Chapter 22

Constructions

POINTS TO REMEMBER

1. **Length of Direct Common Tangents :**

$$= \sqrt{d^2 - (r_1 - r_2)^2}$$ where d is the difference between the centres of two circles and r_1, r_2 are the radii of the circles.

2. **Length of Inverse Common Tangents :**

$$= \sqrt{d^2 - (r_1 - r_2)^2}$$ where d is the difference between the centres of the two circles and r_1, r_2 are the radii of the circles.

3. **Angle at the Centre of a Regular Polygon :**

$$= \frac{360°}{n}$$ where n is the number of sides of the regular polygon.

4. **Circumcircle of a Triangle :**

The circle passing through the vertices of a triangle is called the circumcircle of the triangle.

5. **Circumcentre of a Triangle :**

The point of intersection of the right bisectors of any two sides of a triangle is called its circumcentre.

6. **Incircle of a Triangle :**

A circle inside a triangle touching its sides, is called the incircle of a triangle.

7. **Incentre of a Triangle :**

The point of intersection of the bisectors of any two angles of a triangle is called the incentre of the triangle.

EXERCISE 22 (A)

Q.1. Draw a circle of radius 3 cm. Take a point P on it. Using ruler and compasses only construct a tangent to the circle at the point P.

Sol. *Steps of Construction :*

(*i*) Draw a circle with O as centre and radius 3 cm.

(*ii*) Take a point P on it.

(*iii*) Join OP.
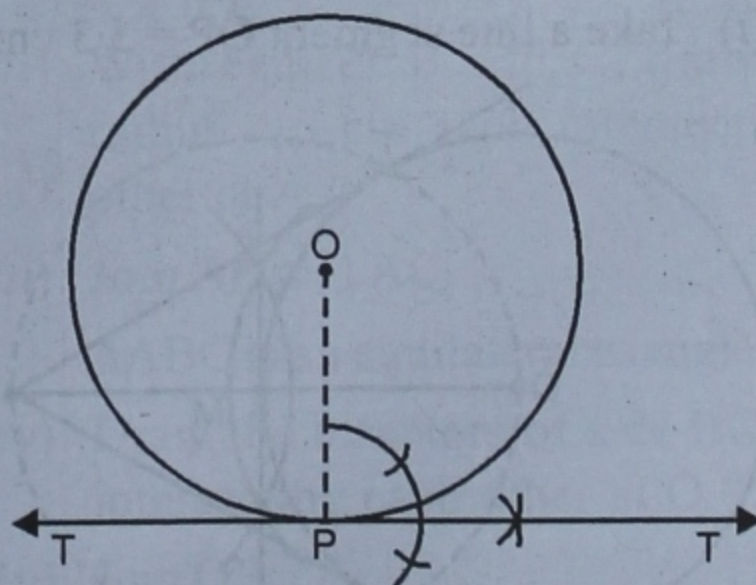
(*iv*) At P draw a perpendicular PT on OP and produce it both sides.

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Then, T'PT is the required tangent to the circle.

Q.2. Draw a circle of radius 3·4 cm. Take a point P on it. Without using the centre of the circle, construct a tangent to the circle at the point P.

Sol. *Steps of Construction :*

(*i*) Draw a circle with radius 3·4 cm.

(*ii*) Take a point P on it.

(*iii*) Take two more points Q and R on it.

(*iv*) Join QP and PR and RQ.
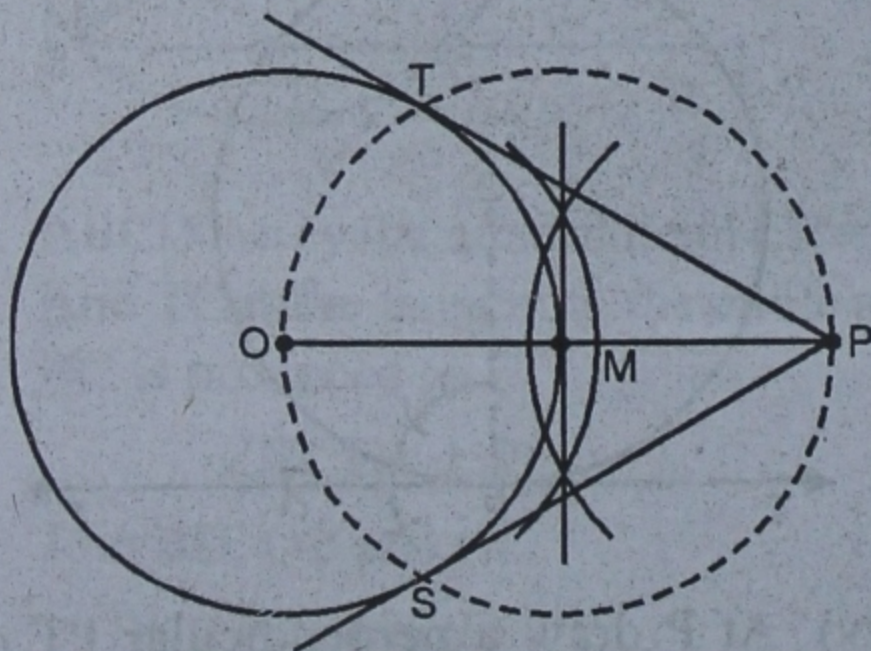
(*v*) Draw ∠QPT = ∠PRQ.

(*vi*) Produce TP to T'.

Then T'PT is the required tangent to the circle.
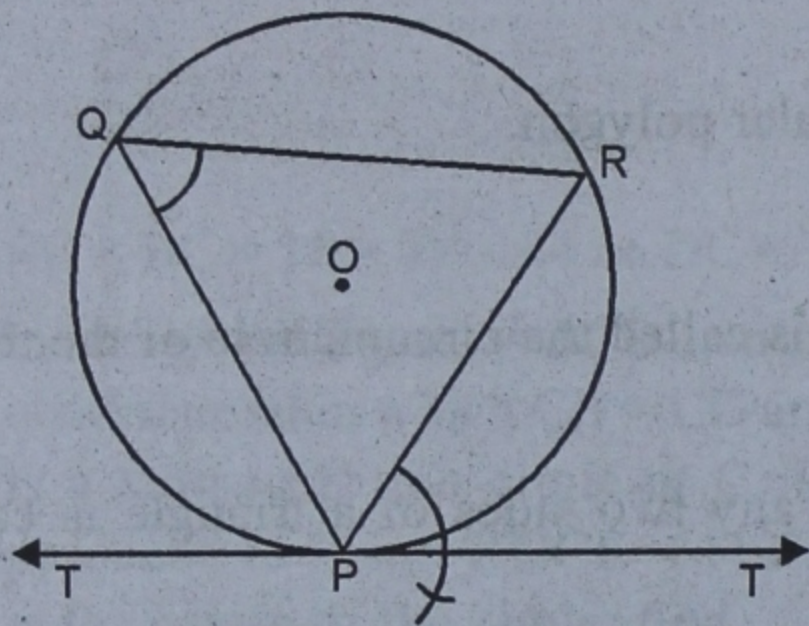
Q.3. Draw a circle of radius 2·7 cm. Mark its centre as O. Take a point P at distance of 5·3 cm from O. From the point P, draw two tangents to the circle. Measure the length of each.

Sol. *Steps of Construction :*

(*i*) Take a line segment OP = 5·3 cm.

(*ii*) At O, with a radius of 2·7 cm, draw a circle.

(*iii*) Bisect OP at M.

(*iv*) With centre M and OP as diameter, draw at circle intersecting the given circle at T and S.

(*v*) Join PT and PS. PT and PS are the required tangents to the circle.

On measuring them, PT = PS = 4·7 cm.

Q.4. Draw a circle of radius 4 cm. Mark its centre as C and mark a point D such that CD = 7 cm. Using ruler and compasses only but not using the centre of the circle, construct two tangents from D.

Sol. *Steps of Construction :*

(*i*) Draw a circle of radius 4 cm with centre C.

