2 \times y)$$

$$= 105a^3x^5y^6 \quad (\text{Ans.})$$

(ii) Multiplication of a polynomial and a monomial :

Multiplication of $4x^2y - 3xy^2 + 4xy$ and $2xy$

$$= 2xy \times (4x^2y - 3xy^2 + 4xy)$$

$$= (2xy \times 4x^2y) - (2xy \times 3xy^2) + (2xy \times 4xy)$$

$$= 8x^3y^2 - 6x^2y^3 + 8x^2y^2$$

Multiply each term of the polynomial by the monomial

In the same way :

$$(a) 5x^2(3x^3y^2 - 2xy^3 + y^5)$$

$$= (5x^2 \times 3x^3y^2) - (5x^2 \times 2xy^3) + (5x^2 \times y^5)$$

$$= 15x^5y^2 - 10x^3y^3 + 5x^2y^5$$

$$(b) -3a(4a^2 - 8a^3 + 7a^4)$$

$$= -12a^3 + 24a^4 - 21a^5$$

(iii) Multiplication of two binomials :**Steps :**

1. Multiply each term of the first binomial by each term of the other one.
2. In the product obtained, combine the like terms.

$$\begin{aligned}
 \text{(i) Multiplication of } x + 3 \text{ and } x + 5 &= (x + 3) \cdot (x + 5) \\
 &= x \cdot (x + 5) + 3 \cdot (x + 5) \\
 &= x \cdot x + x \cdot 5 + 3 \cdot x + 15 \\
 &= x^2 + 5x + 3x + 15 \\
 &= \mathbf{x^2 + 8x + 15}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } (3x - 2y)(5x + 3y) &= 3x(5x + 3y) - 2y(5x + 3y) \\
 &= 15x^2 + 9xy - 10xy - 6y^2 \\
 &= 15x^2 - xy - 6y^2
 \end{aligned}$$

Alternative method :**Steps :**

1. Write the binomials one below the other.
2. Multiply the first term of the binomial in the lower line with each term of the binomial in the upper line.
3. Multiply the second term of the binomial in the lower line with each term of the binomial in the upper line; then place the like terms one below the other.
4. Add the like terms column-wise.

\therefore Multiplication of $x + 3$ and $x + 5$ is :

$$\begin{array}{r}
 x + 3 \\
 \times x + 5 \\
 \hline
 x^2 + 3x \\
 + 5x + 15 \\
 \hline
 \mathbf{x^2 + 8x + 15}
 \end{array}$$

$$[\because x \cdot (x + 3) = x^2 + 3x]$$

$$[\because 5 \cdot (x + 3) = 5x + 15]$$

On adding :

(Ans.)

Example 11 :

Multiply : $3x - 4y$ and $4x + 5y$

Solution :

$$\begin{aligned}
 (3x - 4y) \cdot (4x + 5y) &= 3x \cdot (4x + 5y) - 4y \cdot (4x + 5y) \\
 &= 12x^2 + 15xy - 16xy - 20y^2 = \mathbf{12x^2 - xy - 20y^2} \quad \text{(Ans.)}
 \end{aligned}$$

Alternative method :

$$\begin{array}{r}
 3x - 4y \\
 \times 4x + 5y \\
 \hline
 12x^2 - 16xy \\
 + 15xy - 20y^2 \\
 \hline
 \mathbf{12x^2 - xy - 20y^2}
 \end{array}$$

$$[\because 4x \cdot (3x - 4y) = 12x^2 - 16xy]$$

$$[\because 5y \cdot (3x - 4y) = 15xy - 20y^2]$$

On adding :

(Ans.)

