CHAPTER 5

FRACTIONS

5.1 BASIC CONCEPT

If a certain quantity of rice is divided into four equal parts, each part so obtained is said to be one-fourth $\left(\frac{1}{4}\right)$ of the whole quantity of the rice.

Similarly, if an apple is divided into five equal parts, each part is one-fifth $\left(\frac{1}{5}\right)$ of the whole apple. Now, if two parts of these 5 equal parts are eaten, three parts are left, and we say three-fifths $\left(\frac{3}{5}\right)$ of the apple is left.

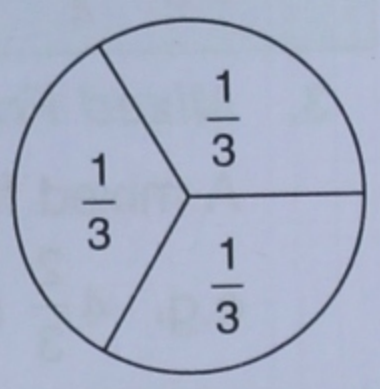
The numbers $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{3}{5}$ discussed above, each representing a part of the whole quantity, are called **fractions**.

∴ ***A fraction is a quantity that expresses a part of the whole.***

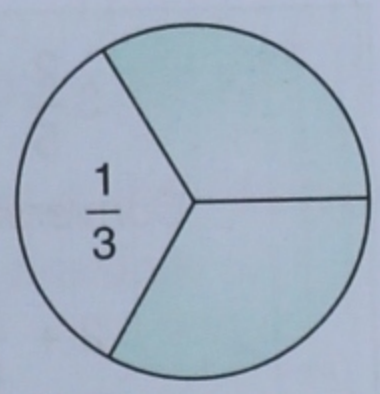
To make the concept of fractions more clear :

Draw a circle with any suitable radius.

Divide the circle into three equal parts (sectors).



If two parts of the three equal parts be shaded, we say $\frac{2}{3}$ (two-thirds) of the circle is shaded and $\frac{1}{3}$ (one-third) of the circle is not.



In the fraction $\frac{a}{b}$, ***a*** is the **numerator** of the fraction and ***b*** is its **denominator**.

$$\therefore \text{ FRACTION} = \frac{\text{NUMERATOR}}{\text{DENOMINATOR}}$$

Thus, in fraction $\frac{7}{11}$, numerator = 7 and denominator = 11.

1. The **numerator** and the **denominator** are also known as the terms of a fraction.
2. Every fraction must be expressed in its lowest terms. In other words, the terms of a fraction must not have any common factor except 1(one).

   Fractions $\frac{3}{7}$, $\frac{15}{11}$ and $\frac{7}{10}$, are in their lowest terms, because the terms of each of these fractions have only 1 (one) as common factor.
3. **5 out of 7** means a given quantity is divided into seven equal parts and five of these equal parts are taken.

   Thus, 5 out of 7 = $\frac{5}{7}$.

5.2 TYPES OF FRACTIONS

1. *Proper Fraction :*

A fraction whose ***numerator*** is ***less*** than its ***denominator*** is called a ***proper fraction***, e.g. $\frac{4}{5}, \frac{3}{7}, \frac{101}{235}, \frac{4}{7}, \frac{9}{14}$, etc.

2. *Improper Fraction :*

A fraction whose ***numerator*** is ***greater than or equal to*** its ***denominator*** is called an ***improper fraction***.

e.g. (i) $\frac{7}{5}, \frac{25}{12}, \frac{181}{62}$, etc. — Numerator is greater than denominator

(ii) $\frac{3}{3}, \frac{4}{4}, \frac{5}{5}$, etc. — Numerator is equal to denominator

If the numerator and the denominator of a fraction are equal, the value of the fraction is unity (1).

e.g. $\frac{4}{4} = 1, \frac{-3}{-3} = 1$, etc.

3. *Mixed Fraction :*

A mixed fraction consists of *two* parts : (i) an integer and (ii) a proper fraction.

e.g. $4\frac{2}{3}$ is a mixed fraction, consisting of an integer (4) and a proper fraction $\left(\frac{2}{3}\right)$.

$3\frac{2}{5} = 3 + \frac{2}{5}, 8\frac{5}{6} = 8 + \frac{5}{6}, -2\frac{1}{8} = -\left(2 + \frac{1}{8}\right)$ *and so on.*

Conversely,

$2 + \frac{3}{8} = 2\frac{3}{8}, 7 + \frac{4}{9} = 7\frac{4}{9}, -8 - \frac{5}{6} = -\left(8 + \frac{5}{6}\right) = -8\frac{5}{6}$ *and so on.*

4. *Like and Unlike Fractions :*

Two or more fractions with the ***same denominator*** but ***different numerators*** are called ***like fractions***.

e.g. $\frac{3}{5}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{7}{5}$, etc. are like fractions.

Two or more fractions with different ***denominators*** are called ***unlike fractions***.

e.g. $\frac{5}{9}, \frac{7}{8}, \frac{3}{4}, \frac{1}{3}$, etc.

5. *Equivalent Fractions :*

If two or more ***fractions*** have the ***same value***, they are called ***equivalent*** or ***equal fractions***.

e.g.the fractions $\frac{1}{3}, \frac{3}{9}, \frac{6}{18}$ and $\frac{9}{27}$ are equivalent fractions as $\frac{1}{3} = \frac{3}{9} = \frac{6}{18} = \frac{9}{27}$.