(*ii*) Take a point D such that CD = 7 cm.
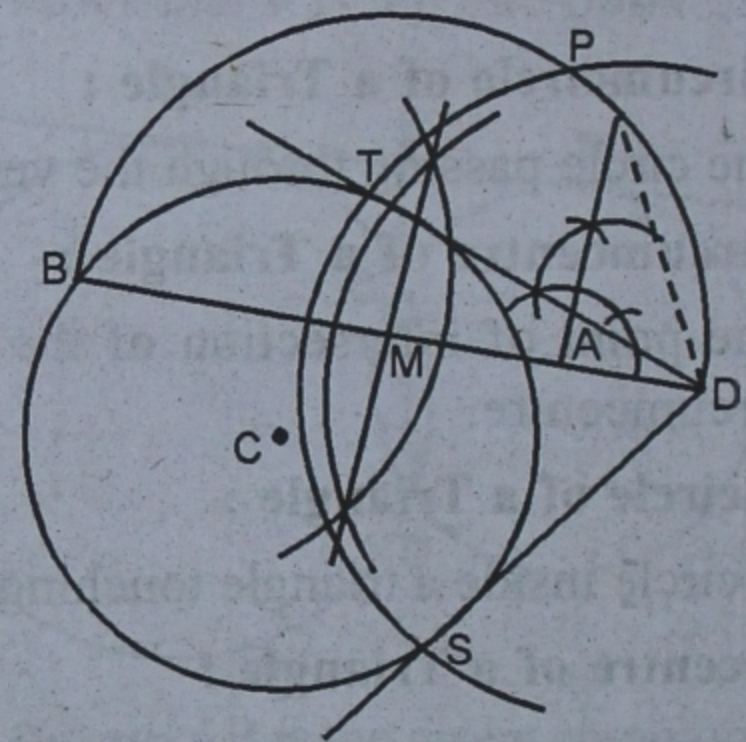
(*iii*) Through D, draw a secant AB intersecting the circle at A and B.

(*iv*) With DB as diameter and M, the mid-point of DB as centre, draw a semi-circle.

(*v*) From A, draw a perpendicular meeting the semi-circle at P.

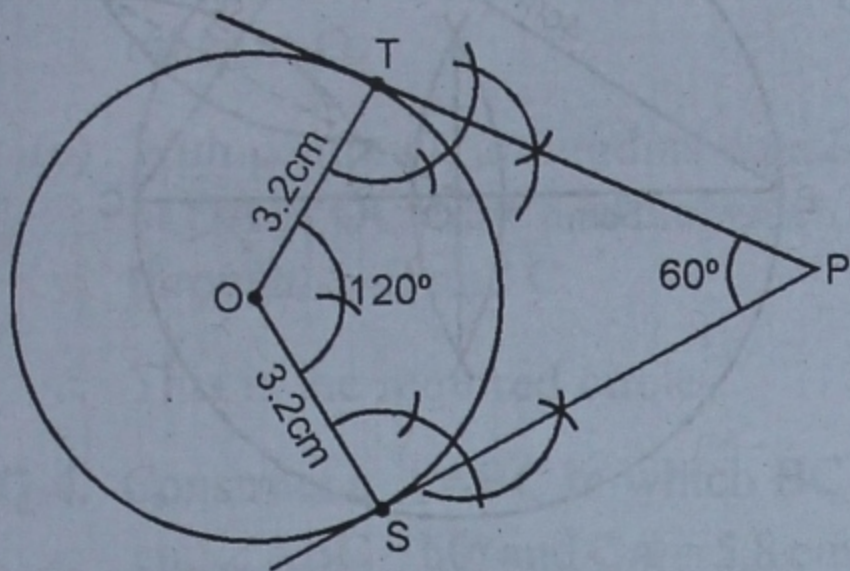(*vi*) With D as centre and DP as radius draw an arc intersecting the circle at T and S.

(*vii*) Join DT and DS.

DT and DS are the required tangents to the given circle.

Q.5. Draw a circle of radius 3·2 cm. Draw two tangents to it inclined at an angle of 60° with each other.

Sol. *Steps of Construction :*

(i) Draw a circle with centre O and radius 3·2 cm.

(ii) Draw a radius OS.

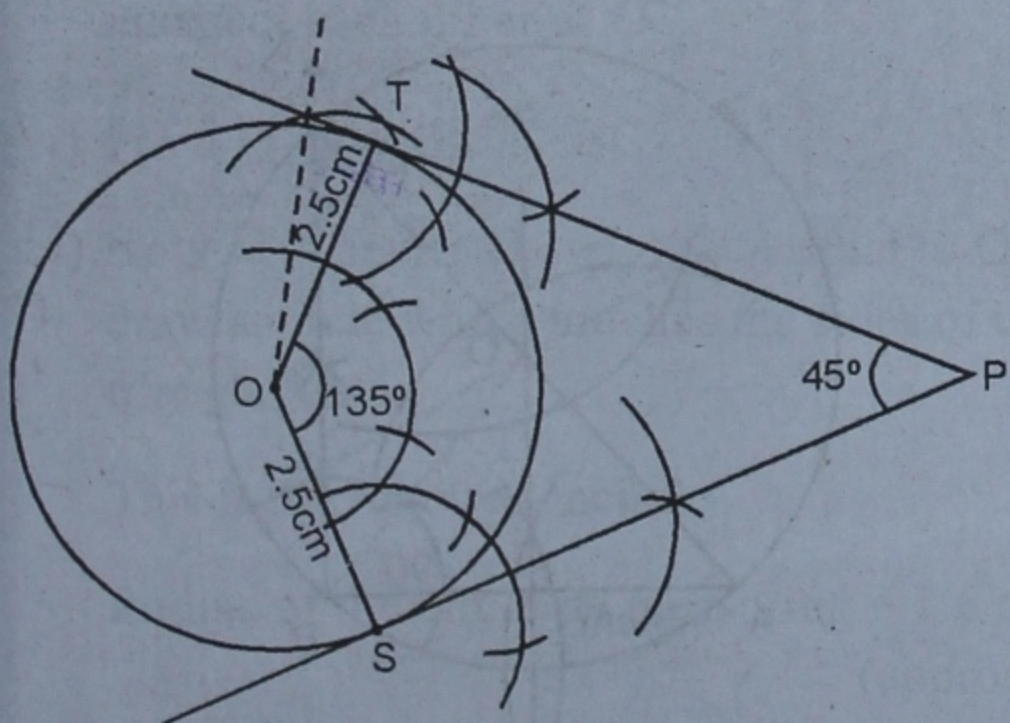(iii) At O, draw angle of (180° – 60°) = 120° on OT.

(iv) At S and T, draw perpendiculars meeting each other at P.

Then, PT and PS are the required tangents inclined at an angle of 60°.

Q.6. Draw a circle of radius 2·5 cm. Draw two tangents to it inclined at an angle of 45° to each other.

Sol. *Steps of Construction :*

(i) Draw a circle with centre O and radius 2·5 cm.

(ii) Draw a radius OS.

(iii) With OS and at O, draw another radius so that ∠TOS = (180° – 45°) 135° with each other.

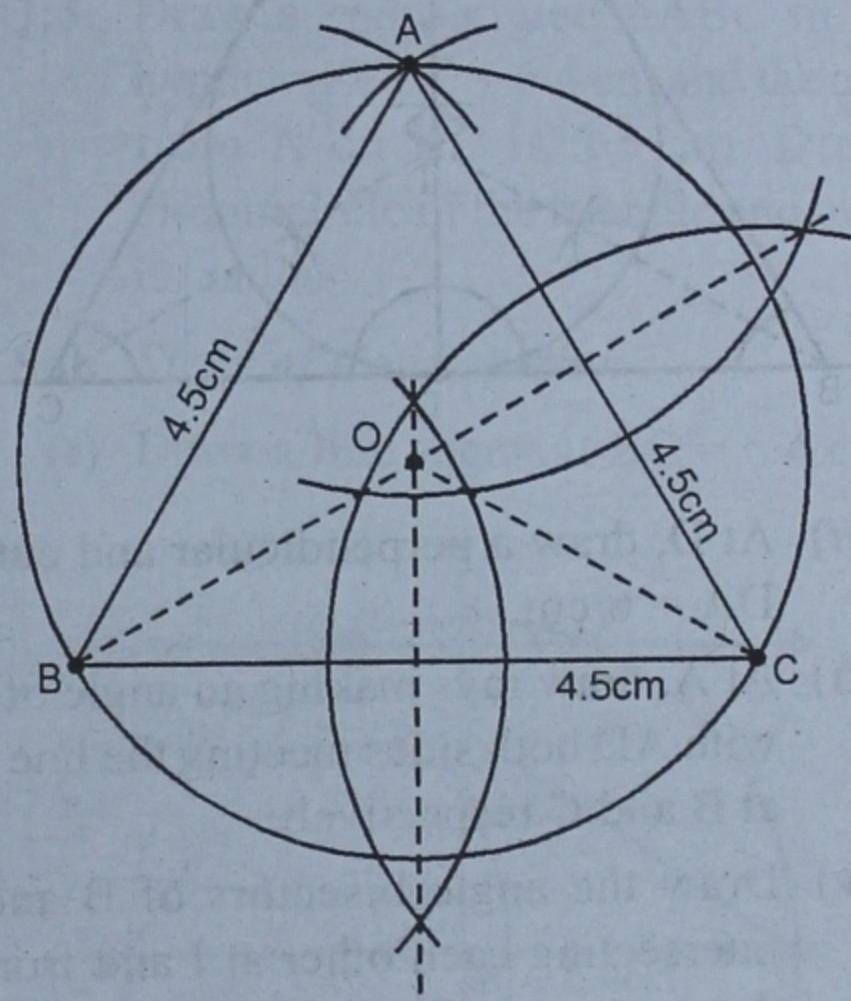(iv) At S and S, draw perpendiculars meeting each other at P.

