Chapter 20

Angle Properties of A Circle

POINTS TO REMEMBER

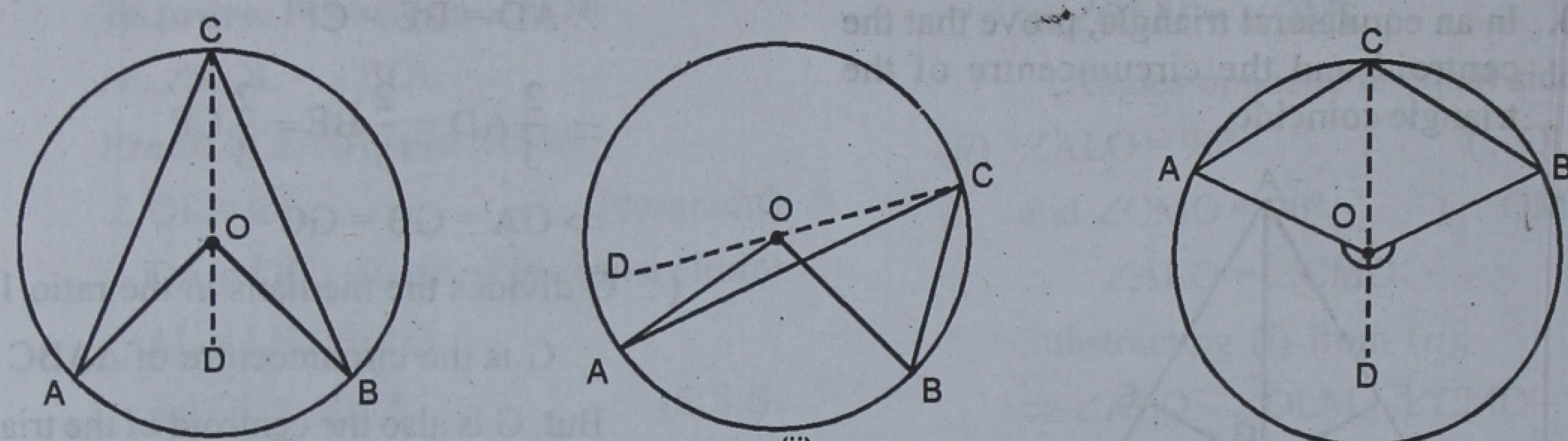
1. Some Important Theorems.

Theorem 1. *The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

Given. A circle with centre O and an arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at any point C on the remaining part of the circle.

To prove. $\angle AOB = 2\angle ACB$.

Construction. Join CO and produce it to some point D.



(i) (ii) (iii)

Proof.

Statement	Reason
1. In ΔAOC,	
$OA = OC$	Radii of the same circle.
$\Rightarrow \angle OAC = \angle OCA$ ...(I)	Angles opposite to equal sides of a Δ are equal.
2. $\angle AOD = \angle OCA + \angle OCA$	Ext. angle of a Δ = Sum of its int. opp. $\angle s$.
$= \angle OCA + \angle OCA$	Using (I).
$= 2\angle OCA$ ...(II)	
3. Similarly,	
$\angle BOD = 2\angle OCB$ ...(III)	
4. In figure (i),	
$\angle AOD + \angle BOD = 2\angle OCA + 2\angle OCB$	Adding corresponding sides of (II) and (III).
$= 2(\angle OCA + \angle OCB) = 2\angle ACB$	
$\therefore$ **$\angle AOB = 2\angle ACB$.**	

In Figure (iii), $\angle AOD + \angle BOD = 2\angle OCA + 2\angle OCB$ $= 2\angle ACB$ $\therefore$ ***Reflex*** $\boldsymbol{\angle AOB = 2\angle ACB}$.	Adding the corresponding sides of (II) and (III).
In Figure (ii), $\angle BOD - \angle AOD = 2\angle OCB - 2\angle OCA$ $= 2(\angle OCB - \angle OCA) = 2\angle ACB$ $\therefore \boldsymbol{\angle AOB = 2\angle ACB}$	Subtracting the corresponding sides of (III) and (II).

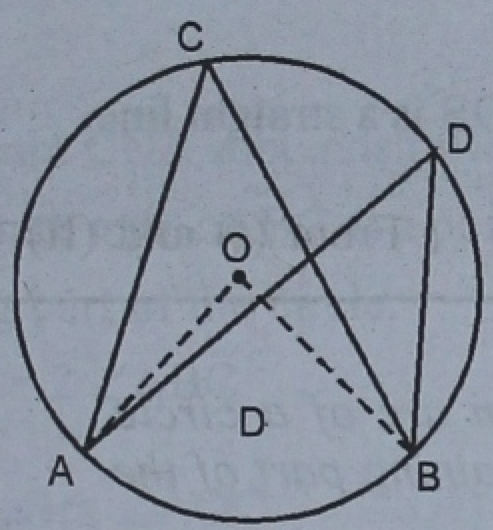
Hence, $\angle AOB = 2\angle ACB$.

Theorem 2. *Angles in the same segment of a circle are equal.*

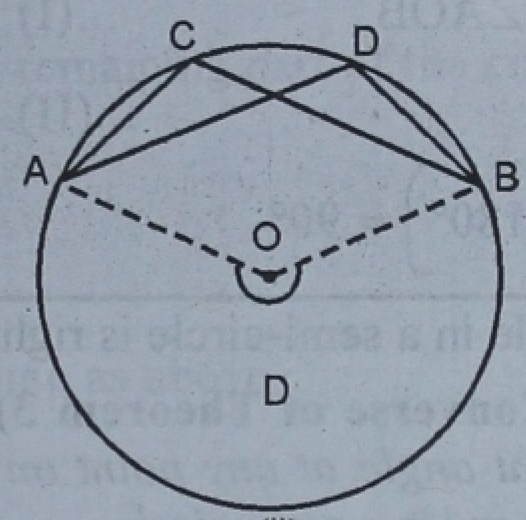
Given. A circle with centre O and two angles $\angle ACB$ and $\angle ADB$ in the same segment of the circle.

To prove. $\angle ACB = \angle ADB$.

Construction. Join OA and OB.



(i)

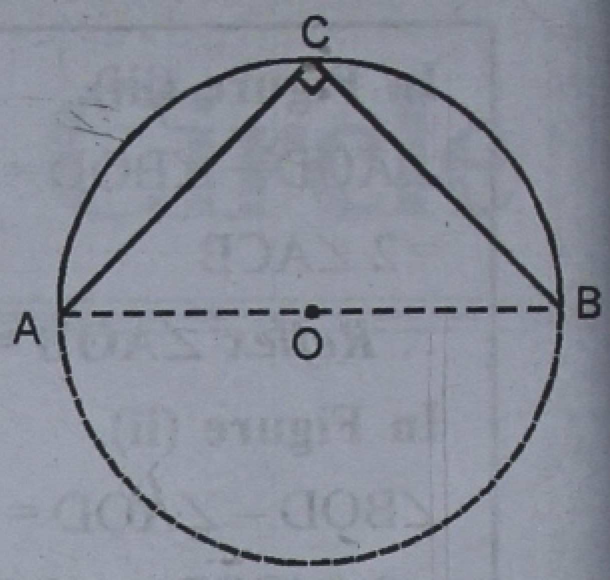


(ii)

Proof.

Statement	Reason
In Fig. (I) :	
1. Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at a point C of the remaining part of the circle. $\therefore \angle AOB = 2\angle ACB$ ...(I)	Angle at the centre is double the angle at any point on remaining part of the circle.
2. Arc AB subtends $\angle AOB$ at the centre and $\angle ADB$ at a point D on the remaining part of the circle. $\therefore \angle AOB = 2\angle ADB$ ...(II)	Same as above.
3. $2\angle ACB = 2\angle ADB$ $\therefore \angle ACB = \angle ADB$	From (I) and (II).
4. Similarly, in Fig. (II) : $\angle ACB = \angle ADB = \frac{1}{2}$ reflex $\angle AOB$ $\therefore \angle ACB = \angle ADB$.	

Hence, the angles in the same segment of a circle are equal.



Theorem 3. *The angle in a semi-circle is a right angle.*

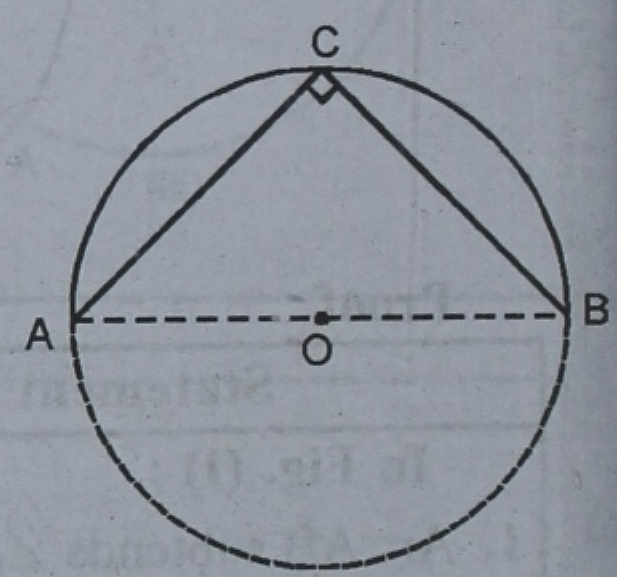
Given. A semi-circle ACB of a circle with centre O.

To prove. $\angle ACB = 90^\circ$.

Proof.

Statement	Reason
1. Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at a point C on the remaining part of the circle.	
$\therefore \angle AOB = 2\angle ACB$	Angle at the centre is double the angle at any point on remaining part of the circle.
$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB$ ...(I)	
2. $\angle AOB = 180^\circ$ ...(II)	AOB is a straight line.
3. $\angle ACB = \left(\frac{1}{2} \times 180^\circ\right) = 90^\circ$	From (I) and (II).

Hence, the angle in a semi-circle is right angle.



Theorem 4 (Converse of Theorem 3). *If an arc of a circle subtends a right angle at any point on the remaining part of the circle, then the arc is a semi-circle.*

Given. A circle with centre O and an arc AB subtending $\angle ACB$ at a point C on the remaining part of the circle such that $\angle ACB = 90^\circ$.

To prove. Arc AB is a semi-circle.

Construction. Join OA and OB.

Proof.

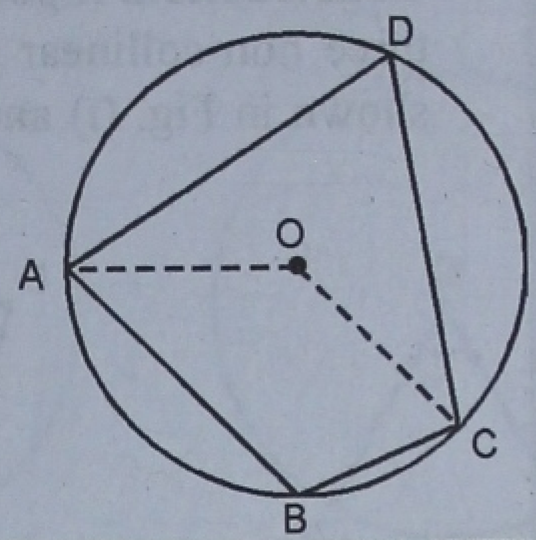
Statement	Reason
1. Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at a point C on the remaining part of the circle.	
$\therefore \angle AOB = 2\angle ACB$ ...(I)	Angle at the centre is double the angle at a point on the remaining of the circle.
2. $\angle ACB = 90^\circ$ ...(II)	Given.
3. $\therefore \angle AOB = (2 \times 90^\circ) = 180^\circ$	From (I) and (II).
$\Rightarrow$ AOB is a straight line.	
$\Rightarrow$ AOB is a diameter	Chord AB passes through centre O.
$\Rightarrow$ Arc AB is a semi-circle.	

Hence, arc AB is a semi-circle.

Theorem 5. *The opposite angles of a quadrilateral inscribed in a circle are supplementary.*

OR

The sum of the opposite angles of a cyclic quadrilateral is 180°.



Given. A quadrilateral ABCD inscribed in a circle with centre O.

To prove. $\angle ADC + \angle ABC = 180°$

and $\angle BAD + \angle BCD = 180°$

Construction. Join OA and OC.

Proof.

Statement	Reason
1. Arc ABC subtends ∠AOC at the centre and ∠ADC at a point D on the remaining part of the circle. $\therefore \angle AOC = 2\angle ADC$ $\Rightarrow \angle ADC = \frac{1}{2}\angle AOC$ ...(I)	Angle at the centre is double the angle at any point on remaining part of the circle.
2. Similarly, major arc CDA subtends reflex ∠AOC at the centre and ∠ABC at a point B on the remaining part of the circle. $\therefore$ reflex $\angle AOC = 2\angle ABC$ $\Rightarrow \angle ABC = \frac{1}{2}$ reflex $\angle AOC$ ...(II)	Same as above
3. Adding (I) and (II), we get $\angle ADC + \angle ABC = \frac{1}{2}\angle AOC + \frac{1}{2}$ reflex $\angle AOC$ $= \frac{1}{2}(\angle AOC + \text{reflex } \angle AOC)$ $= \left(\frac{1}{2} \times 360°\right) = 180°$ $\therefore \angle ADC + \angle ABC = 180°$	(∠AOC + reflex ∠AOC) = sum of the angle around a point O = 360°
4. Similarly, $\angle BAD + \angle BCD = 180°$.	

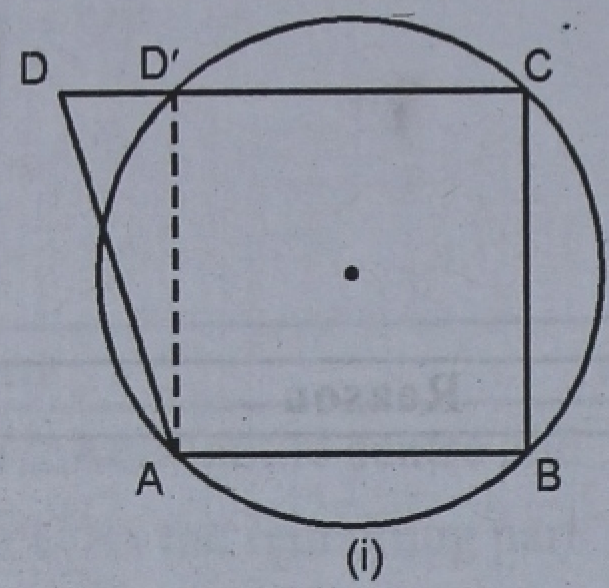
Hence, the opposite angles of a cyclic quadrilateral arc supplementary

Theorem 6. (*Converse of Theorem 5*). *If a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.*

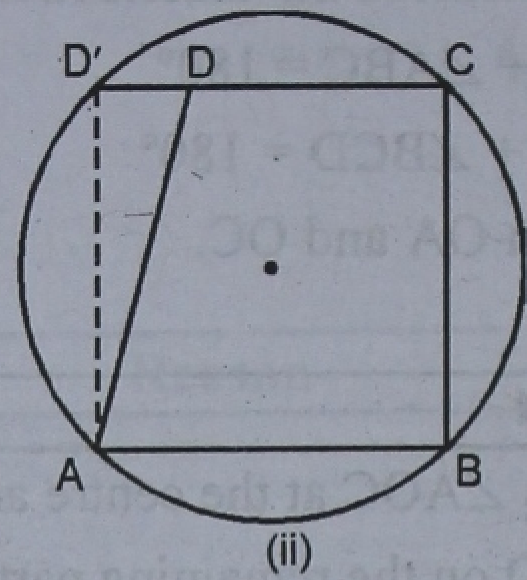
Given. A quadrilateral ABCD is which $\angle B + \angle D = 180°$.

To prove. ABCD is a cyclic quadrilateral.

Construction. If possible, let ABCD be not a cyclic quadrilateral. Draw a circle passing through three non-collinear points A, B, C. Suppose this circle meets CD or CD produced at D′, as shown in Fig. (*i*) and Fig. (*ii*) respectively, Join D′A.



(i)



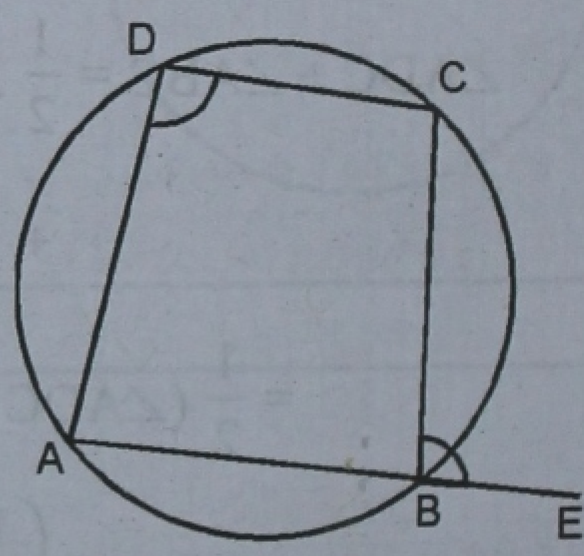
(ii)

Proof.

Statement	Reason
1. ∠B + ∠D = 180°	Given.
2. ∠B + ∠D′ = 180°	ABCD′ is a cyclic quadrilateral and so its opposite ∠s are supplementary.
3. ∠B + ∠D = ∠B + ∠D′ ⇒ ∠D = ∠D′	From 1 and 2.
4. But, this is not possible ∴ Our supposition is wrong.	An exterior angle of a triangle is never equal to its int. opp. angle.

Hence, ABCD is a cyclic quadrilateral.

Theorem 7. *The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.*



Given. A cyclic quadrilateral whose side AB is produced to a point E.

To prove. ∠CBE = ∠ADC

Proof.

