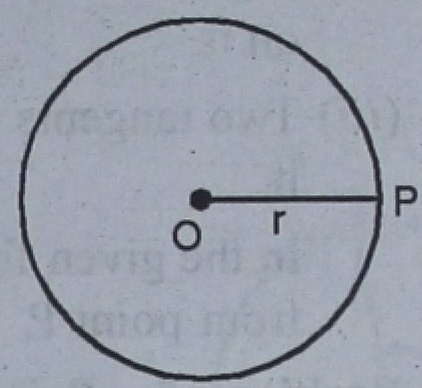
Chapter 19

Chord Properties Of A Circles

POINTS TO REMEMBER

1. **Circle.** *A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point remains constant.*



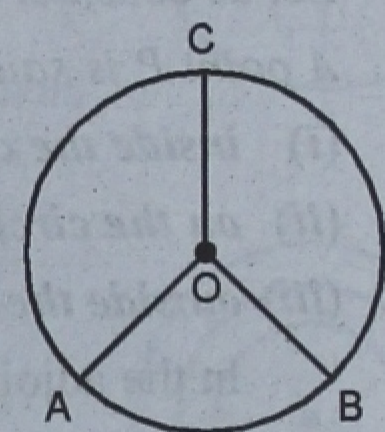
The fixed point is called the **centre** and the constant distance is called the **radius** of the circle.

The given figure consists of a circle with centre O and radius equal to *r* units.

2. **Circumference.** *The perimeter of a circle is called its circumference.*

Circumference = $2\pi r$.

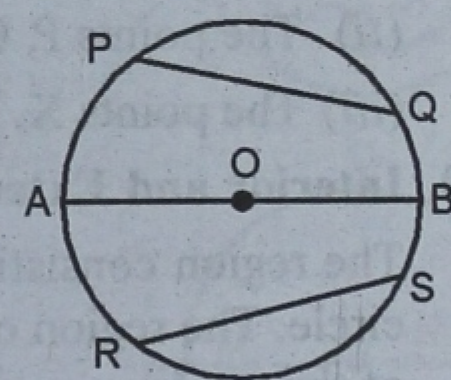
3. **Radius.** *A line segment joining the centre and a point on the circle is called its radius.*



The plural of radius is **radii.**

In the given figure, OA, OB and OC are the radii of a circle.

4. **Chord.** *A line segment joining any two points on a circle is called a chord of the circle.*



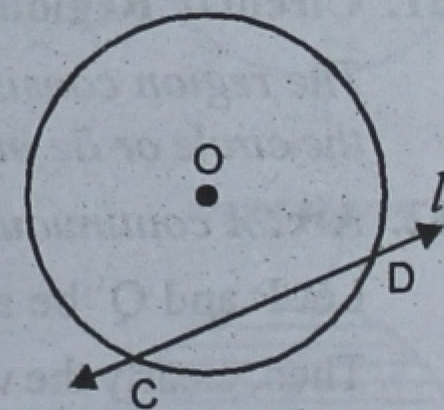
5. **Diameter.** *A chord of the circle passing through the centre of a circle is called its diameter.*

In the adjoining figure, AOB is a diameter of a circle with centre O.

Diameter is the largest chord of a circle.

All diameters of a circle are equal in length.

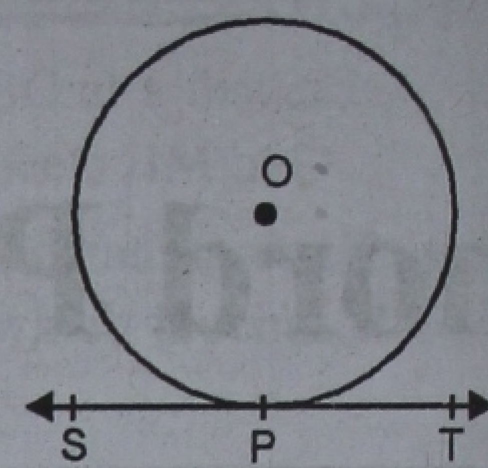
Diameter = 2 × Radius.



6. **Secant.** *A line which intersects a circle in two distinct points is called a secant of the circle.*

In the given figure, the line *l* cuts the circle in two points C and D. Then, *l* is a secant of the circle.

7. Tangent. *A line that intersects the circle in exactly one point is called a tangent to the circle.*



The point at which the tangent intersects the circle is called its **point of contact.** In the given figure, SPT is a tangent at the point P of the circle with centre O. Clearly, P is the point of contact of the tangent with the circle.

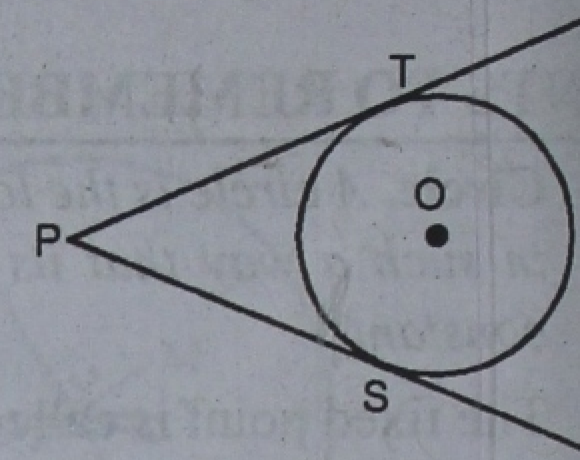
8. Facts about Tangents :

(*i*) No tangent can be drawn to a circle through a point inside it.

(*ii*) One and only one tangent can be drawn to a circle at a point on it.

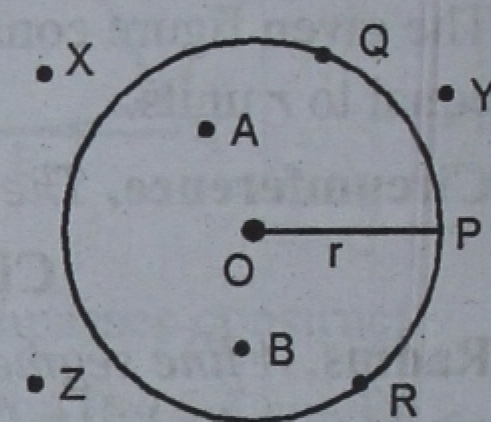
(*iii*) Two tangents can be drawn to a circle from a point outside it.

In the given figure, PT and PS are the tangents to the circle from point P.



9. Position of a Point With Respect to a Circle

Let us consider a circle with centre O and radius *r*.



A point P is said to lie.

(*i*) ***inside the circle, if* OP < *r* ;**

(*ii*) ***on the circle, if* OP = *r*.**

(*iii*) ***outside the circle , if* OP > *r*.**

In the adjoining figure of a circle with centre O and radius *r*.

(*i*) The points A, O, B lie inside the circle ;

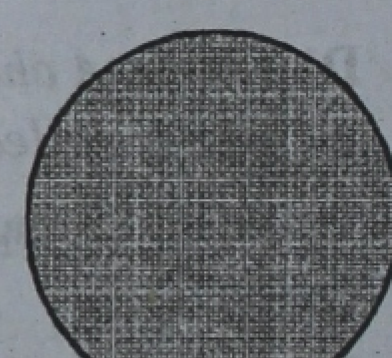
(*ii*) The points P, Q, R lie on the circle ;

(*iii*) The points X, Y, Z lie outside the circle.

10. Interior and Exterior of a Circle :

The region consisting of all those points which lie inside a circle, is called the **interior** of the circle. The region consisting of all those points which lie outside a circle, is called the **exterior** of the circle.

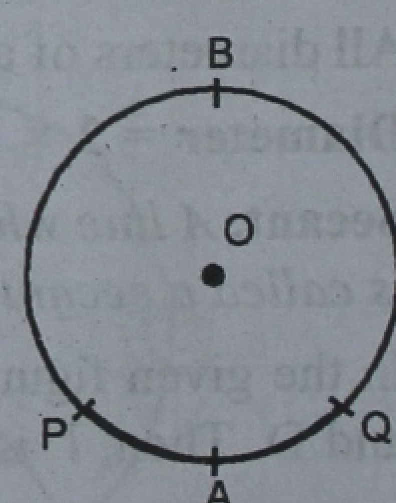
11. Circular Region or Circular Disc :



The region consisting of all those points which are either on the circle or lie inside the circle , is called the circular region.

12. Arc. *A continuous piece of a circle is called an arc of the circle.*

Let P and Q be any two points on a circle with centre O.



Then, clearly the whole circle has been divided into two pieces, namely **arc PAQ** and **arc QBP**, to be denoted by $\widehat{PAQ}$ and $\widehat{QBP}$ respectively.

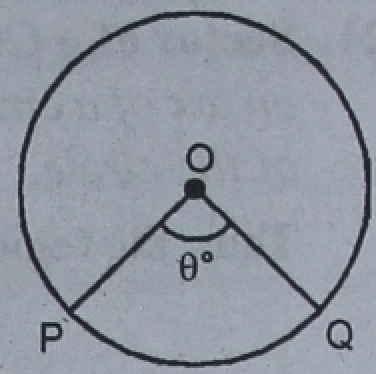
We may denote them by $\widehat{PQ}$ and $\widehat{QP}$ respectively.

An arc **PQ** is called a **minor arc** or a **major arc,** according as length.

$(\widehat{PQ})$ < *length* $(\widehat{QP})$ or *length* $(\widehat{PQ})$ > *length* $(\widehat{QP})$

13. Central Angle. *An angle subtended by an arc at the centre of a circle is called its central angle.*

In the given figure, central angle of $\overset{\frown}{PQ} = \angle POQ = \theta$



14. Degree Measure of an Arc

Let $\overset{\frown}{PQ}$ be an arc of a circle with centre O.

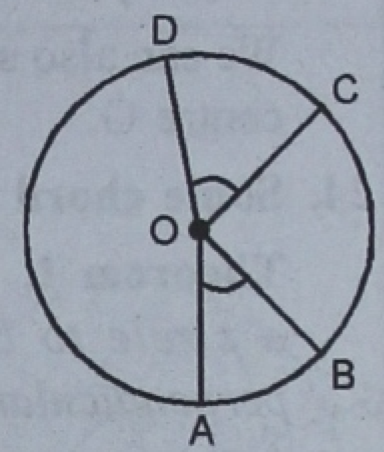
If $\angle POQ = \theta°$, we say that the degree measure of $\overset{\frown}{PQ}$ is $\theta°$ and we write, $(\overset{\frown}{PQ}) = \theta°$.

If $m(\overset{\frown}{PQ}) = \theta°$, *then* $m\,\overset{\frown}{QP} = (360 - \theta)°$.

Degree measure of a circle is 360°.

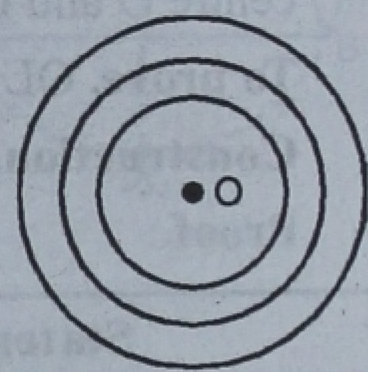
15. Congruent Arcs. Two arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are said to be congruent, if they have same degree measure.