(v) TP and SP are the required tangents inclined at an angle of 45°.

EXERCISE 22 (B)

Q.1. Using ruler and compasses only, draw an equilateral triangle of side 4·5 cm and draw its circumscribed circle. Measure the radius of the circle.

Sol. *Steps of Construction :*

(i) Draw a line segment BC = 4·5 cm.

(ii) With centres B and C, draw arcs of radius 4·5 cm each intersecting each other at A.

(iii) Join AB and AC.

ΔABC is an equilateral triangle.

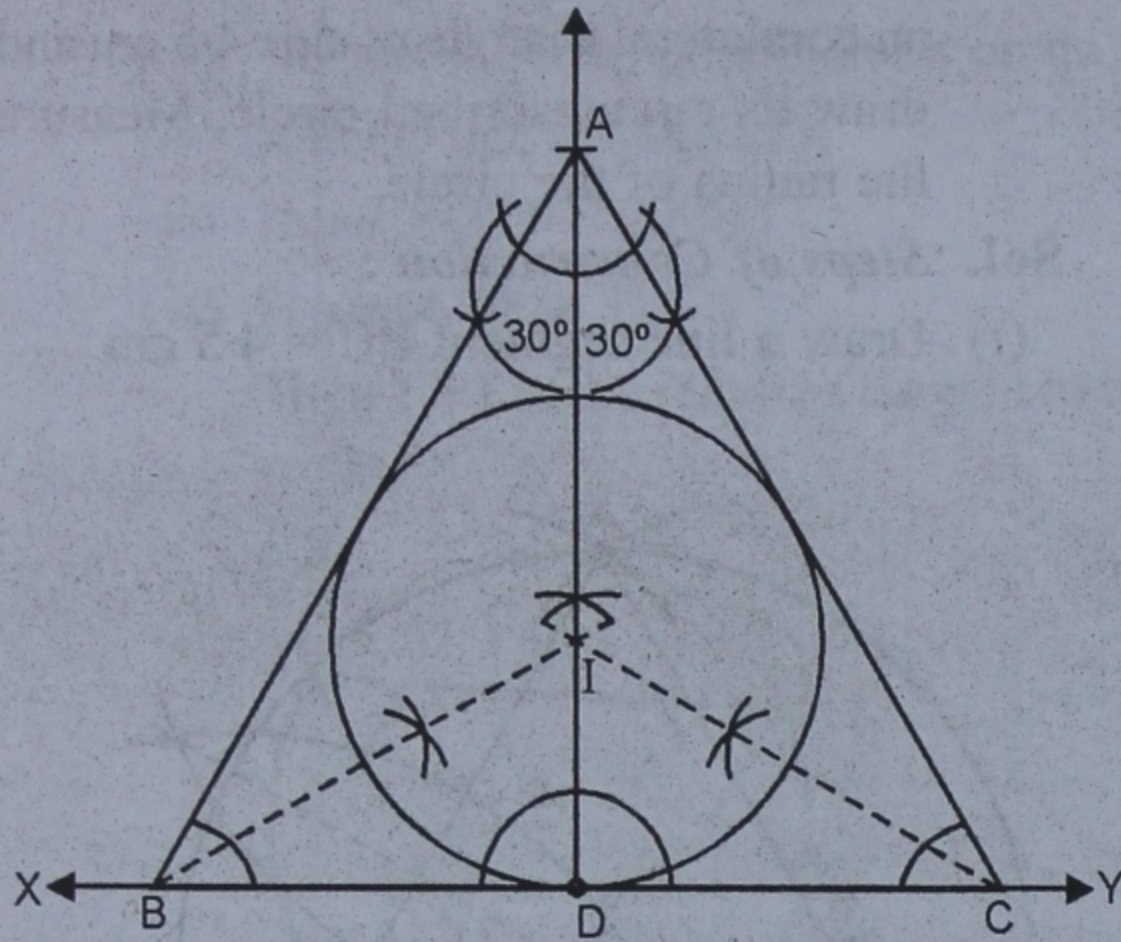(iv) Draw the bisectors of side BC and AC intersecting each other at O.

(v) Join OB.

(vi) With centre O and radius OB, draw a circle which passes through A, B and C.

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This is the required circle and on measuring its radius, it is 2·6 cm.

Q.2. Using ruler and compasses only, draw an equilateral triangle of height 6 cm and draw its inscribed circle. Measure the radius of the circle.

Sol. *Steps of Construction :*

(*i*) Draw a line XY and take a point D on it.

(*ii*) At D, draw a perpendicular and cut off DA = 6 cm.

(*iii*) At A, draw rays making an angle of 30° with AD both sides meeting the line XY at B and C respectively.

(*iv*) Draw the angle bisectors of B and C intersecting each other at I and from I, draw a perpendicular I D on BC.

(*v*) With I as centre and I D as radius, draw a circle which to makes the sides of the triangle internally. This is the required incircle and its radius, on measuring it is 2 cm.

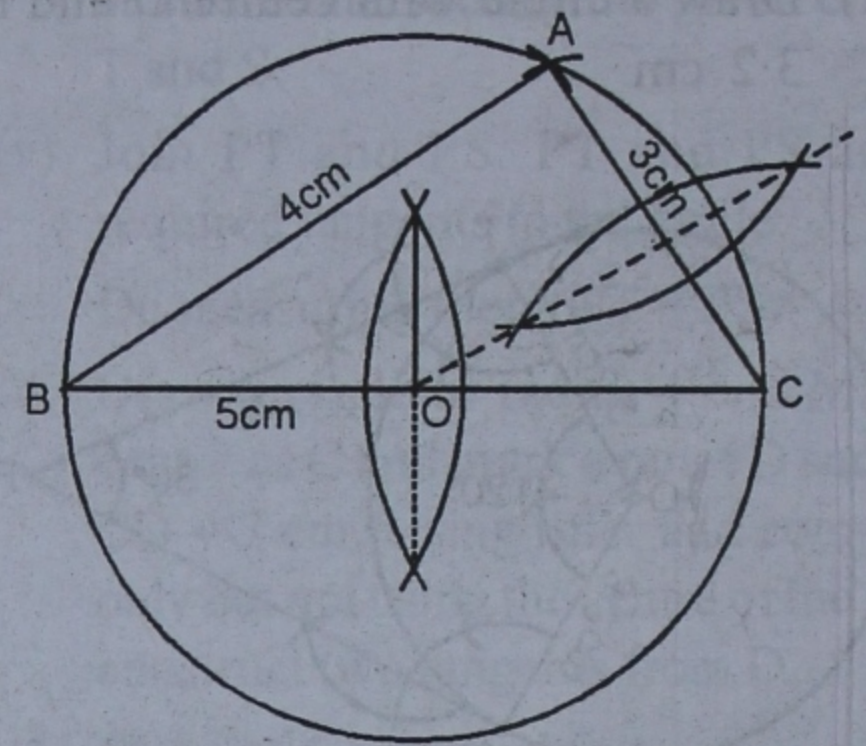**Q.3.** (*i*) Construct a triangle with sides 5 cm, 4 cm and 3 cm. Draw its circumcircle and measure its radius.

(*ii*) Using a ruler and a pair of compasses only, construct :

(a) a triangle ABC, given AB = 4 cm, BC = 6 cm and ∠ABC = 90°.

(b) a circle which passes through the points A, B and C and mark its centre as O.

Sol. (*i*) **Steps of Construction :**

(a) Draw a line segment BC = 5 cm.

(b) With centre B and radius 4 cm, and with centre C and radius 3 cm, draw arcs intersecting each other at A.

(c) Join AB and AC.