The value of a fraction does not change if its numerator and denominator are both *multiplied or divided by the same non-zero number.*

e.g. $\frac{4}{7}$ and $\frac{4\times2}{7\times2}$ *i.e.* $\frac{4}{7}$ and $\frac{8}{14}$ are equivalent fractions.

Also, $\frac{15}{20}$ and $\frac{15\div5}{20\div5}$ *i.e.* $\frac{15}{20}$ and $\frac{3}{4}$ are equivalent fractions.

5.3 CONVERTING A MIXED FRACTION INTO AN IMPROPER FRACTION

Multiply the integral part by the denominator and add the numerator to the product. The result so obtained is the numerator of the required improper fraction.

The denominator of the required fraction will be the same as the denominator of the given mixed fraction.

Thus, for the mixed fraction $3\frac{7}{15}$,

the **required improper fraction** $= \frac{\text{Integral part} \times \text{Denominator} + \text{Numerator}}{\text{Denominator}}$

$= \frac{(3\times15)+7}{15} = \frac{45+7}{15} = \frac{52}{15}$

Similarly, $5\frac{3}{4} = \frac{5\times4+3}{4} = \frac{20+3}{4} = \frac{23}{4}$;

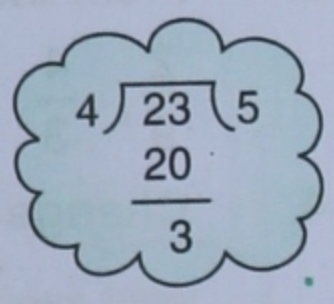
$7\frac{5}{6} = \frac{7\times6+5}{6} = \frac{42+5}{6} = \frac{47}{6}$ and so on.

5.4 CONVERTING AN IMPROPER FRACTION INTO A MIXED FRACTION

Divide the numerator by the denominator. The quotient of this division is the integral part and the remainder obtained is the numerator of the required mixed fraction.

Of course, the denominator will remain the same.

Thus, $\frac{23}{4}$ = Quotient $\frac{\text{Remainder}}{\text{Denominator}} = 5\frac{3}{4}$



On dividing 23 by 4, quotient = 5 and remainder = 3.

Similarly, $\frac{37}{8}$ = Quotient $\frac{\text{Remainder}}{\text{Denominator}} = 4\frac{5}{8}$

$\frac{41}{9} = 4\frac{5}{9}$, $\frac{73}{12} = 6\frac{1}{12}$ and so on.

5.5 CONVERTING UNLIKE FRACTIONS INTO LIKE FRACTIONS

Steps :

1. Find the L.C.M. of the denominators of all the given fractions.
2. Multiply the numerator and the denominator of each fraction by a same suitable number so that the denominator of each fraction becomes equal to the L.C.M. obtained in step 1.

***Example* 1 :**

Convert $\frac{3}{7}$, $\frac{4}{5}$ and $\frac{1}{3}$ into like fractions.

***Solution* :**

L.C.M. of denominators 7, 5 and 3 = 105 [Step 1]

Now, $\frac{3}{7} = \frac{3\times15}{7\times15} = \frac{45}{105}$; $\frac{4}{5} = \frac{4\times21}{5\times21} = \frac{84}{105}$ and $\frac{1}{3} = \frac{1\times35}{3\times35} = \frac{35}{105}$ [Step 2]

$\therefore$ $\frac{3}{7}$, $\frac{4}{5}$ and $\frac{1}{3} = \frac{45}{105}$, $\frac{84}{105}$ and $\frac{35}{105}$ respectively **(Ans.)**

EXERCISE 5(A)

1. For each expression given below, **write a fraction :**

(i) 2 out of 7 = ... (ii) 5 out of 17 = ... (iii) three-fifths = ...

2. Fill in the blanks :

(i) $\frac{5}{8}$ is ..... fraction. (ii) $\frac{8}{5}$ is ..... fraction.

(iii) $\frac{-15}{-15}$ is ..... fraction. (iv) The value of $\frac{5}{5}$ = ...

(v) The value of $\frac{5}{-5}$ = ... (vi) $3\frac{3}{10}$ is ..... fraction.

(vii) $\frac{2}{15}$ and $\frac{7}{15}$ are ..... fractions. (viii) $\frac{23}{12}$ and $\frac{23}{15}$ are ..... fractions.

(ix) $\frac{6}{15}$ and $\frac{28}{70}$ are ..... fractions. (x) $\frac{8}{24}$ and $\frac{8}{32}$ are not ..... fractions.

(xi) $3\frac{2}{13} = \frac{3\times13+.....}{13}$ = ... (xii) $-4\frac{3}{5}$ = ... = ...

3. From the following fractions, separate (i) **proper fractions** and (ii) **improper fractions :**

$\frac{2}{9}$, $\frac{4}{3}$, $\frac{7}{15}$, $\frac{11}{20}$, $\frac{20}{11}$, $\frac{18}{23}$, $\frac{27}{35}$.