$\overset{\frown}{AB} \cong \overset{\frown}{CD} \Leftrightarrow m(\overset{\frown}{AB}) = m(\overset{\frown}{CD}) \Leftrightarrow \angle AOB = \angle COD$.



16. Congruent Circles. *Two circles of equal radii are said to be congruent.*

17. Concentric Circles. *Circles having same centre but different radii are called concentric circles.*

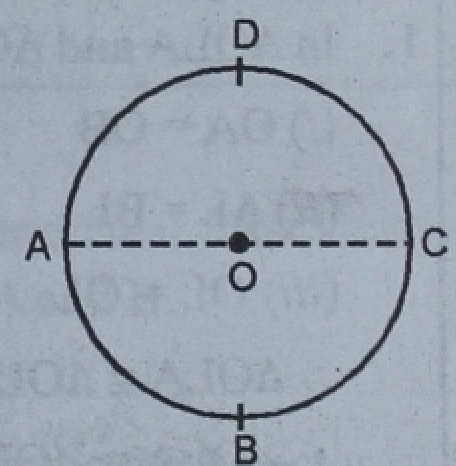


18. Semi-Circle. *A diameter divides a circle into two equal arcs. Each of these two arcs is called a semi-circle.*

The degree measure of a semi-circle is 180°.

An arc whose length is less than the arc of a semi-circle is called a **minor arc,** otherwise it is called a **major arc.**

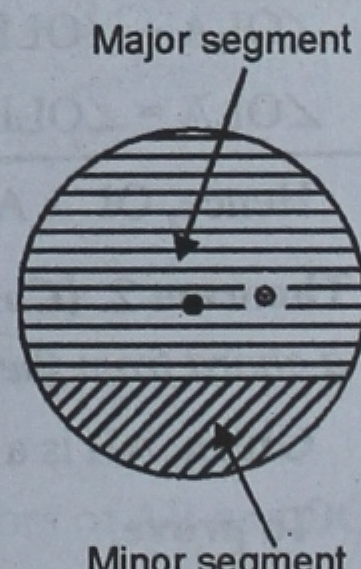
In the given figure of a circle with centre O, $\overset{\frown}{ABC}$ as well as $\overset{\frown}{ADC}$ is a semi-circle.



19. Segment. *A segment is a part of a circular region bounded by an arc and a chord, including the arc and the chord.*

The segment containing the minor arc is called a **minor segment,** while the other one is **major segment.**

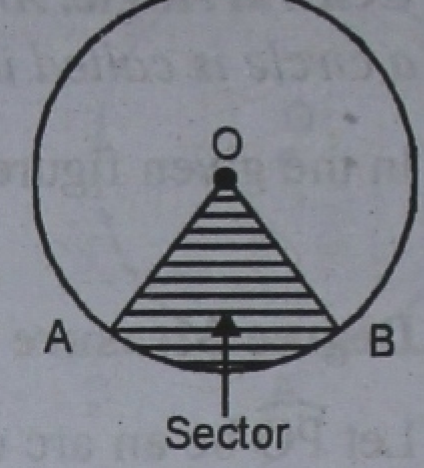
The centre of the circle lies in the major segment.



20. Alternate Segments of a Circle. *The minor and major segments of a circle are called* **alternate segments** *of each other.*

21. Sector of a Circle : *The part of the plane region enclosed by an arc of a circle and its two bounding radii is called a* **sector** *of the circle.*

Thus, the region OABO is the sector of a circle with centre O.

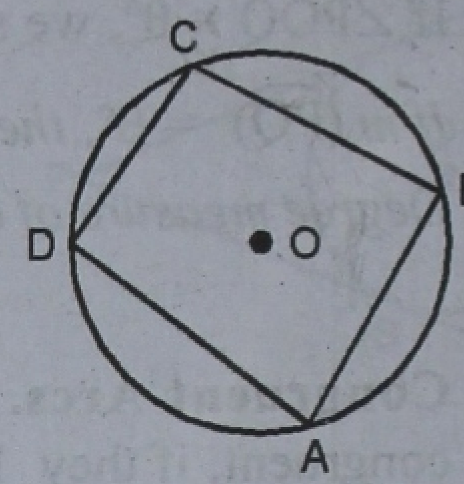


22. Quadrant. *One-fourth of a circular disc is called a quadrant.*

23. Cyclic Quadrilateral. *If all the four vertices of a quadrilateral lie on a circle, then such quadrilateral is called a cyclic quadrilateral.*

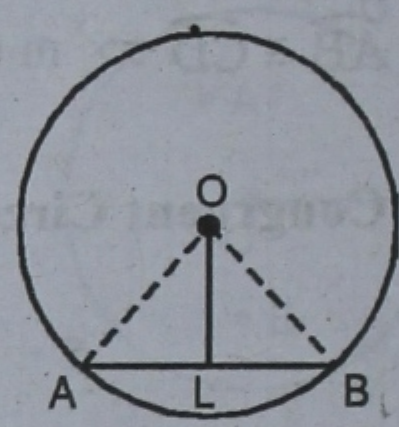
If four points lie on a circle, they are said to be **concyclic.**

We can also say that quad. ABCD is inscribed in a circle with centre O.



24. Some chord properties of a circle (Theorems)

Theorem 1. *The straight line drawn from the centre of a circle to bisect a chord, which is not a diameter, is perpendicular to the chord.*



Given. AB is a chord, other than the diameter of a circle with centre O and OL bisects AB.

To prove. OL ⊥ AB.

Construction. Join OA and OB.

Proof.

Statement	Reason
1. In ΔOLA and ΔOLB,	
(*i*) OA = OB	Radii of the same circle
(*ii*) AL = BL	Given, OL biects AB
(*iii*) OL = OL	common
∴ ΔOLA ≅ ΔOLB	SSS – Axiom of congruence
∴ ∠OLA = ∠OLB ...(I)	c.p.c.t.
2. ∠OLA + ∠OLB = 180° ...(II)	ALB is a straight line
3. ∠OLA = ∠OLB = 90°	From (I) and (II).

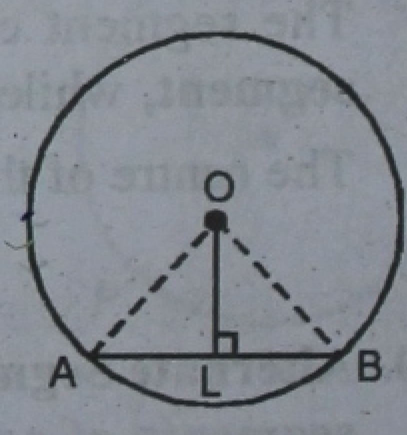
Hence, OL ⊥ AB.

Theorem 2. (*Converse of Theorem 1*). *The perpendicular to a chord from the centre of a circle bisects the chrod.*

Given. AB is a chord of a circle with centre O and OL ⊥ AB.

To prove. LA = LB.

Construction. Join OA and OB.

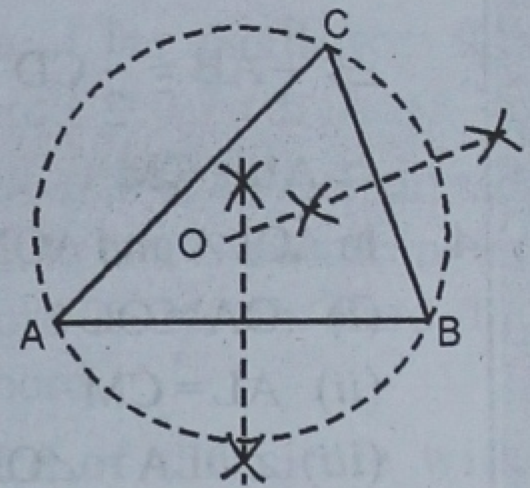


Proof.

Statement	Reason
1. In ΔOLA and ΔOLB,	
(*i*) OA = OB	Radii of the same circle
(*ii*) $\angle OLA = \angle OLB$	Each equal to 90°, since $OL \perp AB$
(*iii*) OL = OL	Common
$\therefore \Delta OLA \cong \Delta OLB$	RHS-axiom of congruency
$\therefore$ LA = LB	c.p.c.t.

Hence, LA = LB.

Theorem 3. *One and only one circle can be drawn, passing through three non-collinear points.*



Given. Three non-collinear points A, B, C.

To prove. One and only one circle can be drawn, passing through A, B and C.

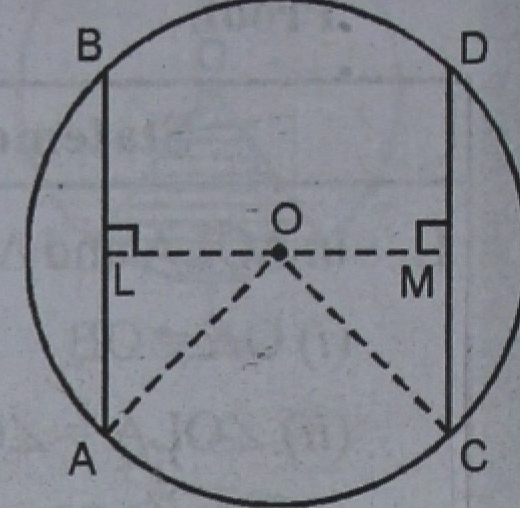
Construction. Join AB and BC. Draw the perpendicular biectors of AB and BC, meeting at a point O.

Proof.

Statement	Reason
1. O lies on the perpendicular bisector of AB $\therefore$ OA = OB ...(*i*)	Each point on perpendicular bisector of AB is equidistant from A and B.
2. O lies on the perpendicular bisector of BC. $\therefore$ OB = OC ...(*ii*)	Each point on perpendicular bisector of BC is equidistant from B and C.
3. OA = OB = OC $\therefore$ O is the equidistant from A, B and C $\Rightarrow$ Any circle drawn with centre O and radius OA will pass through B and C also.	From (*i*) and (*ii*)
4. O is the only point equidistant from A, B and C.	Perpendicular bisectors of AB and BC cut each other at point O only.

Hence, one and only one circle can be drawn to pass through three non-collinear points A, B,

C. **Theorem 4.** *Equal chords of a circle are equidistant from the centre.*



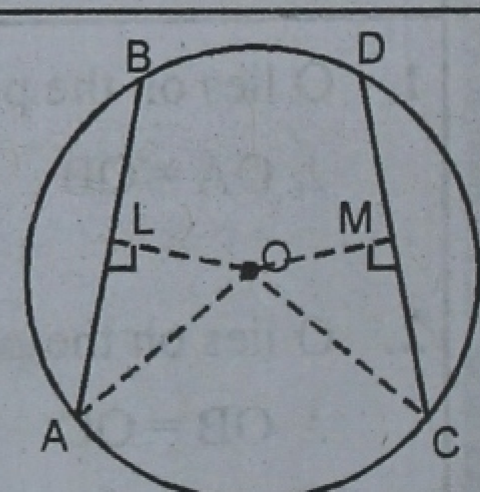
Given. A circle with centre O in which chord AB = chord CD ; OL ⊥ AB and OM ⊥ CD.

To prove. OL = OM.

Proof.