(d) Draw the perpendicular bisectors of sides AC and BC intersecting each other at O.
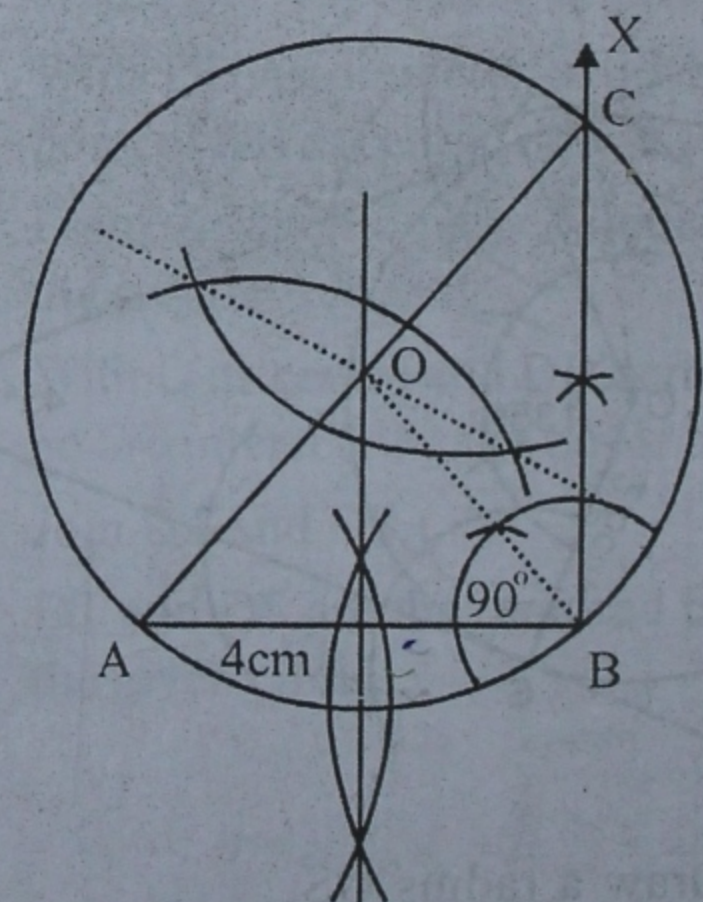
(e) We see that O lies on the side BC.

(f) With centre O and radius OB, draw a circle which will pass through A, B and C respectively.

On measuring the radius of the circle, it is 2·5 cm.

(*ii*) In △ABC, AB = 4 cm, BC = 6 cm
∠ABC = 90°

Steps of construction :

(a) Draw a line segment AB = 4 cm

(b) At B, draw a ray B x making an angle of 90° and cut off BC = 6 cm.

(c) Join AC,

(d) Draw the perpendicular bisectors of sides AB and AC which intersect each other at O.

(e) With centre O, and radius equal to OB or OA or OC draw a circle which passes through A, B and C.

∴ This is the required circle.

Q.4. Construct a △ABC in which BC = 6.4 cm, ∠ABC = 60° and CA = 5.8 cm. Draw the inscribed circle of △ABC. Use ruler and compasses only for your construction. Measure and record the radius of the incircle. (2007)

Sol.

Steps of construction :

(i) Draw a line segment BC = 6.4 cm.

(ii) At B draw a ray BX making an angle of 60°.

(iii) From C, draw an arc of radius 5.8 cm which intersects the ray BX at A.

(iv) Join AC.

(v) Draw the bisectors of ∠B and ∠C which intersect each other at O.

(vi) From O, drw OL ⊥ BC.

(vii) Now with centre O and radius equal to OL, draw a circle which touches the sides of the triangle ABC.
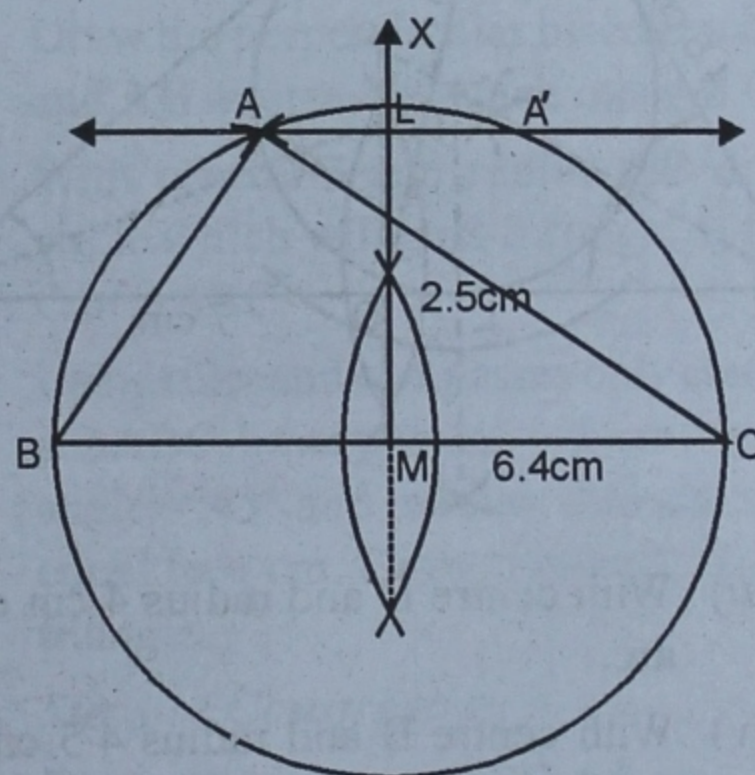
This is the required incircle

Radius of the circle on measuring = 1.6 cm (approx.)
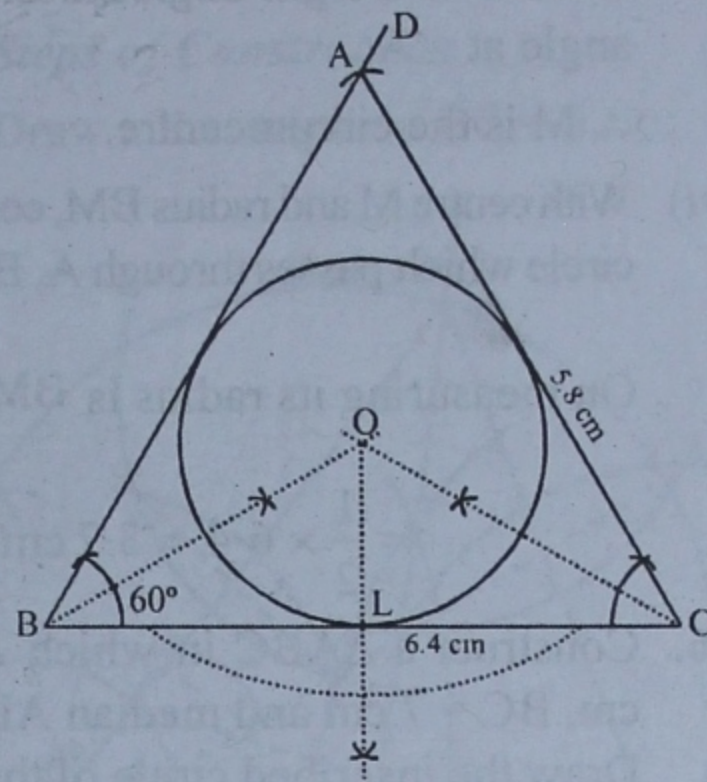
Q.5. Draw a right-angled △ABC in which hypotenuse BC = 6·4 cm and the altitude from A on BC is 2·5 cm. Draw the circumcircle of the triangle and measure its radius.

Sol. *Steps of Construction :*

(i) Draw a line segment BC = 6·4 cm.

(ii) Draw its perpendicular bisector MX and cut off ML = 2·5 cm.

(iii) With centre M and diameter BC, draw a semi-circle.

(iv) Through L, draw a line parallel to BC intersecting the semi-circle at A and A′.

(v) Join AB and AC.

∵ ABC is a right-angled triangle, right angle at ∠A.

∴ M is the circumcentre.

(vi) With centre M and radius BM, complete the circle which passes through A, B and C.