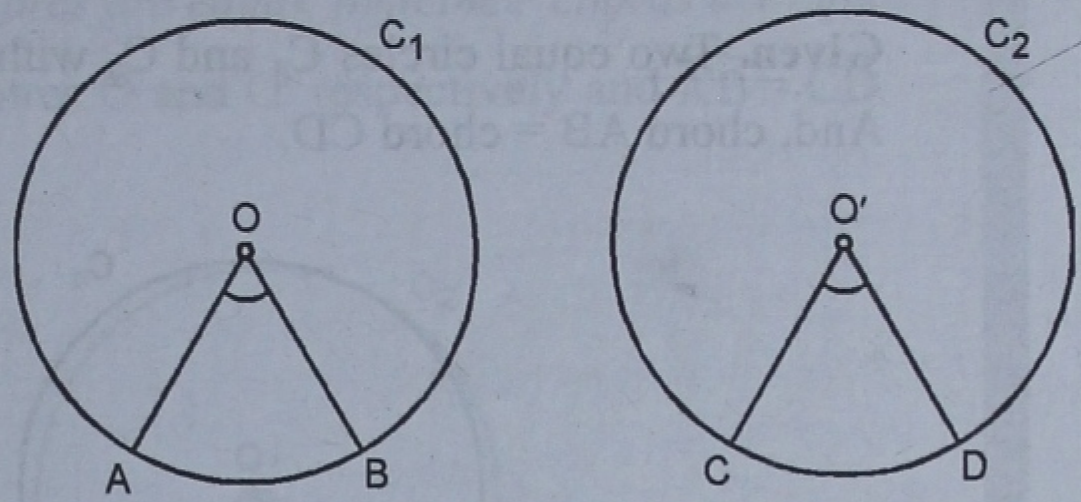
Statement	Reason
1. ∠ABC + ∠ADC = 180°	ABCD is a cyclic quadrilateral and so the sum of its opp. ∠s is 180°.
2. ∠ABC + ∠CBE = 180°	ABE is a straight line.
3. ∠ABC + ∠ADC = ∠ABC + ∠CBE	From 1 and 2.
⇒ ∠ADC = ∠CBE	∠ABC is common to both sides.

Hence, the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

2. Arc Properties of Circles (Theorems) :

Theorem 1. *In equal circles (or in the same circle), if two arcs subtend equal angles at the centre, they are equal.*

Given. Two equal circles C_1 and C_2 with O and O′ as their centres respectively. $\overset{\frown}{AB}$ subtends ∠AOB and $\overset{\frown}{CD}$ subtends ∠CO′D such that ∠AOB = ∠CO′D.



To Prove : $\overset{\frown}{AB} = \overset{\frown}{CD}$

Proof.

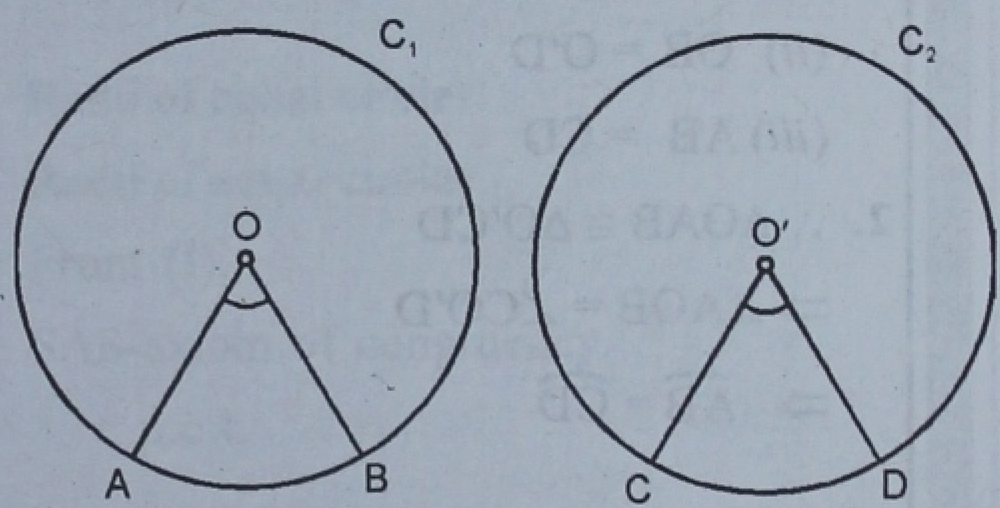
Statement	Reason
1. Place circle C_1 on circle C_2 such that O falls on O′ and OA falls along O′C.	
2. Then, A falls on C and OB falls along O′D.	OA = O′C (Radii of equal circles). ∠AOB = ∠CO′D (Given).
3. Clearly, B falls on D. ∴ AB completely coincides with CD.	OB = O′D (Radii of equal circles). A falls on C, B falls on D and AB falls along CD, as circles are equal.

Hence, $\overset{\frown}{AB} = \overset{\frown}{CD}$.

Theorem 2. (*Converse of Theorem 1*).

In equal circles (or in the same circle), if two arcs are equal, they subtend equal angles at the centre.

Given. Two equal circles C_1 and C_2 with O and O′ as their respective centres such that $\overset{\frown}{AB} = \overset{\frown}{CD}$.



To prove. ∠AOB = ∠CO′D.

To Proof.

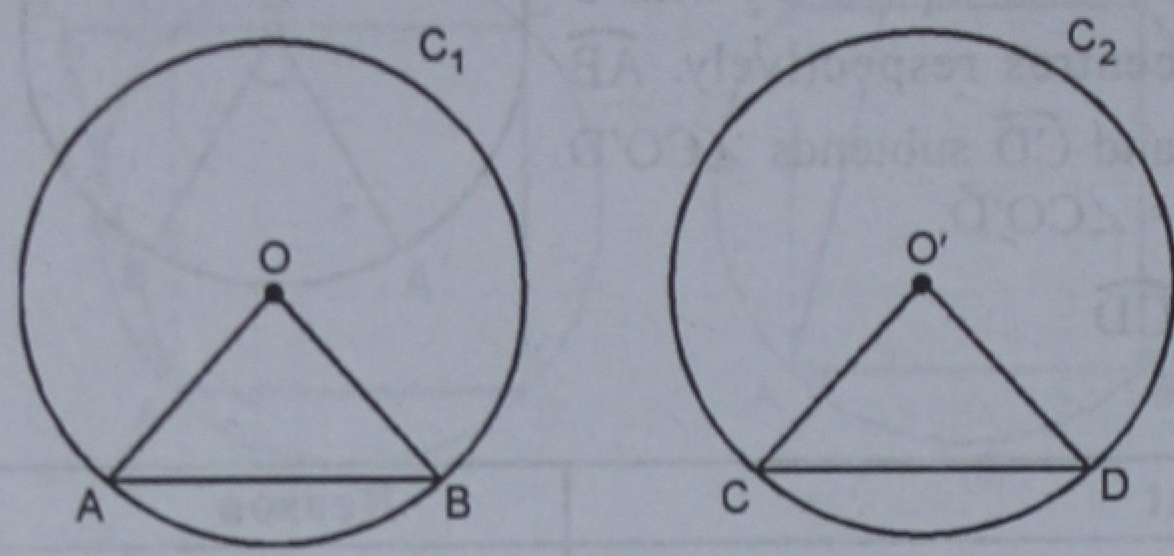
Statement	Reason
1. Place circle C_1 on circle C_2 such that A falls on C, AO falls along CO′ and $\overset{\frown}{AB}$ falls on $\overset{\frown}{CD}$.	
2. Then, O falls on O′ and B falls on D. ∴ OB falls on O′D.	AO = CO′ (Radii of equal circles) and AB = CD (Given).
3. Sector AOB completely coincides with sector CO′D. ∴ ∠AOB = ∠CO′D.	A falls on C, O falls on O′ and B falls on D.

Hence, ∠AOB = ∠CO′D.

Theorem 3. *In equal circles (or in the same circle), if two chords are equal, they cut off equal arcs.*

Given. Two equal circles C_1 and C_2 with centres O and O′ respectively.

And, chord AB = chord CD.



To prove. $\overset{\frown}{AB} = \overset{\frown}{CD}$

Proof.

Statement	Reason
Case I. When $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are Minor Arcs	
1. In ΔOAB and ΔO′CD,	
(*i*) OA = O′C	Radii of equal circles.
(*ii*) OB = O′D	Radii of equal circles.
(*iii*) AB = CD	Given.
2. ∴ ΔOAB ≅ ΔO′CD	SSS-axiom of congruency.
⇒ ∠AOB = ∠CO′D	c.p.c.t.
⇒ $\overset{\frown}{AB} = \overset{\frown}{CD}$ ...(I)	In equal circles, two arcs subtending equal ∠s at the centre, are equal.
Case II. When $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are Major Arcs	
In this case, BA and DC are Minor Arcs.	
∴ AB = CD ⇒ $\overset{\frown}{BA} = \overset{\frown}{DC}$	Chord AB = chord BA, chord CD = chord DC
⇒ $\overset{\frown}{BA} = \overset{\frown}{DC}$	Result being true for Minor Arcs
⇒ $\overset{\frown}{AB} = \overset{\frown}{CD}$	Equal arcs subtracted from equal circles give equal arcs.
Case III. When AB and CD are diameters	
In this case, $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are semi-circles.	
∴ $\overset{\frown}{AB} = \overset{\frown}{CD}$	Semi-circles of equal circles are equal.

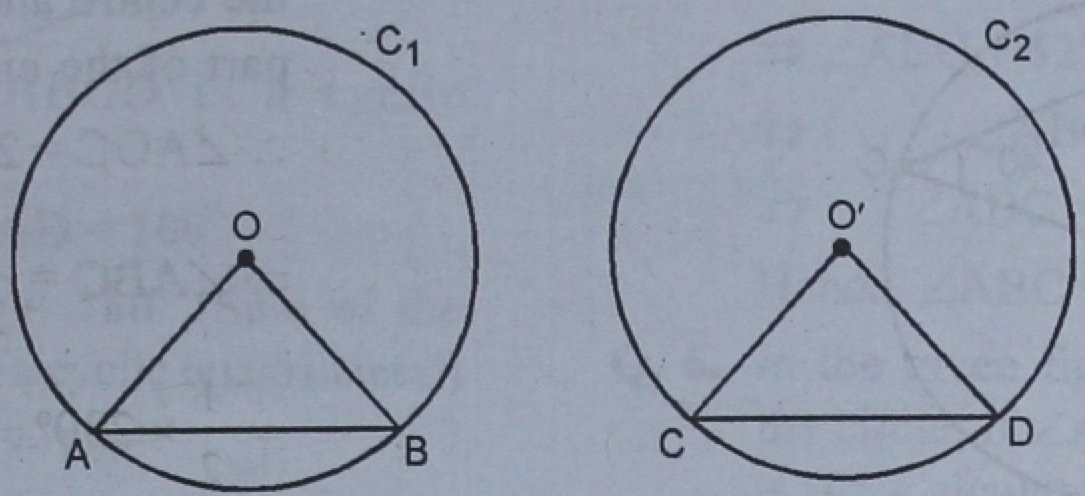
Hence, chord AB = chord CD ⇒ AB = CD.

Theorem 4 (*Converse of Theorem 3*).

In equal circles (or in the same circle), if two arcs are equal, then their chords are equal.

Given. Two equal circles C_1 and C_2 with centres O and O′ respectively and $\overset{\frown}{AB} = \overset{\frown}{CD}$.

To prove. Chord AB = chord CD.



Construction. Join OA, OB, O′C and O′D.

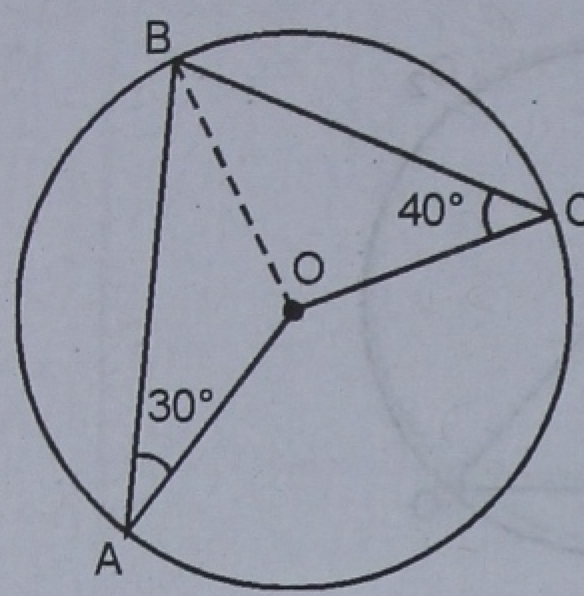
Proof.

Statement	Reason
Case 1. When $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are Minor Arcs	
1. AB = CD ⇒ ∠AOB = ⇒ ∠CO′D...(I)	Equal arcs of equal circles subtend equal angles at the centre.
2. In ΔOAB and ΔO′CD,	
(*i*) OA = O′C	Radii of equal circles.
(*ii*) OB = O′D	Radii of equal circles.
(*iii*) ∠AOB = ∠CO′D	From (I).
∴ ΔOAB ≅ ΔO′CD	SAS-axiom of congruency.
⇒ Chord AB = chord CD ...(II)	c.p.c.t.
Case 2. When $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are Major Arcs	
In this case, $\overset{\frown}{BA}$ and $\overset{\frown}{DC}$ are minor arcs.	
Now, $\overset{\frown}{AB} = \overset{\frown}{CD} \Rightarrow \overset{\frown}{BA} = \overset{\frown}{DC}$	
$\Rightarrow \overset{\frown}{BA} = \overset{\frown}{DC}$	
⇒ BA = DC	
⇒ AB = CD	Chord BA = chord AB, and chord DC = chord CD.
Case 3. When $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are semi-circles	
In this case, AB and CD are diameters.	
∴ AB = CD.	Diameters of equal circles are equal.

Hence, in all the cases, $\overset{\frown}{AB} = \overset{\frown}{CD}$ ⇒ chord AB = chord CD.

EXERCISE – 20 (A)

Q. 1. In the given figure, O is the centre of the circle ; $\angle OAB = 30°$ and $\angle OCB = 40°$. Calculate $\angle AOC$.



Sol. Join OB.

Now in ΔAOB,

$OA = OB$ (Radii of the same circle)

$\therefore \angle OAB = \angle OBA$

(Opposite angles to equal sides)

$\therefore \Rightarrow \angle OBA = 30°$ ($\because \angle OAB = 30°$)

Similarly, in ΔOBC,

$OB = OC$

$\therefore \Rightarrow \angle OBC = \angle OCB = 40°$

Adding, we get

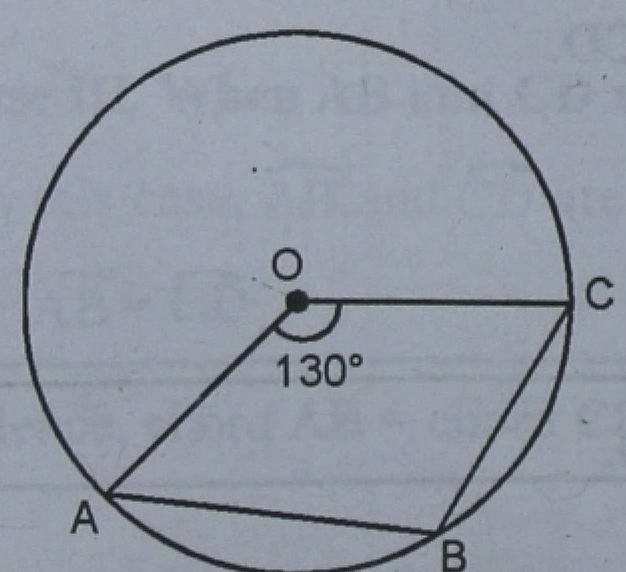
$\angle OBA + \angle OBC = 30° + 40° = 70°$

Now, arc AC subtends $\angle AOC$ at the centre of the circle and $\angle ABC$ at the remaining part of the circle.

$\therefore \angle AOC = 2\angle ABC$

$= 2 \times 70° = 140°$ **Ans.**

Q. 2. In the given figure, O is the centre of the circle and $\angle AOC = 130°$. Find $\angle ABC$.



Sol. In the figure,

$\angle AOC = 130°$

$\therefore$ Reflex $\angle AOC = 360° - 130° = 230°$

Now, major arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle.

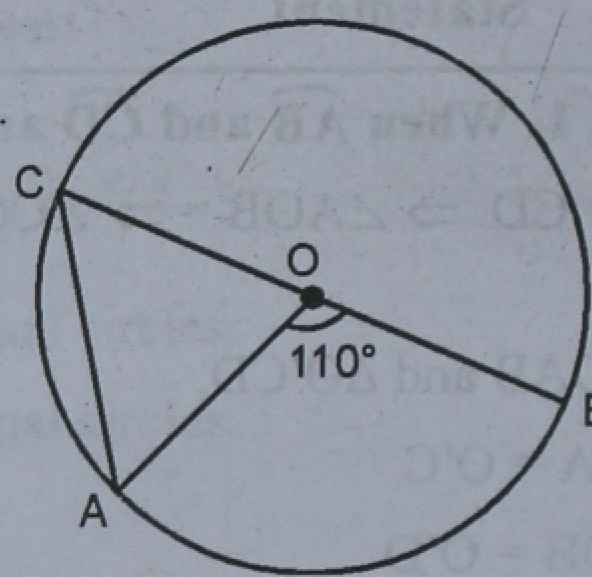
$\therefore \angle AOC = 2\angle ABC.$

$\Rightarrow \angle ABC = \frac{1}{2}\angle AOC$

$= \frac{1}{2} \times 230° = 115°$ **Ans.**

Q. 3. In the given figure, O is the centre of the circle and $\angle AOB = 110°$. Calculate :

(*i*) $\angle ACO$ (*ii*) $\angle CAO$.



Sol. In the figure,

$\angle AOB = 110°$

(*i*) Now, arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle

$\therefore \angle AOB = 2\angle ACB$

$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 110° = 55°$

or $\angle ACO = 55°$

(*ii*) Now in ΔOAC,

$OA = OC$ (Radii of the same circle)

$\therefore \angle CAO = \angle ACO$

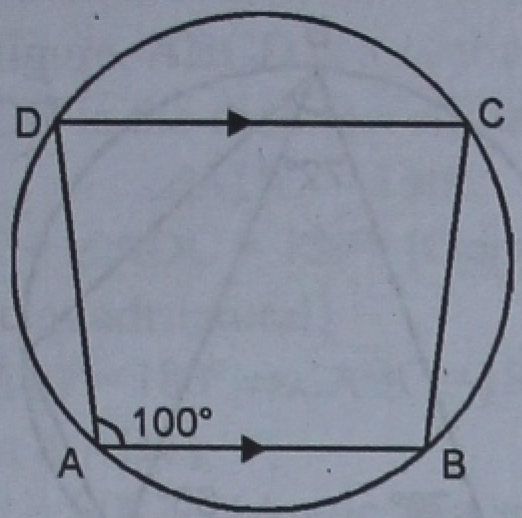
(Angles opposite to equal sides)

$= 55°$

Q. 4. In the given figure, AB || DC and $\angle BAD = 100°$. Calculate :

(*i*) $\angle BCD$ (*ii*) $\angle ADC$

(*iii*) $\angle ABC$.



Sol. In the figure, ABCD is a cyclic quadrilateral.