Statement	Reason
1. $AL = \frac{1}{2}AB$ ...(I)	Perpendicular from centre bisects the chrod.
2. $CM = \frac{1}{2}CD$ ...(II)	Perpendicular from centre bisects the chord.
3. Now, AB = CD	Given.
$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$	Halves of equals are equal.
⇒ AL = CM ...(III)	From (I) and (II).
4. In ΔOLA and ΔOMC,	
(*i*) OA = OC	Radii of the same circle.
(*ii*) AL = CM	From (III).
(*iii*) ∠OLA = ∠OMC	Each equal to 90°, as OL ⊥ AB and OM ⊥ CD.
∴ ΔOLA ≅ ΔOMC	RHS–axiom of congruency of Δ*s*.
∴ OL = OM	c.p.c.t.
Hence, the chords AB and CD are equidistant from the centre O.	

Theorem 5 (Converse of Theorem 4). *Chords of a circle that are equidistant from the centre of the circle, are equal.*



Given. AB and CD are two chords of a circle with centre O ; OL ⊥ AB, OM ⊥ CD and OL = OM.

To prove. AB = CD

Construction. Join OA and OC.

Proof.

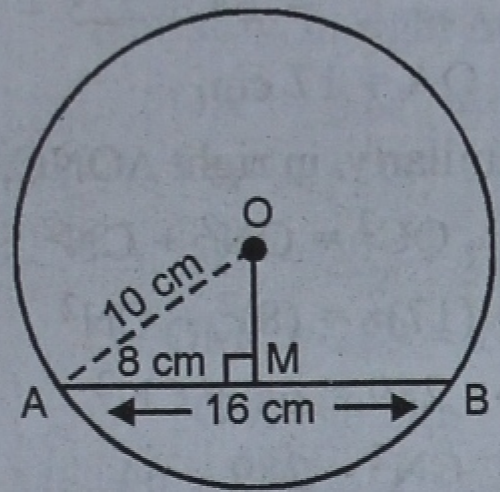
Statement	Reason
1. In ΔOLA and ΔOMC,	
(*i*) OL = OM	Given.
(*ii*) OA = OC	Radii of the same circle.
(*iii*) ∠OLA = ∠OMC	Each equal to 90°, as OL ⊥ AB and OM ⊥ CD.
∴ ΔOLA ≅ ΔOMC	RHS-axiom of congruency of Δ*s*.
∴ AL = CM	c.p.c.t.
$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$	Perpendicular from centre bisects the chord.
⇒ AB = CD	Doubles of equals are equal.

Hence, chord AB = chord CD.

EXERCISE 19

Q. 1. A chord of length 16 cm is drawn in a circle of radius 10 cm. Calculate the distance of the chord from the centre of the circle.

Sol. In a circle with centre O and radius OA = 10 cm., AB is a chord. OM ⊥ AB.



AB = 16 cm and OA = 10 cm

∵ OM ⊥ AB

∴ M is the mid-point of AB

$\Rightarrow AM = \frac{16}{2} = 8\,\text{cm}.$

Now, in right ΔOAM,

$OA^2 = AM^2 + OM^2$ (Pythagoras Theorem)

$\Rightarrow (10)^2 = (8)^2 + OM^2$

$\Rightarrow 100 = 64 + OM^2$

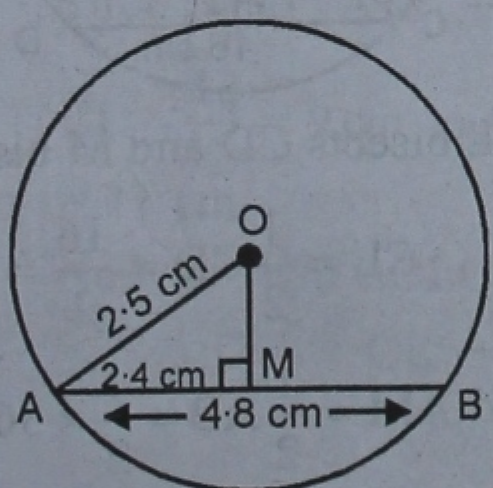
$\Rightarrow OM^2 = 100 - 64 = 36 = (6)^2$

∴ OM = 6 cm.

Hence, distance of the chord from the centre = 6 cm. **Ans.**

Q. 2. A circle of radius 2·5 cm has a chord of length 4·8 cm. Find the distance of the chord from the centre of the circle.

Sol. In a circle with centre O and radius OA = 2·5 cm.



AB is chord of the circle OM ⊥ AB.

∴ M is the mid-point

∵ AB = 4·8 cm.

$\therefore AM = \frac{4\cdot 8}{2} = 2\cdot 4\,\text{cm}.$

Now, in right ΔOAM,

$OA^2 = AM^2 + OM^2$ (Pythagoras Theorem)

$\Rightarrow (2\cdot 5)^2 = (2\cdot 4)^2 + OM^2$

$\Rightarrow 6\cdot 25 = 5\cdot 76 + OM^2$

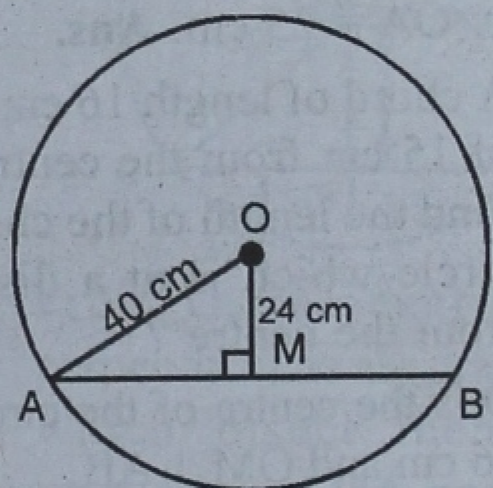
$\Rightarrow OM^2 = 6\cdot 25 - 5\cdot 76 = 0\cdot 49 = (0\cdot 7)^2.$

∴ OM = 0·7 cm.

Hence, distance of the chord from the centre of the circle = 0·7 cm. **Ans.**

Q. 3. The radius of a circle is 40 cm and the length of perpendicular drawn from its centre to a chord is 24 cm. Find the length of the chord.

Sol. O is the centre of the circle with OA = 40 cm as radius, AB is the chord and OM ⊥ AB



∴ M bisects AB at M

$\therefore AM = \frac{1}{2}AB$, OM = 24 cm.

Now, in right ΔOAM,

$OA^2 = AM^2 + OM^2$ (Pythagoras Theorem)

$\Rightarrow (40)^2 = AM^2 + (24)^2$

$\Rightarrow 1600 = AM^2 + 576$

$\Rightarrow AM^2 = 1600 - 576 = 1024 = (32)^2$

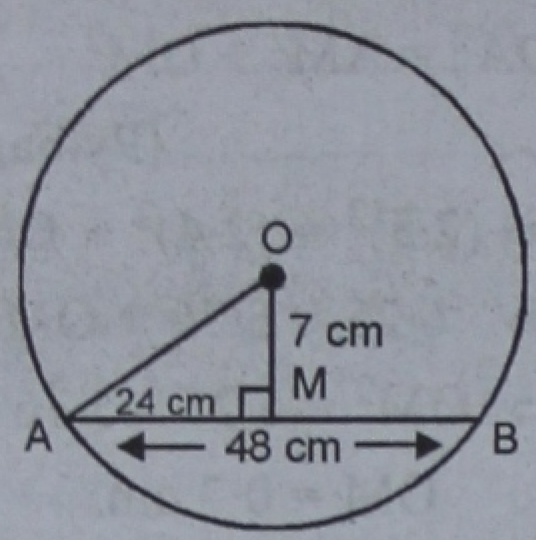
∴ AM = 32 cm.

and AB = 2AM = 2 × 32 = 64 cm.

∴ Length of the chord = 64 cm. **Ans.**

Q. 4. A chord of length 48 cm is drawn at a distance of 7 cm from the centre of the circle. Calculate the radius of the circle.

Sol. O is the centre of the circle with OA as radius.



AB is the chord and OM ⊥ AB

∴ AM = MB or $AM = \frac{1}{2}AB$

and OM = 7 cm.

Now, in right ΔOAM,

$OA^2 = AM^2 + OM^2$ (Pythagoras Theorem)

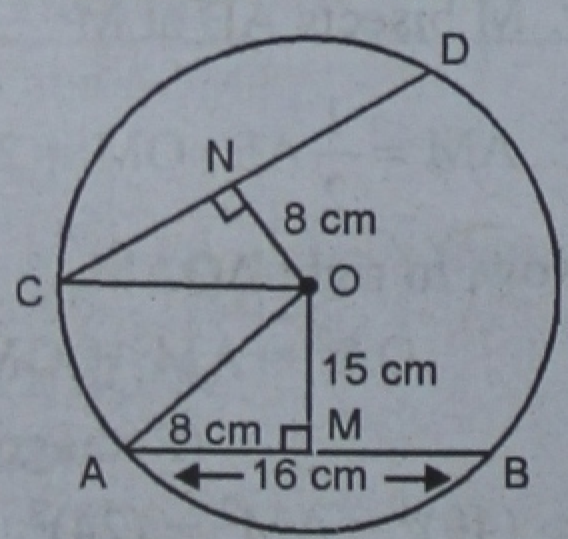
$= (24)^2 + (7)^2 = 576 + 49 = 625 = (25)^2$

∴ OA = 25 cm. **Ans.**

Q. 5. A chord of length 16 cm is at a distance of 15 cm from the centre of the circle. Find the length of the chord of the same circle which is at a distance of 8 cm from the centre ?

Sol. O is the centre of the circle chord AB = 16 cm and OM ⊥ AB.

Such that OM = 15 cm.



$$AM = \frac{1}{2}AB = 16 \times \frac{1}{2} = 8\,cm.$$

OA and OC are the radii of the circle

CD is another chord and ON ⊥ CD

Such that ON = 8 cm and $CN = \frac{1}{2}CD$

Now, in right ΔOAM,

$OA^2 = AM^2 + OM^2$ (Pythagoras Theorem)

$= (8)^2 + (15)^2$

$= 64 + 225 = 289 = (17)^2$

∴ OA = 17 cm.

Similarly, in right ΔONC,

$OC^2 = ON^2 + CN^2$

$\Rightarrow (17)^2 = (8)^2 + CN^2$ (∵ OA = OC)

$\Rightarrow 289 = 64 + CN^2$

$\Rightarrow CN^2 = 289 - 64$

$\Rightarrow CN^2 = 225 = (15)^2$

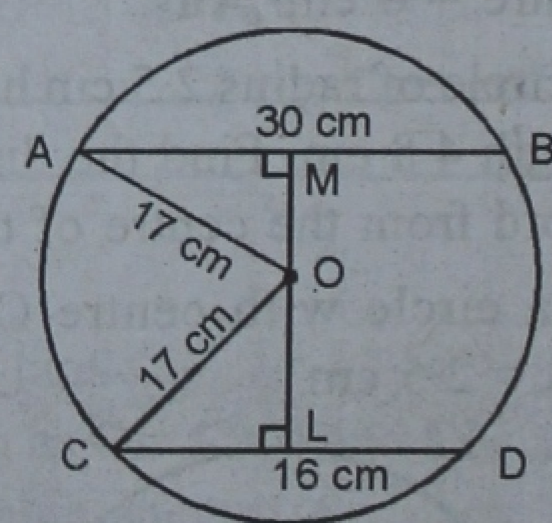
∴ CN = 15 cm.