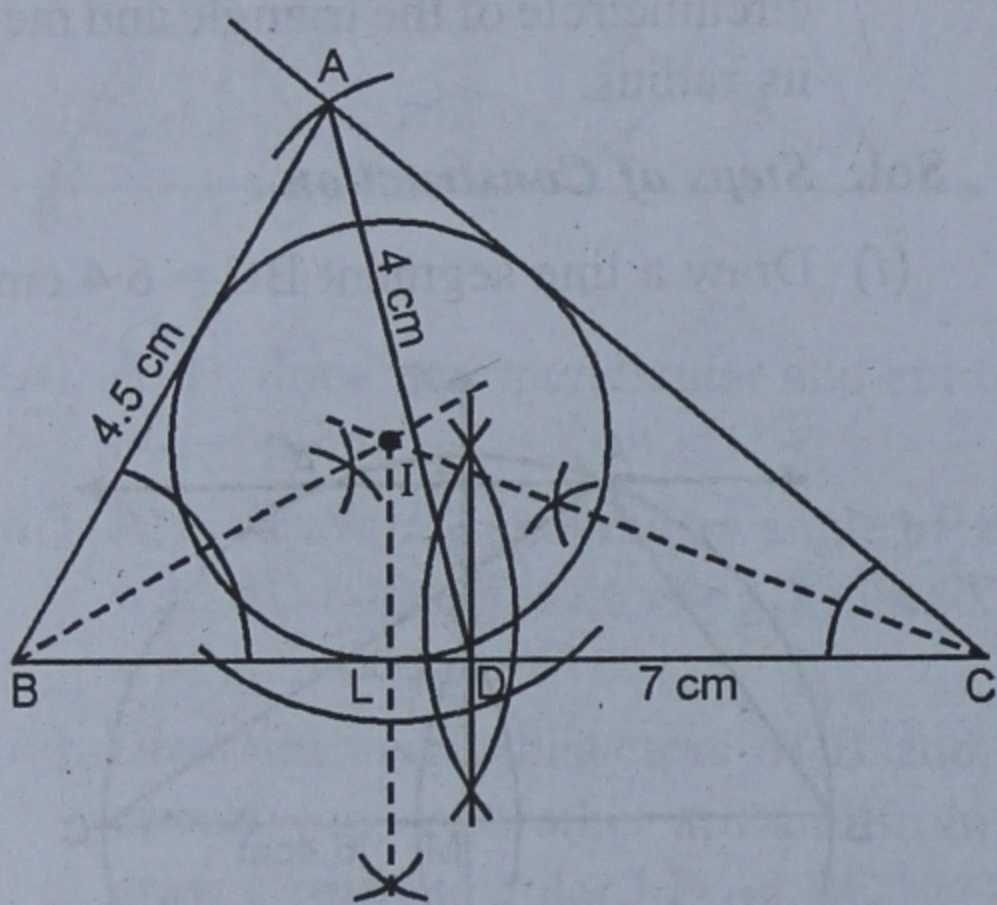On measuring its radius is $BM = \dfrac{1}{2}BC$

$$= \dfrac{1}{2} \times 6 \cdot 4 = 3 \cdot 2 \text{ cm}.$$

Q.6. Construct a ΔABC in which AB = 4·5 cm, BC = 7 cm and median AD = 4 cm. Draw the inscribed circle of the triangle and measure its radius.

Sol. *Steps of Construction :*

(i) Draw a line segment BC = 7 cm.

(ii) Find its mid-point D.

(iii) With centre D and radius 4 cm draw an arc.

(iv) With centre B and radius 4·5 cm, draw another arc intersecting the first arc at A.

(v) Join AB, AC and AD.

(vi) Draw the bisectors of ∠B and ∠C intersecting each other at I.

(vii) From I, draw IL ⊥ BC.

(viii) With centre I and radius IL, draw a circle touching the sides of the ΔABC.

On measuring its radius IL, it is = 1·5 cm.

Q.7. Using ruler and compasses only, construct a ΔPQR in which QR = 6 cm, ∠Q = 60° and ∠R = 75°. Draw the circumcircle of the triangle.

**Sol.** *Steps of Construction :*

(i) Draw a line segment QR = 6 cm.
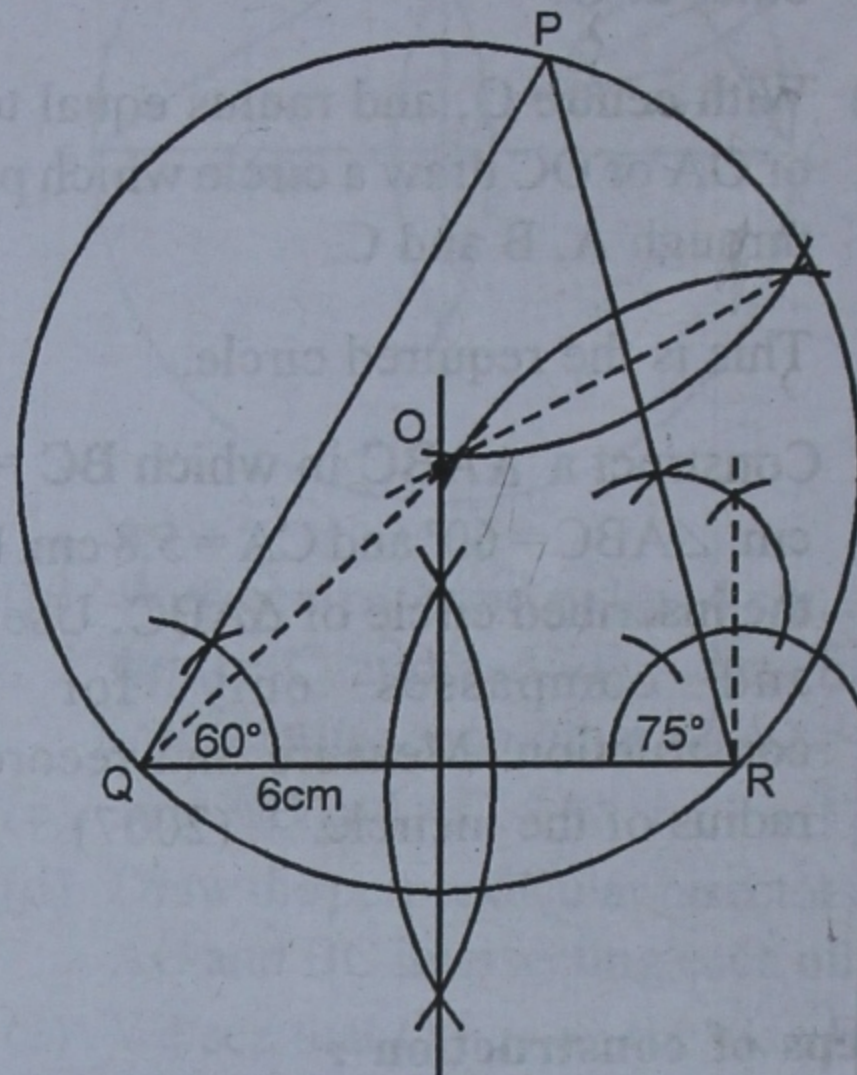
(ii) At Q draw a ray making an angle of 60° and at R, a ray making an angle of 75° which intersect each other at P.

(iii) Draw the perpendicular bisector of side QR and PR intersecting each other at O.

(iv) With centre O and radius OQ, draw a circle. It will pass through P, Q and R.

Q.8. Using compasses and ruler, construct a ΔABC in which AB = 6 cm, BC = 7 cm, AC = 5 cm. Draw a circle inside the triangle touching its sides.

Sol. *Steps of Construction :*

(i) Draw a line segment BC = 7 cm.

(ii) At B, with a radius 6 cm and at C with a radius of 5 cm, draw arcs intersecting each other at A.

(iii) Join AB and AC.