AB || DC and ∠BAD = 100°

(*i*) ∠BAD + ∠BCD = 180° (Sum of the opposite angles of a cyclic quadrilateral)

⇒ 100° + ∠BCD = 180°

⇒ ∠BCD = 180° – 100° = 80°

(*ii*) ∵ DC || AB

∴ ∠BAD + ∠ADC = 180°

(Sum of angles on the same side of a transversal)

⇒ 100° + ∠ADC = 180°

⇒ ∠ADC = 180° – 100° = 80°

(*iii*) ∠ABC + ∠ADC = 180° (Sum of opposite angles of a cyclic quadrilateral)

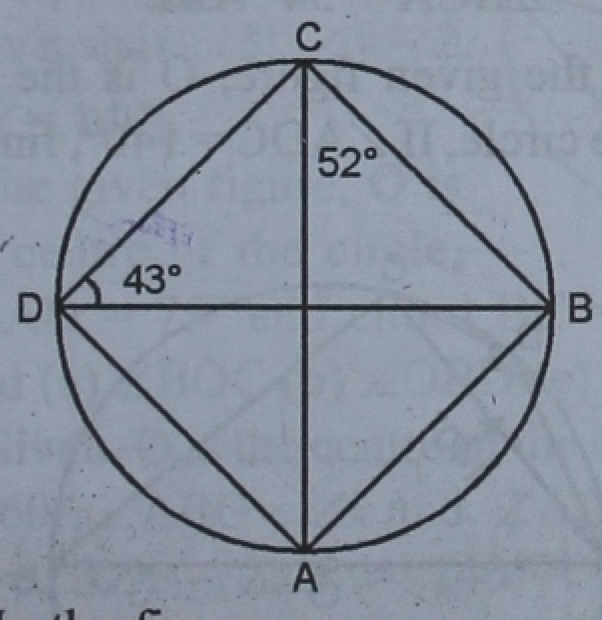
⇒ ∠ABC + 80° = 180°

⇒ ∠ABC = 180° – 80° = 100° **Ans.**

Q. 5. In the given figure, ∠ACB = 52° and ∠BDC = 43°. Calculate :

(*i*) ∠ADB (*ii*) ∠BAC

(*iii*) ∠ABC.



Sol. In the figure,

(*i*) ∠ADB = ∠ACB

(Angles in the same segment)

= 52° (∵ ∠ACB = 52°)

(*ii*) ∠BAC = ∠BDC

(Angles in the same segment)

= 43° (∵ ∠BDC = 43°)

(*iii*) In ΔABC,

∠ABC + ∠BCA + ∠BAC = 180°

(Angles of a triangle)

⇒ ∠ABC + 52° + 43° = 180°

⇒ ∠ABC + 95° = 180°

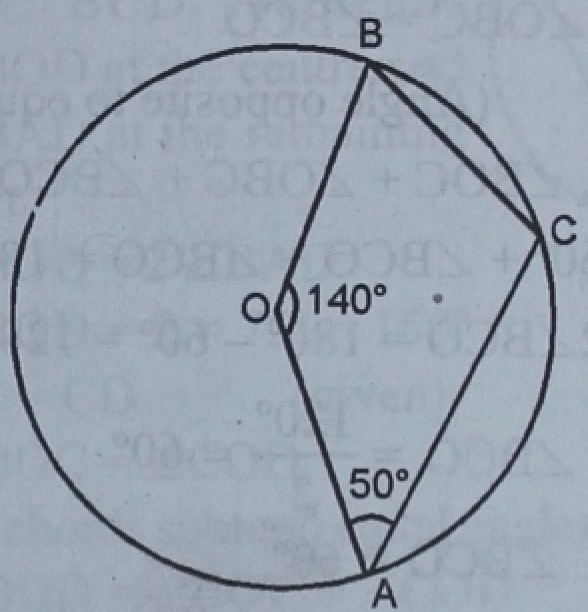
⇒ ∠ABC = 180° – 95° = 85°

Hence, ∠ABC = 85° **Ans.**

Q. 6. In the given figure, O is the centre of the circle, If ∠AOB = 140° and ∠OAC = 50°, Calculate :

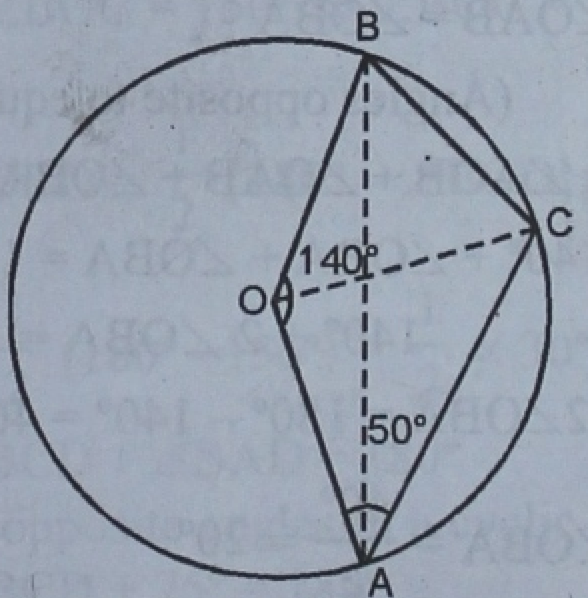
(*i*) ∠ABC (*ii*) ∠BCO

(*iii*) ∠OAB (*iv*) ∠BCA



Sol. O is the centre of the circle

∠AOB = 140°, ∠OAC = 50°



Join OC and AB

In ΔOAC,

OA = OC (Radii of the same circle)

∴ ∠OCA = ∠OAC = 50°

(∵ ∠OAC = 50°)

But in ΔAOC,

$\angle AOC + \angle OAC + \angle ACO = 180^\circ$

(Angles of a triangle)

$\Rightarrow \angle AOC + 50^\circ + 50^\circ = 180^\circ$

$\Rightarrow \angle AOC + 100^\circ = 180^\circ$

$\Rightarrow \angle AOC = 180^\circ - 100^\circ = 80^\circ$

$\therefore \angle BOC = 140^\circ - 80^\circ = 60^\circ$

(*i*) Now, arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle

$\therefore \angle AOC = 2\angle ABC$

$\Rightarrow \angle ABC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 80^\circ = 40^\circ$

(*ii*) In ΔOBC, OB = OC

(Radii of the same circle)

$\therefore \angle OBC = \angle BCO$

(Angle opposite to equal sides)

But, $\angle BOC + \angle OBC + \angle BCO = 180^\circ$

$\Rightarrow 60^\circ + \angle BCO + \angle BCO = 180^\circ$

$\Rightarrow 2\angle BCO = 180^\circ - 60^\circ = 120^\circ$

$\Rightarrow \angle BCO = \frac{120^\circ}{2} = 60^\circ$

$\Rightarrow \angle BCO = 60^\circ$.

(*iii*) In ΔOAB,

OB = OA (Radii of the same circle)

$\therefore \angle OAB = \angle OBA$

(Angles opposite to equal sides)

But, $\angle AOB + \angle OAB + \angle OBA = 180^\circ$

$\Rightarrow 140^\circ + \angle OBA + \angle OBA = 180^\circ$

$\Rightarrow 140^\circ + 2\angle OBA = 180^\circ$

$\Rightarrow 2\angle OBA = 180^\circ - 140^\circ = 40^\circ$

$\therefore \angle OBA = \frac{40^\circ}{2} = 20^\circ$

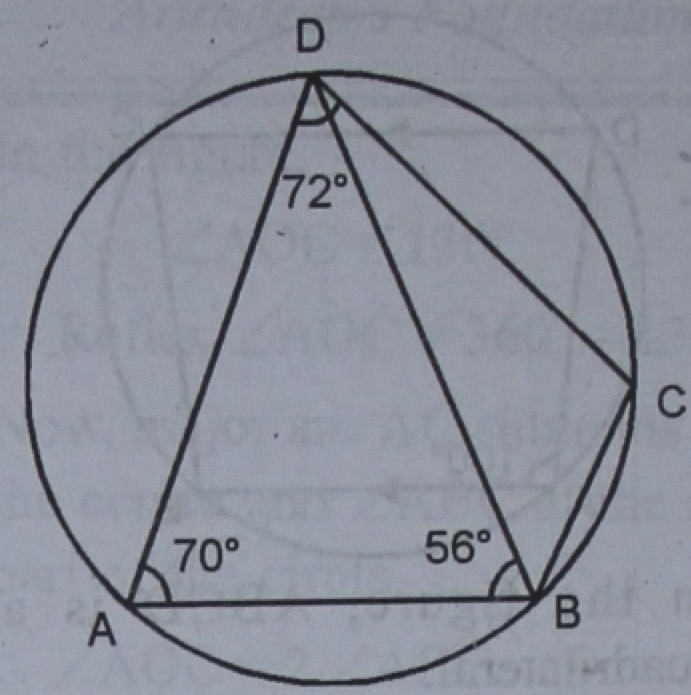
(*iv*) $\angle BCA = \angle OCB + \angle ACO$

$= 60^\circ + 50^\circ = 110^\circ$

Q. 7. In the given figure, $\angle BAD = 70^\circ$, $\angle ABD = 56^\circ$, $\angle ADC = 72^\circ$. Calculate :

(*i*) $\angle BDC$ (*ii*) $\angle BCD$

(*iii*) $\angle BCA$.



Sol. In the figure;

ABCD is a cyclic quadrilateral

$\angle BAD = 70^\circ, \angle ABD = 56^\circ$

and $\angle ADC = 72^\circ$

Join AC

(*i*) $\angle BDC = \angle ADC - \angle ADB$

$= \angle ADC - \{180^\circ - \angle DAB - \angle ABD\}$

$= 72^\circ - (180^\circ - 70^\circ - 56^\circ)$

$= 72^\circ - 180^\circ + 70^\circ + 56^\circ$

$= 198^\circ - 180^\circ = 18^\circ$

(*ii*) $\angle BCD = 180^\circ - \angle BAD$

{$\because$ ABCD is a cyclic quadrilateral}

$= 180^\circ - 70^\circ = 110^\circ$

(*iii*) $\angle BCA = \angle ADB$

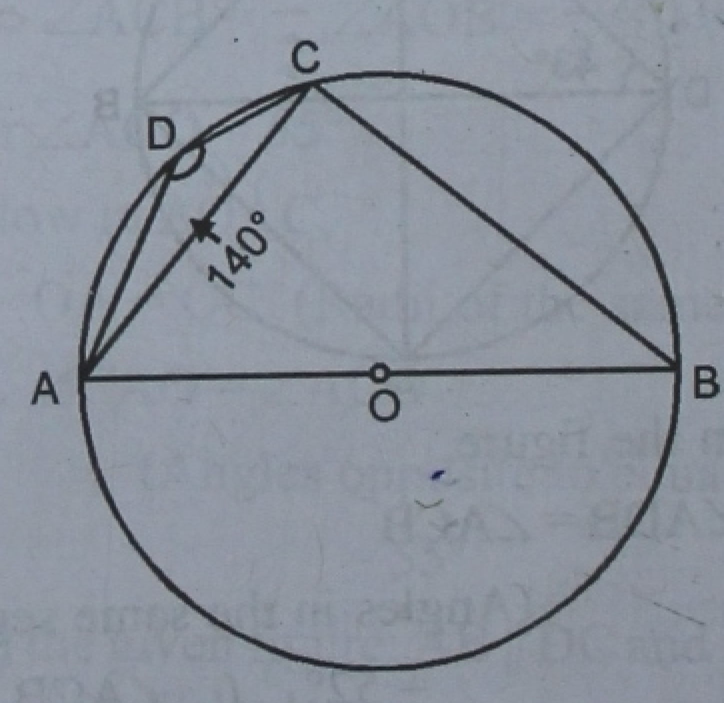
(Angles in the same segment)

But $\angle ADB = \angle ADC - \angle BDC$

$= 72^\circ - 18^\circ = 54^\circ$

$\therefore \angle BCA = 54^\circ$ **Ans.**

Q. 8. In the given figure, O is the centre of the circle. If $\angle ADC = 140^\circ$, find $\angle BAC$.



Sol. In the figure, ABCD is a cyclic quadrilateral and AOB is the diameter of the circle.

$\angle ADC = 140^\circ$

$\because \angle ABC + \angle ADC = 180^\circ$ {Opposite angles of a cyclic quadrilateral}

$\Rightarrow \angle ABC + 140^\circ = 180^\circ \Rightarrow \angle ABC = 180^\circ - 140^\circ = 40^\circ$

Now in Δ ABC,

$\angle ACB = 90^\circ$ (Angle in a semi-circle)

$\angle ABC = 40^\circ$ (proved)

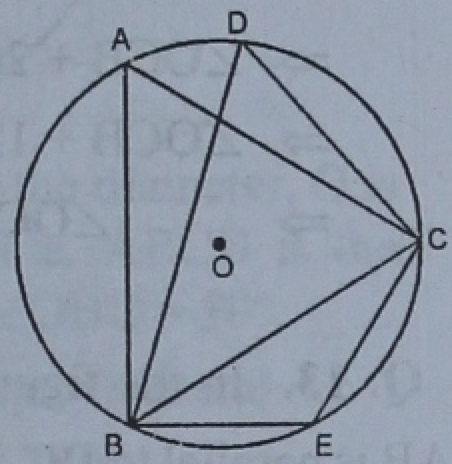
But $\angle BAC + \angle ACB + \angle ABC = 180^\circ$ (Angles of a triangle)

$\Rightarrow \angle BAC + 90^\circ + 40^\circ = 180^\circ$

$\Rightarrow \angle BAC + 130^\circ = 180^\circ \Rightarrow \angle BAC = 180^\circ - 130^\circ$

$\therefore \angle BAC = 50^\circ$ **Ans.**

Q. 9. In the given figure, O is the centre of the circle and ΔABC is equilateral. Find (*i*) $\angle BDC$ (*ii*) $\angle BEC$.



Sol. In the figure,

O is the centre of the circle and ΔABC is an equilateral triangle

(*i*) $\therefore \angle A = \angle ABC = \angle ACB = 60^\circ$

$\angle BAC = \angle BDC$ (Angles in the same segment)

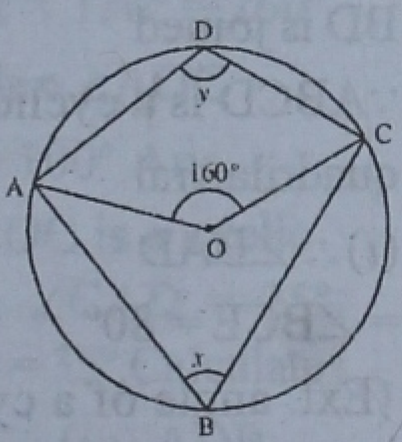
$\therefore \angle BDC = 60^\circ$

(*ii*) $\because$ ABEC is a cyclic quadrilateral

$\therefore \angle A + \angle BEC = 180^\circ \Rightarrow 60^\circ + \angle BEC = 180^\circ$

$\Rightarrow \angle BEC = 180^\circ - 60^\circ = 120^\circ$ **Ans.**

Q. 10. (*i*) In the figure, O is the centre of the circle and $\angle AOC = 160^\circ$. Prove that : $3\angle y - 2\angle x = 140^\circ$.



(*ii*) In the given figure, O is the centre of the circle, $\angle BAD = 75^\circ$ and chord BC = chord CD. Find (a) $\angle BOC$ (b) $\angle OBD$ (c) $\angle BCD$.

Sol. (*i*) **Given.** O is the centre of the circle $\angle AOC = 160^\circ$, $\angle ABC = \angle x$ and $\angle ADC = \angle y$.

To Prove. $3\angle y - 2\angle x = 140^\circ$

Proof. $\because \angle AOC$ + reflex $\angle ADC = 360^\circ$ (Angles at a point)

$\Rightarrow 160^\circ$ + Reflex $\angle ADC = 360^\circ$

$\Rightarrow$ Reflex $\angle ADC = 360^\circ - 160^\circ = 200^\circ$

Now arc. ADC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle

$\therefore \angle AOC = 2x \Rightarrow 2x = 160^\circ$

$\Rightarrow x = \dfrac{160^\circ}{2} = 80^\circ$

Similarly, reflex $\angle ADC = 2y$

$\Rightarrow 2y = 200^\circ \Rightarrow y = \dfrac{200^\circ}{2} = 100^\circ$

Now, L.H.S. $= 3\angle y - 2\angle x$

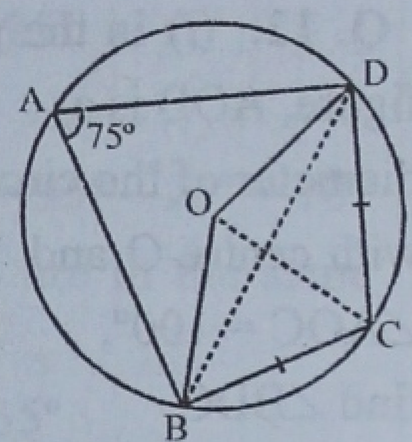
$= 3 \times 100^\circ - 2 \times 80^\circ$

$= 300^\circ - 160^\circ = 140^\circ$ = R.H.S.

Hence Proved.

(*ii*) In the figure, O is the centre of the circle, $\angle BAD = 75^\circ$, chord BC = chord CD

Join BD, OC



$\because$ arc BCD subtends $\angle BOD$ at the centre and $\angle BAD$ at the remaining part

$\therefore \angle BOD = 2\angle BAD$

$\therefore \angle BOD = 2 \times 75^\circ = 150^\circ$

$\because BC = CD$ (given)

$\therefore \angle BOC = \angle COD$

{Equals chords subtend equal angles at the centre}

$\therefore \angle BOD = \angle BOC + \angle COD$

$= \angle BOC + \angle BOC = 2\angle BOC$

$\Rightarrow 2\angle BOC = 150^\circ \Rightarrow \angle BOC = \dfrac{150^\circ}{2} = 75^\circ$

(b) $\angle OBD = \dfrac{1}{2}[180^\circ - \angle BOD]$

$= \dfrac{1}{2}(180^\circ - 150^\circ) = \dfrac{1}{2} \times 30^\circ = 15^\circ$

(c) $\angle BCD + \angle BAD = 180^\circ$

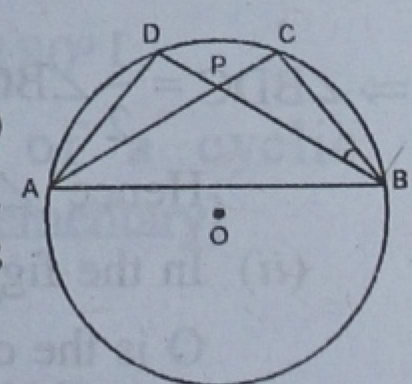
{opposite angles of a cyclic quadrilateral}

$\Rightarrow \angle BCD + 75^\circ = 180^\circ$

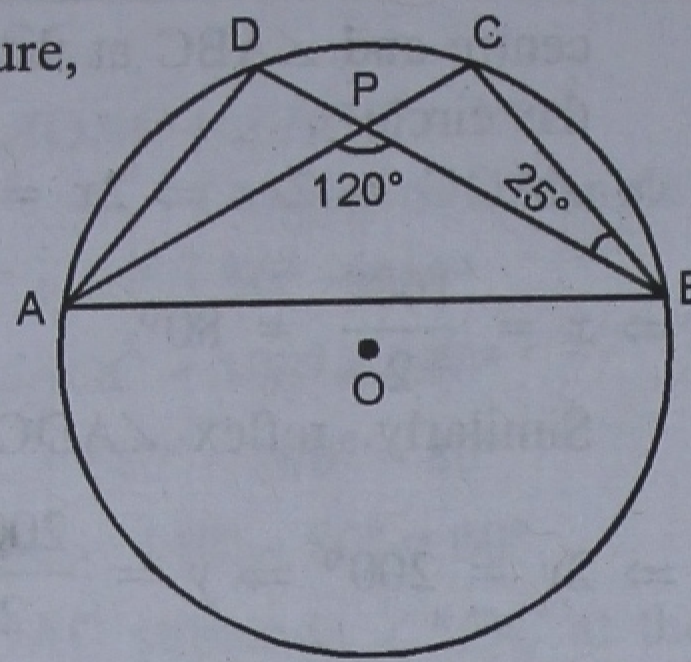
$\Rightarrow \angle BCD = 180^\circ - 75^\circ$

$= 105^\circ$

Q. 11. In the given figure, O is the centre of the circle. If $\angle CBD = 25^\circ$ and $\angle APB = 120^\circ$, find $\angle ADB$.