4. **Change** the following mixed fractions **to improper fractions :**

(i) $2\frac{1}{5}$ (ii) $3\frac{1}{4}$ (iii) $7\frac{1}{8}$ (iv) $2\frac{1}{11}$

5. **Change** the following improper fractions **to mixed fractions :**

(i) $\frac{100}{17}$ (ii) $\frac{81}{11}$ (iii) $-\frac{209}{7}$ (iv) $-\frac{113}{15}$

6. **Change** the following groups of fractions **to like fractions :**

(i) $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{4}$, $\frac{1}{6}$ (ii) $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{3}{10}$ (iii) $\frac{2}{7}$, $\frac{7}{8}$, $\frac{5}{14}$, $\frac{9}{16}$

5.6 REDUCING A FRACTION TO ITS LOWEST TERMS

A fraction is said to be in its lowest terms if its numerator and denominator have no common factor other than 1, *i.e.* the numerator and the denominator are co-prime.

To reduce a fraction to its lowest terms :

(i) find the H.C.F. of its numerator and denominator.

(ii) divide each term of the fraction by the H.C.F. obtained in step (i).

For example :

Consider the fraction $\frac{48}{60}$.

As the H.C.F. of 48 and 60 is 12, divide both numerator and denominator by 12.

Thus, $\frac{48}{60} = \frac{48 \div 12}{60 \div 12} = \frac{4}{5}$, ***which is the fraction in its lowest terms.***

Similarly, $\frac{45}{75} = \frac{45 \div 15}{75 \div 15} = \frac{3}{5}$

H.C.F. of 45 and 75 is 15

If both the terms of a fraction are divided or multiplied by the same number, the value of the fraction remains unchanged.

Alternative method :

First express each term of the given fraction as a product of prime factors; then cancel the common factors.

e.g. $\frac{48}{60} = \frac{\cancel{2}\times\cancel{2}\times 2\times 2\times\cancel{3}}{\cancel{2}\times\cancel{2}\times\cancel{3}\times 5} = \frac{2\times 2}{5} = \frac{4}{5}$.

In the same way, $\frac{45}{75} = \frac{\cancel{3}\times 3\times\cancel{5}}{\cancel{3}\times\cancel{5}\times 5} = \frac{3}{5}$ and so on.

5.7 COMPARING FRACTIONS

Comparing fractions means comparing their values to find out which of them is greater or smaller.

Example 2 :

Which fraction is greater, $\frac{3}{8}$ or $\frac{5}{12}$?

Solution :

Step 1 :

1. Convert the given fractions to like fractions.
2. The fraction with the greater numerator is greater.

Since the L.C.M. of the denominators 8 and 12 = 24,

$\therefore \quad \frac{3}{8} = \frac{3\times 3}{8\times 3} = \frac{9}{24}$ and $\frac{5}{12} = \frac{5\times 2}{12\times 2} = \frac{10}{24}$.

If two fractions have the same denominator, the greater fraction has the greater numerator

Step 2 :

See the numerators of these like fractions. The numerator 10 is greater.

$\therefore \quad \frac{10}{24}$ *i.e.* $\frac{5}{12}$ **is greater.** **(Ans.)**

Alternative method :

Fractions can also be compared by making the numerators equal and then comparing the denominators. In this case, ***the fraction with the smaller denominator is greater.***

Since the L.C.M. of the numerators 3 and 5 = 15,

$\therefore \quad \frac{3}{8} = \frac{3\times5}{8\times5} = \frac{15}{40}$ and $\frac{5}{12} = \frac{5\times3}{12\times3} = \frac{15}{36}$.

For the same numerator, the fraction is greater if the denominator is smaller.

$\therefore \quad \frac{15}{36}$ *i.e.* $\frac{5}{12}$ **is greater.** **(Ans.)**

Example 3 :

Which of the given fractions is smaller, $\frac{8}{15}$ or $\frac{12}{25}$?

Solution :

First method : *By making the denominators equal :*

Since the L.C.M. of denominators 15 and 25 = 75,

$\therefore \quad \frac{8}{15} = \frac{8\times5}{15\times5} = \frac{40}{75}$ and $\frac{12}{25} = \frac{12\times3}{25\times3} = \frac{36}{75}$

Hence $\frac{36}{75}$ *i.e.* $\frac{12}{25}$ **is smaller.** **(Ans.)**

With equal denominators, the fraction with the smaller numerator is smaller.

Second method : *By making numerators equal :*

Since the L.C.M. of numerators 8 and 12 = 24,

$\therefore \quad \frac{8}{15} = \frac{8\times3}{15\times3} = \frac{24}{45}$ and $\frac{12}{25} = \frac{12\times2}{25\times2} = \frac{24}{50}$

Hence $\frac{24}{50}$ *i.e.* $\frac{12}{25}$ **is smaller.** **(Ans.)**

With equal numerators, the fraction with the bigger denominator is smaller.

Example 4 :

Compare the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{12}$ and $\frac{9}{16}$ by writing them in the descending order.

Solution :

Step 1 :

Since the L.C.M. of the denominators 3, 4, 12 and 16 = 48,

$\therefore \quad \frac{2}{3} = \frac{2\times16}{3\times16} = \frac{32}{48}$, $\quad \frac{3}{4} = \frac{3\times12}{4\times12} = \frac{36}{48}$,

$\frac{5}{12} = \frac{5\times4}{12\times4} = \frac{20}{48}$ and $\frac{9}{16} = \frac{9\times3}{16\times3} = \frac{27}{48}$.

Making the denominators equal.

Step 2 :

See the numerators of these like fractions. The fraction with the largest numerator is largest.

The largest numerator is 36. $\therefore \quad \frac{36}{48}$ *i.e.* $\frac{3}{4}$ is the largest fraction.

The smallest numerator is 20. $\therefore \quad \frac{20}{48}$ *i.e.* $\frac{5}{12}$ is the smallest fraction.