EXERCISE 13(C)

1. Fill in the blanks :

- | | | | |
|--------|---------------------------------|-----|---|
| (i) | $6 \times 3 = \dots\dots\dots$ | and | $6x \times 3x = \dots\dots\dots$ |
| (ii) | $6 \times 3 = \dots\dots\dots$ | and | $6x^2 \times 3x^3 = \dots\dots\dots$ |
| (iii) | $5 \times 4 = \dots\dots\dots$ | and | $5x \times 4y = \dots\dots\dots$ |
| (iv) | $4 \times 7 = \dots\dots\dots$ | and | $4ax \times 7x = \dots\dots\dots$ |
| (v) | $6 \times 2 = \dots\dots\dots$ | and | $6xy \times 2xy = \dots\dots\dots$ |
| (vi) | $12 \times 4 = \dots\dots\dots$ | and | $12ax^2 \times 4ax = \dots\dots\dots$ |
| (vii) | $1 \times 8 = \dots\dots\dots$ | and | $a^2xy^2 \times 8a^3x^2y = \dots\dots\dots$ |
| (viii) | $15 \times 3 = \dots\dots\dots$ | and | $15x \times 3x^5y^2 = \dots\dots\dots$ |

2. Fill in the blanks :

- | | | | |
|-------|---|--------|--|
| (i) | $4x \times 6x \times 2 = \dots\dots\dots$ | (ii) | $3ab \times 6ax = \dots\dots\dots$ |
| (iii) | $x \times 2x^2 \times 3x^3 = \dots\dots\dots$ | (iv) | $5 \times 5a^3 = \dots\dots\dots$ |
| (v) | $6 \times 6x^2 \times 6x^2y^2 = \dots\dots\dots$ | (vi) | $-8x \times -3x = \dots\dots\dots$ |
| (vii) | $-5 \times -3x \times 5x^2 = \dots\dots\dots$ | (viii) | $8 \times -4xy^2 \times 3x^3y^2 = \dots\dots\dots$ |
| (ix) | $-4x \times 5xy \times 3z = \dots\dots\dots$ | | |
| (x) | $5x \times 2x^2y \times -7y^3 \times 2x^3y^2 = \dots\dots\dots$ | | |

3. Find the value of :

- | | | | | | |
|------|-------------------------|------|--------------------------|-------|---------------------|
| (i) | $3x^3 \times 5x^4$ | (ii) | $5a^2 \times 7a^7$ | (iii) | $3abc \times 6ac^3$ |
| (iv) | $a^2b^2 \times 5a^3b^4$ | (v) | $2x^2y^3 \times 5x^3y^4$ | (vi) | $abc \times bcd$ |

4. Multiply :

- | | | | | | |
|-------|--------------------------|--------|---------------------|-------|----------------------|
| (i) | $a + b$ by ab | (ii) | $3ab - 4b$ by $3ab$ | (iii) | $2xy - 5by$ by $4bx$ |
| (iv) | $4x + 2y$ by $3xy$ | (v) | $x^2 - x$ by $2x$ | (vi) | $1 + 4x$ by x |
| (vii) | $9xy^2 + 3x^2y$ by $5xy$ | (viii) | $6x - 5y$ by $3axy$ | | |

5. Multiply :

- | | | | | | |
|------|----------------------------|------|-----------------------------|-------|---------------------------|
| (i) | $-x + y - z$ and $-2x$ | (ii) | $xy - yz$ and x^2yz^2 | (iii) | $2xyz + 3xy$ and $-2y^2z$ |
| (iv) | $-3xy^2 + 4x^2y$ and $-xy$ | (v) | $4xy$ and $-x^2y - 3x^2y^2$ | | |

6. Multiply :

- | | | | |
|-----|-------------------------|------|----------------------------|
| (i) | $3a + 4b - 5c$ and $3a$ | (ii) | $-5xy$ and $-xy^2 - 6x^2y$ |
|-----|-------------------------|------|----------------------------|

7. Multiply :

- | | | | | | |
|-------|-----------------------|--------|-------------------------|-------|------------------------|
| (i) | $x + 2$ and $x + 10$ | (ii) | $x + 5$ and $x - 3$ | (iii) | $x - 5$ and $x + 3$ |
| (iv) | $x - 5$ and $x - 3$ | (v) | $2x + y$ and $x + 3y$ | (vi) | $3x - 5y$ and $x + 6y$ |
| (vii) | $x + 9y$ and $x - 5y$ | (viii) | $2x + 5y$ and $2x + 5y$ | | |