Sol. In the figure,
O is the centre of the circle
$\angle CBD = 25^\circ$ and
$\angle APB = 120^\circ$



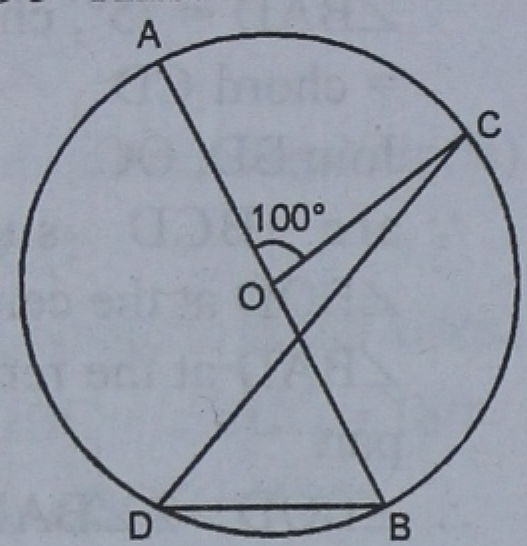
In ΔCPB,
Ext. $\angle APB$
$= \angle CBD + \angle BCP$
$\Rightarrow 120^\circ = 25^\circ + \angle BCP \Rightarrow \angle BCP = 120^\circ - 25^\circ = 95^\circ$

But, $\angle BCP$ or $\angle BCA$ and $\angle ADB$ are in the same segment of a circle.

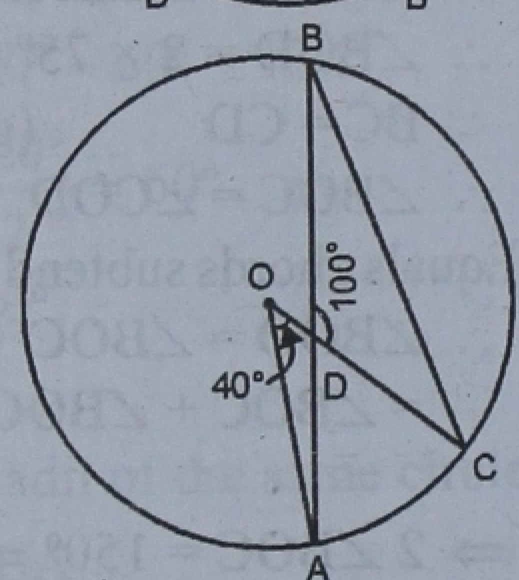
$\therefore \quad \angle ADB = \angle BCA = 95^\circ$

Hence, $\angle ADB = 95^\circ$ **Ans.**

Q. 12. (*i*) In the given figure, AOB is a diameter of the circle with centre O and $\angle AOC = 100^\circ$, find $\angle BDC$.



(*ii*) In the given figure, O is the centre of the circle :
$\angle AOD = 40^\circ$ and
$\angle BDC = 100^\circ$.
Find $\angle OCB$.



Sol.(*i*) In the figure,
AOB is the diameter of the circle with centre O,
$\angle AOC = 100^\circ$

But, $\angle AOC + \angle BOC = 180^\circ$ (A linear pair)

$\Rightarrow \quad 100^\circ + \angle BOC = 180^\circ$

$\Rightarrow \angle BOC = 180^\circ - 100^\circ = 80^\circ$

Now, arc BC subtends $\angle BOC$ at the centre and $\angle BDC$ at the remaining part of the circle.

$\therefore \angle BOC = 2\angle BDC$

$\Rightarrow \angle BDC = \frac{1}{2}\angle BOC \Rightarrow \angle BDC = \frac{1}{2} \times 80^\circ = 40^\circ$

Hence $\angle BDC = 40^\circ$ **Ans.**

(*ii*) In the figure,
O is the centre of the circle
$\angle AOD = 40^\circ$ or $\angle AOC = 40^\circ$
and $\angle BDC = 100^\circ$

Arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle

$\therefore \angle AOC = 2\angle ABC$

$\Rightarrow \angle ABC = \frac{1}{2}\angle AOC \Rightarrow \angle ABC = \frac{1}{2} \times 40^\circ = 20^\circ$

Now, in ΔDBC,
$\angle DCB + \angle DBC + \angle BDC = 180^\circ$
(Angles of a triangle)

$\Rightarrow \angle OCB + \angle ABC + \angle BDC = 180^\circ$

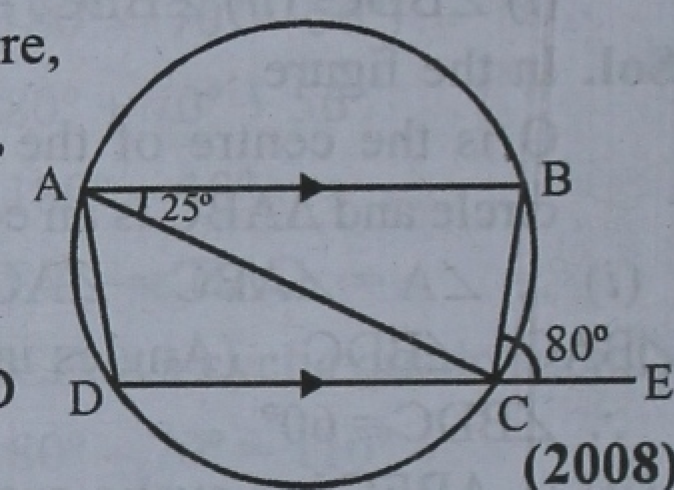
$\Rightarrow \angle OCB + 20^\circ + 100^\circ = 180^\circ$

$\Rightarrow \angle OCB + 120^\circ = 180^\circ$

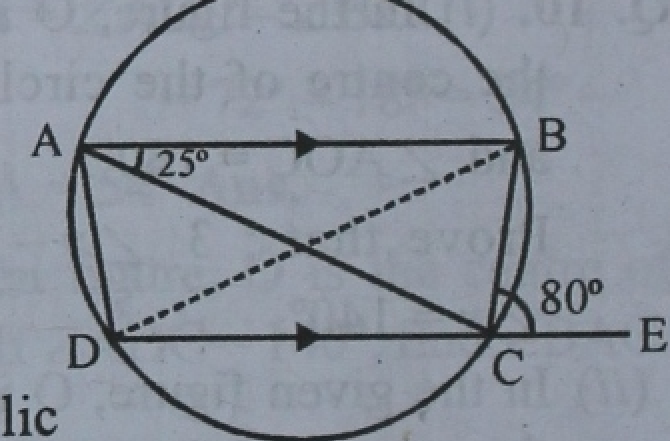
$\Rightarrow \quad \angle OCB = 180^\circ - 120^\circ = 60^\circ$

$\therefore \quad \angle OCB = 60^\circ$ **Ans.**

Q. 13. In the figure, AB is parallel to DC, $\angle BCE = 80^\circ$ and $\angle BAC = 25^\circ$. Find :
(*i*) $\angle CAD$ (*ii*) $\angle CBD$
(*iii*) $\angle ADC$ **(2008)**



Sol. In the figure, AB || DC
$\angle BCE = 80^\circ$ and $\angle BAC = 25^\circ$
BD is joined
$\because$ ABCD is a cyclic quadrilateral



(*i*) $\therefore \angle BAD$
$= \angle BCE = 80^\circ$
{Ext. angle of a cyclic quadrilateral is equal to its interior opposite angle}

$\Rightarrow \angle BAC + \angle CAD = 80^\circ$

$\Rightarrow 25^\circ + \angle CAD = 80^\circ \Rightarrow \angle CAD = 80^\circ - 25^\circ$

$\Rightarrow \angle CAD = 55^\circ$

(*ii*) $\angle CBD = \angle CAD = 55^\circ$
(Angles in the same segment)
{From (*i*)}

(*iii*) $\because$ AB || DC (given)
and AD is its transversal

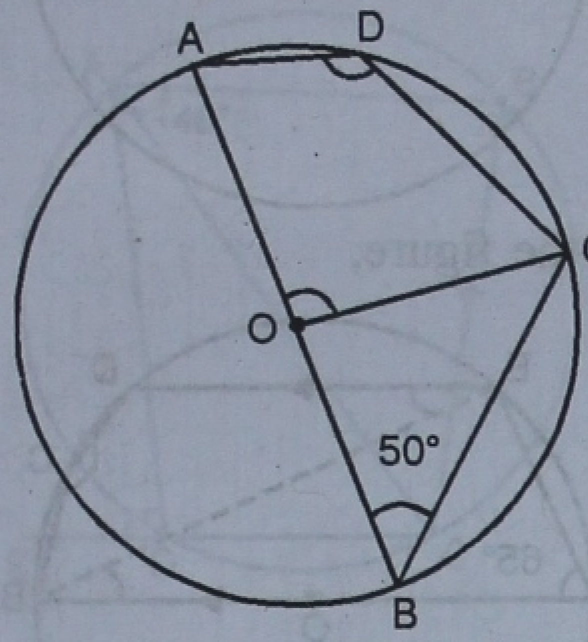
$\therefore \angle BAD + \angle ADC = 180^\circ$ (co-interior angles)

⇒ 80° + ∠ADC = 180°

⇒ ∠ADC = 180° – 80° = 100°

Q. 14. In the given figure, O is the centre of the circle and ∠OBC = 50°. Calculate :

(*i*) ∠ADC (*ii*) ∠AOC.



Sol. In the figure, AOB is the diameter, O is the centre of the circle, ABCD is the cyclic quadrilateral, ∠OBC = 50°.

(*i*) ∵ ABCD is a cyclic quadrilateral

∴ ∠ABC + ∠ADC = 180°

⇒ ∠OBC + ∠ADC = 180°

⇒ 50° + ∠ADC = 180° ⇒ ∠ADC = 180° – 50°

∴ ∠ADC = 130°

(*ii*) Major arc AC, subtends reflex ∠AOC at the centre and ∠ADC at the remaining part of the circle

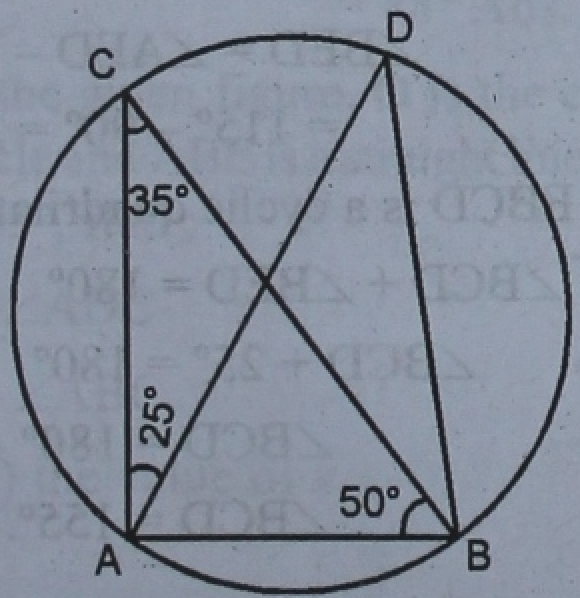
∴ Reflex ∠AOC = 2 ∠ADC = 2 × 130° = 260°

∴ ∠AOC = 360° – Reflex ∠AOC

= 360° – 260° = 100° **Ans.**

Q.15. In the given figure, ABDC is a cyclic quadrilateral in which ∠CAD = 25°, ∠ABC = 50° and ∠ACB = 35°. Calculate :

(*i*) ∠CBD (*ii*) ∠DAB (*iii*) ADB



Sol. In the figure,

∠CAD = 25°, ∠ABC = 50° and ∠ACB = 35°

(*i*) ∵ ∠CAD and ∠CBD are in the same segment of a circle

∴ ∠CBD = ∠CAD = 25°

(∵ ∠CAD = 25°)

(*ii*) In ΔABC,

∠ACB + ∠ABC + CAB = 180°

(Sum of angles of a triangle)

⇒ 35° + 50° + ∠CAB = 180°

⇒ 85° + ∠CAB = 180°

⇒ ∠CAB = 180° – 85° = 95°

⇒ ∠CAD + ∠DAB = 95°

⇒ 25° + ∠DAB = 95°

∴ ∠DAB = 95° – 25° = 70°

(*iii*) ∵ ∠ADB and ∠ACB are in the same segment

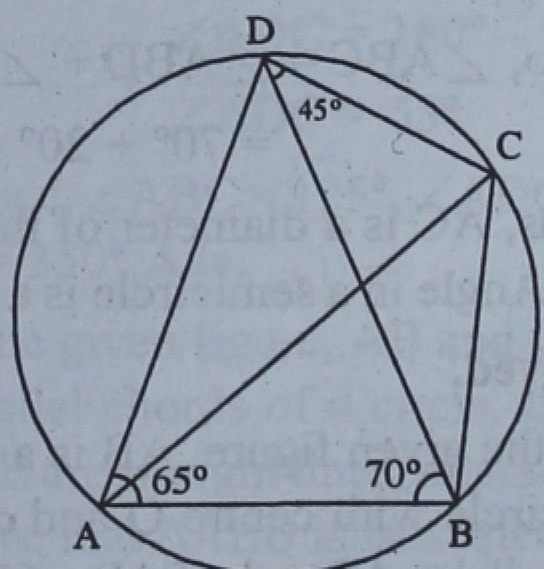
∴ ∠ADB = ∠ACB = 35°

(∵ ∠ACB = 35°) **Ans.**

Q. 16. In the figure, ∠BAD = 65°, ∠ABD = 70° and ∠BDC = 45°.

Find : *(i)* ∠BCD *(ii)* ∠ADB

Hence, show that AC is a diameter.



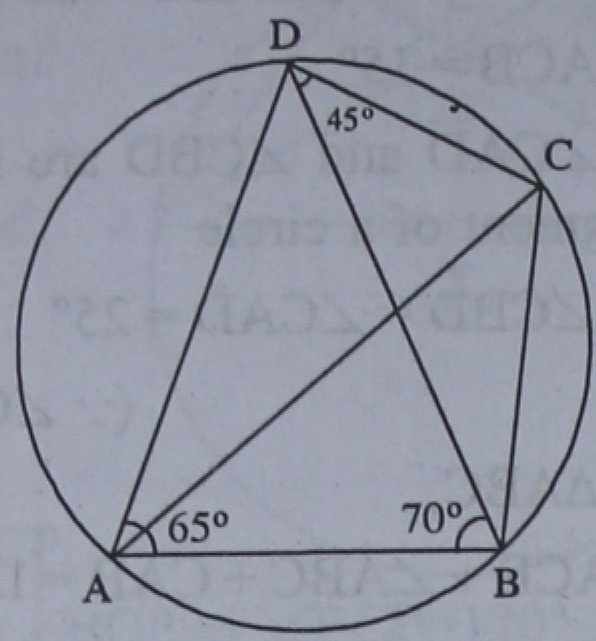
Sol. *(i)* ABCD is a cyclic quadrilateral

∴ ∠BAD + ∠BCD = 180°

[Opposite angles of a cyclic quadrilateral are supplementary]

⇒ ∠65° + ∠BCD = 180°

⇒ ∠BCD = 180° – 65° = 115° **Ans.**



(ii) In Δ ABD, we have

$\angle BAD + \angle ABD + \angle ADB = 180^\circ$

[Angle sum property of a Δ]

$\Rightarrow 65^\circ + 70^\circ + \angle ADB = 180^\circ \Rightarrow \angle ADB = 180^\circ - 65^\circ - 70^\circ$

$\Rightarrow \angle ADB = 180^\circ - 135^\circ \Rightarrow \angle ADB = 45^\circ$ **Ans.**

In Δ BCD, we have

$\angle BDC + \angle BCD + \angle CBD = 180^\circ$

[Angle sum property of a Δ]

$\Rightarrow 45^\circ + 115^\circ + \angle CBD = 180^\circ$

[From (i), $\angle BCD = 115^\circ$]

$\Rightarrow \angle CBD = 180^\circ - 45^\circ - 115^\circ$

$\Rightarrow \angle CBD = 180^\circ - 160^\circ \Rightarrow \angle CBD = 20^\circ$

Also, $\angle ABC = \angle ABD + \angle CBD$

$= 70^\circ + 20^\circ = 90^\circ$

Thus, AC is a diameter of the circle.

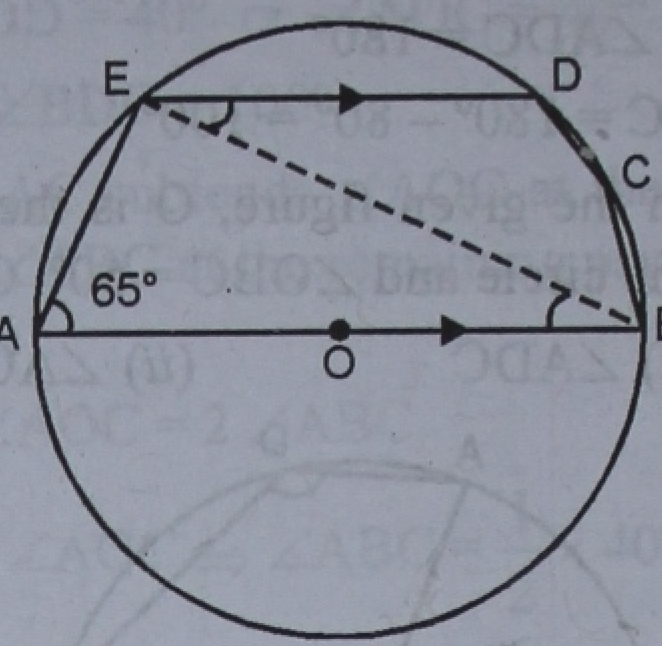
[$\because$ Angle in a semicircle is a right angle]

Proved.