Thus, the given fractions in descending order of value are :

$\frac{36}{48}, \frac{32}{48}, \frac{27}{48}$ and $\frac{20}{48}$ *i.e.* $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$

i.e. $\frac{3}{4} > \frac{2}{3} > \frac{9}{16} > \frac{5}{12}$ (Ans.)

Alternative method : (Comparing fractions by making the numerators equal)

Taking the same fractions as given in the above example,

i.e. $\frac{2}{3}, \frac{3}{4}, \frac{5}{12}$ and $\frac{9}{16}$,

The L.C.M. of the numerators 2, 3, 5 and 9 = 90

$\therefore \frac{2}{3} = \frac{2\times45}{3\times45} = \frac{90}{135}$, $\frac{3}{4} = \frac{3\times30}{4\times30} = \frac{90}{120}$,

$\frac{5}{12} = \frac{5\times18}{12\times18} = \frac{90}{216}$ and $\frac{9}{16} = \frac{9\times10}{16\times10} = \frac{90}{160}$.

Making the numerators equal.

We know that, with the numerators being the same, the fraction with the smallest denominator is the biggest fraction and the fraction with the largest denominator is the smallest fraction.

Thus, $\frac{90}{120}$ is the biggest fraction and $\frac{90}{216}$ is the smallest fraction.

And so, $\frac{90}{120}, \frac{90}{135}, \frac{90}{160}$ and $\frac{90}{216}$ are in the descending order of value,

i.e. $\frac{3}{4}, \frac{2}{3}, \frac{9}{16}$ and $\frac{5}{12}$ are in the descending order of value,

i.e. $\frac{3}{4} > \frac{2}{3} > \frac{9}{16} > \frac{5}{12}$ (Ans.)

For the example given above, the fraction can be written in ascending order as $\frac{5}{12} < \frac{9}{16} < \frac{2}{3} < \frac{3}{4}$.

EXERCISE 5(B)

1. Reduce the given fractions to their lowest terms :

(i) $\frac{8}{10}$ (ii) $\frac{50}{75}$ (iii) $\frac{18}{81}$

(iv) $\frac{40}{120}$ (v) $\frac{105}{70}$

2. State whether true or false :

(i) $\frac{2}{5} = \frac{10}{15}$ (ii) $\frac{35}{42} = \frac{5}{6}$ (iii) $\frac{5}{4} = \frac{4}{5}$

(iv) $\frac{7}{9} = 1\frac{1}{7}$ (v) $\frac{9}{7} = 1\frac{1}{7}$

3. Which fraction is greater ?

(i) $\frac{3}{5}$ or $\frac{2}{3}$ (ii) $\frac{5}{9}$ or $\frac{3}{4}$ (iii) $\frac{11}{14}$ or $\frac{26}{35}$

4. **Which** fraction **is smaller** ?

(i) $\frac{3}{8}$ or $\frac{4}{5}$ (ii) $\frac{8}{15}$ or $\frac{4}{7}$ (iii) $\frac{7}{26}$ or $\frac{10}{39}$

5. Arrange the given fractions in **descending order** of magnitude :

(i) $\frac{5}{16}, \frac{13}{24}, \frac{7}{8}$ (ii) $\frac{4}{5}, \frac{7}{15}, \frac{11}{20}, \frac{3}{4}$ (iii) $\frac{5}{7}, \frac{3}{8}, \frac{9}{11}$

6. Arrange the given fractions in **ascending order** of magnitude :

(i) $\frac{9}{16}, \frac{7}{12}, \frac{1}{4}$ (ii) $\frac{5}{6}, \frac{2}{7}, \frac{8}{9}, \frac{1}{3}$ (iiii) $\frac{2}{3}, \frac{5}{9}, \frac{5}{6}, \frac{3}{8}$

7. I bought one dozen bananas and ate five of them. What fraction of the total number of bananas was left ?

8. Insert the symbol '=' or '>' or '<' between each of the pairs of fractions given below :

(i) $\frac{6}{11}$ ..... $\frac{5}{9}$ (ii) $\frac{3}{7}$ ..... $\frac{9}{13}$ (iii) $\frac{56}{64}$ ..... $\frac{7}{8}$ (iv) $\frac{5}{12}$ ..... $\frac{8}{33}$

9. Out of 50 identical articles, 36 are broken. Find the fraction of :
   (i) the total number of articles and the articles broken.
   (ii) the remaining articles and total number of articles.

5.8 FUNDAMENTAL OPERATIONS ON FRACTIONS

The four fundamental operations are *addition, subtraction, multiplication* and *division*.

5.9 ADDITION AND SUBTRACTION

Steps :

1. If any of the given fractions is in mixed form, convert it into an improper fraction.
2. Convert the fractions obtained in step 1 into equivalent fractions.
3. Keeping the denominator same, as obtained in step 2, combine the numerators of the equivalent fractions and obtain a single fraction.
4. Reduce, if required, the fraction so obtained into its lowest terms and then into a mixed fraction.