8. Multiply :

- | | | | |
|-------|---|------|--|
| (i) | $3abc$ and $-5a^2b^2c$ | (ii) | $x - y + z$ and $-2x$ |
| (iii) | $2x - 3y - 5z$ and $-2y$ | (iv) | $-8xyz + 10x^2yz^3$ and xyz |
| (v) | xyz and $-13xy^2z + 15x^2yz - 6xyz^2$ | (vi) | $4abc - 5a^2bc - 6ab^2c$ and $-2abc^2$ |

9. Find the product of :

- | | | | |
|-------|----------------------------------|------|--------------------------------|
| (i) | $xy - ab$ and $xy + ab$ | (ii) | $2abc - 3xy$ and $2abc + 3xy$ |
| (iii) | $a + b - c$ and $2a - 3b$ | (iv) | $5x - 6y - 7z$ and $2x + 3y$ |
| (v) | $5x - 6y - 7z$ and $2x + 3y + z$ | (vi) | $2a + 3b - 4c$ and $a - b - c$ |

Example 12 :

Multiply :

- (i) $\frac{3}{5}x^2y$ and $\frac{5}{6}x^3y^5$ (ii) $\frac{3}{7}ab^3$ and $-\frac{14}{15}a^2b$
 (iii) $-2m^2n^2$ and $\frac{7}{16}mn^4$ (iv) $-5ax^3$ and $-\frac{2}{5}a^3x$.

Solution :

- (i) $\frac{3}{5}x^2y \times \frac{5}{6}x^3y^5 = \left(\frac{3}{5} \times \frac{5}{6}\right) \times x^{2+3} \times y^{1+5} = \frac{1}{2}x^5y^6$ (Ans.)
 (ii) $\frac{3}{7}ab^3 \times -\frac{14}{15}a^2b = \left(\frac{3}{7} \times -\frac{14}{15}\right) \times a^{1+2} \times b^{3+1}$
 $= -\frac{2}{5}a^3b^4$ (Ans.)
 (iii) $-2m^2n^2 \times \frac{7}{16}mn^4 = \left(-2 \times \frac{7}{16}\right) \times m^{2+1}n^{2+4}$
 $= -\frac{7}{8}m^3n^6$ (Ans.)
 (iv) $-5ax^3 \times -\frac{2}{5}a^3x = \left(-5 \times -\frac{2}{5}\right) \times a^{1+3} \times x^{3+1}$
 $= 2a^4x^4$ (Ans.)

The product of any term/expression with zero (0) is always equal to zero.

For example :

- (i) $3x^2 \times 0 = 0$ (ii) $0 \times (-7xy^2z) = 0$ (iii) $4m^2 \times 0 \times 2n^5 = 0$
 (iv) $6a^3b^2 \times 2m^2x^2 \times 0 \times 7a^3m^2 = 0$ and so on.

EXERCISE 13(D)

1. Multiply :

- (i) 6 and $\frac{3}{2}x$ (ii) $2\frac{1}{3}m$ and 9 (iii) $5ab$ and $1\frac{3}{5}$
 (iv) $\frac{3}{8}xy$ and $3\frac{3}{7}$ (v) $-\frac{2}{5}x$ and $\frac{5}{4}y$ (vi) $\frac{8}{7}m^2$ and $-\frac{7}{16}mn$
 (vii) $-\frac{6}{7}am^2$ and $-\frac{14}{9}a^3$ (viii) 0 and $-\frac{5}{6}xyz$ (ix) $\frac{8}{15}x^2y^3$ and 0
 (x) $-\frac{9}{16}m^3y^2$ and $3\frac{1}{5}y^7m^6$ (xi) $\frac{4}{25}x^2y^3$ and $5\frac{5}{19}x^4y$