Hence, length of the chord CD = 2 × CN

= 2 × 15 = 30 cm **Ans.**

Q. 6. Two parallel chords of lengths 30 cm and 16 cm are drawn on the opposite sides of the centre of the circle of radius 17 cm. Find the distance between the chords.

Sol. O is the centre of the circle. AB and CD are the chords of the circle drawn on the same side of the centre. AB = 30 cm and CD = 16 cm. Radius of the circle = 17 cm. OA = OC radii of the circle OL ⊥ CD and OM ⊥ AB.



∴ L bisects CD and M bisects AB.

$$\therefore \quad CL = \frac{1}{2}CD = \frac{16}{2} = 8\,cm.$$

and $AM = \frac{1}{2}AB = \frac{1}{2} \times 30 = 15\,cm.$

Now, in right ΔOCL,

$$OC^2 = CL^2 + OL^2$$

(Pythagoras Theorem)

$\Rightarrow (17)^2 = (8)^2 + OL^2$

$\Rightarrow 289 = 64 + OL^2$

$\Rightarrow OL^2 = 289 - 64 = 225 = (15)^2$

$\therefore$ OL = 15 cm.

Similarly, in right ΔOAM,

$$OA^2 = AM^2 + OM^2$$

$\Rightarrow (17)^2 = (15)^2 + OM^2$

$\Rightarrow 289 = 225 + OM^2$

$\Rightarrow OM^2 = 289 - 225 = 64 = (8)^2$

$\therefore$ OM = 8 cm.

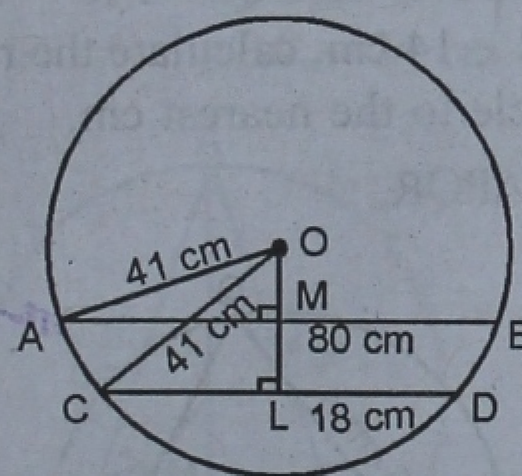
Now, ML = OL + OM = 15 + 8 = 23 cm.

Hence, the distance between the two chords is 23 cm. **Ans.**

Q. 7. Two parallel chords of the lengths 80 cm and 18 cm are drawn on the same side of the centre of the a circle of radius 41 cm. Find the distance between the chrods.

Sol. A circle with centre O and radius 41 cm.

Two chords AB and CD are parallel and drawn on the same side of the centre. AB = 80 cm and CD = 18 cm. From O, OL $\perp$ CD which intersects AB at M.



M and L bisects AB and CD respectively

Such that AM $= \frac{80}{2} = 40$ cm and

CL $= \frac{18}{2} = 9$ cm and radius OA = OC = 41 cm.

Now, in right ΔOLC,

$$OC^2 = CL^2 + OL^2$$

(Pythagoras Theorem)

$\Rightarrow (41)^2 = (9)^2 + OL^2$

$\Rightarrow 1681 = 81 + OL^2$

$\Rightarrow OL^2 = 1681 - 81 = 1600 = (40)^2$

$\therefore$ OL = 40 cm.

Similarly, in right ΔOAM,

$$OA^2 = AM^2 + OM^2$$

$\Rightarrow (41)^2 = (40)^2 + OM^2$

$\Rightarrow 1681 = 1600 + OM^2$

$\Rightarrow OM^2 = 1681 - 1600 = 81 = (9)^2$

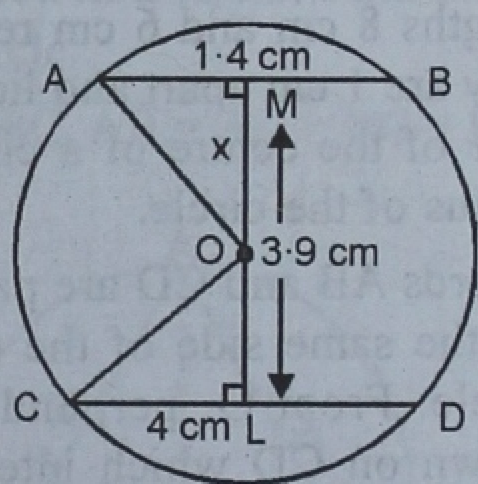
$\therefore$ OM = 9 cm.

Now, ML = OL – OM

= 40 – 9 = 31 cm **.Ans.**

Q. 8. Two parallel chords AB and CD are 3·9 cm apart and lie on the opposite sides of the centre of a circle. If AB = 1·4 cm. CD = 4 cm, find the radius of the circle.

Sol. Length of chord AB = 1·4 cm. and CD = 4 cm.



From O, the centre of the circle, perpendiculars are drawn on AB and CD which bisect them at M and L respectively. OA and OC are joined. LM = 3·9 cm.

Let OM = x cm., then OL = $(3{\cdot}9 - x)$ cm.

In right ΔOAM,

$$OA^2 = AM^2 + OM^2$$

(Pythagoras Theorem)

$$= \left(\frac{1{\cdot}4}{2}\right)^2 + x^2$$

$$= (0{\cdot}7)^2 + x^2 \quad ...(i)$$

Similarly, in right ΔOCL,

$$OC^2 = CL^2 + OL^2$$

$= \left(\frac{4}{2}\right)^2 + (3\cdot9 - x)^2$

$= (2)^2 + (3\cdot9 - x)^2$ ...(ii)

But OA = OC

(Radii of the same circle)

$\therefore (0\cdot7)^2 + x^2 = (2)^2 + (3\cdot9 - x)^2$

$\Rightarrow 0\cdot49 + x^2 = 4 + 15\cdot21 + x^2 - 7\cdot8\,x$

$\Rightarrow x^2 - x^2 + 7\cdot8\,x = 4 + 15\cdot21 - 0\cdot49$

$\Rightarrow 7\cdot8\,x = 19\cdot21 - 0\cdot49 = 18\cdot72$

$\therefore \quad x = \frac{18\cdot72}{7\cdot8} = 2\cdot4$

$\therefore$ OM = 2·4 cm.

Now from (i),

$OA^2 = (0\cdot7)^2 + x^2 = 0\cdot49 + (2\cdot4)^2$

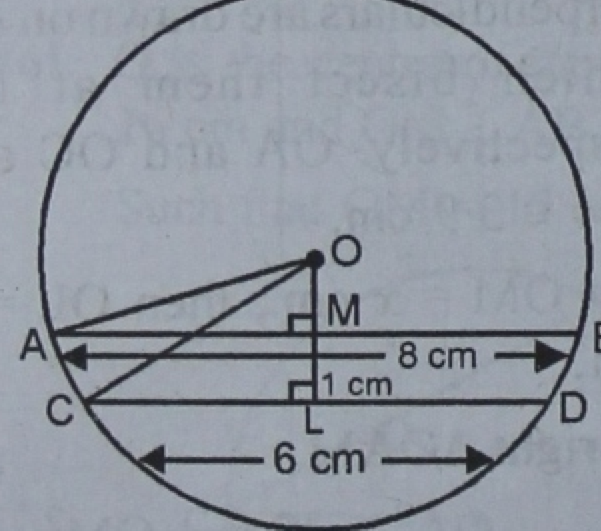
$= 0\cdot49 + 5\cdot76 = 6\cdot25 = (2\cdot5)^2$

$\therefore$ OA = 2·5 cm.

Hence, radius of the circle = 2·5 cm. **Ans.**

Q.9. AB and CD are two parallel chords of lengths 8 cm and 6 cm respectively. If they are 1 cm apart and lie on the same side of the centre of a circle, find the radius of the circle.

Sol. Chords AB and CD are parallel and lie on the same side of the centre of the circle. From O, perpendicular OL is drawn on CD which intersects AB at M. OA and OC are joined.



Now, AB = 8 cm, CD = 6 cm.

And LM = 1 cm.

Let OM = x cm, then OL = $(1 + x)$ cm.

OA and OC are joined

$AM = \frac{1}{2}AB = \frac{8}{2} = 4\,\text{cm}$ and

$CL = \frac{1}{2}CD = \frac{6}{2} = 3\,\text{cm}$.

Now, in right ΔOCL,

$OC^2 = CL^2 + OL^2$

(Pythagoras Theorem)

$= (3)^2 + (1 + x)^2 = 9 + (1 + x)^2$ ...(i)

Similarly, in right ΔOAM,

$OA^2 = AM^2 + OM^2$

$= (4)^2 + x^2 \Rightarrow 16 + x^2$ ...(ii)

But OA = OC (Radii of the same circle)

$\therefore$ From (i) and (ii),

$9 + (1 + x)^2 = 16 + x^2$

$\Rightarrow 9 + 1 + x^2 + 2x = 16 + x^2$

$\Rightarrow x^2 + 2x - x^2 = 16 - 9 - 1$

$\Rightarrow 2x = 6 \quad \Rightarrow x = \frac{6}{2} = 3$

$\therefore$ OM = 3 cm.

Now, from (ii),

$OA^2 = 16 + x^2 = 16 + (3)^2 = 16 + 9$

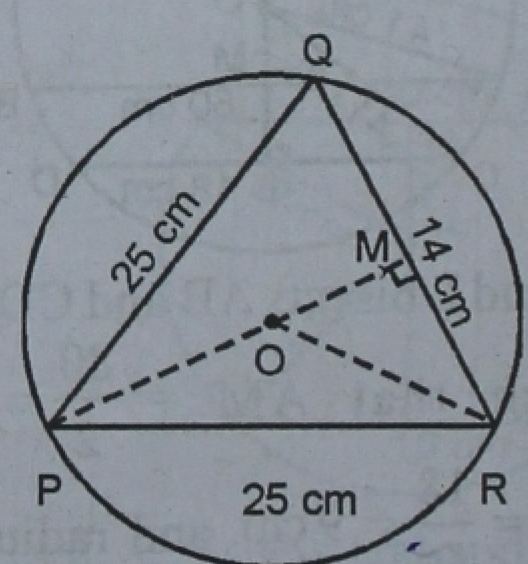
$= 25 = (5)^2$

$\therefore$ OA = 5.

Hence, radius of the circle = 5 cm. **Ans.**

Q. 10. PQR is an isosceles triangle inscribed in a circle. If PQ = PR = 25 cm and QR = 14 cm, calculate the radius of the circle to the nearest cm.

Sol. In ΔPQR,



$\because$ PQ = PR = 25 cm.

From P, draw a perpendicular PM which passes through O.

It bisects QR also. Join OR.

$\therefore$ RM $= \frac{1}{2}$QR $= \frac{1}{2} \times 14 = 7$ cm.

Now, in right ΔPRM,

$$PR^2 = PM^2 + RM^2$$

(Pythagoras Theorem)

$\Rightarrow (25)^2 = PM^2 + (7)^2$

$\Rightarrow 625 = PM^2 + 49$

$PM^2 = 625 - 49 = 576 = (24)^2$

$\therefore$ PM = 24 cm.

Let PO = OR = r cm.