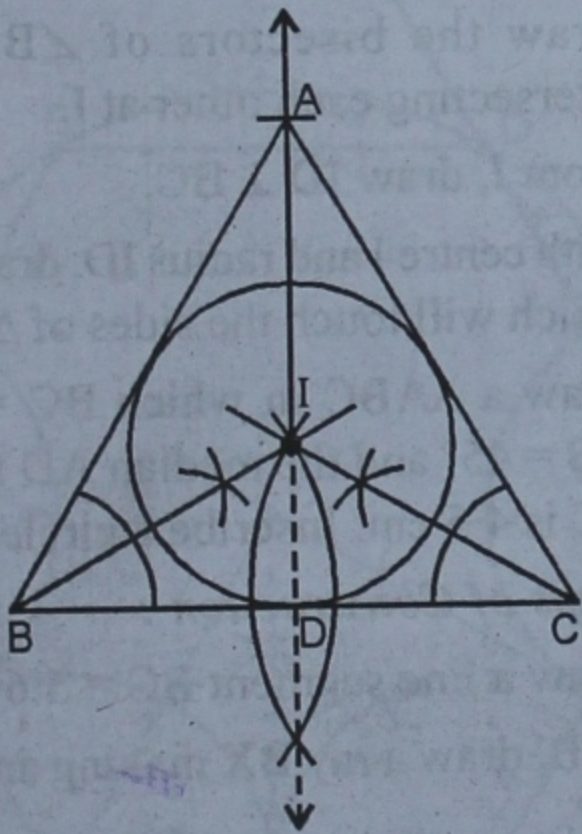(iv) Draw the bisectors of ∠B and ∠C intersecting each other at I.
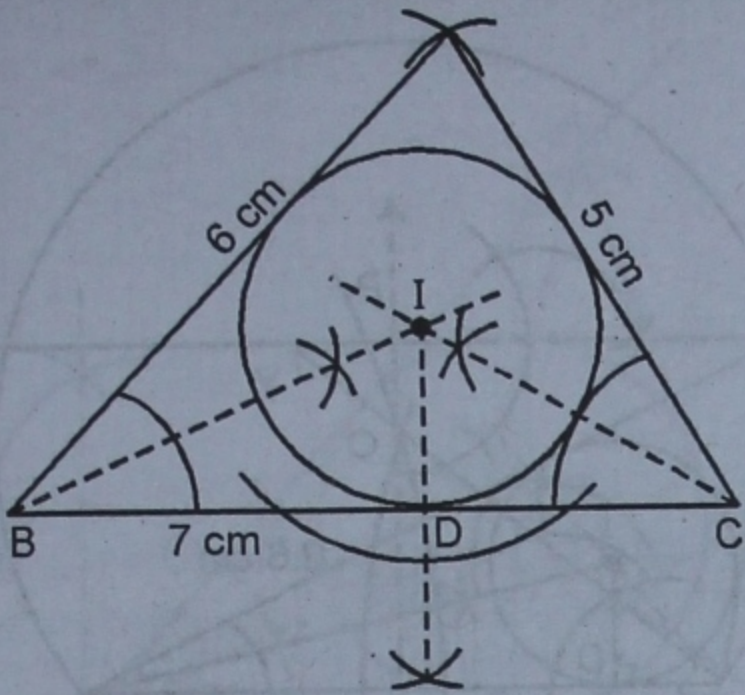
(v) From I, draw a perpendicular ID on BC.

(vi) With centre I and radius ID, draw a circle which will touch the sides of the ΔABC.

Q.9. Draw an isosceles ΔABC in which base BC = 6 cm and the altitude from vertex to the base is 4 cm. Draw its inscribed circle.

Sol. *Steps of Construction :*

(i) Draw a line segment BC = 6 cm.

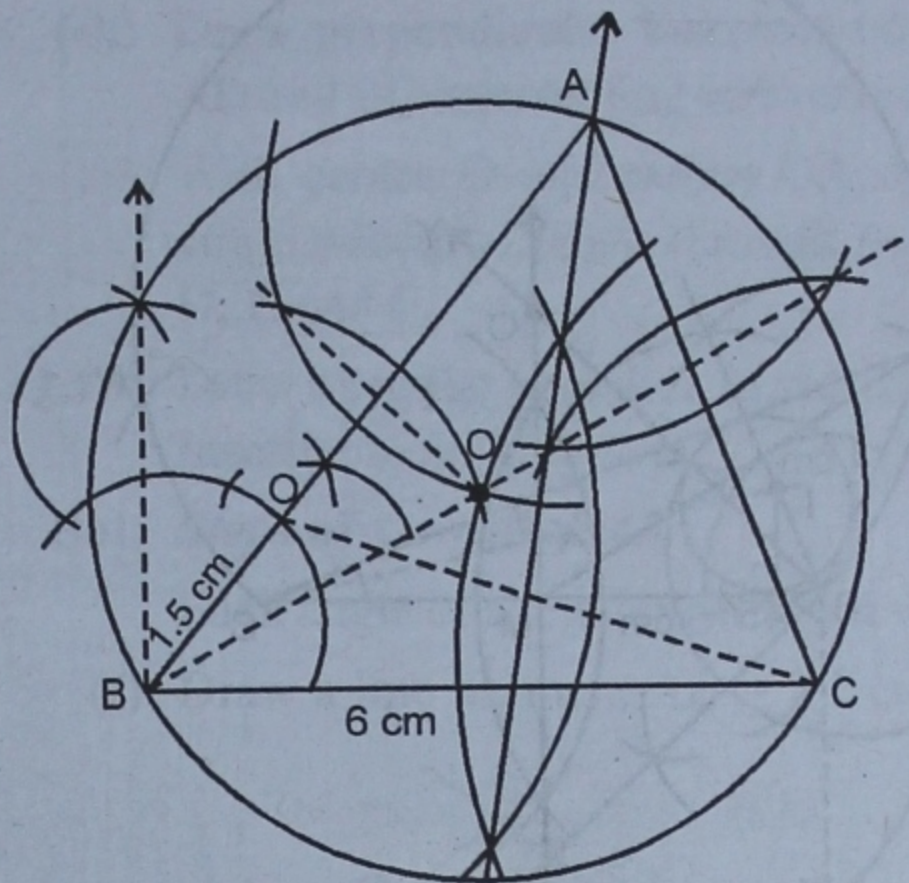(ii) Draw the perpendicular bisector and cut off DA = 4 cm.

(iii) Join AB and AC.

(iv) Draw the bisector of ∠B and ∠C intersecting each other at I.

(v) With I as centre and ID as radius draw a circle which will touch its sides.

Q.10. Draw a ΔABC in which BC = 6 cm, ∠B = 45° and (AB − AC) = 1·5 cm. Draw

the circumcircle of the triangle. Use ruler and compasses only.

**Sol.** *Steps of Construction :*

(i) Draw a line segment BC = 6 cm.

(ii) At B, draw a ray making an angle of 45° and cut off BD = 1·5 cm.

(iii) Join DC and draw its perpendicular bisector which intersects BD produced at A.

(iv) Join AC.

(v) Draw the perpendicular bisectors of AC and AB intersecting each other at O.

(vi) With centre O and radius OB draw a circle which will pass through A, B and C of ΔABC.

Q.11. Using ruler and compasses only construct a ΔABC having base = 5 cm, vertical angle = 45° and median through vertex equal to 4 cm. Draw the incircle of the triangle.

Sol. *Steps of Construction :*

(i) Draw a line segment BC = 5 cm.

(ii) Draw a ray BX of 45° at B below the base BC.

(iii) At B, with BX draw another ray BY making an angle of 90°.