2. Multiply :

- (i) $5x$, $3x^2$ and x^3 (ii) 2, $8m^2$ and $4m^5$ (iii) 0, $8ax$ and $\frac{7}{8}bx$
 (iv) $-\frac{4}{5}y^3$, 0 and $\frac{5}{8}y^4$ (v) $2mn$, $-3mn$ and $5mn$ (vi) $7ab$, $-2ab$ and $-ab$
 (vii) $2\frac{1}{2}x$, $1\frac{1}{2}x^2$ and 8 (viii) $-\frac{2}{5}ab$, $2\frac{1}{2}a^2$ and $4b^2$ (ix) $\frac{2}{3}xy$, $5\frac{1}{2}x^2$ and $\frac{3}{11}y^2$

3. Multiply :

(i) $\frac{2}{3}xy - 2\frac{1}{2}x^2 + 1\frac{1}{6}y^2$ and $6xy$

(ii) $mn^3 + 2\frac{3}{5}m^2n - \frac{2}{5}m^3$ by $-10m^2n^2$

(iii) $\frac{1}{2}ax - \frac{3}{2}bx^2 - 2\frac{1}{2}cx^3$ by $4ax^2$

4. Evaluate :

(i) $(\frac{3}{5}ax^2y) \times (-15a^2y)$

(ii) $(2\frac{1}{3}xy) \times (1\frac{2}{7}x^2) \times (\frac{2}{3}xy^2)$

(iii) $(-\frac{5}{12}b^2cx) \times (-4\frac{4}{5}bc^2x^3)$

(iv) $(5ab^2) \times (-1\frac{1}{5}a^2b) \times (-\frac{1}{3}ax)$

(v) $(-3ab) \times (-2a^2b) \times (-5ab^3)$

(vi) $(-2x) \times (-4x^2) \times (-5x^3) \times (-x^4)$

(vii) $(-x^2y) \times (xy^3) \times (-x^5y^2)$

13.4 DIVISION

(i) Division of a monomial by a monomial :

(a) $10ab$ divide by $5a = \frac{10ab}{5a} = \frac{2 \times 5 \times a \times b}{5 \times a} = 2b$

i.e. when $10ab$ is divided by $5a$, the quotient is $2b$.

(b) $21ayz$ divided by $7az$

$$= 21ayz \div 7az = \frac{21ayz}{7az} = \frac{3 \times 7 \times a \times y \times z}{7 \times a \times z} = 3y$$

(c) Division of $12m^5$ by $4m^3$

$$= 12m^5 \div 4m^3$$

$$= \frac{3 \times 4 \times m \times m \times m \times m \times m}{4 \times m \times m \times m}$$

$$= 3 \times m \times m = 3m^2$$

Write each term in its expanded form and then cancel the terms that are common to the numerator and the denominator.

Using exponents (quotient law) :

(i) $\frac{a^m}{a^n} = a^{m-n}$, if $m > n$ and

(ii) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, if $n > m$

$$\therefore \frac{x^7}{x^3} = x^{7-3} = x^4, \quad \frac{x^3}{x^7} = \frac{1}{x^{7-3}} = \frac{1}{x^4},$$

$$\frac{x^5y^3}{x^2y^8} = \frac{x^{5-2}}{y^{8-3}} = \frac{x^3}{y^5} \quad \text{and so on}$$

$$\begin{aligned}\therefore 12m^5 \div 4m^3 &= \frac{12m^5}{4m^3} \\ &= 3 \times m^{5-3} = 3 \times m^2 = 3m^2\end{aligned}$$

(d) Division of $35a^3b^5$ by $5a^6b^2$.

$$\begin{aligned}&= 35a^3b^5 \div 5a^6b^2 = \frac{35a^3b^5}{5a^6b^2} = \frac{5 \times 7 \times b^{5-2}}{5 \times a^{6-3}} \quad [\text{Using exponents}] \\ &= \frac{7b^3}{a^3}\end{aligned}$$