$\therefore$ OM = $(24 - r)$ cm.

Similarly, in right ΔORM,

$OR^2 = OM^2 + RM^2$

$\Rightarrow r^2 = (24 - r)^2 + RM^2$

$\Rightarrow r^2 = 576 + r^2 - 48r + 49$

$\Rightarrow r^2 - r^2 + 48r = 576 + 49$

$\Rightarrow r^2 - r^2 + 48r = 625$

$\Rightarrow 48r = 625 \quad \Rightarrow r = \frac{625}{48}$

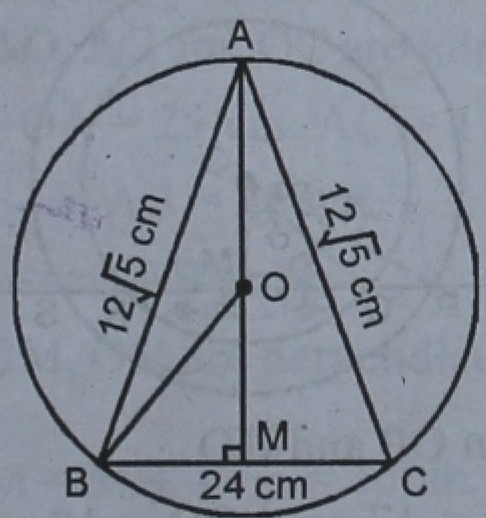
$\Rightarrow \quad r = 13$

$\therefore$ Radius of the circle = 13 cm. **Ans.**

Q. 11. An isosceles ΔABC is inscribed in a circle. If AB = AC = $12\sqrt{5}$ cm and BC = 24 cm., find the radius of the circle.

Sol. In ΔABC, AB = AC = $12\sqrt{5}$ cm.

BC = 24 cm.



From A, draw a perpendicular. Which passes through O, the centre of the circle and bisects BC at M. Join OB. Let OA = OB = r, then OM = AM – AO

BM $= \frac{1}{2}$BC $= \frac{1}{2} \times 24 = 12$ cm.

Now, in right Δ ABM,

$$AB^2 = AM^2 + BM^2$$

(Pythagoras Theorem)

$\Rightarrow (12\sqrt{5})^2 = AM^2 + (12)^2$

$\Rightarrow \quad 144 \times 5 = AM^2 + 144$

$\Rightarrow \quad AM^2 = 720 - 144 = 576$

$\Rightarrow \quad AM^2 = (24)^2 \Rightarrow AM = 24$ cm.

$\therefore \quad$ OM = AM – AO = $(24 - r)$ cm.

Similarly, in right ΔOBM,

$OB^2 = BM^2 + OM^2$

$\Rightarrow r^2 = (12)^2 + (24 - r)^2$

$= 144 + 576 + r^2 - 48r$

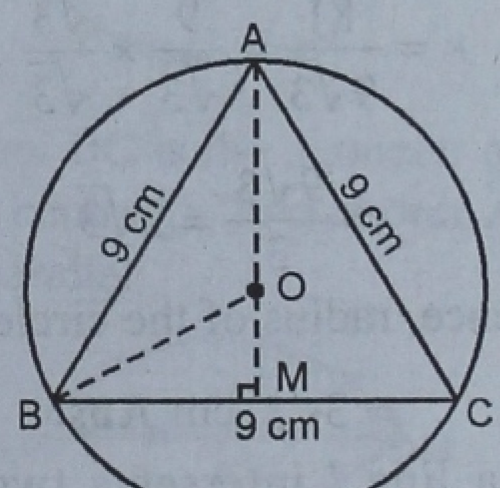
$\Rightarrow r^2 - r^2 + 48r = 144 + 576$

$\Rightarrow 48r = 720 \quad \Rightarrow r = \frac{720}{48} = 15$

$\therefore$ Radius of the circle = 15 cm. **Ans.**

Q. 12. An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.

Sol. In ΔABC, AB = AC = BC = 9 cm.



From A, draw a perpendicular which passes through O, the centre of the circle and meets BC in M and bisects BC. Join OB. Let OA = OB = r.

Now BM $= \frac{1}{2}$BC $= \frac{1}{2} \times 9 = \frac{9}{2}$ cm.

In right ΔABM,

$$AB^2 = BM^2 + AM^2$$

(Pythagoras Theorem)

$\Rightarrow \quad (9)^2 = \left(\frac{9}{2}\right)^2 + AM^2$

$$\Rightarrow \quad 81 = \frac{81}{4} + AM^2$$

$$\Rightarrow AM^2 = 81 - \frac{81}{4} = \frac{324-81}{4}$$

$$\Rightarrow AM^2 = \frac{243}{4} = \left(\frac{3 \times 81}{4}\right) cm$$

$$\therefore AM = \sqrt{\frac{3 \times 81}{4}} = \frac{9}{2}\sqrt{3} \text{ cm.}$$

$$\therefore OM = AM - AO = \left(\frac{9}{2}\sqrt{3} - r\right) \text{ cm.}$$

Similarly, in right ΔOBM,

$$OB^2 = BM^2 + OM^2$$

$$\Rightarrow r^2 = \left(\frac{9}{2}\right)^2 + \left(\frac{9}{2}\sqrt{3} - r\right)^2$$

$$\Rightarrow r^2 = \frac{81}{4} + \frac{243}{4} + r^2 - 9\sqrt{3}\,r$$

$$\Rightarrow r^2 - r^2 + 9\sqrt{3}\,r = \frac{81}{4} + \frac{243}{4}$$

$$\Rightarrow \quad 9\sqrt{3}\,r = \frac{324}{4} = 81$$

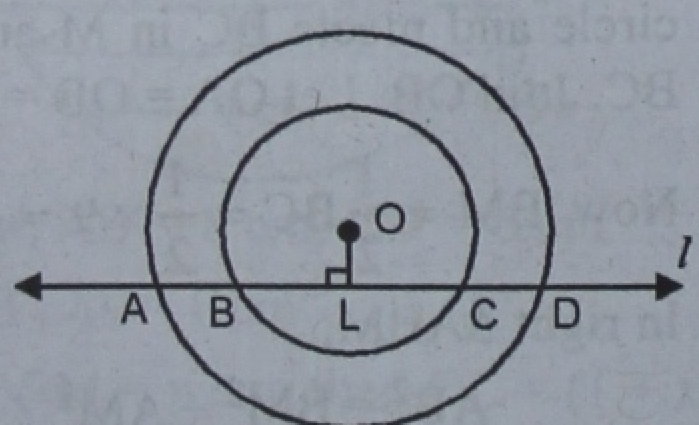
$$\Rightarrow r = \frac{81}{9\sqrt{3}} = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{9\sqrt{3}}{3} = 3\sqrt{3}$$

Hence, radius of the circle

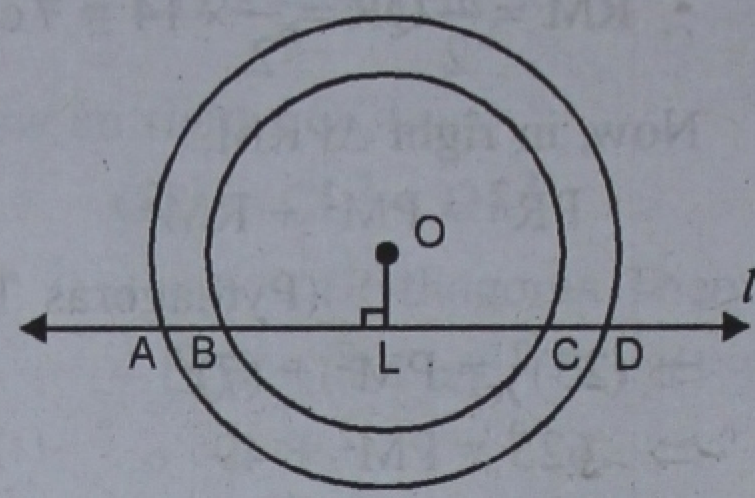
$= 3\sqrt{3}$ cm **Ans.**

Q. 13. If a line l intersects two concentric circles at the points A, B, C and D, as shown in the figure, prove that AB = CD.



Sol. Given. A line l intersects two concentric circles with centre O at A, B, C and D.

To prove. AB = CD.



Construction. From O, draw a perpendicular OL to l.

Proof. In bigger circle,

AD is the chord, $OL \perp AD$

$\therefore$ L bisects AD

$\Rightarrow AL = LD$ ...(*i*)

Similarly, in smaller circle,

BC is the chord and $OL \perp BC$

$\therefore BL = LC$ ...(*ii*)

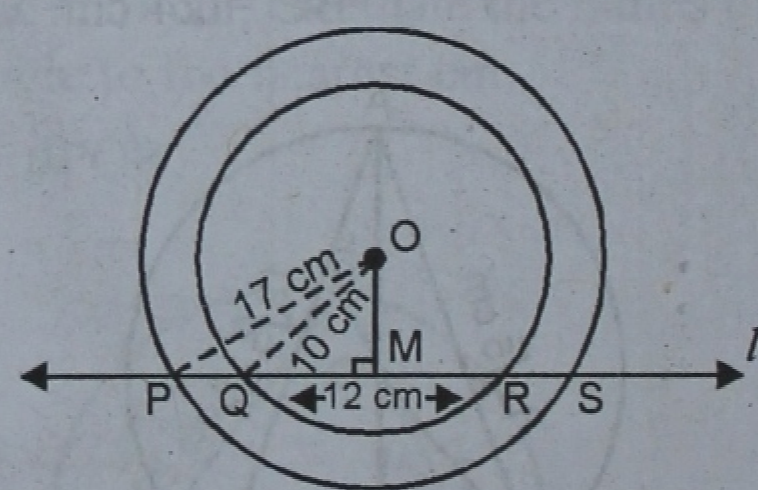
Subtracting (*ii*) from (*i*)

$AL - BL = LD - LC \Rightarrow AB = CD$.

Hence proved.

Q. 14. The radii of two concentric circles are 17 cm. and 10 cm. A line segment PQRS cuts the larger circle at P and S and the smaller circle at Q and R. If QR = 12 cm., find the length of PQ.

Sol. Draw $OM \perp l$



Join OP and OQ.

OP = 17 cm., OQ = 10 cm.

QR = 12 cm.