(iv) Draw the perpendicular bisector of BC which intersect the 90° ray at O.
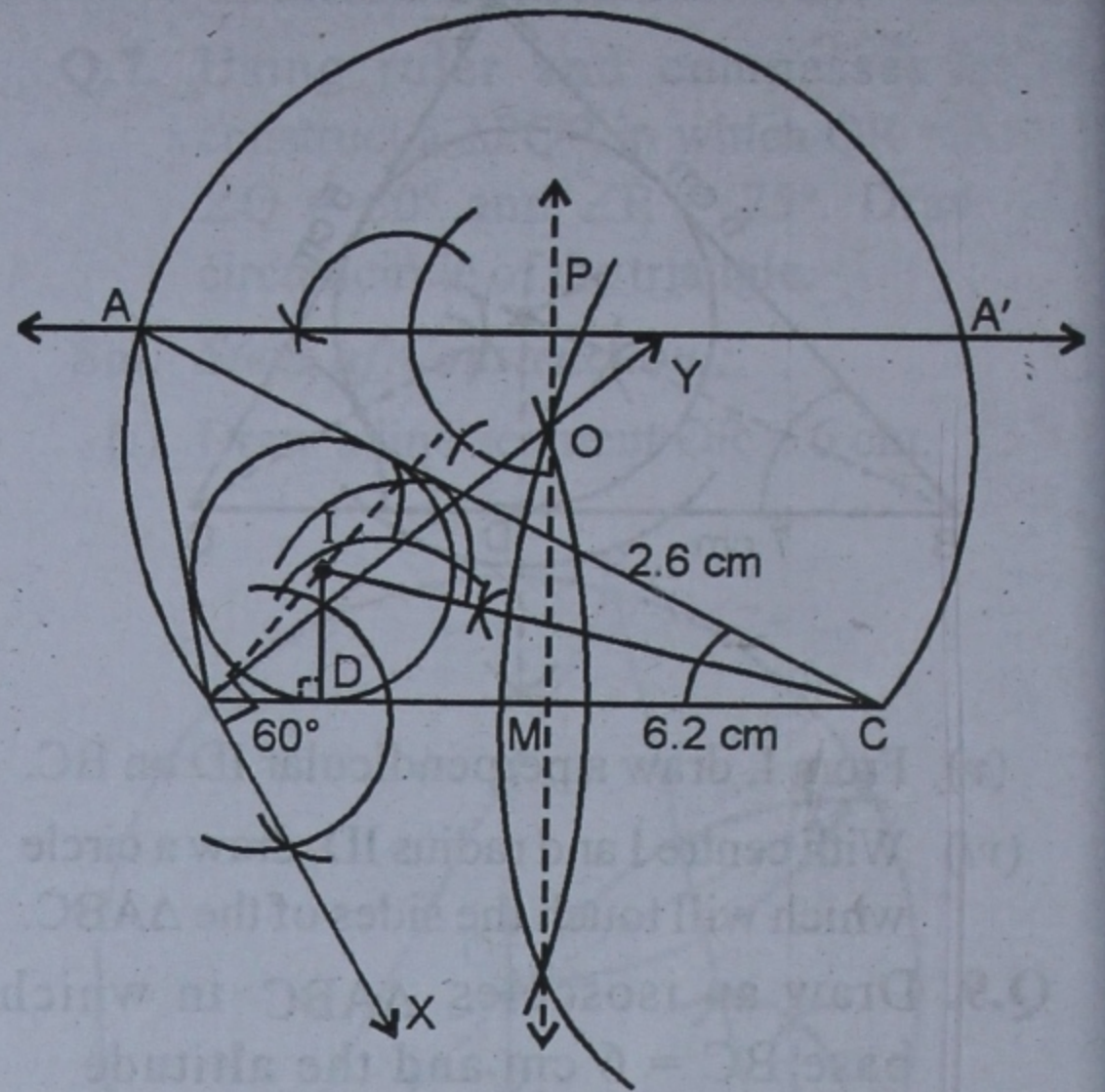
(v) With centre O and radius OB draw a segment on BC.
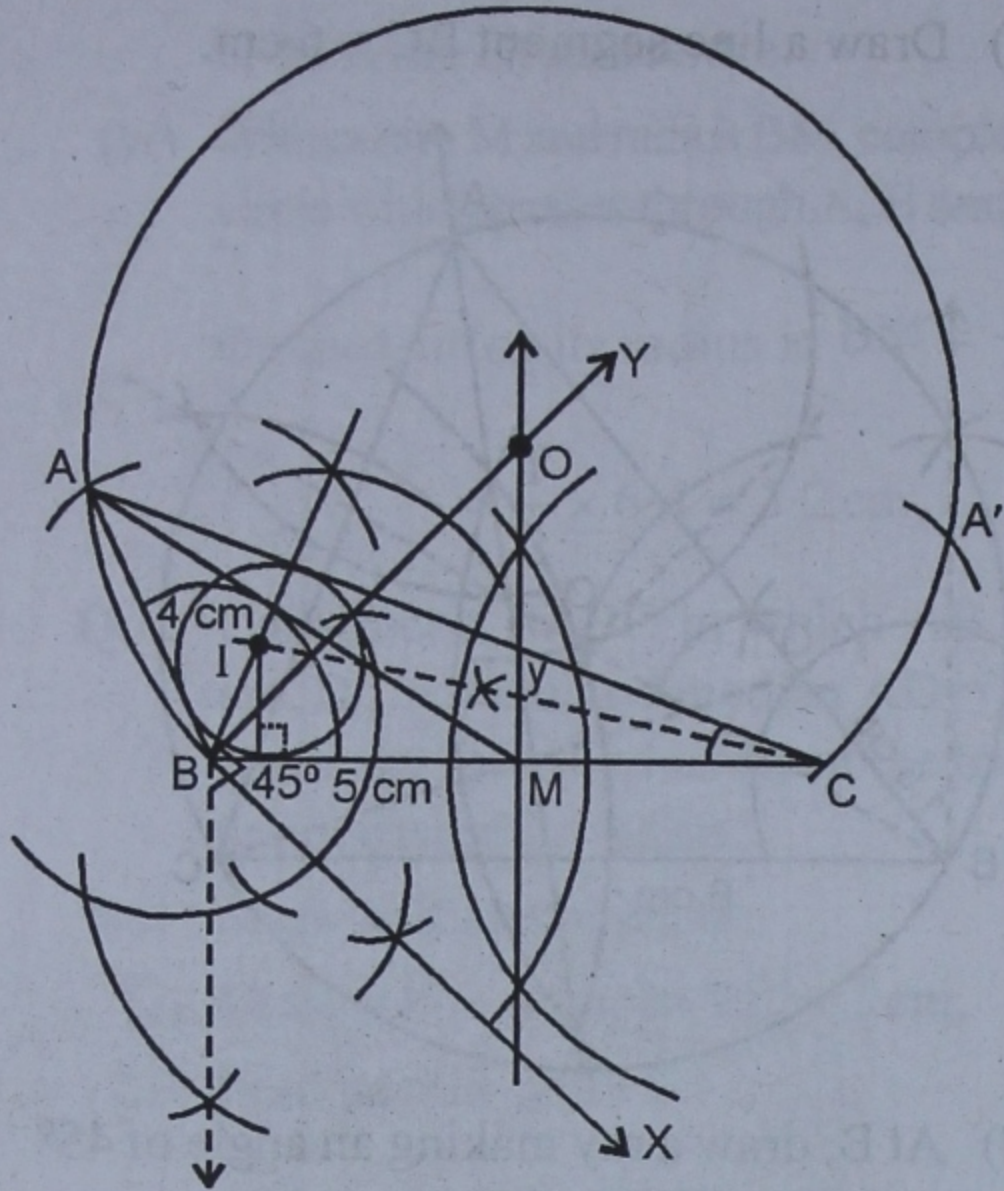
(vi) With centre M, the mid-point of BC and radius 4 cm, draw arc which intersects the segments at A and A'.

(vii) Join AB and AC.

(viii) Draw the bisectors of ∠B and ∠C intersecting each other at I.

(ix) From I, draw a perpendicular ID on BC.

(x) With centre I and radius ID, draw a circle which will touch the sides of the triangle ABC.

Q.12. Using ruler and compasses only, construct a ΔABC in which BC = 6·2 cm, ∠A = 60° and the altitude through A is 2·6 cm. Draw the incircle of the triangle.

Sol. *Steps of Construction :*

(i) Draw a line segment BC = 6·2 cm.

(ii) At B, draw a ray BX below BC making an angle of 60°.

(iii) At B, along AX, draw another ray BY making an angle of 90°.

(iv) Draw the perpendicular bisector of BC making by at O.

(v) With centre O and radius OB, draw a segment on BC.

(vi) Now cut off MP = 2·6 cm.

(vii) From P, draw a line parallel to BC intersecting the segment at A and A'.

(viii) Join AB and AC.

(ix) Draw the bisectors of ∠B and ∠C intersecting each other at I.

(x) From I, draw ID ⊥ BC.
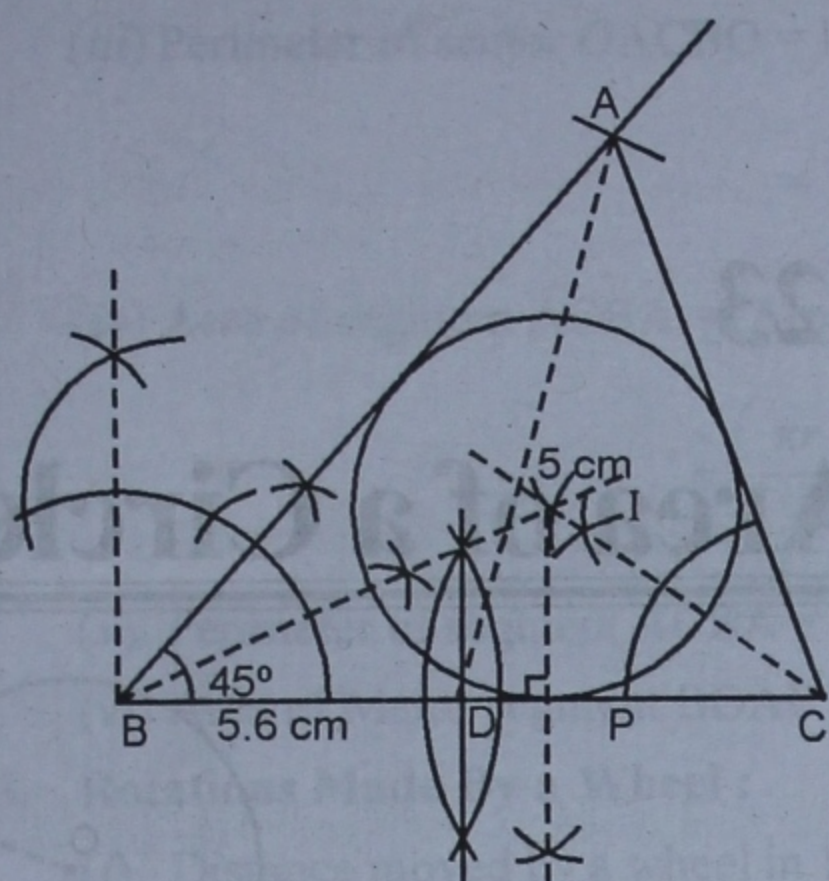
(xi) With centre I and radius ID, draw a circle which will touch the sides of ΔABC.