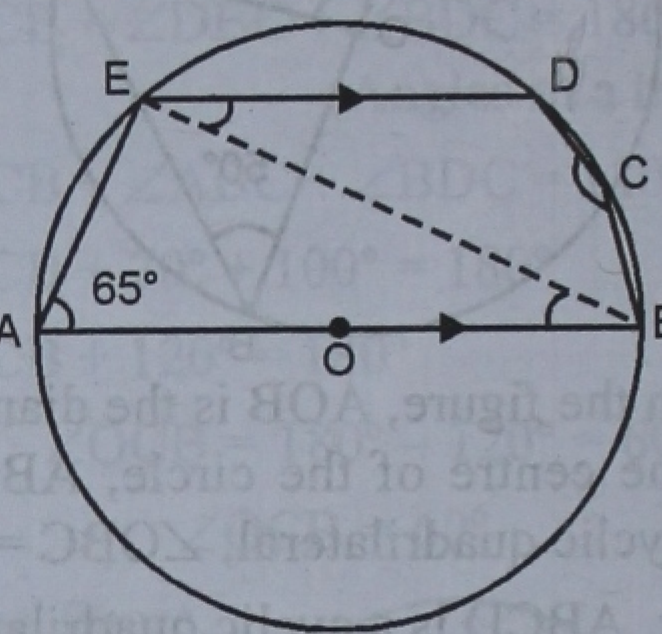
Q. 17. In the given figure, AB is a diameter of a circle with centre O and chord ED is parallel to AB and $\angle EAB = 65^\circ$. Calculate:

(i) $\angle EBA$ *(ii)* $\angle BED$

(iii) $\angle BCD$



Sol. In the figure,



AOB is the diameter of the circle with centre O. Chord ED $\parallel$ AB and $\angle EAB = 65^\circ$. Join EB.

(i) In Δ AEB,

$\angle AEB + \angle EAB + \angle EBA = 180^\circ$

$\Rightarrow 90^\circ + 65^\circ + \angle EBA = 180^\circ$

$\Rightarrow 155^\circ + \angle EBA = 180^\circ$

$\Rightarrow \angle EBA = 180^\circ - 155^\circ = 25^\circ$

$\therefore \angle EBA = 25^\circ$.

(ii) $\because$ ED $\parallel$ AB

$\therefore \angle EAB + \angle AED = 180^\circ$

(Angles on the same side of the transversal)

$\Rightarrow 65^\circ + \angle AED = 180^\circ$

$\Rightarrow \angle AED = 180^\circ - 65^\circ = 115^\circ$

$\therefore \angle BED = \angle AED - \angle AEB$

$= 115^\circ - 90^\circ = 25^\circ$.

(iii) $\because$ EBCD is a cyclic quadrilateral

$\therefore \angle BCD + \angle BED = 180^\circ$

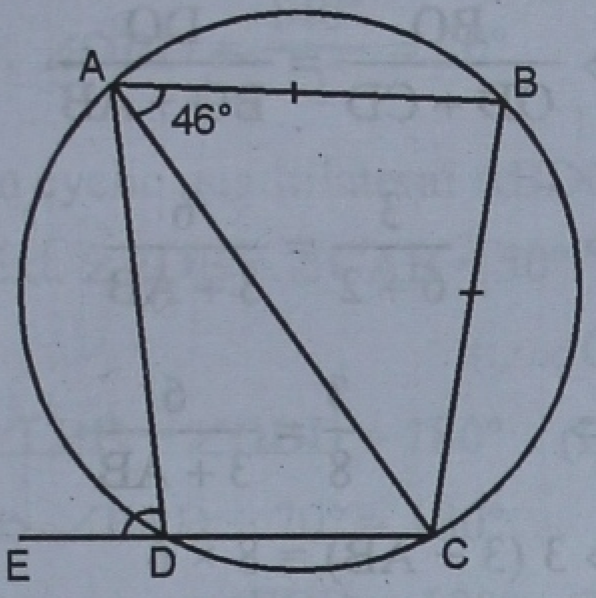
$\Rightarrow \angle BCD + 25^\circ = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 25^\circ$

$\therefore \angle BCD = 155^\circ$.

Q. 18. In the given figure, ABCD is a cyclic quadrilateral whose side CD has been produced to E.

If BA = BC and ∠BAC = 46°, find ∠ADE.



Sol. In the figure,

ABCD is a cyclic quadrilateral. Its side CD is produced to E

BA = BC and ∠BAC = 46°

In ΔABC,

AB = BC

∴ ∠BAC = ∠BCA = 46°

But, ∠ABC + ∠BAC + ∠BCA = 180° (Angles of a triangle)

⇒ ∠ABC + 46° + 46° = 180°

⇒ ∠ABC + 92° = 180°

⇒ ∠ABC = 180° – 92°

∴ ∠ABC = 88°

In cyclic quadrilateral ABCD,

Ext. ∠ADE = Int. opposite ∠ABC

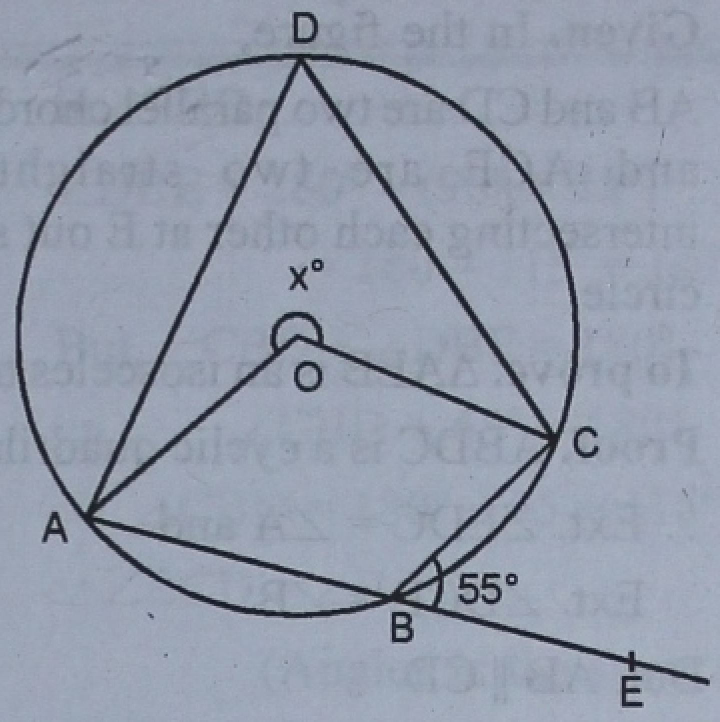
= 88° **Ans.**

Q. 19. In the given figure, O is the centre of a circle and ABE is a straight line. If ∠CBE = 55°, find :

(*i*) ∠ADC

(*ii*) ∠ABC

(*iii*) the value of *x*.



Sol. In the figure,

O is the centre of the circle, ABCD is cyclic quadrilateral. ABE is a straight line and ∠CBE = 55°.

∠ABC + ∠CBE = 180° (Linear pair)

⇒ ∠ABC + 55° = 180°

⇒ ∠ABC = 180° – 55°

⇒ ∠ABC = 125°

Now, major arc ADC subtends reflex ∠AOC at the centre and ∠ABC at the remaining part of the circle

∴ $x = 2 \times 125° = 250°$

(*i*) In cyclic quadrilateral ABCD,

∠ADC + ∠ABC = 180°

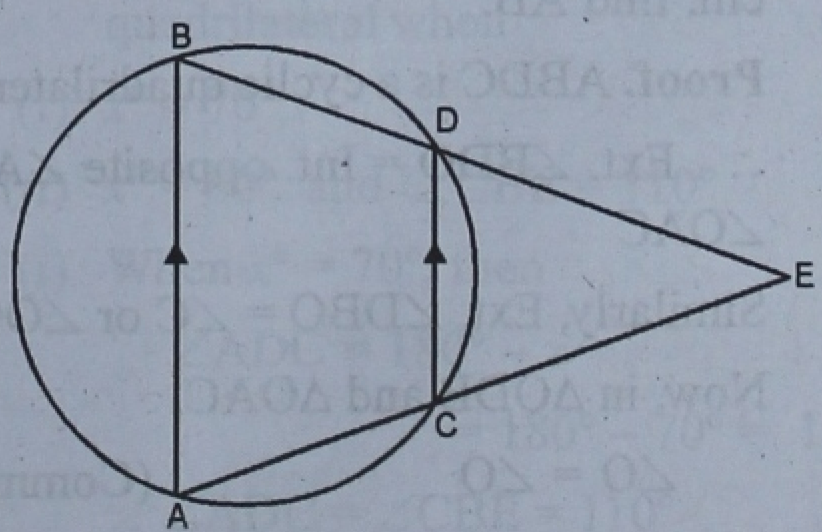
⇒ ∠ADC + 125° = 180°

⇒ ∠ADC = 180° – 125°

∴ ∠ADC = 55°

Hence, ∠ABC = 125°, ∠ADC = 55° and x° = 250° **Ans.**

Q. 20. In the given figure, AB and CD are two parallel chords of a circle. If BDE and ACE are straight lines, intersecting at E, prove that ΔAEB is isosceles.



Sol. Given. In the figure,

AB and CD are two parallel chords. BDE and ACE are two straight lines intersecting each other at E out side the circle.

To prove. ΔAEB is an isosceles triangle.

Proof. ABDC is a cyclic quadrilateral

∴ Ext. ∠EDC = ∠A and

Ext. ∠DCE = ∠B

But, AB ∥ CD

∴ ∠EDC = ∠B (Corresponding angles)

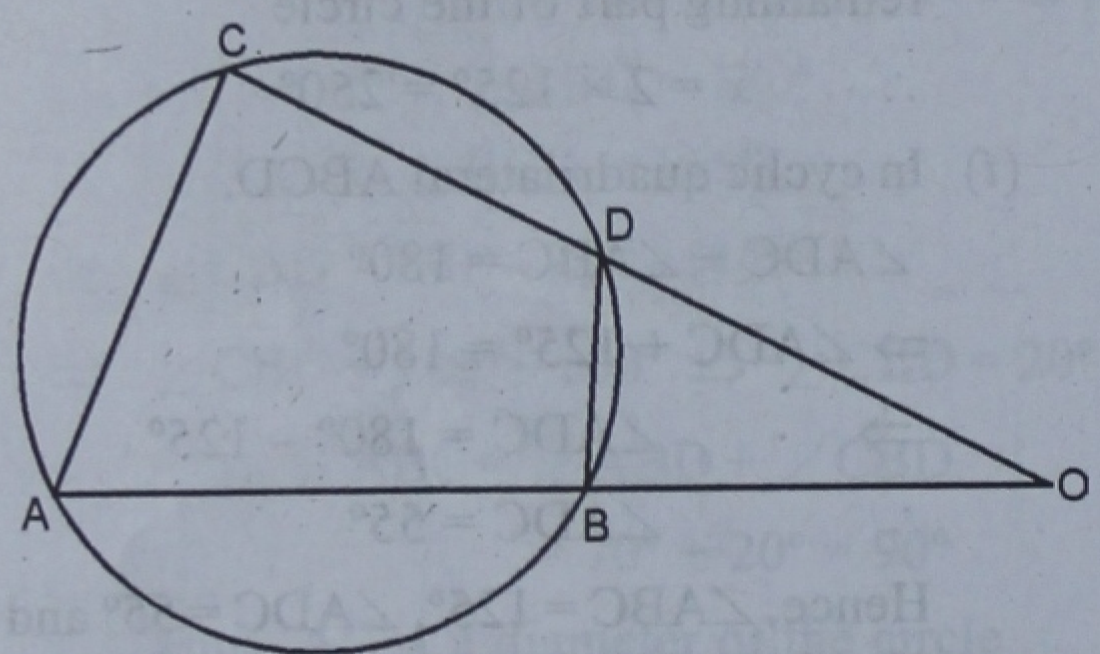
and ∠DCE = ∠A

⇒ ∠B = ∠A

∴ EA = EB

Hence, ΔAEB is an isosceles triangle.

Q. 21. In the given figure, chords AB and CD of a circle are produced to meet at O. Prove that ΔODB and ΔOAC are similar. If BO = 3 cm. DO = 6 cm. and CD = 2 cm, find AB.



Sol. Given. In the figure, two chords AB and CD meet at O on producing.

To prove. (*i*) ΔODB ~ ΔOAC

(*i*) If BO = 3 cm, DO = 6 cm, and CD = 2 cm, find AB.

Proof. ABDC is a cyclic quadrilateral

∴ Ext. ∠BDO = Int. opposite ∠A or ∠OAC

Similarly, Ext. ∠DBO = ∠C or ∠OCA

Now, in ΔODB and ΔOAC,

∠O = ∠O (Common)

∠ODB = ∠OAC (Proved)

∴ ΔODB ~ ΔOAC (A.A. axiom)

$$\therefore \frac{BO}{OC} = \frac{DO}{AO}$$

$$\Rightarrow \frac{BO}{OD + CD} = \frac{DO}{BO + AB}$$

$$\Rightarrow \frac{3}{6+2} = \frac{6}{3 + AB}$$

$$\Rightarrow \frac{3}{8} = \frac{6}{3 + AB}$$

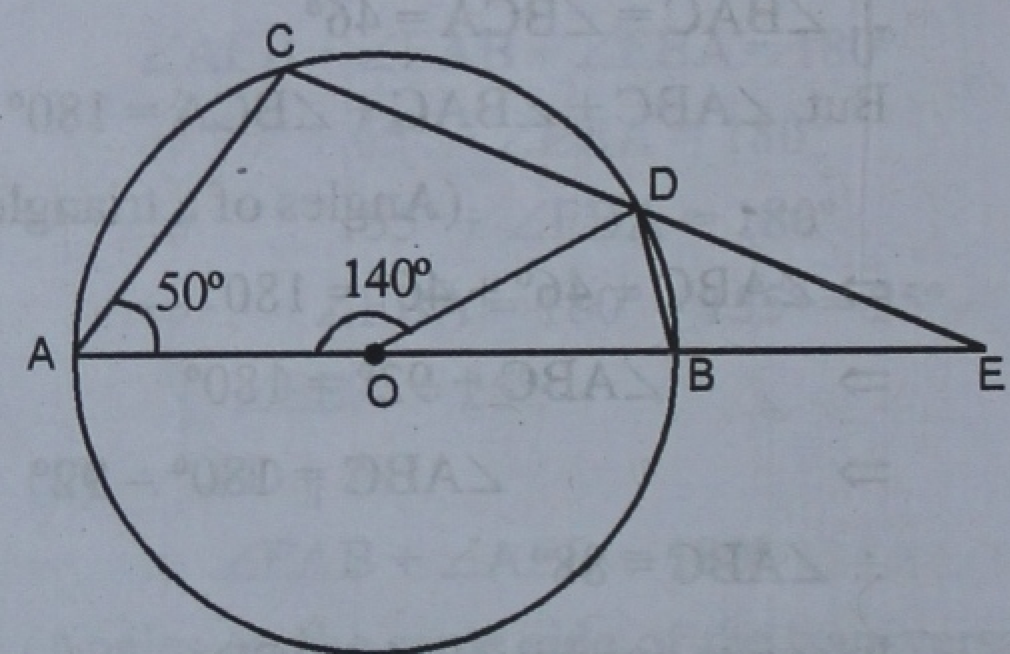
⇒ 3 (3 + AB) = 8 × 6

$$\Rightarrow 3 + AB = \frac{8 \times 6}{3} = 16$$

∴ AB = 16 – 3 = 13 cm. **Ans**

Q. 22. In the given figure, O is the centre of the circle. If ∠AOD = 140° and ∠CAB = 50°. Calculate :

(*i*) ∠EDB (*ii*) ∠EBD



Sol. In the figure, O is the centre of the circle

∠AOD = 140° and ∠CAB = 50°

∠AOD + ∠DOB = 180° (Linear pair)

⇒ 140° + ∠DOB = 180°

⇒ ∠DOB = 180° – 140° = 40°

But, OB = OD (Radii of the same circle)

∴ ∠OBD = ∠ODB

(Angles opposite to equal sides)

But in ΔOBD,

$\angle OBD + \angle ODB + \angle BOD = 180^\circ$

$\Rightarrow \angle OBD + \angle OBD + 40^\circ = 180^\circ$

$\Rightarrow 2\angle OBD = 180^\circ - 40^\circ = 140^\circ$

$\therefore \angle OBD = \frac{140^\circ}{2} = 70^\circ$

(*i*) In cyclic quadrilateral ABDC,

Ext. $\angle EDB = \angle CAB = 50^\circ$

($\because \angle CAB = 50^\circ$)

(*ii*) $\angle EBD + \angle OBD = 180^\circ$ (Linear pair)

$\Rightarrow \angle EBD + 70^\circ = 180^\circ$

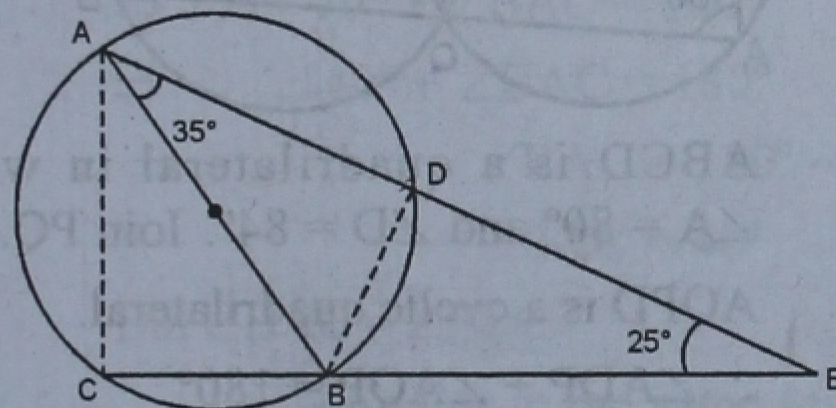
$\Rightarrow \angle EBD = 180^\circ - 70^\circ = 110^\circ$

$\therefore \angle EBD = 110^\circ$ **Ans.**

Q. 23. In the given figure, AB is a diameter of a circle with centre O. If ADE and CBE are straight lines, meeting at E such that $\angle BAD = 35^\circ$ and $\angle BED = 25^\circ$. Find :

(*i*) $\angle DCB$ (*ii*) $\angle DBC$

(*iii*) $\angle BDC$.