$\because OM \perp QR$

$\therefore$ M is mid point of QR

$$\therefore QM = \frac{1}{2} \times 12 = 6 \text{ cm.}$$

Now, in right ΔOQM,

$OQ^2 = QM^2 + OM^2$ (Pythagoras Theorem)

$\Rightarrow (10)^2 = (6)^2 + OM^2$

$\Rightarrow 100 = 36 + OM^2$

$\Rightarrow OM^2 = 100 - 36 = 64$

Similarly, in right ΔOPM,

$OP^2 = PM^2 + OM^2$

$\Rightarrow (17)^2 = PM^{-2} + 64$

$\Rightarrow 289 = PM^2 + 64$

$\Rightarrow PM^2 = 289 - 64 = 225 = (15)^2$

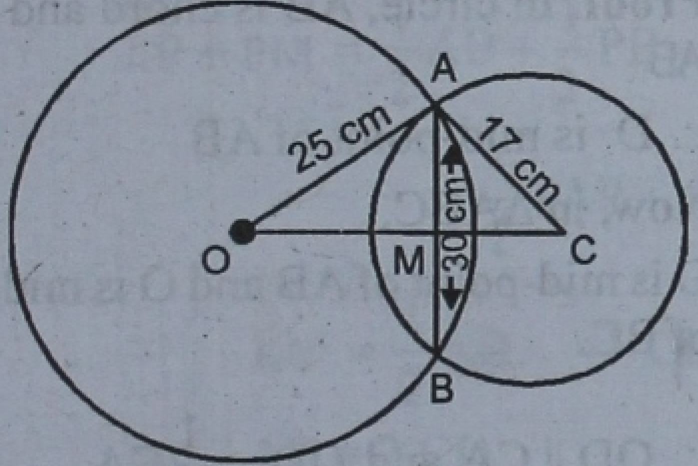
$\therefore$ PM = 15

Now PQ = PM – QM = 15 – 6

= 9 cm. **Ans.**

Q. 15. Two circles of radii 17 cm. and 25 cm intersect each other at two points A and B. If the length of common chord AB of the circles 30 cm, find the distance between the centres of the circles.

Sol. Two circles of radii 17 cm. and 25 cm. intersects each other at A and B.



AB, AO, AC and OC are joined.

Now, OA = 25 cm. AC = 17 cm.

AB = 30 cm.

OC bisects AB at M.

and OM, CM are perpendicular on AB

$\therefore AM = MB = \frac{30}{2} = 15$ cm.

Now, in right ΔAOM,

$AO^2 = OM^2 + AM^2$ (Pythagoras Theorem)

$\Rightarrow (25)^2 = OM^2 + (15)^2$

$625 = OM^2 + 225$

$\Rightarrow OM^2 = 625 - 225 = 400$

$\Rightarrow OM^2 = (20)^2$

$\therefore$ OM = 20 ...(*i*)

Similarly, in right ΔACM,

$AC^2 = CM^2 + AM^2$

$\Rightarrow (17)^2 = CM^2 + (15)^2$

$\Rightarrow 289 = CM^2 + 225$

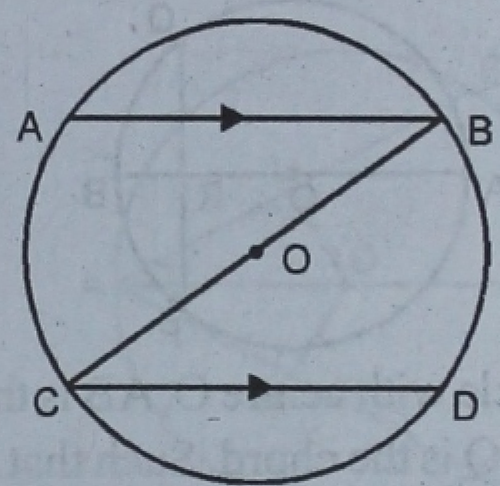
$\Rightarrow CM^2 = 289 - 225 = 64 = (8)^2$

$\therefore$ CM = 8 ...(*ii*)

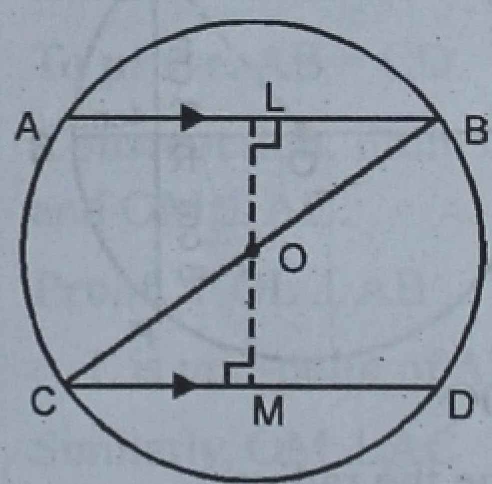
Now, OC = OM + MC = 20 + 8

= 28 cm. **Ans.**

Q. 16. In the adjoining figure, BC is a diameter of a circle with centre O. If AB and CD are two chords such that AB || CD, prove that AB = CD.



Sol. **Given.** BC is the diameter of the circle with centre O. Two chords AB and CD are parallel.



To prove. AB = CD

Construction. Draw OL ⊥ AB and OM ⊥ CD.

Proof. $\because$ OL ⊥ AB

$\therefore$ L is the mid point of AB

Again, OM ⊥ CD

∴ M is mid-point of CD.

Now, in ΔOLB and ΔOMC,

OB = OC (Radii of the same circle)

∠L = ∠M (Each 90°)

∠LBO = ∠OCM (Alternate angles)

∴ ΔOLB ≅ ΔOMC (A.S.A. axiom)

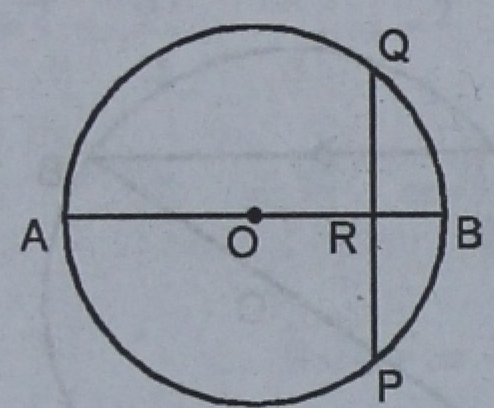
∴ OL = OM (c.p.c.t.)

∴ AB = CD

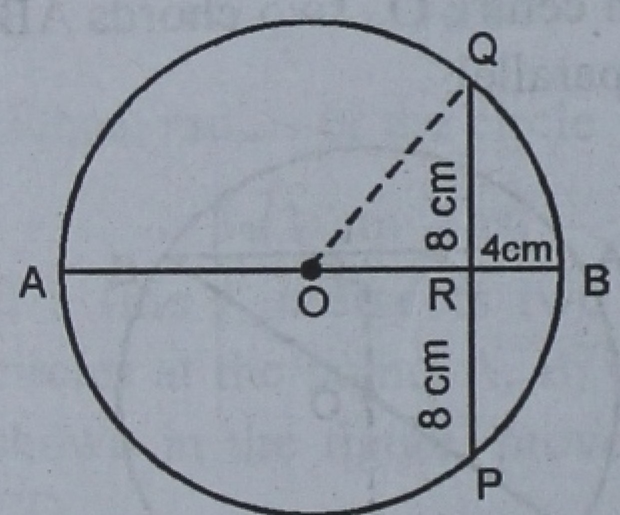
(Equal chords are equidistant from the centre of the circle)

Hence proved.

Q. 17. The adjoining figure shows a circle with centre O in which a diameter AB bisects the chrod PQ at point R. If PR = RQ = 8 cm and RB = 4 cm, find the radius of the circle.



Sol. A circle with centre O, AB is the diameter and PQ is the chord. Such that AB bisects PQ at R. PR = RQ = 8 cm. and RB = 4 cm.



Join OQ.

Let r be the radius

Then OQ = OB = r

And OR = $(r - 4)$

Now, in right Δ ORQ,

$OQ^2 = OR^2 + RQ^2$

(Pythagoras Theorem)

$\Rightarrow r^2 = (r-4)^2 + (8)^2$

$\Rightarrow r^2 = r^2 + 16 - 8r + 64$

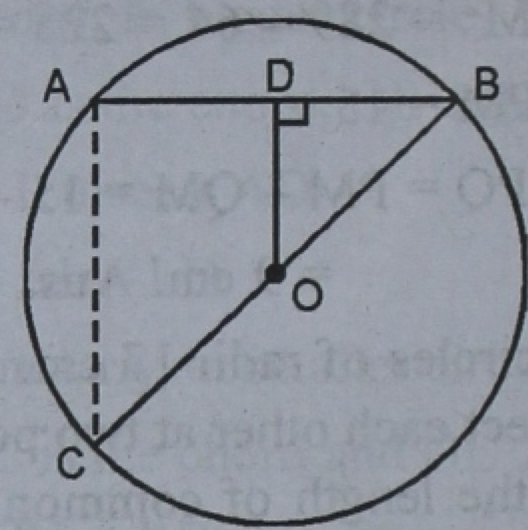
$\Rightarrow r^2 - r^2 + 8r = 16 + 64$

$\Rightarrow r^2 - r^2 + 8r = 80$

$\Rightarrow 8r = 80 \quad \Rightarrow r = \frac{80}{8} = 10$

∴ Radius of the circle = 10 cm. **Ans.**

Q. 18. In the adjoining figure, AB is a chord of a circle with centre O and BC is a diameter. If OD ⊥ AB, show that CA = 2 OD and CA ∥ OD.



Sol. **Given.** AB is the chord of a circle with centre O. BC is the diameter. OD ⊥ AB. AC is joined.

To prove. CA = 2 OD and CA ∥ OD

Proof. In circle, AB is chord and OD ⊥ AB

∴ D is mid-point of AB

Now, in ΔABC,

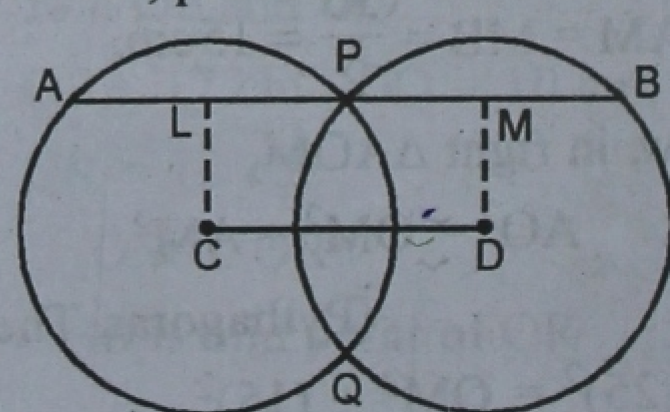
D is mid-point of AB and O is mid-point of BC

∴ OD ∥ CA and OD = $\frac{1}{2}$CA

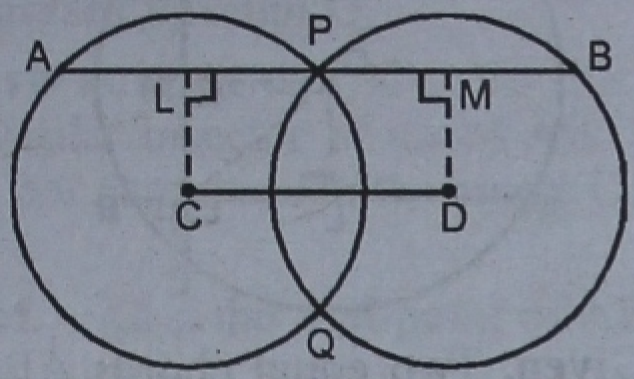
Hence CA = 2 OD and CA ∥ OD

Hence proved.

Q. 19. In the adjoining figure, P is a point of intersection of two circles with centres C and D. If the straight line APB is parallel to CD, prove that AB = 2CD.



Sol. Given. Two circles with centres C and D intersect each other at P and Q. A line APB is parallel to CD.



To prove. AB = 2 CD.

Construction. From C and D, draw perpendiculars CL and DM on AB.

Proof. $\because$ CL || DM (Both perpendiculars on AB)

and AB || CD (given)

$\therefore$ LCDM is a rectangle

$\therefore$ LM = CD

$\because$ CL $\perp$ AB

$\therefore$ L bisects AP

Similarly, M will bisect PB.