For example :

(i) $\frac{3}{4} + \frac{2}{5}$

$= \frac{3\times5}{4\times5} + \frac{2\times4}{5\times4}$ (L.C.M. of 4 and 5 is 20)

$= \frac{15}{20} + \frac{8}{20}$

$= \frac{15+8}{20} = \frac{23}{20} = 1\frac{3}{20}$

(ii) $1\frac{5}{7} - \frac{5}{6}$

$1\frac{5}{7} = \frac{1\times7+5}{7} = \frac{7+5}{7} = \frac{12}{5}$

$= \frac{12}{7} - \frac{5}{6}$

$= \frac{12\times6}{7\times6} - \frac{5\times7}{6\times7}$ (L.C.M. of 7 and 6 is 42)

$= \frac{72}{42} - \frac{35}{42} = \frac{72-35}{42} = \frac{37}{42}$

(iii) $2\frac{2}{5} - 3\frac{3}{4} + 4\frac{1}{2} = \frac{12}{5} - \frac{15}{4} + \frac{9}{2}$

Converting into improper fractions

$= \frac{12\times4}{5\times4} - \frac{15\times5}{4\times5} + \frac{9\times10}{2\times10}$ [L.C.M. of 5, 4 and 2 is 20]

$= \frac{48}{20} - \frac{75}{20} + \frac{90}{20} = \frac{48-75+90}{20} = \frac{63}{20} = 3\frac{3}{20}$

EXERCISE 5(C)

1. **Add** the following fractions :

(i) $1\frac{3}{4}$ and $\frac{3}{8}$ (ii) $\frac{2}{5}, 2\frac{3}{15}$ and $\frac{7}{10}$ (iiii) $1\frac{7}{8}, 1\frac{1}{2}$ and $1\frac{3}{4}$

(iv) $3\frac{3}{4}, 2\frac{1}{6}$ and $1\frac{5}{8}$ (v) $2\frac{8}{9}, \frac{11}{18}$ and $3\frac{5}{6}$ (vi) $3\frac{1}{8}, 5\frac{5}{12}$ and $\frac{5}{16}$

2. **Simplify :**

(i) $1\frac{11}{12} - \frac{13}{16}$ (ii) $2\frac{3}{4} - 1\frac{5}{6}$ (iii) $2\frac{5}{7} + \frac{3}{14} - \frac{13}{21}$

(iv) $3\frac{5}{6} - \frac{1}{6} - 1\frac{1}{12}$ (v) $6 + \frac{3}{10} - 1\frac{8}{15}$ (vi) $1\frac{3}{4} + 2\frac{5}{7} - 1\frac{3}{14}$

(vii) $4 + 3\frac{1}{8} - 3\frac{1}{6}$ (viii) $6 - 3\frac{1}{2} - 2\frac{1}{5}$ (ix) $1\frac{5}{8} - 2\frac{1}{6} + 3\frac{3}{4}$

(x) $3\frac{1}{2} + 1\frac{2}{3} - 2\frac{1}{4}$ (xi) $4\frac{3}{5} - 2\frac{7}{9} - 1\frac{2}{15} - \frac{2}{45}$

5.10 MULTIPLICATION AND DIVISION

Steps for multiplication :

1. See that each given fraction is in proper form or improper form, *i.e.* no fraction is in mixed form.
2. Multiply the numerators of the fractions together to get the numerator of the resulting fraction, and also multiply the denominators of the fractions together to get the denominator of the resulting fraction
3. If required, reduce the resulting fraction obtained in step (2) into its lowest terms and then into a mixed fraction.

***Example* 5 :**

Simplify :

(i) $\frac{3}{4} \times 5$ (ii) $2\frac{2}{5} \times \frac{5}{18}$

Solution :

(i) $\frac{3}{4} \times 5 = \frac{3}{4} \times \frac{5}{1} = \frac{3\times5}{4\times1} = \frac{15}{4} = 3\frac{3}{4}$ (Ans.)

(ii) $2\frac{2}{5} \times \frac{5}{18} = \frac{12}{5} \times \frac{5}{18} = \frac{12\times5}{5\times18} = \frac{\cancel{2}\times2\times\cancel{3}\times\cancel{5}}{\cancel{5}\times\cancel{3}\times3\times\cancel{2}} = \frac{2}{3}$ (Ans.)

For division :

Multiply the dividend (the first fraction) ***by the reciprocal of the divisor*** (the second fraction).

1. Reciprocal of $\frac{5}{7} = \frac{7}{5}$, reciprocal of $\frac{8}{15} = \frac{15}{8}$ and so on.
2. Since, $5 = \frac{5}{1}$; reciprocal of $5 = \frac{1}{5}$. Similarly, reciprocal of $8 = \frac{1}{8}$, reciprocal of $15 = \frac{1}{15}$ and so on.

For example :

$$\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{10}{9} = 1\frac{1}{9}$$

Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$

5.11 COMBINED OPERATIONS OF MULTIPLICATION AND DIVISION

In such cases, *first the operation of division is completed and then of multiplication.*

Example 6 :

Simply : $\frac{3}{8} \div \frac{4}{7} \times \frac{1}{2}$

Solution :

Since, $\frac{3}{8} \div \frac{4}{7} = \frac{3}{8} \times \frac{7}{4} = \frac{21}{32}$

Reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$

$\therefore \quad \frac{3}{8} \div \frac{4}{7} \times \frac{1}{2} = \frac{21}{32} \times \frac{1}{2} = \frac{21 \times 1}{32 \times 2} = \frac{21}{64}$ **(Ans.)**

or, directly : $\frac{3}{8} \div \frac{4}{7} \times \frac{1}{2} = \frac{3}{8} \times \frac{7}{4} \times \frac{1}{2} = \frac{3 \times 7 \times 1}{8 \times 4 \times 2} = \frac{21}{64}$ **(Ans.)**

5.12 USING ' of ' ALONG WITH MULTIPLICATION AND DIVISION

Apart from Addition, Subtraction, Multiplication and Division, there is one more operation, termed 'of'.