More examples :

$$(i) \frac{24a^4b^6}{8a^9b^5} = \frac{3 \times 8 \times b^{6-5}}{8 \times a^{9-4}} = \frac{3b}{a^5}$$

$$(ii) -45m^3n^5 \div 9mn^2 = \frac{-45m^3n^5}{9mn^2} = -5m^2n^3$$

$$(iii) -15a^5b^7x^2 \div -5a^2b^3x^8 = \frac{-15a^5b^7x^2}{-5a^2b^3x^8} = \frac{3a^3b^4}{x^6} \text{ and so on.}$$

(ii) Division of a polynomial by a monomial :

Divide each term of the polynomial by the monomial :

(i) Division of $12x^5 - 9x^3$ by $3x^2$

$$= \frac{12x^5}{3x^2} - \frac{9x^3}{3x^2} = 4x^3 - 3x$$

(ii) Division of $15x^2y^3 - 21x^3y^4 + 18x^4y^2$ by $3x^2y^2$

$$= \frac{15x^2y^3}{3x^2y^2} - \frac{21x^3y^4}{3x^2y^2} + \frac{18x^4y^2}{3x^2y^2} = 5y - 7xy^2 + 6x^2$$

Example 13 : Divide : $24x^3y^3 + 30x^4y^5 - 12x^5y^4$ by $-6x^2y^3$

Solution :

$$\begin{aligned}\frac{24x^3y^3 + 30x^4y^5 - 12x^5y^4}{-6x^2y^3} &= \left(\frac{24x^3y^3}{-6x^2y^3} \right) + \left(\frac{30x^4y^5}{-6x^2y^3} \right) - \left(\frac{12x^5y^4}{-6x^2y^3} \right) \\ &= (-4x) + (-5x^2y^2) - (-2x^3y) \\ &= -4x - 5x^2y^2 + 2x^3y \quad \text{(Ans.)}\end{aligned}$$

EXERCISE 13(E)

1. Divide :

(i) $3a$ by a

(ii) $15x$ by $3x$

(iii) $16m$ by 4

(iv) $20x^2$ by $5x$

(v) $30p^2$ by $10p^2$

(vi) $14a^3b^3$ by $2a^2$

(vii) $18pqr^2$ by $3pq$

(viii) 100 by $50b$

2. Simplify :

- (i) $2x^5 \div x^2$ (ii) $6a^8 \div 3a^3$ (iii) $20xy \div -5xy$
 (iv) $-24a^2b^2c^2 \div 6ab$ (v) $-5x^2y \div xy^2$ (vi) $40p^3q^4r^5 \div 10p^3q$
 (vii) $-64x^4y^3z \div 4x^3y^2z$ (viii) $35xy^5 \div 7x^2y^4$

3. Divide :

- (i) $-\frac{3m}{4}$ by $2m$ (ii) $-15p^6q^8$ by $-5p^6q^7$ (iii) $-21m^5n^7$ by $14m^2n^2$
 (iv) $36a^4x^5y^6$ by $4x^2a^3y^2$ (v) $20x^3a^6$ by $5xy$ (vi) $\frac{28a^2b^3}{c^2}$ by $4abc$
 (vii) $\frac{2a^2}{9b^2}$ by $\frac{3b}{2a}$ (viii) $\frac{-5 \cdot 5x^2}{y}$ by $\frac{11x}{y}$ (ix) $\frac{64x^2y^2}{z^2}$ by $\frac{8xy}{z}$

4. Simplify :

- (i) $\frac{-15m^5n^2}{-3m^5}$ (ii) $\frac{35x^4y^2}{-15x^2y^2}$ (iii) $\frac{-24x^6y^2}{6x^6y}$

5. Divide :

- (i) $9x^3 - 6x^2$ by $3x$ (ii) $6m^2 - 16m^3 + 10m^4$ by $-2m$
 (iii) $15x^3y^2 + 25x^2y^3 - 36x^4y^4$ by $5x^2y^2$
 (iv) $36a^3x^5 - 24a^4x^4 + 18a^5x^3$ by $-6a^3x^3$.