$\therefore LP = \frac{1}{2}AP$ and $PM = \frac{1}{2}PB$

Adding, we get

$LP + PM = \frac{1}{2}AB + \frac{1}{2}PB$

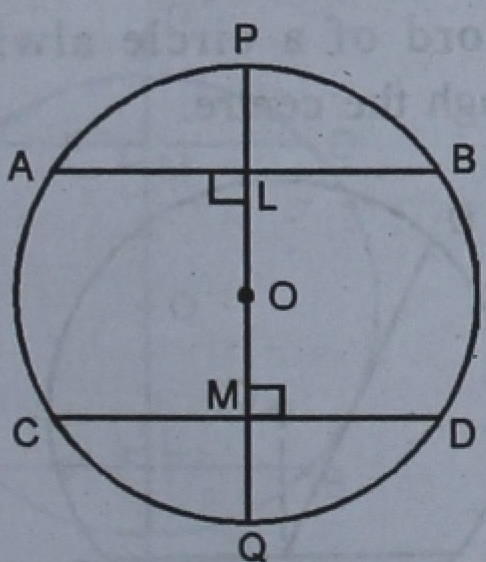
$= \frac{1}{2}(AP + PB)$

$\Rightarrow LM = \frac{1}{2}AB \Rightarrow CD = \frac{1}{2}AB$

$\Rightarrow AB = 2\,CD$

Hence proved.

Q. 20. If a diameter of a circle bisects each of the two chords of a circle, then prove that the chords are parallel.



Sol. Given. Diameter POQ bisects two chords AB and CD at L and M respectively.

To prove. AB || CD

Proof. $\because$ L is mid-point of AB and OL is joined.

$\therefore$ OL $\perp$ AB

Similarly, OM $\perp$ CD

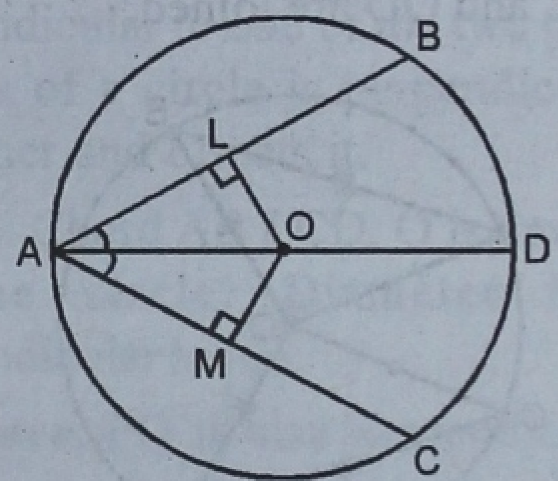
Now, $\angle ALO = \angle DMO$ (each 90°)

But these are alternate interior angles.

$\therefore$ AB || CD

Hence proved.

Q. 21. If two chords of a circle are equally inclined to the diameter through their point of intersection, prove that the chords are equal.



Sol. Given. Two chords AB and AC are equally inclined to diameter AD *i.e.* $\angle BAD = \angle CAD$.

To prove. AB = CD.

Construction. From O, draw OL $\perp$ AB and OM $\perp$ AC.

Proof. $\because$ OL $\perp$ AB

$\therefore$ L is mid-point of AB

Similarly, OM $\perp$ AC

$\therefore$ M is mid-point of AC

Now, in ΔALO and ΔAMO,

AO = AO (Common)

$\angle L = \angle M$ (each 90°)

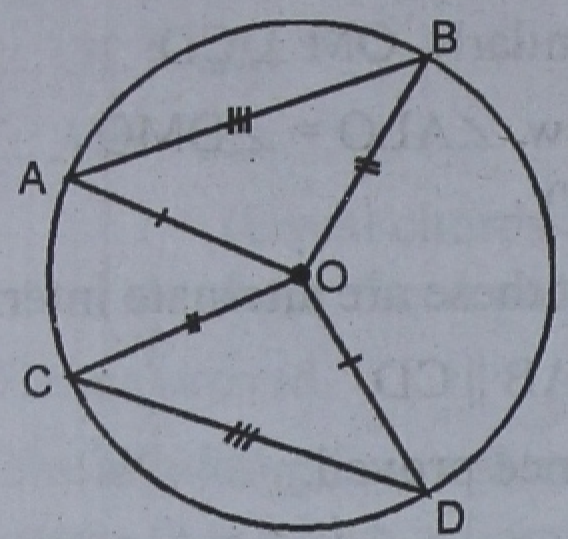
$\angle BAD$ or $\angle LAO = \angle MAO$ (given)

∴ ΔALO ≅ ΔAMO (S.A.A. axiom)

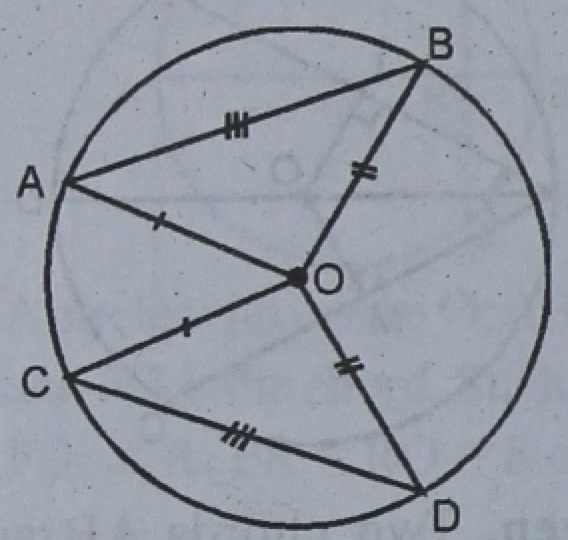
∴ OL = OM (c.p.c.t)

Hence, AB = AC (Equal chords are equidistant from the centre of the circle)

Q. 22. Show that equal chords of a circle subtend equal angles at the centre of the circle.



Sol. Given. In a circle chord AB = chord CD AB subtends ∠AOB and chord CD subtends ∠COD at the centre. OA, OB, OC and OD are joined.



To prove. ∠AOB = ∠COD

Proof. In ΔOAB and ΔOCD

OA = OC (Radii of the same circle)

OB = OD (given)

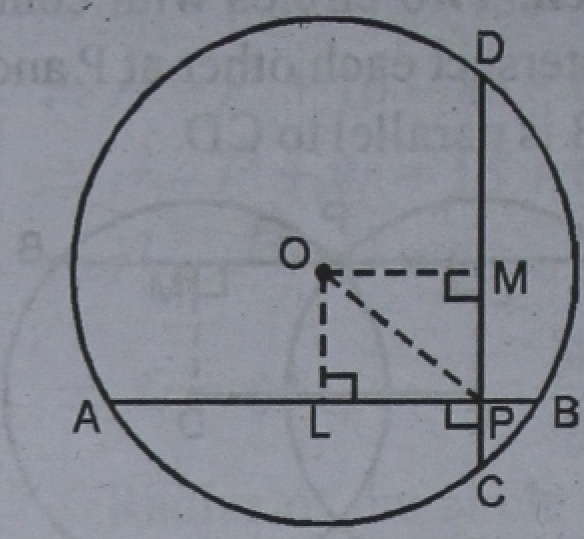
AB = CD (given)

∴ ΔOAB ≅ ΔOCD (S.S.S. axiom)

∴ ∠AOB = ∠COD (c.p.c.t.)

Hence proved.

Q. 23. In the given figure, equal chords AB and CD of circle with centre O, cut at right angels at P. If L and M are mid-points of AB and CD respectively, prove that OLPM is a square.



Sol. Given. Two equal chords AB and CD of a circle with centre O, intersect each other at right angle. L and M are the mid-points of AB and CD respectively.

To prove. OLPM is a square.

Construction. Join OP

Proof. ∵ OL ⊥ AB and OM ⊥ CD

and AB = CD (given)

∴ OL = OM

(Equal chords are equidistant from the centre)

Now, in ΔOLP and ΔOMP,

LP = MP (half of equal chords)

OL = OM (proved)

OP = OP (common)

∴ ΔOLP ≅ ΔOMP (S.S.S. axiom)

∴ LP = PM (c.p.c.t.)

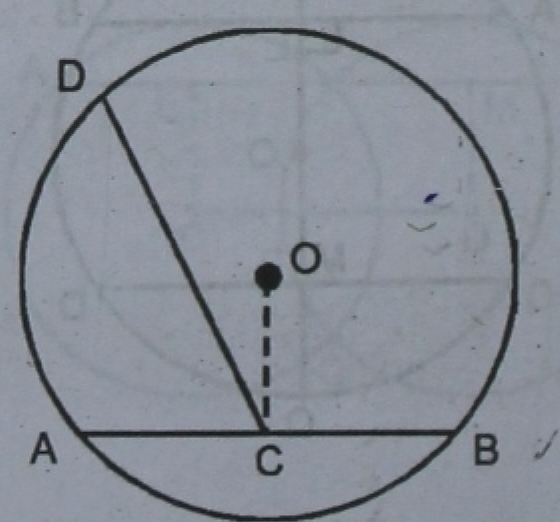
But, OL = OM (proved)

∵ Each angle of quad. OLPM is a right- angle.

∴ OLPM is a square.

Hence proved.

Q. 24. Prove that the perpendicular bisector of a chord of a circle always passes through the centre.



Sol. **Given.** A circle with centre O. AB is its chord.

To prove. The perpendicular bisector of AB passes through O.

Construction. Let CD be the perpendicular bisector of chord AB. Let it does not pass through the centre O. Join OC.

Proof. ∵ C is the mid-point of AB and OC is joined.

∴ ∠OCA = 90° ...(*i*)

But, CD is perpendicular bisector of AB

∴ ∠ACD = 90° ...(*ii*)

From (*i*) and (*ii*),

∠ACD = ∠OCA

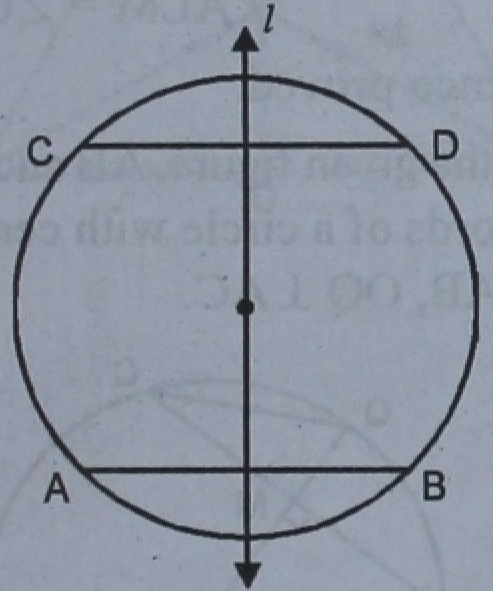
But it is not possible as ∠ACD is a part of ∠OCA

∴ CO and CD will coincide each other.