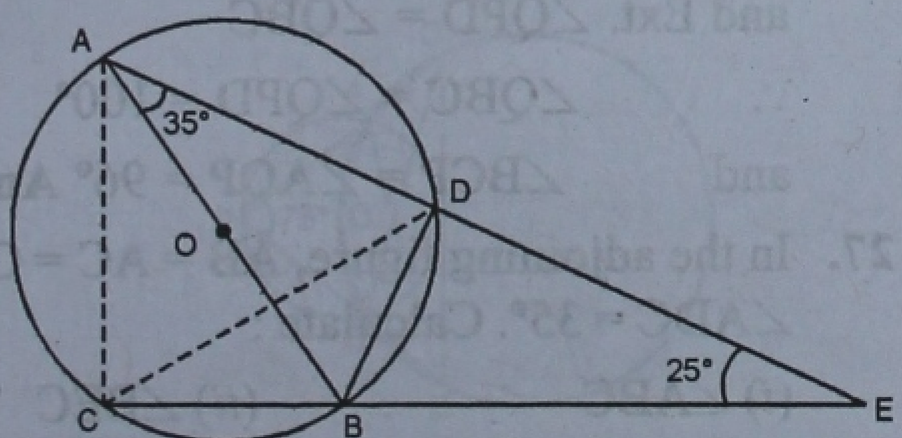
Sol. In the figure,

AB is the diameter of circle with centre O. ADE and CBE are straight lines meeting each other at E. $\angle BAD = 35^\circ$ and $\angle BED = 25^\circ$. Join BD, CA and CD.

In ΔABD,

$\because \angle ADB = 90^\circ$ (Angle in a semi-circle)

$\therefore \angle BDE = 180^\circ - 90^\circ = 90^\circ$



In ΔBED,

$\angle DBE = 180^\circ - (90^\circ + 25^\circ)$

$= 180 - 115^\circ = 65^\circ$

But, $\angle CBD + \angle DBE = 180^\circ$

$\Rightarrow \angle CBD + 65^\circ = 180^\circ$

$\Rightarrow \angle CBD = 180^\circ - 65 = 115^\circ$

$\because \angle BCD = \angle BAD$,

(Angles in the same segment)

$\therefore \angle BCD = 35^\circ$ ($\because \angle BAD = 35^\circ$)

Now, in ΔCBD,

$\angle DCB + \angle DBC + \angle BDC = 180^\circ$

(Angles of a triangle)

$\Rightarrow 35^\circ + 115^\circ + \angle BDC = 180^\circ$

$\Rightarrow 150^\circ + \angle BDC = 180^\circ$

$\Rightarrow \angle BDC = 180^\circ - 150^\circ$

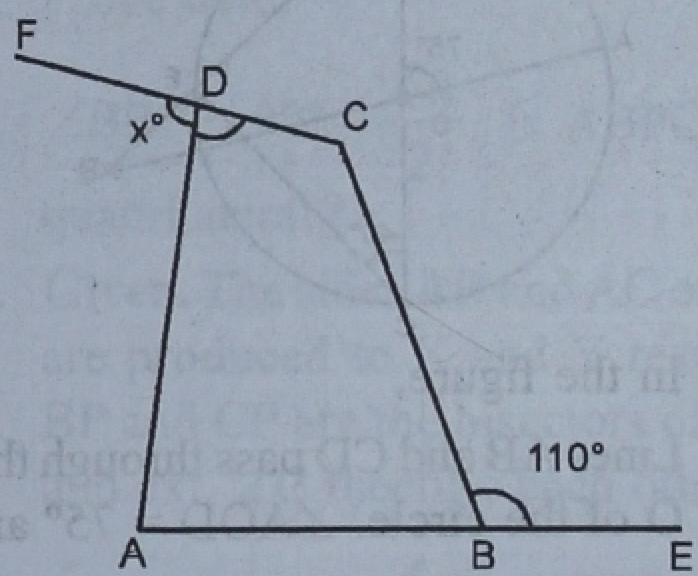
$\therefore \angle BDC = 30^\circ$

Hence (*i*) $\angle DCB = 35^\circ$

(*ii*) $\angle DBC = 115^\circ$ and

(*iii*) $\angle BDC = 30^\circ$. **Ans.**

Q. 24. In the given figure, find whether the points A, B, C, D are concyclic, when (*i*) $x = 70$ (*ii*) $x = 80$.



Sol. Points A, B, C and D forms a quadrilateral when

(*i*) $x^\circ = 70^\circ$,

(*ii*) $x^\circ = 80^\circ$ and $\angle CBE = 110^\circ$

(*i*) When $x^\circ = 70^\circ$, then

$\angle ADC = 180^\circ - x$

$= 180^\circ - 70^\circ = 110^\circ$

$\therefore \angle ADC = \angle CBE = 110^\circ$

$\therefore \angle ABC = 180° - 110° = 70°$

$\because \angle ABC + \angle ADC = 70° + 110° = 180°$

Or the sum of the opposite angles of a quadrilateral. is 180°

$\therefore$ ABCD is cyclic quadrilateral

Hence A, B, C and D are concyclic.

(*ii*) When $x° = 80°$

then $\angle ADC = 180° - x$

$= 180° - 80° = 100°$

and $\angle ABC = 180° - 110° = 70°$

$\therefore \angle ADC + \angle ABC = 100° + 70° = 170°$

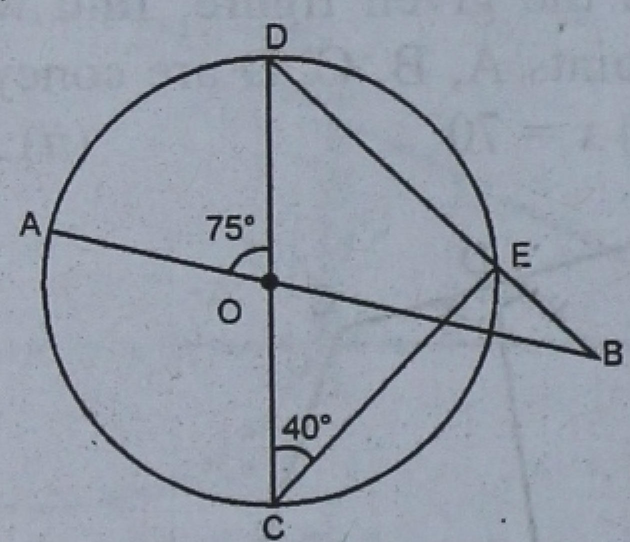
$\because$ Sum of opposite angles of a quadrilateral is not equal to 180°

$\therefore$ ABCD is not a cyclic quadrilateral.

Hence A, B, C and D are not concyclic.

Q. 25. In the given figure, the straight lines AB and CD pass through the centre O of the circle. If $\angle AOD = 75°$ and $\angle OCE = 40°$, find :

(*i*) $\angle CDE$ (*ii*) $\angle OBE$.



Sol. In the figure,

Lines AB and CD pass through the centre O of the circle. $\angle AOD = 75°$ and $\angle OCE = 40°$.

(*i*) $\angle CED = 90°$ (Angle in a semi-circle)

Now, in ΔCDE,

$\angle CDE + \angle CED + \angle ECD = 180°$

(Angles of a triangle)

$\Rightarrow \angle CDE + 90° + 40° = 180°$

$\Rightarrow \angle CDE + 130° = 180°$

$\Rightarrow \angle CDE = 180° - 130°$

$\Rightarrow \angle CDE = 50°$

(*ii*) Now, in ΔOBD,

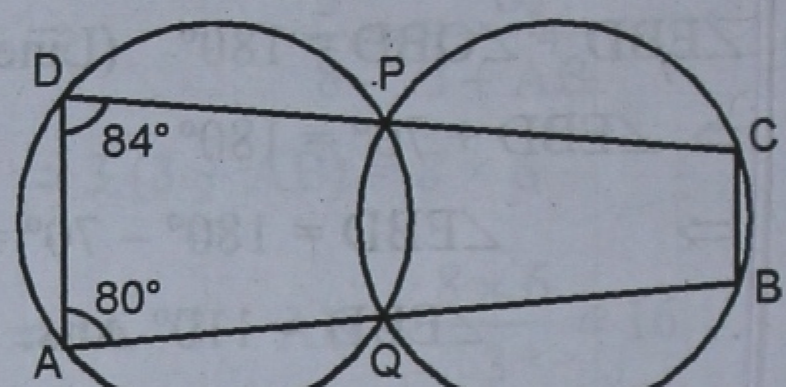
Ext. $\angle DOA = \angle CDE + \angle OBD$

$\Rightarrow 75° = 50° + \angle OBD$

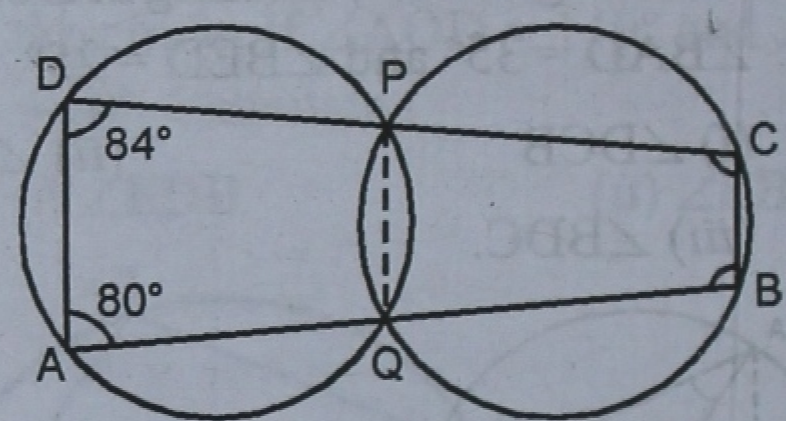
$\Rightarrow \angle OBD = 75° - 50° = 25°$ **Ans.**

Q. 26. In the given figure, the two circle intersect at P and Q. If $\angle A = 80°$ an $\angle D = 84°$, Calcualte :

(*i*) $\angle QBC$ (*ii*) $\angle BCP$.



Sol. In the figure, two circles intersect eac other at P and Q



ABCD is a quadrilateral in whic $\angle A = 80°$ and $\angle D = 84°$. Join PQ.

AQPD is a cyclic quadrilateral.

$\therefore \angle ADP + \angle AQP = 180°$

$\Rightarrow 84° + \angle AQP = 180°$

$\Rightarrow \angle AQP = 180° - 84° = 96$

Similarly, $\angle QAD + \angle QPD = 180°$

$\Rightarrow 80° + \angle QPD = 180°$

$\therefore \angle QPD = 180° - 80° = 100°$

Now, in cyclic quadrilateral QBCP,

Ext. $\angle AQP$ = Int. opposite $\angle BCP$

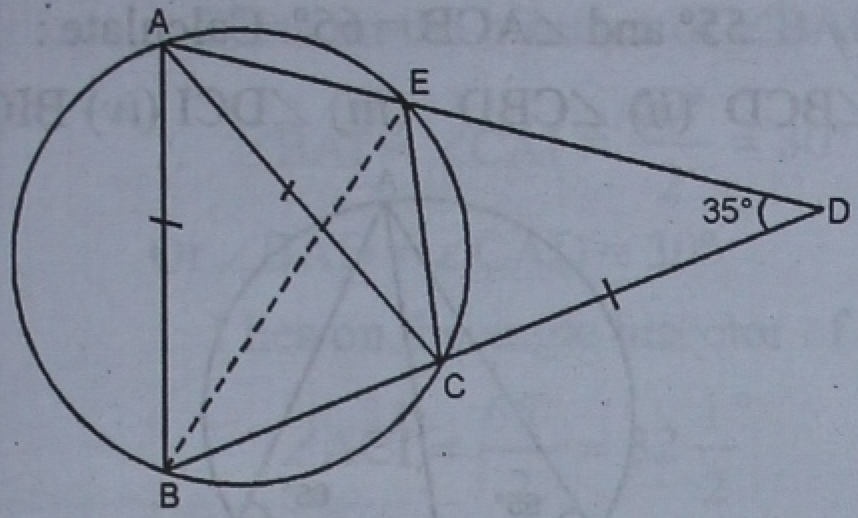
and Ext. $\angle QPD = \angle QBC$

$\therefore \angle QBC = \angle QPD = 100°$

and $\angle BCP = \angle AQP = 96°$ **Ans.**

Q. 27. In the adjoining figure, AB = AC = CD $\angle ADC = 35°$. Calculate :

(*i*) $\angle ABC$ (*ii*) $\angle BEC$



Sol. In the figure,

$AB = AC = CD, \angle ADC = 35°$

$\because \quad AC = CD$

$\therefore \angle CAD = \angle ADC = 35°$

Now, in Δ ACD,

(*i*) Ext. $\angle ACB = \angle CAD + \angle ADC$

$= 35° + 35° = 70°$

$\because \quad AB = AC$

$\therefore \angle ABC = \angle ACB = 70°$

(*ii*) But in ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$

(Angles of a triangle)

$\Rightarrow 70° + 70° + \angle BAC = 180°$

$\Rightarrow \quad 140° + \angle BAC = 180°$

$\Rightarrow \quad \angle BAC = 180° - 140°$

$\therefore \quad \angle BAC = 40°$

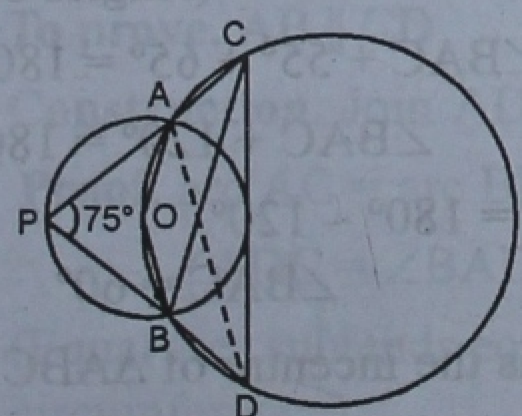
But $\angle BAC = \angle BEC$

(Angles in the same segment)

$\therefore \angle BEC = 40°$ **Ans.**

Q. 28. In the adjoining figure, two circles intersect at A and B. The centre of the smaller circle is O and lies on the circumference of the larger circle. If PAC and PBD are straight lines and $\angle APB = 75°$, find :

(*i*) $\angle AOB$ (*ii*) $\angle ACB$ (*iii*) $\angle ADB$.



Sol. Two circles intersect each other at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. PAC and PBD are two lines and $\angle APB = 75°$.

(*i*) Arc AB of smaller circle subtends $\angle AOB$ at the centre and $\angle APB$ at the remaining part of the circle.

$\therefore \angle AOB = 2\angle APB$

$= 2 \times 75° = 150°$

$\because$ OBDA is a cyclic quadrilateral

$\therefore \angle ADB + \angle AOB = 180°$

$\Rightarrow \quad \angle ADB + 150° = 180°$

$\Rightarrow \quad \angle ADB = 180° - 150° = 30°$

But $\angle ADB = \angle ACB$

(Angles in the same segment)

$\therefore \angle ACB = 30°$ Hence

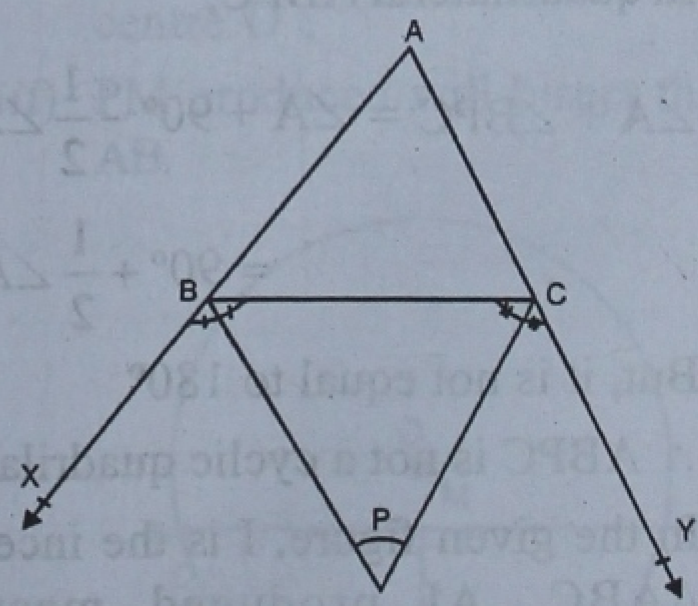
(*i*) $\angle AOB = 150°$ (*ii*) $\angle ACB = 30°$ and

(*iii*) $\angle ADB = 30°$ **Ans.**

Q. 29. The exterior angles B and C in ΔABC are bisected to meet at a point P. Prove that :

$\angle BPC = 90° - \frac{\angle A}{2}$. Is ABPC a cyclic quadrilateral ?

Sol. **Given.** The sides AB and AC of a ΔABC are produced to X and Y respectively. BP and CP are the bisectors of Ext. $\angle B$ and ext. $\angle C$ meeting each other at P.



To prove. (*i*) $\angle BPC = 90^\circ - \frac{\angle A}{2}$

(*ii*) Is ABPC a cyclic quadrilateral ?

Proof : In ΔABC,

Ext. $\angle B$ = Interior $\angle C + \angle A$

Ext. $\angle C$ = Interior $\angle B + \angle A$

or $\angle CBP = \frac{1}{2}(\angle C + \angle A)$

$= \frac{1}{2}\angle C + \frac{1}{2}\angle A$

and $\angle BCP = \frac{1}{2}(\angle B + \angle A)$

$= \frac{1}{2}\angle B + \frac{1}{2}\angle A$

Adding, we get

$\angle CBP + \angle BCP = \frac{1}{2}\angle C + \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}A$

$= \frac{1}{2}(\angle A + \angle B + \angle C) + \frac{1}{2}\angle A$

$= \frac{1}{2} \times 180^\circ + \frac{1}{2}\angle A \quad = 90^\circ + \frac{1}{2}\angle A$

But in ΔBPC,

$\angle BPC = 180^\circ - (CBP + \angle BCP)$

$= 180^\circ - \left[90^\circ + \frac{1}{2}\angle A\right]$

$= 180^\circ - 90^\circ - \frac{1}{2}\angle A \quad = 90^\circ - \frac{1}{2}\angle A$

(*ii*) In quadrilateral ABPC,

$\angle A + \angle BPC = \angle A + 90^\circ - \frac{1}{2}\angle A$

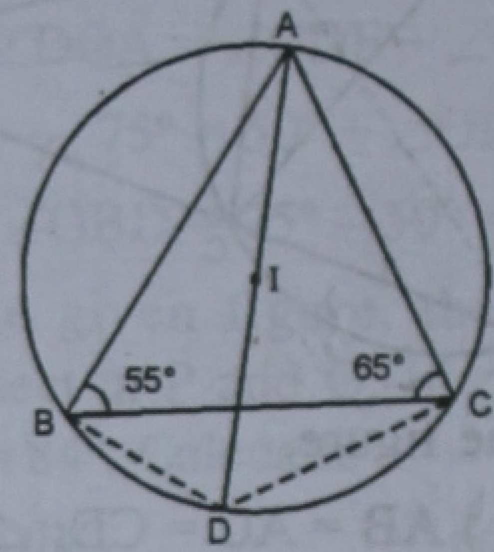
$= 90^\circ + \frac{1}{2}\angle A$

But, it is not equal to 180°

$\therefore$ ABPC is not a cyclic quadrilateral.