The word 'of' written in between two fractions or numbers is to be worked out just as if it were a multiplication sign, but it is to come even before division and multiplication.

For example :

(i) $\frac{3}{2}$ of $\frac{3}{4} \div \frac{9}{2}$

$= \frac{9}{8} \div \frac{9}{2}$

$= \frac{9}{8} \times \frac{2}{9} = \frac{1}{4}$

[Operating 'of' we get : $\frac{3}{2}$ of $\frac{3}{4} = \frac{9}{8}$]

(ii) $\frac{5}{6}$ of $\frac{3}{4} \div \frac{7}{8} \times 1\frac{1}{2}$

$= \frac{5}{8} \div \frac{7}{8} \times \frac{3}{2}$

[As, $\frac{5}{6}$ of $\frac{3}{4} = \frac{5}{6} \times \frac{3}{4} = \frac{15}{24} = \frac{5}{8}$]

$= \frac{5}{8} \times \frac{8}{7} \times \frac{3}{2} = \frac{5 \times \cancel{8} \times 3}{\cancel{8} \times 7 \times 2} = \frac{15}{14} = 1\frac{1}{14}$

5.13 USING 'BODMAS'

"BODMAS" is the acronym (abbreviation) formed by taking the initial letters of the six operations, where :

'B' stands for **"BRACKET"**

'O' stands for **"OF"**

'D' stands for **"DIVISION"**

'M' stands for **"MULTIPLICATION"**

'A' stands for **"ADDITION"**

'S' stands for **"SUBTRACTION"**

Fractions inside brackets are to be operated (combined) first

While simplifying an expression involving three or more operations, the order of operations must be the same as in the order of letters used in 'BODMAS'

For example :

(i) $1\frac{1}{2} \times \frac{1}{12} \div \frac{5}{4} = \frac{3}{2} \times \frac{1}{12} \times \frac{4}{5}$

Using BODMAS; division is done first

$$= \frac{3 \times 1 \times 4}{2 \times 12 \times 5} = \frac{1}{10}$$

(ii) $\frac{1}{3} + \frac{7}{9} \div \left(\frac{7}{10} \times 1\frac{1}{4}\right)$

Using BODMAS; bracket is simplified first.

$$= \frac{1}{3} + \frac{7}{9} \div \left(\frac{7}{10} \times \frac{5}{4}\right)$$

$$= \frac{1}{3} + \frac{7}{9} \div \frac{7}{8} \qquad \left[\frac{7}{10} \times \frac{5}{4} = \frac{7 \times 5}{10 \times 4} = \frac{7}{8}\right]$$

$$= \frac{1}{3} + \frac{7}{9} \times \frac{8}{7} = \frac{1}{3} + \frac{8}{9} = \frac{3+8}{9} = \frac{11}{9} = 1\frac{2}{9}$$

Example 7 :

Simplify : $\left(\frac{2}{3}+\frac{5}{9}\right)$ of $\frac{9}{22} \div \frac{2}{3} \times \frac{4}{5} - \frac{1}{5}$

Solution :

$= \frac{11}{9}$ of $\frac{9}{22} \div \frac{2}{3} \times \frac{4}{5} - \frac{1}{5}$ [Removing 'bracket', we get : $\frac{2}{3}+\frac{5}{9} = \frac{6+5}{9} = \frac{11}{9}$]

$= \frac{1}{2} \div \frac{2}{3} \times \frac{4}{5} - \frac{1}{5}$ [On operating 'of', we get : $\frac{11}{9}$ of $\frac{9}{22} = \frac{11}{9} \times \frac{9}{22} = \frac{1}{2}$]

$= \frac{1}{2} \times \frac{3}{2} \times \frac{4}{5} - \frac{1}{5} = \frac{1 \times 3 \times 4}{2 \times 2 \times 5} - \frac{1}{5} = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$ **(Ans.)**

EXERCISE 5(D)

1. **Simplify :**

(i) $\frac{3}{7} \times \frac{2}{5}$ (ii) $\frac{4}{9} \times \frac{3}{5}$ (iii) $\frac{5}{12} \times 8$

(iv) $\frac{7}{6}$ of $\frac{3}{14}$ (v) $3\frac{3}{8} \times 3\frac{6}{7}$ (vi) $\frac{1}{2}$ of $\frac{1}{3} \times \frac{3}{4}$

(vii) $\frac{3}{7} \times \frac{5}{9} \times 4\frac{1}{5}$ (viii) $1\frac{1}{3} \times 1\frac{2}{7}$ of $1\frac{1}{4}$

2. **Simplify :**

(i) $\frac{2}{3} \div 1\frac{1}{5}$ (ii) $4\frac{1}{2} \div \frac{4}{9}$ (iii) $1 \div \frac{2}{5}$

(iv) $\frac{4}{9} \div \frac{4}{9}$ (v) $2\frac{1}{3} \div 1\frac{3}{4}$ (vi) $2\frac{2}{3} \times 3\frac{1}{2} \div 2\frac{4}{9}$

3. **Simplify :**

(i) $\frac{1}{4}$ of $2\frac{2}{7} \div \frac{3}{5}$ (ii) $1\frac{1}{4} \times \frac{1}{2} \div 1\frac{1}{3}$ (iii) $6\frac{1}{7} \times 0 \times 5\frac{3}{8}$