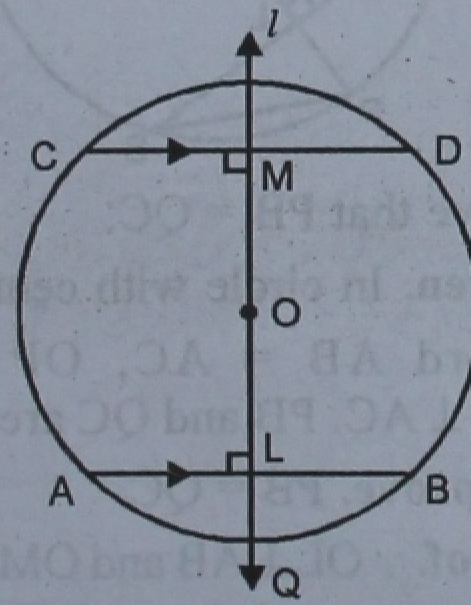
Hence, the perpendicular bisector of chord AB passes through the centre O of the circle.

Hence proved.

Q. 25. AB and CD are two parallel chords of a circle and a line *l* is the perpendicular bisector of AB. Show that *l* is the perpendicular bisector of CD also.



Sol.



Given. Two chords AB and CD are parallel *i.e.* AB || CD

A line *l* is perpendicular bisector of AB and passes through CD at M.

To prove. *l* is perpendicular bisector of CD.

Proof. Since the perpendicular bisector of a chord of a circle passes through the centre of the circle and *l* is the perpendicular bisector of AB

∴ *l* passes through O.

But AB || CD (given)

∴ *l* is perpendicular to CD.

∵ *l* passes through the centre O of the circle.

∴ *l* is also perpendicular bisector of CD.

Hence proved.

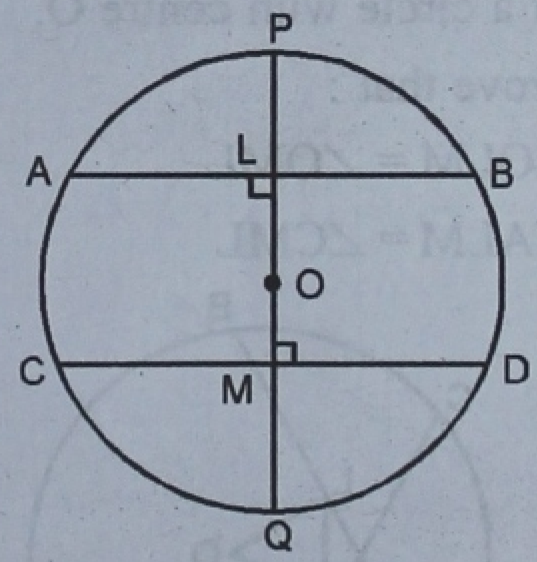
Q. 26. Prove that the diameter of a circle perpendicular to one of the two parallel chords of a circle is perpendicular to the other and bisects it.

Sol. **Given.** Chord AB || CD. O is the centre of the circle. Diameter PQ is perpendicular to AB.

To prove. PQ is also perpendicular to CD.

Proof. ∵ PQ ⊥ AB

∴ ∠ALO = 90°



∵ AB || CD

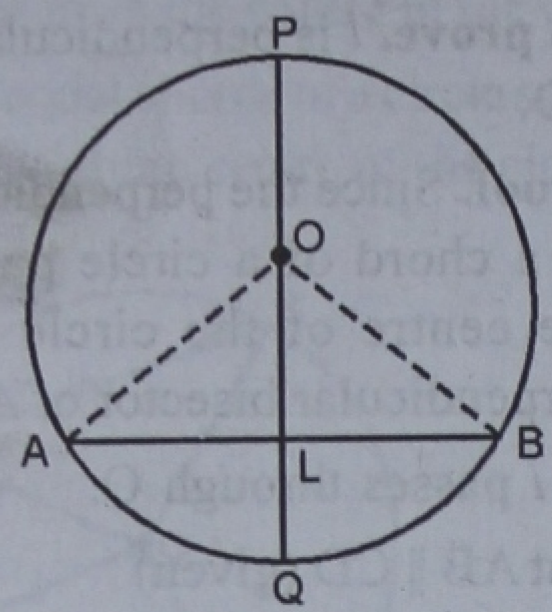
∴ ∠OMD = ∠ALO (Alternate angle)

∴ ∠OMD = 90°

Hence, PQ ⊥ CD.

Hence proved.

Q. 27. Prove that a diameter of a circle, which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.



Sol. Given. AB is the chord of the circle with centre O and diameter PQ bisects AB at L *i.e.* AL = LB, OA, OB are joined.

To prove. PQ bisects ∠AOB

i.e. ∠AOL = ∠BOL

Proof. In ΔAOL and ΔBOL,

∴ OL = OL (common)

OA = OB (Radii of the same circle)

AL = LB (given)

∴ ΔAOL ≅ ΔBOL (S.S.S. axiom)

∴ ∠AOL = ∠BOL (c.p.c.t.)

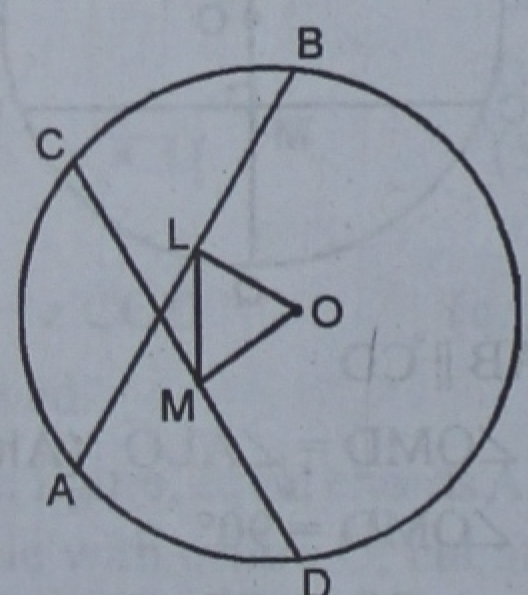
Hence proved.

Q. 28. In the given figure, L and M are mid-points of two equal chords AB and CD of a circle with centre O.

Prove that :

(*i*) ∠OLM = ∠OML

(*ii*) ∠ALM = ∠CML



Sol. Given. L and M are the mid-points of chords AB and CD respectively. AB = CD, LM is joined.

To prove.

(*i*) ∠OLM = ∠OML

(*ii*) ∠ALM = ∠CML

Proof. ∵ L is the mid-point of AB and OL is joined

∴ OL ⊥ AB

Similarly, we can prove that

OM ⊥ CD

∵ AB = CD (given)

∴ OL = OM

(Equal chords are equidistant from the centre)

(*i*) Now, in ΔOLM,

OL = OM (proved)

∴ ∠OLM = ∠OML

(Angles opposite to equal sides) ...(*i*)

(*ii*) ∠ALO = 90° (∵ OL ⊥ AB)

and ∠CMO = 90° (∴ OM ⊥ CD)

∴ ∠ALO = ∠CMO ...(*ii*)

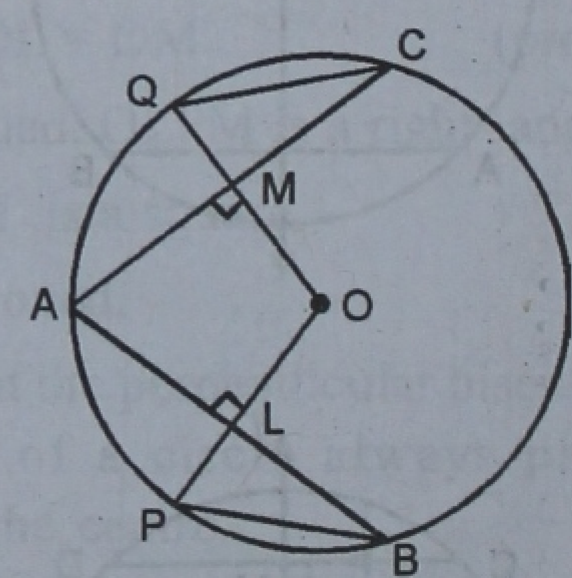
Substracting (*i*) from (*ii*),

⇒ ∠ALO – ∠OLM = ∠CMO – ∠OML

⇒ ∠ALM = ∠CML

Hence proved.

Q. 29. In the given figure, AB and AC are equal chords of a circle with centre O and OP ⊥ AB, OQ ⊥ AC.



Prove that PB = QC.

Sol. Given. In circle with centre O, chord AB = AC, OP ⊥ AB and OQ ⊥ AC. PB and QC are joined.

To prove. PB = QC

Proof. ∵ OL ⊥ AB and OM ⊥ AC (given)

∵ L and M are the mid-points of AB and AC

∴ BL = CM (Half of equal chords)

∵ AB = AC (given)

∴ OL = OM

(Equal chords are equidistant from the centre)

But, OP = OQ (Radii of the same circle)

∴ OP – OL = OQ – OM

∴ LP = MQ

Now, in ΔLPB and ΔMQC,

LB = MC (Half of equal chords)

LP = MQ (Proved)

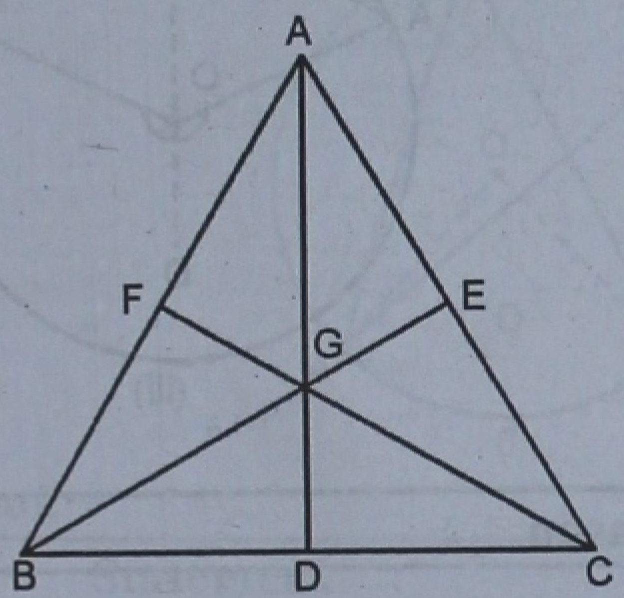
∠PLB = ∠QMC (Each 90°)

∴ ΔLPB ≅ ΔMQC (S.A.S axiom)

∴ BP = QC. (c.p.c.t.)

Hence proved.

Q. 30. In an equilateral triangle, prove that the centroid and the circumcentre of the triangle coincide.



Sol. Given. In ΔABC, AB = BC = CA

To prove. Centroid and circumcentre coincide.

Proof. Let AD, BE and CF be the medians which intersect at G. *i.e.* G is the centroid.

Now, in ΔBEC and ΔBFC,

BC = BC (common)

∠B = ∠C (Each 60°)

CE = BF (Half of equal sides)

∴ Δ BEC ≅ Δ BFC (S.A.S axiom)

∴ BE = CF (c.p.c.t.)

Similarly, we can prove that

AD = BE

∴ AD = BE = CF

$$\Rightarrow \frac{2}{3}AD = \frac{2}{3}BE = \frac{2}{3}CF$$

⇒ GA = GB = GC

(∵ G divides the medians in the ratio 1 : 2)

∴ G is the circumcentre of ΔABC

But, G is also the centroid of the triangle ABC.

Hence, centroid and circumcentre coincide.

Hence proved.