Q.13. Draw a ΔABC in which BC = 5·6 cm, ∠B = 45° and the median AD from A to BC is 4·5 cm. Inscribe a circle in it.

Sol. *Steps of Construction :*

(i) Draw a line segment BC = 5·6 cm.

(ii) At B, draw a ray BX making an angle of 45°.

(iii) Find the mid-point D of BC.

(iv) With centre D and radius 4·5 cm, draw an arc which intersects BX at A.

(v) Join AD and AC.

(vi) Draw the bisectors of ∠B and ∠C intersecting each other at I.
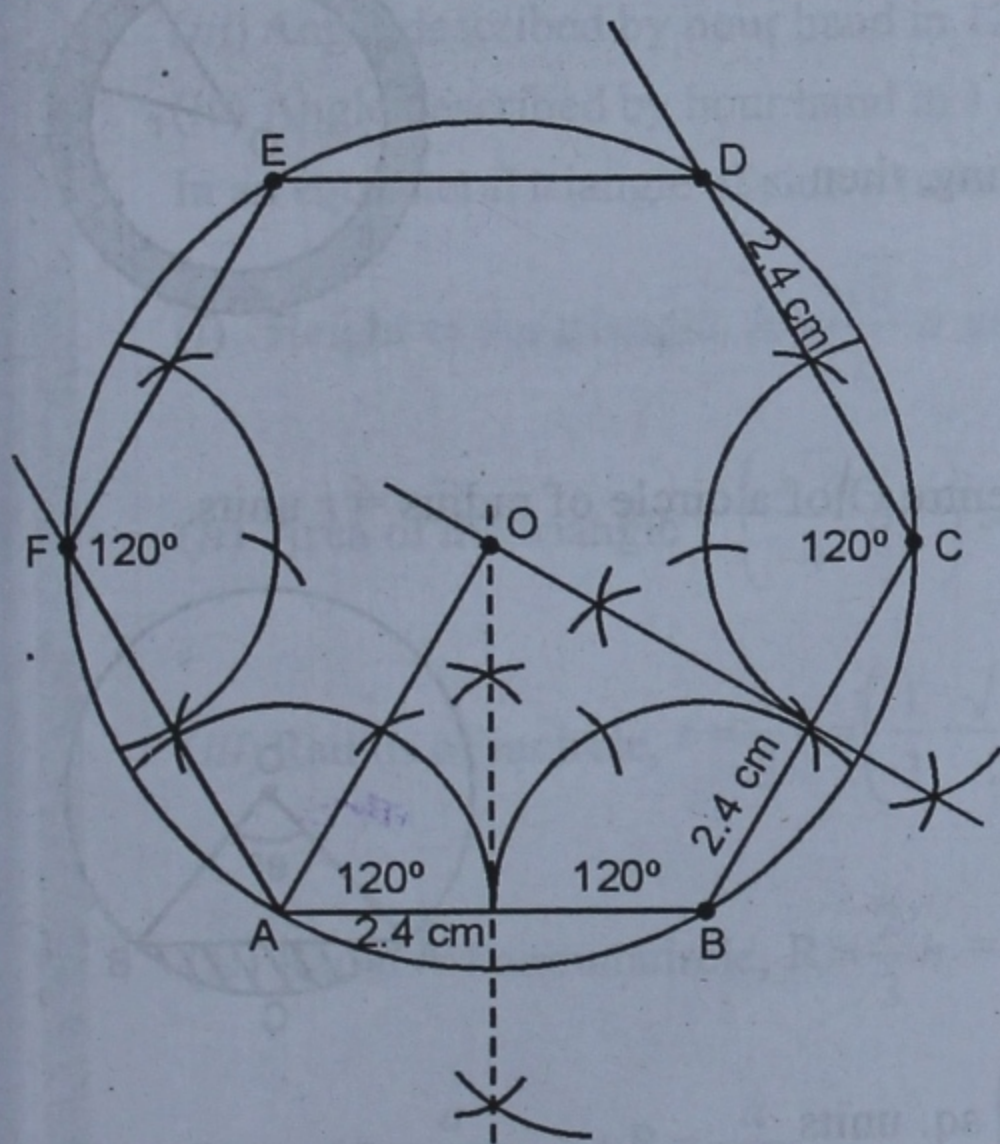
(vii) From I, draw IP ⊥ BC.

(viii) With centre I and radius IP, draw a circle which will touch the sides of the triangle ABC.

Q.14. Draw a regular hexagon of side 2·4 cm. Circumscribe a circle to it.

Sol. *Steps of Construction :*

Here, number of sides (x) = 6

∴ Angle at the centre $= \dfrac{360°}{6} = 60°$

And included angle between two adjacent sides = 120°.

(i) Draw a line segment AB = 2·4 cm.

(ii) At A and B, draw rays making angle of 120° on each and cut off AF = BC = 2·4 cm. ABCDEF is a regular hexagon.

(iii) Draw perpendicular bisectors of sides AB and BC intersecting each other at O.

(iv) With centre O and radius OA draw a circle which will pass through A, B, C, D, E and F.

Q.15. Draw a regular pentagon of side 2·2 cm. Inscribe a circle in it.

Sol. *Steps of Construction :*

Each angle of a regular pentagon = 108°

(i) Draw a line segment AB = 2·2 cm.
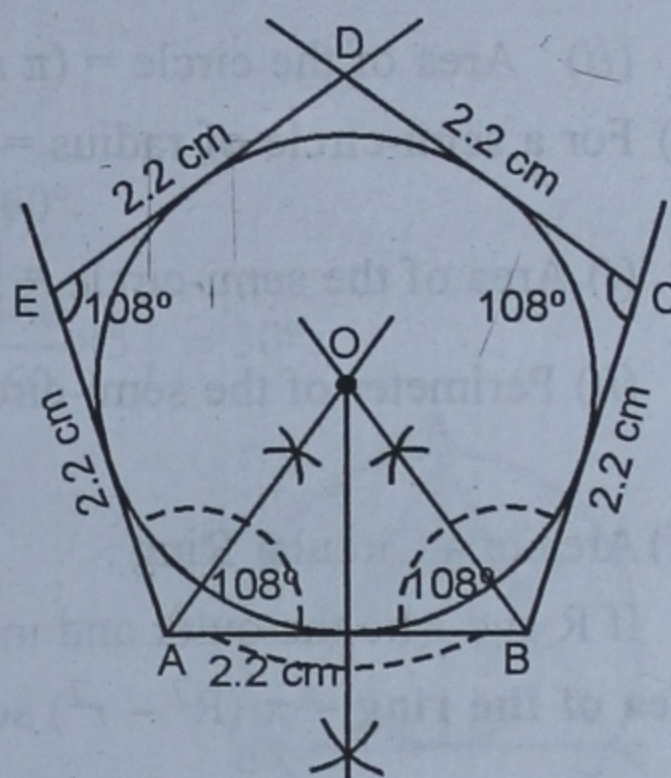
(ii) At A and B, draw rays making an angle of 108° and cut off AE = BC = 2·2 cm.

(iii) Similarly, at C and E, draw rays making an angle of 108° making each other at D.

Now, ABCDE is the required pentagon.

(iv) Draw the bisectors of ∠A and ∠B intersecting each other at O.

(v) From O, draw OL ⊥ AB.

(vi) With centre O and radius OL, draw a circle touching the sides of the regular pentagon.