(iv) $\frac{3}{4} \times 1\frac{1}{3} \div \frac{3}{7}$ of $2\frac{5}{8}$ (v) $2\frac{1}{4} \div \frac{2}{7}$ of $1\frac{1}{3} \times \frac{2}{3}$ (vi) $\left(\frac{3}{7} \div \frac{1}{2}\right)$ of $1\frac{1}{7}$

(vii) $\left(1\frac{7}{8} \div 1\frac{1}{2}\right)$ of $\left(8\frac{1}{3} \div 1\frac{1}{2}\right)$ (viii) $\frac{1}{3}$ of $60 \div 60$

4. **Simplify :**

(i) $5 - \left(\frac{8}{11} - 3\frac{3}{11}\right)$ (ii) $\frac{1}{2} \div \left(\frac{7}{8} - \frac{3}{5}\right)$

(iii) $2\frac{1}{3} \div \left(5\frac{1}{2} + 3\frac{3}{4}\right)$ (iv) $\left(3\frac{7}{8} - 3\frac{3}{5}\right) \div \frac{1}{2}$

(v) $\frac{4}{7} \div \left(\frac{1}{3} \times 2\frac{4}{5}\right)$ (vi) $\frac{3}{4} \div \left(\frac{1}{6} \div \frac{1}{2}\right)$

(vii) $\left(\frac{1}{4} - \frac{1}{6}\right)$ of $\left(\frac{2}{3} - \frac{5}{12}\right) \times \left(\frac{5}{8} - \frac{7}{12}\right)$

5. **Simplify :**

(i) $\left(\frac{1}{2} + \frac{1}{3}\right) \div \left(\frac{1}{4} - \frac{1}{6}\right)$ (ii) $\left(\frac{24}{35} \div \frac{6}{7} + \frac{5}{9}\right) \times \frac{3}{4}$

(iii) $\frac{3}{4}$ of $6\frac{1}{8} - \frac{2}{3}$ of $2\frac{1}{4}$ (iv) $\frac{7}{30}$ of $\left(\frac{1}{3} + \frac{7}{15}\right) \div \left(\frac{5}{6} - \frac{3}{5}\right)$

(v) $2\frac{1}{2} - 3\frac{1}{2} \times 1\frac{3}{4} + 2\frac{1}{2}$ (vi) $4\frac{5}{7}\left(3\frac{1}{8} \div \frac{11}{12}\right)$

(vii) $\frac{2}{5}$ of $\left(\frac{1}{7} - \frac{1}{12}\right)$ of $1\frac{2}{5}$ (viii) $\left(\frac{1}{2} - \frac{1}{3}\right)\left(\frac{3}{4} - \frac{4}{5}\right) \div \left(\frac{1}{2} - \frac{2}{5} + \frac{1}{7}\right)$

(ix) $\frac{5}{6} - \frac{3}{5}\left(\frac{1}{3} + \frac{2}{11}\right)$

(x) $4\frac{2}{3} \div \left(3 - \frac{1}{2}\right) + \left(\frac{2}{5} \div 1\frac{1}{5}\right)$

(xi) $\frac{1}{2}$ of $40 + 1\frac{3}{4}$ of $2\frac{2}{9} + 2\frac{1}{5} \times 0$

5.14 PROBLEMS INVOLVING FRACTIONS

Example 8 :

A man earns ₹ 7,500 per month. If he saves $\frac{1}{4}$ of his earning, find :

(a) his savings per month (b) his expenditure per month.

Solution :

(a) **Savings per month** = $\frac{1}{4}$ of his earning

= $\frac{1}{4}$ of ₹ 7,500 = ₹ $\frac{1}{4} \times 7{,}500$ = **₹ 1,875** **(Ans.)**

(b) His **expenditure per month** = ₹ 7,500 – ₹ 1,875 = **₹ 5,625** **(Ans.)**

Alternative method :

In fractions, the whole quantity is taken as 1.

Since the man saves $\frac{1}{4}$ of his earnings,

his expenditure = $1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$ of his earnings.

$\Rightarrow$ **Expenditure per month**

= $\frac{3}{4}$ of ₹ 7,500 = **₹ 5,625** **(Ans.)**

Example 9 :

There are 12 dozen bananas in a basket. $\frac{5}{24}$ of them are rotten and $\frac{1}{3}$ of them get eaten. How many bananas are left ?

Solution :

Total number of bananas = 12 dozen = $12 \times 12 = 144$

No. of rotten bananas = $\frac{5}{24}$ of $144 = \frac{5}{24} \times 144 = 30$

No. of bananas eaten = $\frac{1}{3}$ of $144 = \frac{1}{3} \times 144 = 48$

Since $30 + 48 = 78$

$\therefore$ **No. of bananas left** = $144 - 78 =$ **66** **(Ans.)**

Example 10 :

A man spends $\frac{2}{5}$ of his money and is left with ₹ 30. How much did he initially have ?

Solution :

Remember, while solving problems on fractions, the whole quantity is always considered 1.

Since the man spends $\frac{2}{5}$ of his money,

$\therefore$ Money left with him $= \left(1 - \frac{2}{5}\right)$ of his money $= \frac{3}{5}$ of his money

Given : $\frac{3}{5}$ of his initial money $= ₹\ 30$

$\therefore$ **Initially he had** $= ₹\ 30 \times \frac{5}{3} =$ **₹ 50** **(Ans.)**

Example 11 :

After travelling 10 km, Dev found that $\frac{1}{3}$ of his journey was still left. How long was his total journey ?