REVISION EXERCISE (Chapter 13)

1. Add :

- (i) $3xy, -2xy, 4xy$ and $-3xy$ (ii) $2a + b, a + 2b, 5a - b$ and $-a + 5b$
 (iii) $a - y, 2a - 4b, 5b$ and $a + c$ (iv) $2a^2 - 3b^2, 3a^2 - 4b^2$ and $-4a^2 + 8b^2$

2. Subtract :

- (i) $3xy - x^2 - 2y^2$ from $3x^2 + 4y^2 + xy$ (ii) $7a^2 - b^2 - 5ab$ from $2a^2 + 5b^2 + 3ab$

3. From the sum of $4x - 5y - 6z$ and $6x + 5y + 14z$ subtract $2x + 3y - 4z$.

4. Subtract the sum of $13a - 11b + 9c$ and $-3a + 21b$ from $11a - 7b + 10c$.

5. Subtract the sum of $12x - 10y + 8z^2$ and $-5x - 2y + 14z^2$ from the sum of $3x - 5y + 7z^2$ and $2x - 9y + 13z^2$.

6. Simplify :

- (i) $(3x - 4y) - (x + 3y) + (2x - y)$ (ii) $(-8a + 3b) - (5a - 11b) - (-25a + 4b)$

7. Multiply :

- (i) $2a - 3b$ and $2a + 3b$ (ii) $5x$ and $2x - 3y + 4z$
 (iii) $5a + 7b$ and $3a - 8b$ (iv) $5a - 7b$ and $3a + 8b$

8. Divide :

- (i) $-72a^2b^3$ by $9ab^2$ (ii) $-36x^3y^2z^5$ by $-9x^2yz^3$ (iii) $-42a^7b^5$ by $-14a^5b^2$
 (iv) $15x^5 - 21x^6 + 18x^4$ by $3x^2$ (v) $21x^3y^4 + 35x^4y^3 - 49x^2y^5$ by $-7x^2y^3$

9. Simplify :

(i) $(36 xy) \div (9x)$

(ii) $(36 xy) \div (-9x)$

(iii) $(-36 xy) \div (9x)$

(iv) $(-36 xy) \div (-9x)$

(v) $3xy(4x^2y + 2xy^2)$

(vi) $2ab^2(a^3b - 3a^2b^3)$

10. Simplify :

(i) $5xy(3x^2y^3)$

(ii) $-5x^2y(3x^2y^4)$

(iii) $5x^3y^2(-3x^4y^3)$

(iv) $-5x^2y^2(-3x^5y^6)$

(v) $\frac{3xy(-6x^2y^3)}{18x^2y}$

(vi) $\frac{8ab(6a^2b^4)}{-12a^3b^5}$

11. Evaluate :

(i) $10ab - 26ab + 3ab$

(ii) $6xy - 3xy - xy$

(iii) $7x^2y - 5x^2y + 3x^2y - 2x^2y$

(iv) $7x + 8y - 4x - 13y$

(v) $-8m - 16n + 5m - 7n$

12. What should be added to $2m^2 + 3mn + 2n^2$ to get $5n^2 - 3mn$?

13. What should be subtracted from $8x^3 + 3x^2 + 9$ to obtain $4x^3 - 5x^2 + 8x - 4$?

14. Multiply : (i) $(5xy + 3yz)$ and $2xz$

(ii) $\frac{6}{7}x^2y$ and $\left(\frac{35}{6}xy + \frac{14}{3}xy^2\right)$

(iii) $(2x^3 - 3x^2 + 6x - 8)$ and $(-5x^2)$

15. Simplify : (i) $4x^2(3x - 2) + 3x(5x + 6) - 4(2x - 6)$

(ii) $x^2(y^2 - z^2) + y^2(z^2 - x^2) + z^2(x^2 - y^2)$

(iii) $ab(2a^2 - b^3) - 3ab^2(4ab - a^2) + 2a^2b(ab - 5ab^2)$