Q. 30. In the given figure, I is the incentre of ΔABC, AI produced meets the circumcircle of ΔABC at D ; $\angle ABC = 55^\circ$ and $\angle ACB = 65^\circ$. Calculate :

(*i*) $\angle BCD$ (*ii*) $\angle CBD$ (*iii*) $\angle DCI$ (*iv*) BIC.



Sol. I is the incentre of the ΔABC. AD is joined and produced to meet the circle at D. DB, DC; IC and IB are joined.

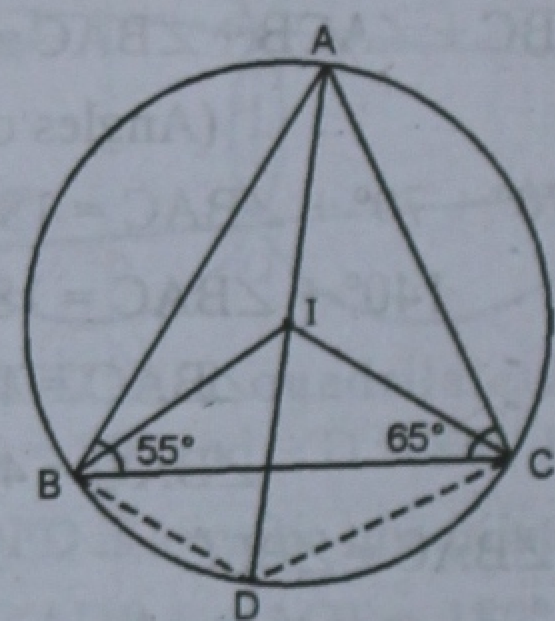
$\angle ABC = 55^\circ$ and $\angle ACB = 65^\circ$

(*i*) $\because$ AD is the diameter.

$\therefore \angle ACD = 90^\circ$ (Angle in a semi-circle)

$\Rightarrow \angle ACB + \angle BCD = 90^\circ \Rightarrow 65^\circ + \angle BCD = 90$

$\Rightarrow \quad \angle BCD = 90^\circ - 65^\circ = 25^\circ$



(*ii*) Similarly, $\angle ABD = 90^\circ$

$\Rightarrow \angle ABC + \angle CBD = 90^\circ$

$\Rightarrow \quad 55^\circ + \angle CBD = 90^\circ$

$\Rightarrow \quad \angle CBD = 90^\circ - 55^\circ = 35^\circ$

(*iii*) In ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

(Angles of a triangle)

$\Rightarrow \angle BAC + 55^\circ + 65^\circ = 180^\circ$

$\Rightarrow \quad \angle BAC + 120^\circ = 180^\circ$

$\Rightarrow \quad \angle BAC = 180^\circ - 120^\circ$

$\angle BAC = 60^\circ$

$\because$ I is the incentre of ΔABC

$\therefore$ I lies on the bisector of $\angle BAC$

$\therefore \quad \angle BAI = \angle CAI = \frac{60^\circ}{2} = 30^\circ$

Or $\angle BAD = \angle CAD = 30^\circ$

$\because$ I lies on the angle bisector of $\angle ACB$

$\therefore \quad \angle ACI = \frac{65^\circ}{2} = 32\frac{1}{2}^\circ$

Now, $\angle DCI = \angle ACD - ACI$

$= 90^\circ - 32\frac{1}{2}^\circ = 57\frac{1}{2}^\circ = 57{\cdot}5^\circ$

(*iv*) $\because$ I lies on the angle bisector of $\angle ABC$

$= \angle IBC = \frac{55^\circ}{2} = 27{\cdot}5^\circ$

Now, in ΔBIC,

$\angle BIC + \angle ICB + \angle IBC = 180^\circ$ (Angles of a triangle)

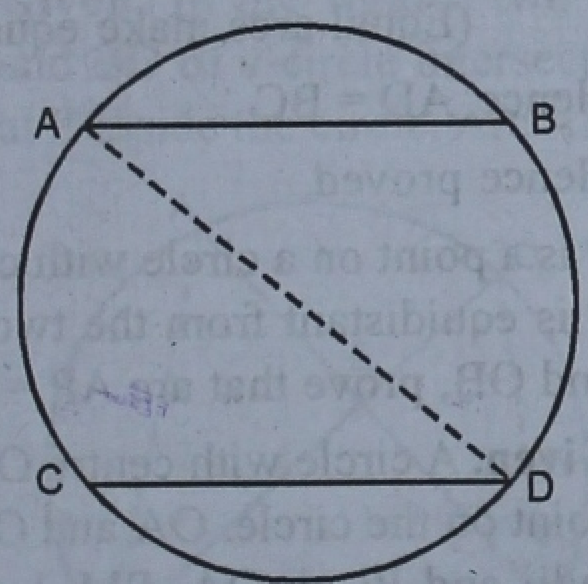
$\Rightarrow \angle BIC + (32{\cdot}5^\circ + 27{\cdot}5^\circ) = 180^\circ$

$\Rightarrow \quad \angle BIC + 60^\circ = 180^\circ$

$\Rightarrow \quad \angle BIC = 180^\circ - 60^\circ = 120^\circ$ **Ans.**

EXERCISE 20 (B)

Q. 1. In the given figure, arc AC and arc BD are two equal arcs of a circle. Prove that chord AB and chord CD are parallel.



Sol. Given. In a circle, arc AC = arc BD. AB and CD arc joined.

To prove. AB ∥ CD

Construction. Join AD.

Proof. arc AC = arc BD. (given)

$\therefore \quad \angle ADC = \angle BAD$

(Equal arcs subtends equal angles at the circumference)

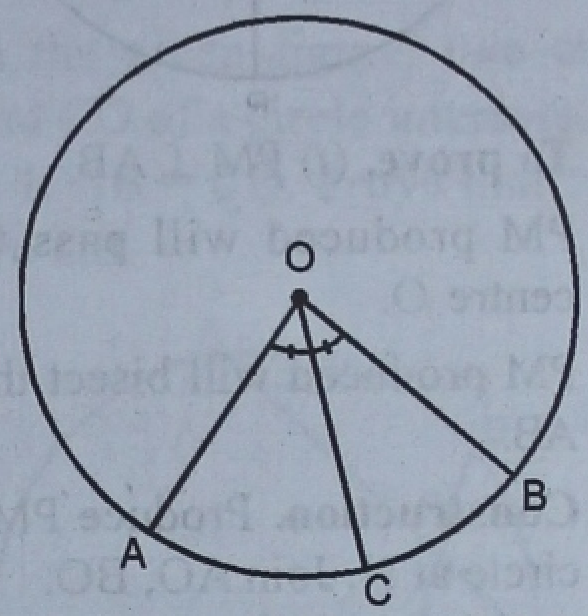
But, these are alternate angles

$\therefore$ AB ∥ CD

Hence proved.

Q. 2. Prove that the angle subtended at the centre of a circle, is bisected by the radius passing through the mid-point of the arc.

Sol. Given. An arc AB of the circle which subtend $\angle AOB$ at the centre. C is the mid-point of arc AB, OC is joined.



To prove. $\angle AOC = \angle BOC$

Proof. $\because$ C is the mid-point of arc AB.

$\therefore$ arc AC = arc BC

But these subtends $\angle AOC$ and $\angle BOC$ at the centre

$\therefore \angle AOC = \angle BOC$

Hence, OC is the bisector of $\angle AOB$

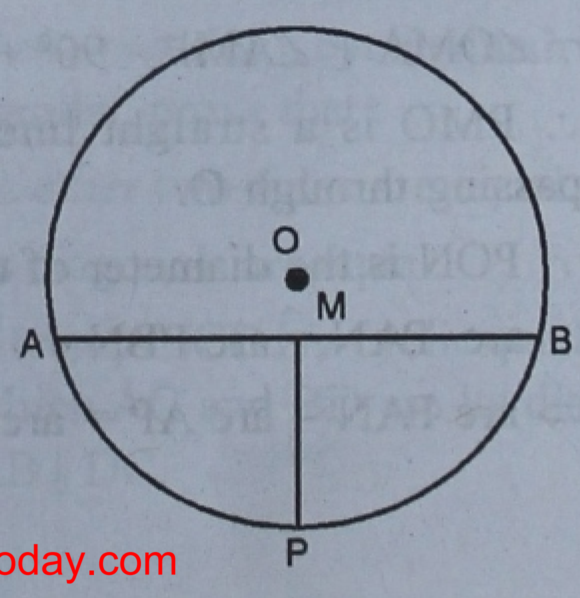
Hence proved.

Q. 3. In the given figure, P is the mid-point of arc APB and M is the mid-point of chord AB of a circle with centre O. Prove that:

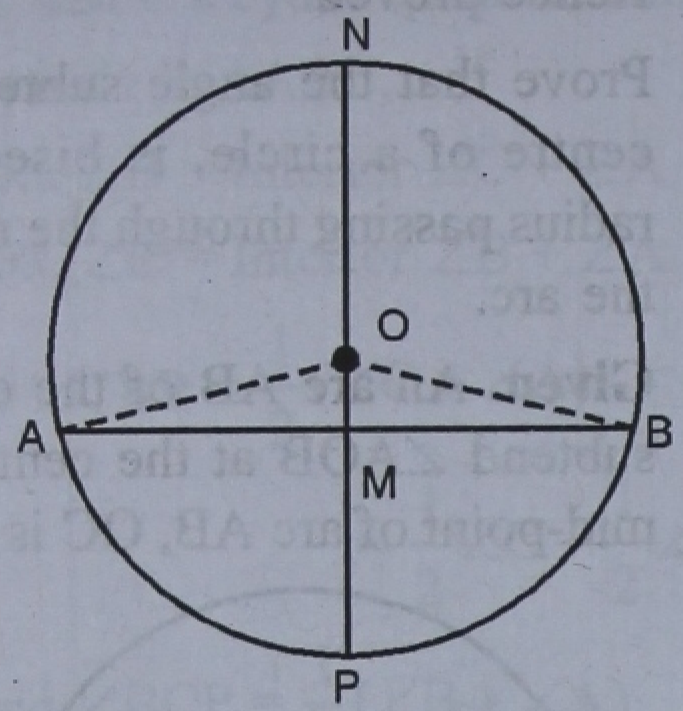
(*i*) PM ⊥ AB ;

(*ii*) PM produced will pass through the centre O ;

(*iii*) PM produced will bisect the major arc AB.



Sol. Given. P is the mid-point of arc APB and M is the mid-point of chord AB of the circle with centre O.



To prove. (*i*) PM ⊥ AB

(*ii*) PM produced will pass through the centre O.

(*iii*) PM produced will bisect the major arc AB.

Construction. Produce PM to join the circle at N. Join AO, BO.

Proof. ∵ P is the mid-point of arc AB

∴ Arc AP = arc PB

∴ ∠AOP = ∠POB

⇒ ∠AOM = ∠BOM

(*i*) Now, in ΔOAM and OBM,

OM = OM (Common)

OA = OB (Radii of the same circle)

∠AOM = ∠BOM (Proved)

∴ ΔOAM ≅ ΔOBM (S.A.S. axiom)

∴ ∠AMO = ∠BMO (c.p.c.t.)

But, ∠AMO + ∠BMO = 180° (Linear pair)

∴ ∠AMO = ∠BMO = 90°

Hence, OM or MP ⊥ AB.

(*ii*) ∵ ∠OMA + ∠AMP = 90° + 90° = 180°

∴ PMO is a straight line. Which is passing through O.

(*iii*) ∵ PON is the diameter of the circle

∴ arc PAN = arc PBN

⇒ arc PAN – arc AP = arc PBN – arc PB (∵ arc AP = arc PB)

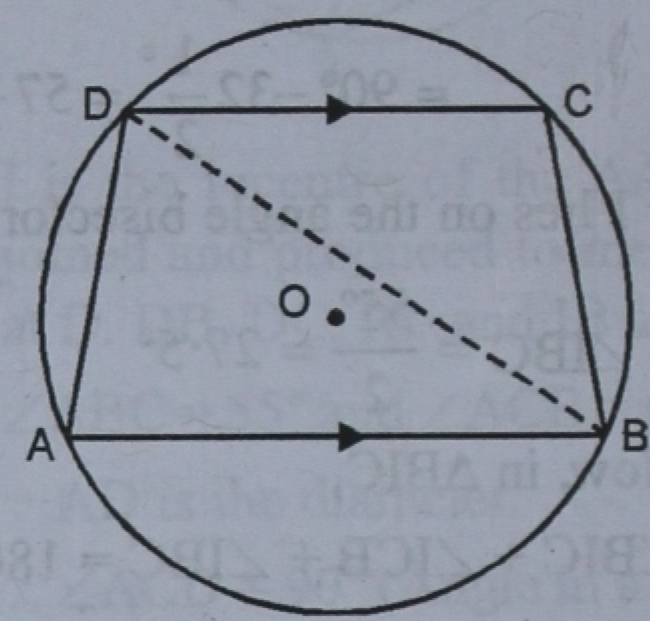
⇒ arc AN = arc BN

Hence, N bisects major arc AB

Hence proved.

Q. 4. Prove that in a cyclic trapezium, the non-parallel sides are equal.

Sol. Given. ABCD is a cyclic trapezium in which AB ∥ DC



To prove. AD = BC

Construction. Join BD

Proof. ∵ AB ∥ DC (given)

∴ ∠ABD = ∠CDB (Alternate angles)

But these are the angles subtended by the arcs AD and BC respectively

∴ arc AD = arc BC

∴ chord AD = chord BC

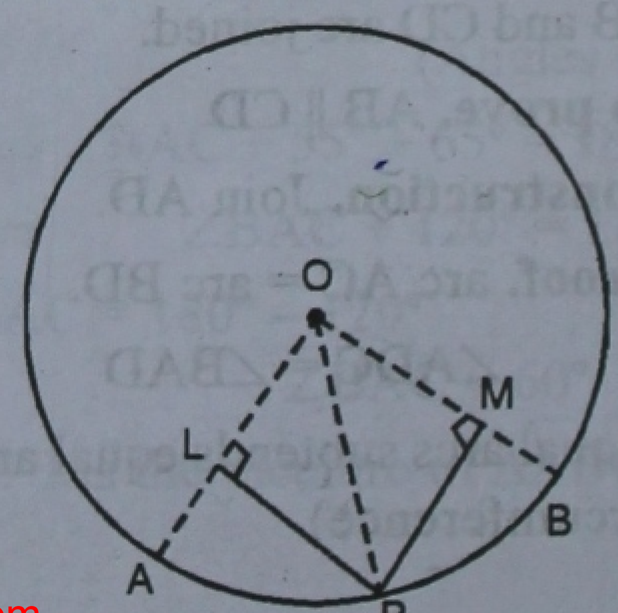
(Equal arcs make equal chords).

Hence, AD = BC

Hence proved.

Q. 5. P is a point on a circle with centre O. If P is equidistant from the two radii OA and OB, prove that arc AP = arc BP.

Sol. Given. A circle with centre O and P is a point on the circle. OA and OB are two radii and PL ⊥ OA, PM ⊥ OB. Such that OL = OM.



To prove. arc AP = arc PB

Construction. Join PO

Proof. In right ΔOLP and ΔOMP,

Hyp. OP = OP (Common)

Side PL = PM (Given)

∴ ΔOLP ≅ ΔOMP (R.H.S. axiom)

∴ ∠LOP = ∠MOP (c.p.c.t.)

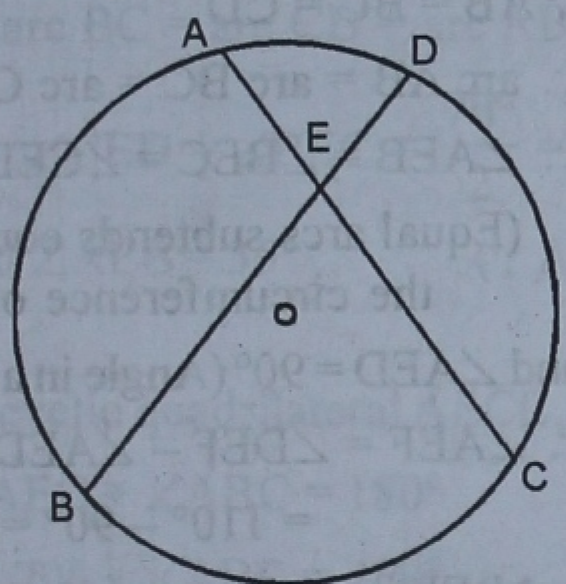
or ∠AOP = ∠BOP

∴ arc AP = arc PB

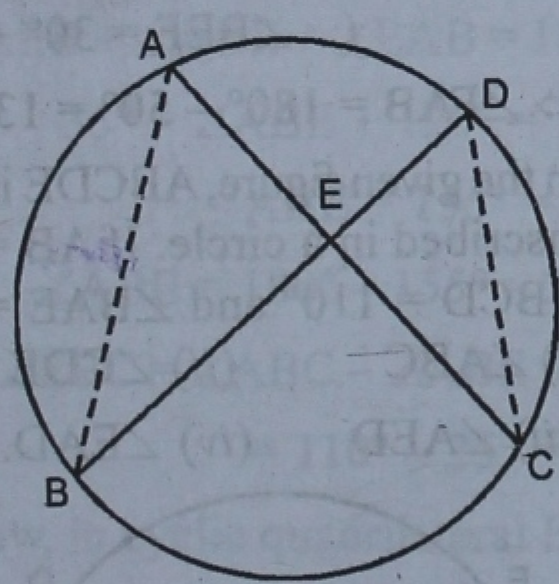
(Equal arcs subtend equal angles at the centre)

Hence proved.

Q. 6. In the given figure, two chords AC and BD of a circle intersect at E. If arc AB = arc CD. prove that : BE = EC and AE = ED.



Sol. Given. In the figure, two chords AC and BD of a circle intersect each other at E inside the circle. Arc AB = arc CD.



To prove. BE = EC and AE = ED

Construction. Join AB and CD.