Solution :

Since $\frac{1}{3}$ of the journey is left,

therefore, $1 - \frac{1}{3} = \frac{2}{3}$ of the journey is completed.

Given : $\frac{2}{3}$ of the total journey $= 10$ km

$\therefore$ **Total journey** $= 10\ \text{km} \times \frac{3}{2} =$ **15 km** **(Ans.)**

EXERCISE 5(E)

1. From a rope $10\frac{1}{2}$ m long, $4\frac{5}{8}$ m is cut off. Find the length of the remaining rope.
2. A piece of cloth is 5 m long. After washing, it shrinks by $\frac{1}{25}$ of its length. What is the length of the cloth after washing ?
3. I bought wheat worth ₹ $12\frac{1}{2}$, rice worth ₹ $25\frac{3}{4}$ and vegetables worth ₹ $10\frac{1}{4}$. If I gave a hundred-rupee note to the shopkeeper; how much did he return to me ?
4. Out of are 500 oranges in a box, $\frac{3}{25}$ are bad and $\frac{1}{5}$ are kept for some guests. How many are left ?
5. An ornament piece is made of gold and copper. Its total weight is 96 g. If $\frac{1}{12}$ of the ornament is copper, find the weight of gold in it.
6. A girl did half of some work on Monday and one-third of it on Tuesday. How much will she have to do on Wednesday in order to complete the work ?
7. A man spends $\frac{3}{8}$ of his money and still has ₹ 720 left with him. How much money did he have at first ?
8. In a school, $\frac{4}{5}$ of the students are boys, and the number of girls is 100. Find the number of boys.
9. After finishing $\frac{3}{4}$ of my journey, I find that 12 km of my journey is covered. How much distance is still left to be covered ?
10. When Ajit travelled 15 km, he found that one-fourth of his journey was still left. What was the full length of the journey ?

11. In a particular month, a man earns ₹ 7,200. Out of this income, he spends $\frac{3}{10}$ on food, $\frac{1}{4}$ on house-rent, $\frac{1}{10}$ on insurance and $\frac{2}{25}$ on holidays. How much did he save in that month ?

Revision Exercise (Chapter 5)

1. Show that $\frac{3}{7}$ lies between $\frac{2}{5}$ and $\frac{5}{7}$.

$\frac{3}{7}$ will lie between $\frac{2}{5}$ and $\frac{5}{7}$ if :

$\frac{2}{5} > \frac{3}{7} > \frac{5}{7}$ or $\frac{2}{5} < \frac{3}{7} < \frac{5}{7}$

On comparing $\frac{2}{5}$, $\frac{3}{7}$ and $\frac{5}{7}$, we find : $\frac{2}{5} < \frac{3}{7} < \frac{5}{7}$

$\therefore$ $\frac{3}{7}$ lies between $\frac{2}{5}$ and $\frac{5}{7}$.

2. Show that $\frac{4}{5}$ lies between $\frac{3}{4}$ and $\frac{5}{6}$.

3. Evaluate :

(i) $3\frac{5}{6} - 1\frac{4}{15} - \left(3\frac{2}{9} - 1\frac{3}{5}\right)$

(ii) $\frac{3}{4}$ of $1\frac{1}{2} \div 4\frac{1}{2}$

(iii) $\frac{5}{6}$ of $\frac{3}{4} \div \frac{7}{8} \times 1\frac{1}{2}$

(iv) $\frac{1}{3} + \frac{7}{9} \div \left(\frac{7}{10} \times 1\frac{1}{4}\right)$

(v) $1\frac{4}{13}$ of $2\frac{2}{7} \div \frac{68}{91} - \left(1\frac{1}{2} - 1\frac{1}{3}\right)$

(vi) $8 - \left\{5\frac{1}{3} - \left(3 - 2\frac{1}{2}\right)\right\}$

4. Mr. Mehra gave one-third of his money to his son, one-fifth of his money to his daughter, and the remaining amount to his wife. If his wife got ₹ 91,000, how much money did Mr. Mehra have originally ?

5. A sum of ₹ 84,000 is divided among three persons : A, B and C. If A gets one-fourth of it and B gets one-fifth of it, how much did C get ?

6. In one hour Rohit walks $3\frac{2}{5}$ km. How much distance will he cover in $2\frac{1}{2}$ hours ?

7. An 84 m long string is cut into pieces, each of length $5\frac{1}{4}$ m. How many pieces are obtained?

8. In buying a ready-made shirt, two-fifths of my pocket money is spent. If ₹ 540 is still left with me, find :
   (i) the money I had before I bought the shirt.
   (ii) the cost of the shirt.

9. Mohan leaves ₹ 1,20,000 to his wife and three children such that two-fifths of this money is given to his wife and the remaining is distributed equally among the children. Find how much each child gets ?

10. Simplify :

(i) $3\frac{5}{8}$ of $2\frac{2}{3} \div 1\frac{3}{8}$ (ii) $\left(1 \div 3\frac{1}{3}\right) \times 3\frac{1}{3}$ of $7\frac{2}{9} - 6$ (iii) $\frac{3}{4} \times 1\frac{1}{3} \div \frac{3}{7}$ of $2\frac{5}{8}$