Proof. ∵ Arc AB = arc CD (Given)

∴ Chord AB = chord CD

Now, in ΔAEB and ΔCED,

AB = CD (Proved)

∠BAE = ∠CDE

∠ABE = ∠ECD

(Angle in the same-segment)

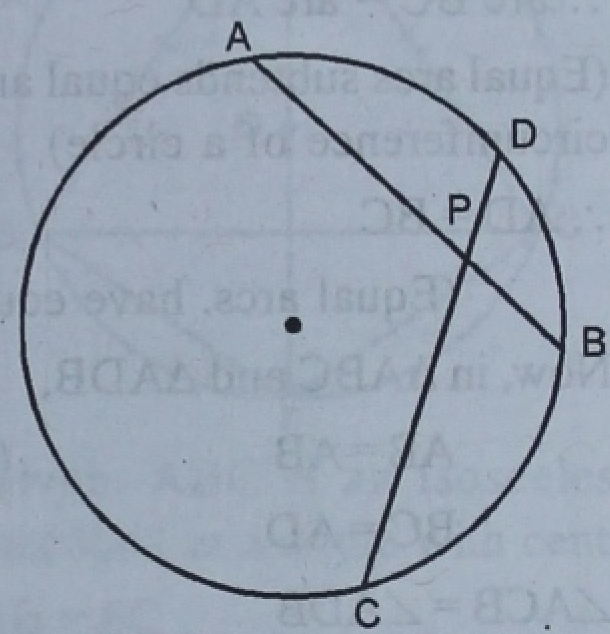
∴ ΔAEB ≅ ΔCED (A.S.A. axiom)

∴ BE = EC (c.p.c.t.)

and AE = ED (c.p.c.t.)

Hence proved.

Q. 7. In the given figure, two chords AB and CD of a circle intersect at a point P. If AB = CD. Prove that : arc AD = arc CB.



Sol. Given. Two chords AB and CD of a circle intersect each other at P inside the circle and AB = CD.

To prove. arc AD = arc CB

Proof. ∵ AB = CD (Given)

∴ Minor arc AB = Minor arc CD

Subtracting arc BD from both sides,

Minor arc AB – arc BD = Minor arc CD – arc BD

⇒ arc AD = arc CB

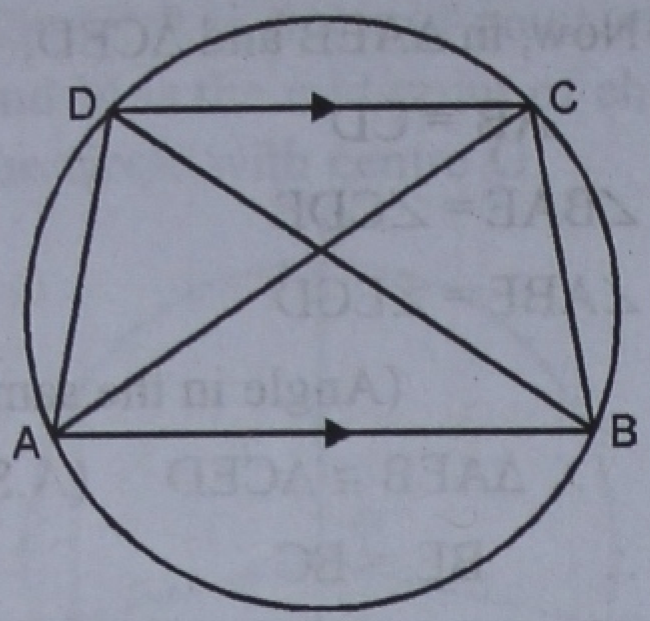
Hence proved.

Q. 8. If two sides of a cyclic quadrilateral are parallel, prove that :

(*i*) its other two sides are equal.

(*ii*) its diagonals are equal.

Sol. Given. ABCD is a cyclic quadrilateral in which AC and BD are its diagonal and AB ∥ DC.



To prove.

(*i*) AD = BC (*ii*) AC = BD.

Proof. In quadrilateral ABCD,

AB ∥ DC (given)

∴ ∠CAB = ∠DCA (Alternate angles)

∴ arc BC = arc AD

(Equal arcs subtends equal angles at the circumference of a circle)

∴ AD = BC

(Equal arcs. have equal chords)

Now, in ΔABC and ΔADB,

AB = AB (Common)

BC = AD (Proved)

∠ACB = ∠ADB

(Angles in the same segment)

∴ ΔABC ≅ Δ ADB (S.A.S. axiom)

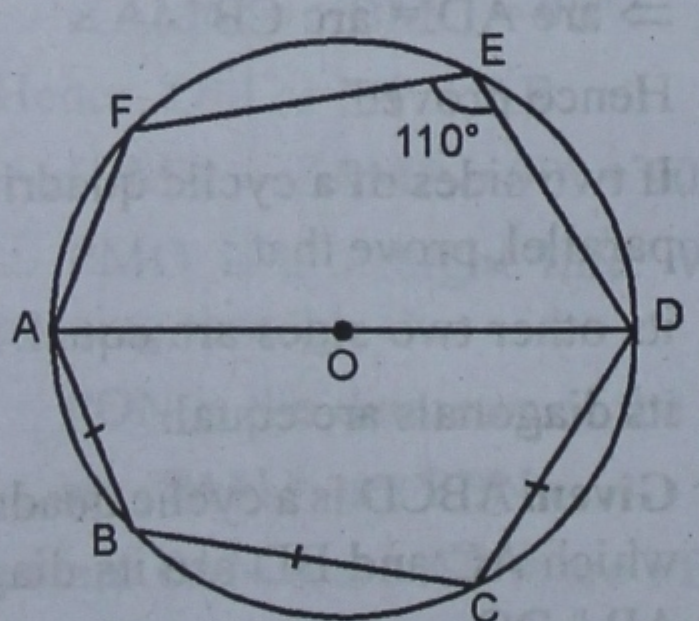
∴ AC = BD. (c.p.c.t.)

Hence proved.

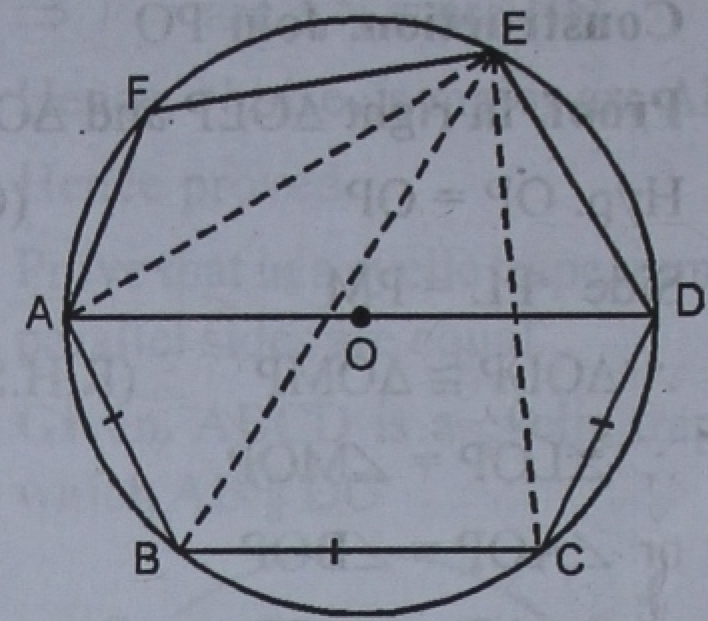
Q. 9. In the given figure, AB, BC and CD are equal chords of a circle with centre O and AD is a diameter.

If ∠DEF = 110°, find

(*i*) ∠AEF (*ii*) ∠FAB.



Sol. Given. In the figure,



Chord AB = chord BC = chord CD

O is the centre of the circle and AD is its diameter and ∠DEF = 110°

To find (*i*) ∠AEF and (*ii*) ∠FAB.

Construction. Join AE, BE and CE

Proof :

∵ AB = BC = CD

∴ arc AB = arc BC = arc CD

∴ ∠AEB = ∠BEC = ∠CED

(Equal arcs subtends equal angles at the circumference of the circle)

and ∠AED = 90° (Angle in a semi-circle)

(*i*) ∴ ∠AEF = ∠DEF – ∠AED

= 110° – 90° = 20°

(*ii*) ∵ ABEF is cyclic quadrilateral

∴ ∠FAB + ∠BEF = 180°

⇒ ∠FAB + 50° = 180°

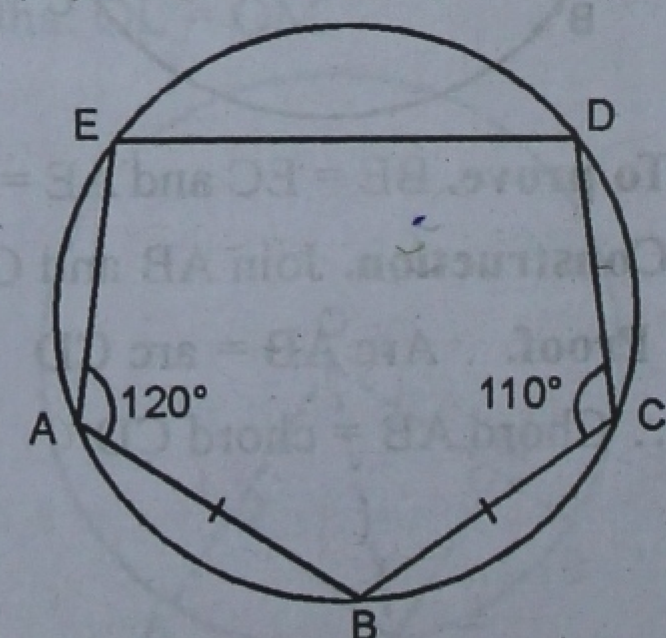
(∵ ∠BEF = 30° + 20° = 50°)

⇒ ∠FAB = 180° – 50° = 130° **Ans.**

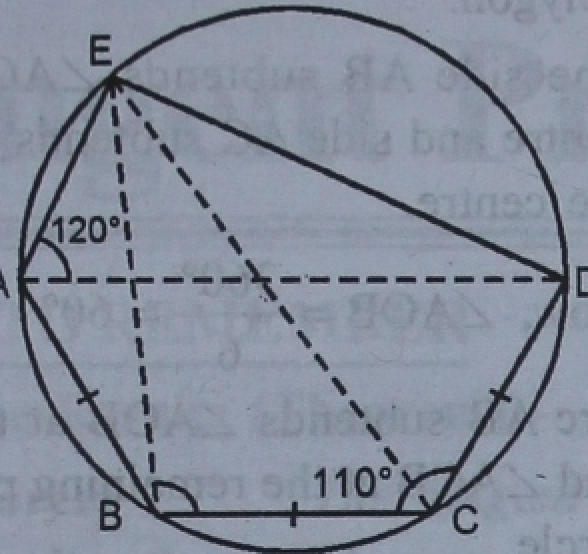
Q. 10. In the given figure, ABCDE is a pentagon inscribed in a circle. If AB = BC = CD, ∠BCD = 110° and ∠BAE = 120°, find

(*i*) ∠ABC (*ii*) ∠CDE

(*iii*) ∠AED (*iv*) ∠EAD.



Sol. ABCDE is a pentagon inscribed in a circle, AB = BC = CD and ∠BCD = 110° and ∠BAE = 120°.



Join BE, CE and AD

(*i*) In cyclic quadrilateral EBCD,

∠BCD = 110°

∴ ∠BED = 180° – 110° = 70°

∵ BC = CD = AB

∴ arc BC = arc CD = arc AB

$\therefore\ \angle CED = \angle BEC = \frac{70^\circ}{2} = 35^\circ$

and ∠AEB = 35° (∵ AB = BC = CD)

(*ii*) In cyclic quadrilateral AECB,

∠AEC + ∠ABC = 180°

⇒ 70° + ∠ABC = 180°

⇒ ∠ABC = 180° – 70° = 110°

In ΔABE,

∠AEB + ∠ABE + ∠EAB = 180°

⇒ 35° + ∠ABE + 120° = 180°

⇒ ABE + 155° = 180°

⇒ ∠ABE = 180° – 155° = 25°

∴ ∠EBC = ∠ABC – ∠ABE

= 110° – 25° = 85°

Now, in cyclic quadrilateral EBCD,

∠EBC + ∠CDE = 180°

⇒ 85° + ∠CDE = 180°

⇒ ∠CDE = 180° – 85°

∴ ∠CDE = 95°

(*iii*) ∠AED = 35° + 35° + 35° = 105°

(*iv*) In cyclic quadrilateral ABCD,

∠DAB + ∠BCD = 180°

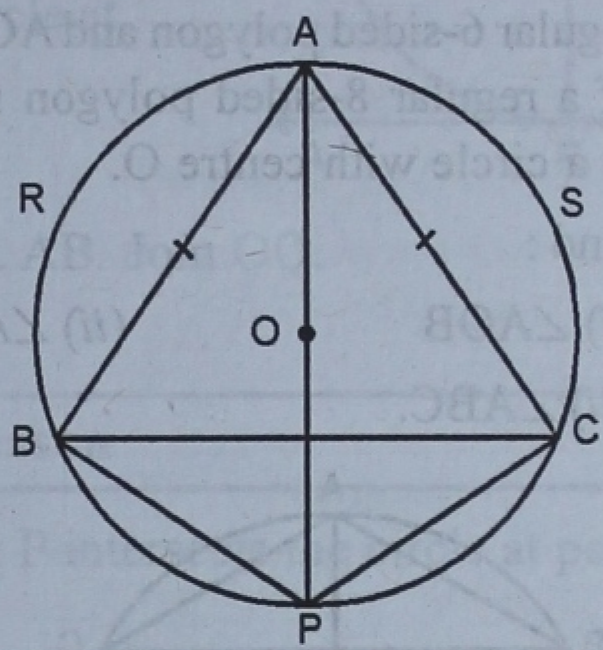
⇒ ∠DAB + 110° = 180°

⇒ ∠DAB = 180° – 110° = 70°

But ∠EAB = 120°

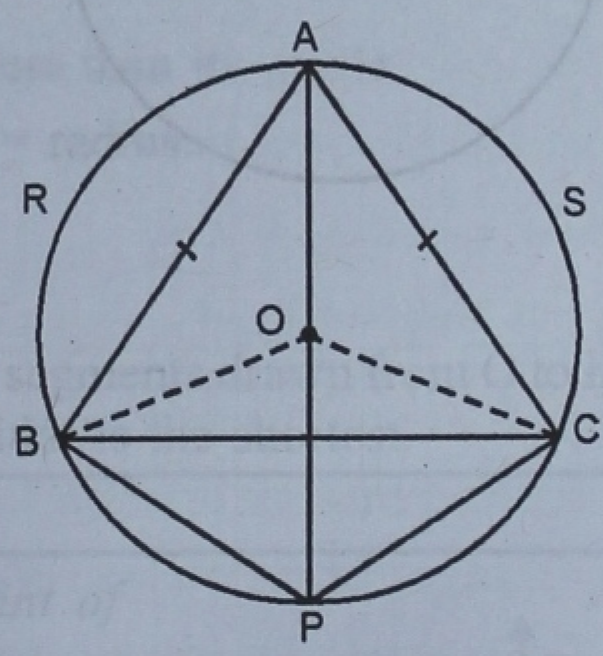
∴ ∠EAD = 120° – 70° = 50° **Ans.**

Q. 11. In the given figure, ΔABC is an isosceles triangle inscribed in a circle with centre O. If AB = AC, prove that : AP bisects ∠BPC.



Sol. Given. ABC is an isosceles triangle inscribed in a circle with centre O.

AB = AC



To prove. AP bisects ∠BPC.

Construction. Join OB and OC.

Proof. ∵ AB = AC

∴ arc AB = arc AC

⇒ ∠AOB = ∠AOC

(Equal arcs subtends equal angles at the centre)

$\because$ Arc AB subtends $\angle AOB$ at the centre and $\angle APB$ at the remaining part of the circle.

$\therefore \quad \angle AOB = 2\ \angle APB$

Similarly, $\angle AOC = 2\ \angle APC$

$\because \quad \angle AOB = \angle AOC$

$\therefore 2\angle APB = 2\angle APC$

$\Rightarrow \quad \angle APB = \angle APC$

$\therefore$ AP is the bisector of $\angle BPC$

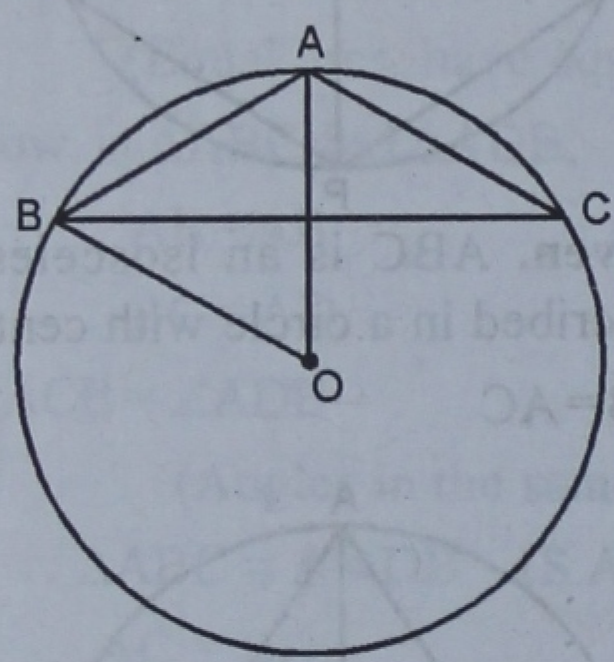
Hence proved.

Q. 12. In the given figure, AB is a side of a regular 6-sided polygon and AC is a side of a regular 8-sided polygon inscribed in a circle with centre O.

Find :

(*i*) $\angle AOB$ (*ii*) $\angle ACB$

(*iii*) $\angle ABC$.



Sol. AB is the side of a regular 6-sided polygon inscribed in a circle with centre O and AC is the side of regular 8-sided polygon.

(*i*) The side AB subtends $\angle AOB$ at the centre and side AC subtends $\angle AOC$ at the centre.

Now, $\angle AOB = \dfrac{360^\circ}{6} = 60^\circ$

(*ii*) Arc AB subtends $\angle AOB$ at the centre and $\angle ACB$ at the remaining part of the circle.

$\therefore \angle AOB = 2\angle ACB$

$\Rightarrow \angle ACB = \dfrac{1}{2}\angle AOB = \dfrac{1}{2} \times 60^\circ = 30^\circ$

(*iii*) $\angle AOC = \dfrac{360^\circ}{8} = 45^\circ$

Now, arc AC subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle

$\therefore \angle AOC = 2\ \angle ABC$

$\Rightarrow \angle ABC = \dfrac{1}{2}\ \angle AOC$

$\Rightarrow \angle ABC = \dfrac{45^\circ}{2} = 22{\cdot}5^\circ$

$= 22^\circ 30'$ **Ans.**